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Theoretical and Numerical Study of Electrohydrodynamic Flow in a Planar, Cylindrical, and Spherical Conduit

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Abstract. This paper analyzes the mathematical model of electrohydrodynamic (EHD) fluid flow in a general conduit (planar, cylindrical and spherical) with an ion drag configuration. The phenomenon is modelled using a nonlinear differential equation. The velocity field is obtained by solving this nonlinear equation using two analytical methods. The effects of the Hartmann electric number and nonlinearity strength are discussed and presented graphically. Additionally, we compare this method with a numerical solution obtained using MATLAB, demonstrating that the proposed approaches are less computational intensive and more efficient for solving the underlined problem.

1. Introduction

The wide range of applications of electrofluid systems in manufacturing and mechanical processes has drawn much research interest over the past decades [1,2]. The effect of electric fields on fluids is currently being studied theoretically to create novel processes [3]. Electrohydrodynamics (EHD) studies the interplay between electrodynamics and hydrodynamics in dielectric fluid flow under an electric field [4,5]. Electrostatic precipitators [6,7], electrical pump designs [8], tokamak reactors [9], thermal microelectronics through electro-gas dynamic pumps [10], electrospray for liquid atomization [11], electro-fluid control of colloidal particles [12], MEMS and inkjet devices [13,14], propulsion of small-scale naval vessels [15], and medical powder [16] are just a few of the many fields in which EHD flows find applications.

The electrohydrodynamic flow of fluid in a circular cylindrical conduit, which was controlled by a nonlinear boundary value problem, was initially addressed by McKee et al. [17] in 1997. They

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studied velocity profiles using the perturbation method. Pallet [18] confirmed the validity and existence of solutions for the EHD fluid flow in the cylindrical conduit in 1999. For more significant values of α, the results obtained by [18] differed qualitatively different from those obtained by [17]. Mastroberardino [19] derived analytical solutions for the EHD flow velocity profile over a range of Hartmann numbers using the homotopy perturbation approach. Panday [20] solved the EHD problem using two semi-analytical methods based on the homotopy asymptotic and an optimum homotopy analysis method. In addition, Khan et al. [21] developed a novel homotopy perturbation technique to examine the EHD flow equation. Moghtadaei [22] used the spectral homotopy analysis method, and Pradip Roul [23, 24] proposed a discrete Adomian decomposition method to solve this problem. Furthermore, by changing the level of nonlinearity ($0 < \alpha < 1$), Gavabari et al. [25] employed the differential transform approach to provide an analytical expression for the velocity profile. Abukhaled and Khuri [26] obtained the analytical solution of EHD flow in a cylindrical conduit using the Green's function method.

Least squares [27], Galerkin collocation [28], optimum B-spline collocation [29, 30], discrete optimized homotopy analysis [31], pseudospectral collocation [32], spectral collocation [33], and DTM-Pade approximation [34] are some of the other simulation techniquest that have been used to solve the EHD flow problem. Although the electrohydrodynamic (EHD) flow of fluids has been researched using various numerical and analytical techniques, standard methods have difficulty approximating the solution due to the singularity and nonlinearity in the mathematical model. The rational function form of the EHD flow equation's nonlinearity in a circular cylindrical conduit presents a major obstacle to obtaining analytical solutions. This work utilizes two efficient semi-analytical approaches (AGM and ADM) to solve the EHD fluid flow in a general conduit.

2. Mathematical formulation of the problem

This section presents the basic equation's theoretical summary. Planar, circular cylindrical, and spherical conduits with insulating walls are depicted in an axisymmetric coordinate system in Figures 1(a-c).

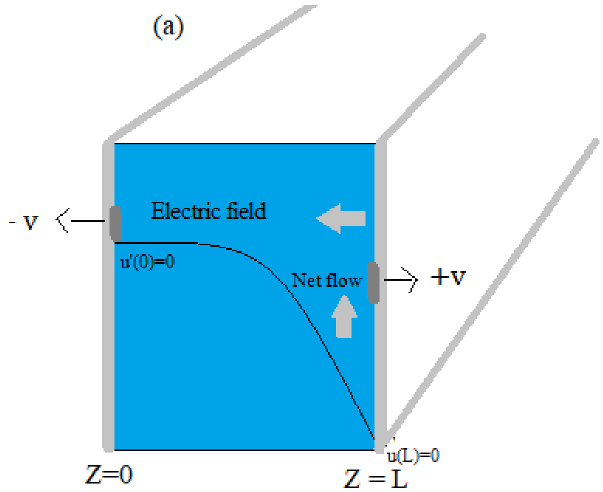
Applying a voltage (V) to a fluid creates an electric field ($\vec{E_0}$). For a single ionized particle, the current density in a dielectric medium is expressed as [20, 35]:

$$(0,0,j(R)) = \rho(R)[(0,0,w(R)) + K\vec{E_0}], \tag{2.1}$$

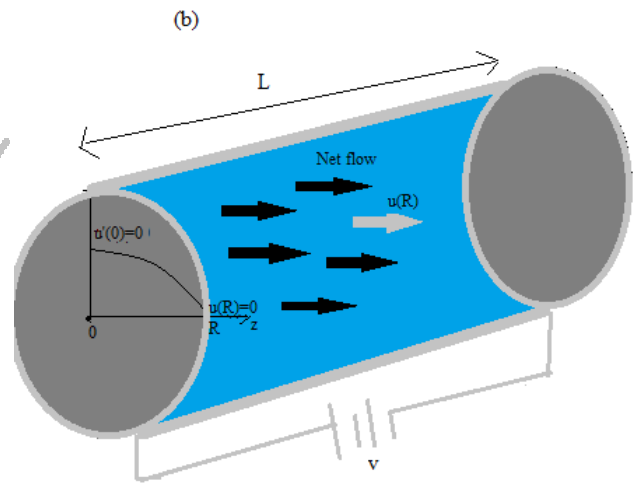
where ρ_f is the charge density of ions in fluid and K represents the ion mobility. By using equation (2.1), the Naiver-Stokes equation reduces to

$$R\frac{\partial p}{\partial z} = R\rho_f E_0 + \mu\frac{d}{dR}\left[R\frac{dw}{dR}\right], \tag{2.2}$$

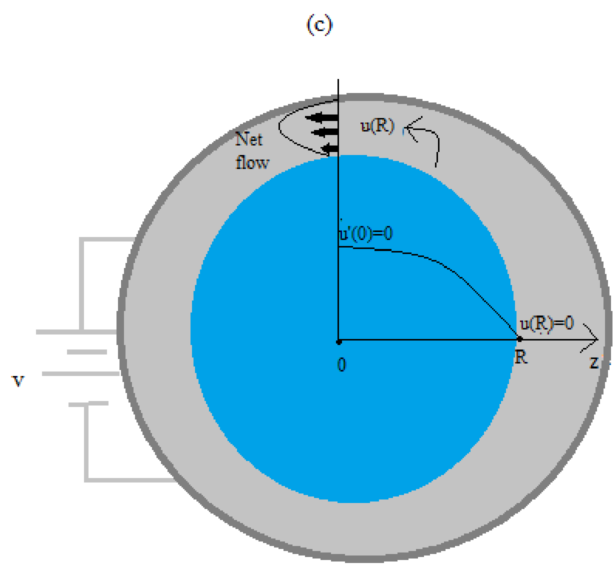
where μ represents the viscosity of the fluid, U and L are the velocity scale and length, the pressure gradient $\left(\frac{\partial p}{\partial z}\right)$ is assumed to be constant. Also ϵ_0 denotes the permittivity constant of free space, t_c is the relation time of charge, t_f is the time of fluid transport, and ρ_0 denotes the charge density at



(A) Amount of BSP in the blood



(B) Amount of BSP in the liver



(C) Amount of BSP in the liver

FIGURE 1. Schematic diagram of the EHD flow in a planar, cylindrical and spherical conduit.

the inlet screen. From equation (1), assuming that $j(z=0)=j_0$ the charge density can be defined as

$$t_c=\frac{\epsilon_0}{K\rho_0}, t_f=\frac{L}{U}, \rho_f(R)=\frac{j_0}{KE_0+u(R)}. \tag{2.3}$$

In order to calculate the velocity profile of the EHD fluid, Equations (2.1)–(2.3) are integrated to obtain a nonlinear differential given by

$$\frac{\partial p}{\partial z}=\frac{j_0E_0}{KE_0+u(R)}+\frac{\mu}{R}\frac{d}{dR}\left[R\frac{dw}{dR}\right], \tag{2.4}$$

with the following boundary conditions defined at the center and wall of a conduit:

$$u'(R) = 0 \text{ at } R = 0, \; u(R) = 0 \text{ at } R = a. \tag{2.5}$$

For any case of conduit, the velocity is bounded at $R = 0$. By introducing the dimensionless variables

$$r = \frac{R}{a} \quad \text{and} \quad w = -\frac{u}{KE_0\alpha}, \tag{2.6}$$

the electrohydrodynamic flow of a fluid in an "ion drag"configuration in a circular general conduit is governed by the following nonlinear second-order ordinary differential equation [35]:

$$\frac{1}{r^n}\frac{d}{dr}\left\{r^n\frac{dw(R)}{dr}\right\} + Ha^2\left(\frac{1-(\alpha+1)w(r)}{1-\alpha w(r)} =\right) = 0. \tag{2.7}$$

The boundary conditions are

$$w'(0) = 0 \tag{2.8}$$

$$w(1) = 0, \tag{2.9}$$

where $w(r)$ represents the velocity of fluid, Ha is the Hartmann number, r is the radial distance from the centre of the conduit and α is the measures of nonlinearity. The effectiveness factor is given by:

$$\eta = \frac{-(n+1)}{Ha^2}\left(\frac{dw}{dr}\right)_{r=1}. \tag{2.10}$$

3. Approximate analytical expression of the fluid velocity

3.1. **Akbari-Ganji method (AGM).** Nonlinear differential equations can be solved using the innovative algebraic AGM method [36–40]. Initially, a solution function consisting of unknown constant coefficients is assumed in the AGM, satisfying the differential equation and the initial conditions. Then, the unknown coefficients are computed using algebraic equations obtained concerning the initial condition and their derivatives. AGM has shown to be effective algebraic approach for obtaining accurate semi-analytic solutions. The analytical expression of fluid velocity for all parameters is obtained using this method as follows (see Appendix A):

$$w(r) = w_2(r^2 - 1), \tag{3.1}$$

where the parameter w_2 is obtained by solving the nonlinear equation

$$2w_2(1+n) + Ha^2\left(1 + \frac{0.9w_2}{1+0.9\alpha w_2}\right) = 0, \tag{3.2}$$

and hence, the effectiveness factor is computed from

$$\eta = \frac{-(n+1)}{Ha^2}\left(\frac{dw}{dr}\right)_{r=1} = \frac{-2(n+1)w_2}{Ha^2}. \tag{3.3}$$

3.2. **Adomian decomposition method (ADM).** The Adomian decomposition method is distinguished by its extensive applicability, easy computation, and fast convergence rate without estimate constraints [41,42]. Using this method, the analytical expression of fluid velocity for planar, cylindrical, and spherical conduit are obtained as follows (see Appendix B)

$$w(r) = \frac{Ha^2}{2(n+1)}(1+r_2) + \frac{(Ha^2)^2(\alpha^2-1)}{2(n+1)} f(r), \tag{3.4}$$

where $f(r)$ is given by

$$f(r) = \left(\frac{r^4}{12} - \frac{r^2}{2} + \frac{5}{12}\right)(\text{for planar conduit}), \tag{3.5}$$

$$f(r) = \left(\frac{r^4}{16} - \frac{r^2}{4} + \frac{3}{16}\right)(\text{for cylindrical conduit}), \tag{3.6}$$

$$f(r) = \left(\frac{r^4}{20} - \frac{r^2}{6} + \frac{7}{60}\right)(\text{for spherical conduit}). \tag{3.7}$$

Furthermore, the effectiveness factor is computed by

$$\eta = \frac{-(n+1)}{Ha^2}\left(\frac{dw}{dr}\right)_{r=1} = \left(\frac{1}{2} + \frac{Ha^2(\alpha^2-1)}{s}\right), \tag{3.8}$$

where $S = 3, 8, 15$ for planar, cylindrical and spherical, respectively.

3.3. **Previous solution.** Using the homotopy perturbation method (HPM), Mastroberardino [19] obtained the following expression for the velocity in a cylindrical conduit:

$$W(r) \approx W_N(r) = \sum_{m=0}^{N} W_m(r) = \sum_{m=0}^{N} W_m^{\sharp}(r) + c_1(r) + c_2, \tag{3.9}$$

where $W_m^{\sharp}(r)$ is a particular solution of $L[W_m(r) - \chi_m W_{m-1}(r)] = hR_m(\bar{W}_{m-1})$, in which

$$\bar{W}_{m-1} = W''_{m-1} + \tfrac{1}{r}W'_{m-1} + H^2[1-\chi_m-(1+\alpha)W_{m-1}] - \alpha\sum_{i=1}^{m-1} W_i W''_{m-1-i} - \tfrac{\alpha}{r}\sum_{i=1}^{m-1} W_i W'_{m-1-i}.$$

Also Khan et al. [21] employed a modified form of the homotopy perturbation method to derive the following expression for the EHD flow of electrical field in a cylindrical conduit:

$$w(r) = a + \frac{Ha^2(-1+\alpha+\alpha a)}{4(1-\alpha a)} r^2 - \frac{Ha^4(-1+\alpha+\alpha a)}{64(1-\alpha a)^3} r^4, \tag{3.10}$$

where a is an unknown parameter. Hasankhani Gavabari [25] determined the velocity distribution in a cylindrical conduit using the Galerkin method, assuming the physical parameters $Ha = 0.9$ and $\alpha = 0.1$, resulting in the following expression:

$$\begin{aligned} w(r) &= 0.1752427561 - 0.1663792639r^2 - 0.000008000646846r^3 \\ &\quad - 0.008704233158r^4 - 0.00002738811379r^5 - 0.0001238703081r^6. \end{aligned} \tag{3.11}$$

The corresponding residual functions, $R(b\check{} f, r)$, are then determined, with the assumption that they must approach zero. This analysis is related to the Electrohydrodynamic (EHD) flow of a fluid in circular cylindrical conduit.

Ghasemi et. al [27] applied the least square method to derive the following solution of the governing differential equations:

$$w(r) = c_1(1-r^2) + c_2(1-r^3) + c_3(1-r^4) + c_4(1-r^5). \tag{3.12}$$

However, calculating the coefficients $c_i, i = 1, 2, \cdots, 5$ from the residual function (3.12) is computationally cumbersome.

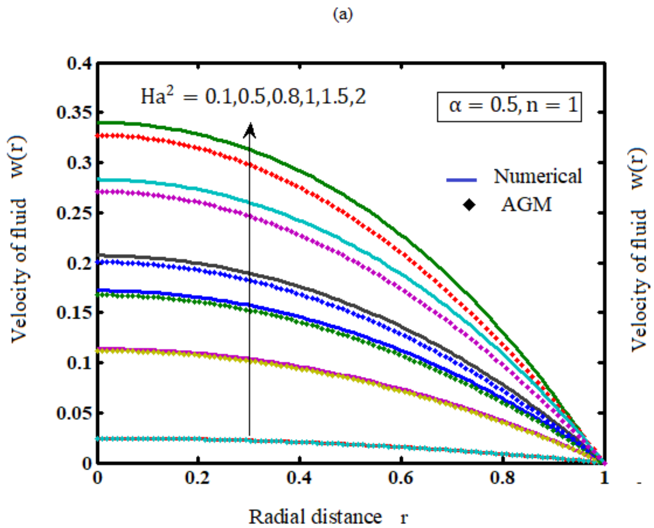
4. Discussion

Equations (3.1) and (3.4) provide a new general analytical result for the velocity profile of a planar, cylindrical, and spherical conduit.

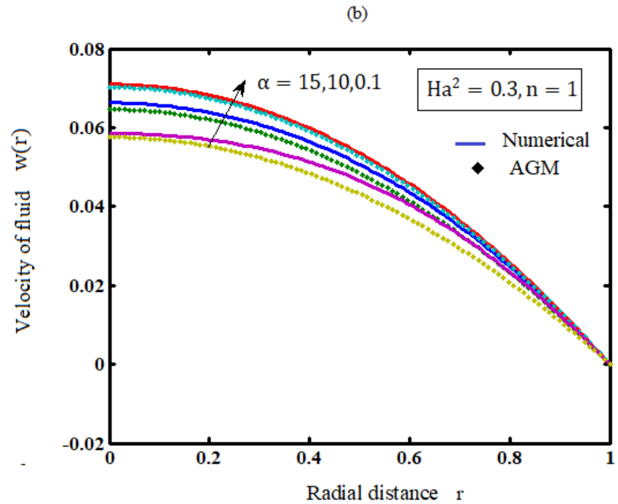
4.1. **Validation of analytical results with numerical simulation.** The general and simple asymptotic expression for the velocity and effectiveness profiles of the EHD flow is derived by solving the strong nonlinear equation. The simple and closed-form result is valid for all Hartmann electric number experimental values and nonlinearity strengths. Our new analytical results and the numerical results (MATLAB) were compared to establish the accuracy of the suggested analytical techniques. Velocity profiles for different Hartmann electric number values and nonlinearity parameter values are illustrated using Figures 1-2 and Tables 1-2. The Tables show that the maximum average deviation between AGM and the numerical result is 3. However, the deviation between ADM and numerical result is 3.5. With the increasing value of α and decreasing value of Ha^2, the problem stiffened, and the absolute errors decreased.

4.2. **Effect of the parameters on velocity profile.** The impact of different parameters, including the Hartmann electric number, nonlinearity strength, and shape factor on velocity, is illustrated in Figures 2(a-c). From Figure 2(a), it can be seen that an increase in the Hartmann electric number causes an increase in the velocity profile. Figure 2(b) shows that an increase in the nonlinearity parameter causes a decrease in the velocity profile of the EHD flow of fluid. The strength of nonlinearity adversely affects the velocity profile in relation to Hartmann electric number. Moreover, a plug flow profile appears for small values with significantly large values of the shape factor. Figure 2(c) shows that the flat conduit has a higher velocity than the other two conduits.

4.3. **Effect of parameters on effectiveness factor.** As shown in Figures 3(a-c), the effectiveness factor is influenced by various parameters, such as the shape factor, nonlinearity intensity, and Hartmann electric number. As the nonlinearity parameter increases, the effectiveness factor of the EHD flow decreases, as illustrated in Figure 3(a). It is evident from Figure 3(b) that the effectiveness factor rises in proportion to the Hartmann electric number. The flat conduit has a higher effectiveness factor than the other two conduits, as shown in Figure 3(c).



(A)



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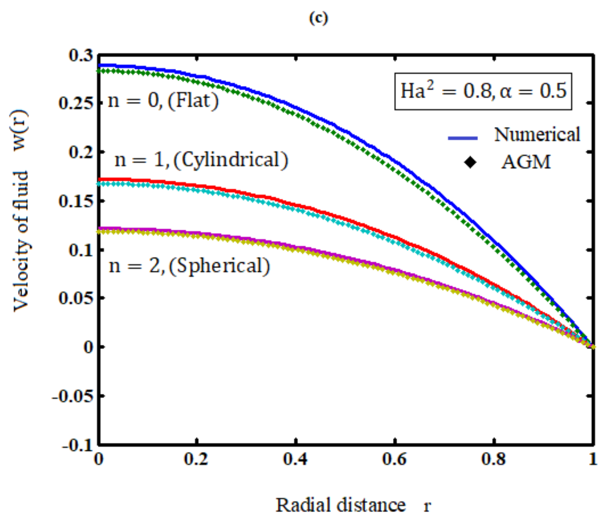

(C)

Figure 2. Comparative analysis between analytical fluid velocity $w(r)$ (Eq. (3.1)) and simulation results for different values of Ha^2, α, and n.

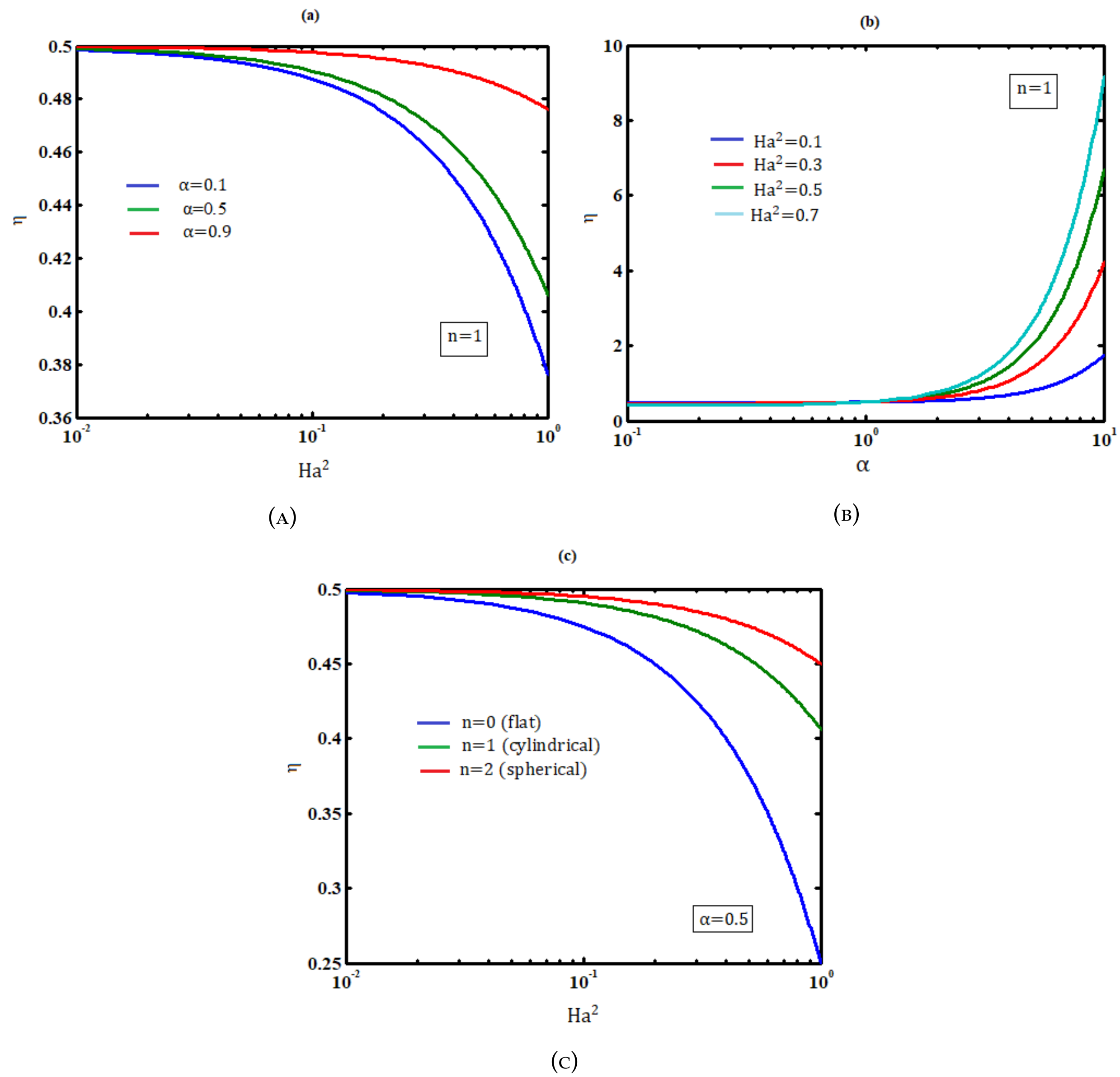

Figure 3. The effectiveness factor curve η (Eq. (3.8)) against different values of Ha^2 and α for the flat, cylindrical, and spherical geometries.

Table 1. Comparison between numerical and analytical results for fluid velocity $w(r)$ for various values of parameter Ha^2 when $\alpha = 0.5, n = 1$.

r	$Ha^2 = 0.1, w_2 = -0.02444$					$Ha^2 = 0.5, w_2 = -0.11176$					$Ha^2 = 1, w_2 = -0.20043$				
	Num.	fluid velocity		Error		Num.	fluid velocity		Error		Num.	fluid velocity		Error	
		AGM	ADM	AGM	ADM		AGM	ADM	AGM	ADM		AGM	ADM	AGM	ADM
		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)
0	0.0245	0.0244	0.0247	0.4081	0.8200	0.1138	0.1117	0.1162	1.8453	2.1090	0.2071	0.2004	0.2148	3.2351	3.7180
0.2	0.0235	0.0234	0.0237	0.4255	0.8511	0.1093	0.1072	0.1116	1.9213	2.1043	0.1992	0.1923	0.2065	3.4638	3.6646
0.4	0.0206	0.0204	0.0206	0.4878	0.4878	0.0957	0.0940	0.0976	1.7764	1.9854	0.1752	0.1690	0.1814	3.5388	3.5388
0.6	0.0156	0.0157	0.0156	0.6410	0.0000	0.0729	0.0717	0.0742	1.6460	1.7832	0.1344	0.1368	0.1387	1.7857	3.1944
0.8	0.0085	0.0085	0.0086	0.0000	1.1765	0.0403	0.0399	0.0400	0.9925	0.7444	0.0751	0.0760	0.0772	1.1984	2.7963
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Average error			0.3271	0.5559	Average error			1.3636	1.4543	Average error			2.2036	2.8195

Table 2. Comparison between numerical and analytical results for fluid velocity $w(r)$ for various values of parameter Ha^2 when $\alpha = 0.1, n = 1$.

r	$Ha^2 = 0.1, w_2 = -0.02445$					$Ha^2 = 0.5, w_2 = -0.112244$					$Ha^2 = 1, w_2 = -0.203385$				
	Num.	fluid velocity		Error		Num.	fluid velocity		Error		Num.	fluid velocity		Error	
		AGM	ADM	AGM	ADM		AGM	ADM	AGM	ADM		AGM	ADM	AGM	ADM
		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)		Eq.(3.1)	Eq.(3.4)	Eq.(3.1)	Eq.(3.4)
0	0.0245	0.0244	0.0245	0.0250	0.0000	0.1142	0.1122	0.1134	1.7513	0.7005	0.2096	0.2034	0.2336	2.9580	11.4504
0.2	0.0235	0.0234	0.0235	0.0240	0.0000	0.1096	0.1077	0.1089	1.7336	0.6387	0.2015	0.1951	0.1959	3.1762	2.7792
0.4	0.0206	0.0204	0.0205	0.9709	0.4854	0.0960	0.0939	0.0954	2.1875	0.6250	0.1772	0.1702	0.1725	3.9503	2.6523
0.6	0.0156	0.0155	0.0155	0.6410	0.6410	0.0730	0.0710	0.0726	2.7397	0.5479	0.1357	0.1387	0.1324	2.2108	2.4318
0.8	0.0085	0.0085	0.0085	0.0000	0.0000	0.0404	0.0389	0.0402	3.7128	0.4950	0.0751	0.0706	0.0741	5.9921	1.3315
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Average error			0.2768	0.1877	Average error			2.0208	0.5012	Average error			3.0479	3.4409

Table 3. Comparison between numerical, previous result and analytical results for fluid velocity $w(r)$ for various values of parameters $Ha^2 = 1$ and $\alpha = 0.5, n = 1$.

r	$Ha^2 = 1, w_2 = -0.20043, a = 0.2034$						
	Numerical	fluid velocity			Error		
		AGM	ADM	Khan et. al. [21]	AGM	ADM	Khan et. al. [21]
		Eq.(3.1)	Eq.(3.4)	Eq.(3.10)	Eq.(3.1)	Eq.(3.4)	Eq.(3.10)
0	0.2071	0.2004	0.2148	0.2034	3.2351	3.7180	1.7866
0.2	0.1992	0.1923	0.2065	0.1955	3.4638	3.6646	1.8574
0.4	0.1752	0.1690	0.1814	0.1722	3.5388	3.5388	1.7321
0.6	0.1344	0.1368	0.1387	0.1344	1.7857	3.1994	0.0000
0.8	0.0751	0.0760	0.0772	0.0835	1.1984	2.7963	11.185
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Average error				2.2036	2.8195	2.7602

Table 4. Comparison between numerical, previous result and analytical results for fluid velocity $w(r)$ for various values of parameters $Ha^2 = 0.9$ and $\alpha = 0.1, n = 1$.

r	$Ha^2 = 0.9, w_2 = -0.186574$						
	Numerical	fluid velocity			Error		
		AGM	ADM	Gavabari et. al. [25]	AGM	ADM	Gavabari et. al. [25]
		Eq.(3.1)	Eq.(3.4)	Eq.(3.11)	Eq.(3.1)	Eq.(3.4)	Eq.(3.11)
0	0.1919	0.1866	0.1874	0.1752	2.7618	2.3450	8.7024
0.2	0.1844	0.1790	0.1803	0.1684	2.7115	2.2234	8.6956
0.4	0.1620	0.1561	0.1585	0.1478	3.1996	2.1605	8.7654
0.6	0.1239	0.1180	0.1215	0.1129	4.7619	1.9370	8.8781
0.8	0.0690	0.0647	0.0679	0.0684	6.2319	1.5942	0.8695
1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	Average error				3.2778	1.7100	5.9852

5. Conclusions

The problem in the form of a rational function of nonlinearity poses a significant challenge for obtaining analytical solutions. In this paper, the nonlinear boundary value problem for the electrohydrodynamic flow of a fluid has been solved analytically and numerically. Analytical expressions for the fluid velocity can be derived using the AGM and ADM methods. The primary result of this work is simple and approximate expressions of the fluid velocity for all values of the dimensionless parameters. The proposed methods provide a more accurate solution to the nonlinear differential equation in EHD fluid flow. This analytical result will be helpful in analyzing the behaviour of the electrohydrodynamic flow of a fluid in an ion-drag configuration in a general conduit.

Appendix A. Analytical expressions of the fluid velocity using Akbari-Ganji method

We can assume that the Akbari-Ganji solution of equation (2.7) is in the following form.

$$w(r) = \sum_{i=0}^{2} w_i r^i = w_0 + w_1 r + w_2 r^2, \tag{A.1}$$

where w_0, w_1 and w_2 are constants. Using the boundary conditions (2.8) and (2.9), we get

$$w_1 = 0, w_0 = -u_2, \tag{A.2}$$

$$w(r) = w_2(r^2 - 1) \tag{A.3}$$

Now define the function F by

$$F(r) = \frac{d^2 w(r)}{dr^2} + \frac{n}{r}\frac{dw(r)}{dr} + Ha^2\left(1 - \frac{w(r)}{1 - \alpha w(r)}\right) = 0 \tag{A.4}$$

Using Eq. (A.1), the equation (A.4) at $r = 0.1$ becomes

$$F(r = 0.1) = 2w_2(1 + n) + Ha^2\left(1 + \frac{0.9w_2}{1 + 0.9\alpha w_2}\right) = 0 \tag{A.5}$$

The parameter w_2 is obtained by solving the nonlinear equation (6.6).

$$2u_2(1 + n) + Ha^2\left(1 + \frac{0.9u_2}{1 + 0.9\alpha u_2}\right) = 0 \tag{A.6}$$

The unknown parameter w_2 can be obtained by solving equation (A.6) using MATLAB or any computer algebra software.

Appendix B. Analytical expressions of the fluid velocity using modified Adomian decomposition method

In order to solve Eq. (2.7) using the modified Adomian decomposition method [43], we write Eq. (2.7) in the following operator form

$$L(u) = Ha^2 N(u) \tag{B.1}$$

where L and N are linear and non-linear terms of Eq. (2.7). Here $N(r) = 1 - \frac{w(r)}{1-\alpha w(r)}$. The operator for cylindrical conduit is defined as

$$L(\cdot) = r^{-1}\frac{d}{dr}r^{1}\frac{d}{dr}r^{0}(\cdot) \tag{B.2}$$

and the inverse operator is given by

$$L^{-1}(\cdot) = r^0 \int_1^r r^{-1} \int_0^r r(\cdot)drdr \tag{B.3}$$

Applying Eq. (B.3) to Eq. (B.1) gives

$$w(r) = Ar + B + Ha^2 L^{-1}\left(1 - \frac{w(r)}{1 - \alpha w(r)}\right), \tag{B.4}$$

where A and B are constants of integration. Also,

$$w(r) = \sum_{i=0}^{\infty} w_i(r) \text{ and } N(w(r)) = \sum_{i=0}^{\infty} A_i \tag{B.5}$$

In view of Eqs. (B.4) and (B.5), we have

$$\sum_{i=0}^{\infty} c_i(r) = Ar + B + Ha^2 L^{-1} \sum_{i=0}^{\infty} A_i. \tag{B.6}$$

The zeroth term is given by

$$w_0(r) = Ar + B, \tag{B.7}$$

and the remaining terms are produced by the recurrence relation

$$w_{i+1}(r) = Ha^2 L^{-1} A_i(r), \quad i \geq 0, \tag{B.8}$$

where

$$A_i(r) = \frac{1}{i!} \frac{d^i}{d\lambda^i} N\left(\sum_{i=0}^{\infty} \lambda^i w_i\right)_{\lambda=0} \tag{B.9}$$

are the Adomian polynomials of $w_1, w_2, \ldots, w_i$. We can find A_i as follows:

$$A_0 = N(w_0) = 1 - \frac{w_0}{1 - \alpha w_0}, \quad A_1 = \left[\frac{d}{d\lambda} N(w_0 + \lambda u_1)\right]_{\lambda=0} = \left(Ha^2\right)^2 \left(\alpha^2 - 1\right) \tag{B.10}$$

Now the polynomials can be generated as

$$w_0(r) = 0, w_1(r) = \frac{Ha^2}{4}(1 - r^2), \quad w_2(r) = \frac{(Ha^2)^2(\alpha^2 - 1)}{4}\left(\frac{x^4}{16} - \frac{x^2}{4} + \frac{3}{16}\right) \tag{B.11}$$

Notice that the fluid velocity for the cylindrical conduit given in Eq. (2.6) is the sum of w_0, w_1, and w_2.

Nomenclature

Symbols	Description
a	Conduit radius
$\vec{E_0}$	Electric field
Ha	Hartmann electric number
j	Current density
K	Ion mobility
L	Length
p	Pressure
r	Dimensionless distance
t_c	Charge relaxation time
t_f	Fluid transport time
U	Velocity scale
V	Velocity
$u(R)$	Fluid velocity

Symbols	Description
$w(r)$	Dimensionless of fluid velocity
z	Axial coordinate
α	Measure of nonlinearity
μ	Viscosity of the dielectric fluid
ρ_f	Free charge density of the ion/fluid medium
ϵ_0	Permittivity constant of free space
ρ_0	Charge density at the inlet screen

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