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Upper and Lower Faint (τ_1, τ_2)-Continuity

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Abstract. This paper is concerned with the notions of upper and lower faintly (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations of upper and lower faintly (τ_1, τ_2)-continuous multifunctions are investigated.

1. Introduction

Stronger and weaker forms of open sets play an important role in the generalization of different forms of continuity. Using different forms of open sets, many authors have introduced and studied various types of continuity for functions and multifunctions. Viriyapong and Boonpok [57] studied some characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets due to Boonpok and Khampakdee [19]. Dungthaisong et al. [34] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [33] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, some characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, almost quasi (τ_1, τ_2)-continuous functions and weakly quasi (τ_1, τ_2)-continuous functions were presented in [51], [53], [2], [49], [13], [15], [8], [23], [28], [29], [3], [4], [5], [40] and [32], respectively. Long and Herrington [42] considered the class of faintly continuous functions. Additionally, several

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characterizations of faintly continuous functions were studied in [43] and [45], respectively. Three weak forms of faint continuity were introduced by Noiri and Popa [44]. Nasef and Noiri [43] introduced and studied three strong forms of faint continuity under the names of strongly faint semi-continuity, strongly faint precontinuity and strongly faint β-continuity. Jafari and Noiri [36] introduced and investigated the concept of faintly α-continuous functions. Chanapun et al. [31] introduced a new class of functions, called faintly (m, μ)-continuous functions and established the relationships between faint (m, μ)-continuity and other related generalized forms of (m, μ)-continuity.

In 2011, Jafari et al. [35] introduced and studied a new class of multifunctions called faintly δ-β-continuous multifunctions. Popa and Noiri [46] introduced and investigated the notion of θ-quasi continuous multifunctions. Laprom et al. [41] introduced and studied the concept of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Viriyapong and Boonpok [59] introduced and investigated the notion of $(\tau_1, \tau_2)\alpha$-continuous multifunctions. Moreover, several characterizations of $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\iota^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, weakly $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, slightly (Λ, sp)-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions, s-$(\tau_1, \tau_2)p$-continuous multifunctions, c-(τ_1, τ_2)-continuous multifunctions and slightly (τ_1, τ_2)-continuous multifunctions were investigated in [24], [20], [26], [21], [58], [6], [7], [25], [9], [11], [10], [16], [22], [12], [38], [17], [14], [54], [18], [47], [39], [52], [48], [55], [37] and [50], respectively. Carpintero et al. [30] introduced and investigated the concept of faintly ω-continuous multifunctions as a generalization of ω-continuous multifunctions due to Zorlutuna [60]. In this paper, we introduce the concepts of upper and lower faintly (τ_1, τ_2)-continuous multifunctions. We also investigate several characterizations of upper and lower faintly (τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [27] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be

$\tau_1\tau_2$-*clopen* [27] if A is both $\tau_1\tau_2$-open and $\tau_1\tau_2$-closed. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [27] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [27] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 2.1. [27] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) *$A \subseteq \tau_1\tau_2$-$Cl(A)$ and $\tau_1\tau_2$-$Cl(\tau_1\tau_2$-$Cl(A)) = \tau_1\tau_2$-$Cl(A)$.*
(2) *If $A \subseteq B$, then $\tau_1\tau_2$-$Cl(A) \subseteq \tau_1\tau_2$-$Cl(B)$.*
(3) *$\tau_1\tau_2$-$Cl(A)$ is $\tau_1\tau_2$-closed.*
(4) *A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-$Cl(A)$.*
(5) *$\tau_1\tau_2$-$Cl(X - A) = X - \tau_1\tau_2$-$Int(A)$.*

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [59] (resp. $(\tau_1, \tau_2)s$-*open* [24], $(\tau_1, \tau_2)p$-*open* [24], $(\tau_1, \tau_2)\beta$-*open* [24]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))))$. The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is said to be $(\tau_1, \tau_2)r$-*closed*, $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*. A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [56] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is called $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [59] of A if $\tau_1\tau_2$-Cl$(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U of X containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [59] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [59] if $(\tau_1, \tau_2)\theta$-Cl$(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [59] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2.2. [59] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2$-$Cl(A) = (\tau_1, \tau_2)\theta$-$Cl(A)$.*
(2) *$(\tau_1, \tau_2)\theta$-$Cl(A)$ is $\tau_1\tau_2$-closed in X.*

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, following [1] we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and $F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}$. In particular,

$$F^-(y) = \{x \in X \mid y \in F(x)\}$$

for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and Lower Faintly (τ_1, τ_2)-Continuous Multifunctions

In this section, we introduce the concepts of upper and lower faintly (τ_1, τ_2)-continuous multifunctions. Moreover, some characterizations of upper and lower faintly (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 3.1. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be:*

(i) *upper faintly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $F(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(U) \subseteq V$;*

(ii) *upper faintly (τ_1, τ_2)-continuous if F has this property at every point of X.*

Recall that a net (x_γ) in a topological space (X, τ) is said to be *eventually* in the set $U \subseteq X$ if there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$.

Definition 3.2. *A sequence (x_n) is called (τ_1, τ_2)-converge to a point x if for every $\tau_1\tau_2$-open set V containing x, there exists an index γ_0 such that for $\gamma \geq \gamma_0$, $x_\gamma \in V$.*

Theorem 3.1. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper faintly (τ_1, τ_2)-continuous;*

(2) *for each $x \in X$ and for every $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^+(V)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(V)$;*

(3) *for each $x \in X$ and for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y such that $x \in F^+(Y - K)$, there exists a $\tau_1\tau_2$-closed set H of X containing $x \in X - H$ and $F^-(K) \subseteq H$;*

(4) *$F^+(V)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(5) *$F^-(K)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*

(6) *$F^-(Y - V)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(7) *$F^+(Y - K)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*

(8) *for each $x \in X$ and for each net (x_γ) which (τ_1, τ_2)-converges to x in X and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^+(V)$, the net (x_γ) is eventually in $F^+(V)$.*

Proof. (1) $\Leftrightarrow$ (2): Obvious.

(2) $\Leftrightarrow$ (3): Let $x \in X$ and K be any $(\sigma_1, \sigma_2)\theta$-closed set of Y such that $x \in F^+(Y - K)$. By (2), there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(Y - K)$. Then, $F^-(K) \subseteq X - U$. Put $H = X - U$. Then, we have H is $\tau_1\tau_2$-closed and $x \in X - H$. The converse is similar.

(1) $\Leftrightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)\theta$-open set of Y and $x \in F^+(V)$. By (1), there exists a $\tau_1\tau_2$-open set U_x of X containing x such that $U_x \subseteq F^+(V)$. Thus, $F^+(V) = \cup_{x \in F^+(V)} U_x$ and hence $F^+(V)$ is $\tau_1\tau_2$-open in X. The converse can be shown similarly.

(4) $\Rightarrow$ (5): Let K be any $(\sigma_1, \sigma_2)\theta$-closed set of Y. Then, $Y - K$ is $(\sigma_1, \sigma_2)\theta$-open in Y and by (4), $F^+(Y - K) = X - F^-(K)$ is $\tau_1\tau_2$-open in X. Thus, $F^-(K)$ is $\tau_1\tau_2$-closed in X.

(5) $\Rightarrow$ (4): It similar to that of (4) $\Rightarrow$ (5).

(4) $\Leftrightarrow$ (6) and (5) $\Leftrightarrow$ (7): It follows from the fact that $F^-(Y - B) = X - F^+(B)$ and $F^+(Y - B) = X - F^-(B)$ for every subset B of Y.

(1) ⇒ (8): Let (x_γ) be a net which (τ_1, τ_2)-converges to x in X and let V be any $(\sigma_1, \sigma_2)\theta$-open set of Y such that $x \in F^+(V)$. Since F is upper faintly (τ_1, τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^+(V)$. Since (x_γ) (τ_1, τ_2)-converges to x, it follows that there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$. Therefore, $x_\gamma \in U \subseteq F^+(V)$ for all $\gamma \geq \gamma_0$. Thus, the net (x_γ) is eventually in $F^+(V)$.

(8) ⇒ (1): Suppose that F is not upper faintly (τ_1, τ_2)-continuous. There exists a point x of X and a $(\sigma_1, \sigma_2)\theta$-open set V of Y with $x \in F^+(V)$ such that $U \not\subseteq F^+(V)$ for each $\tau_1\tau_2$-open set U of X containing x. Let $x_U \in U$ and $x_U \notin F^+(V)$ for each $\tau_1\tau_2$-open set U of X containing x. Then, for each $\tau_1\tau_2$-neighbourhood net (x_U), (x_U) (τ_1, τ_2)-converges to x, but (x_U) is not eventually in $F^+(V)$. This is a contradiction. Thus, F is upper faintly (τ_1, τ_2)-continuous. □

Definition 3.3. *A multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be:*

(i) *lower faintly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $F(x) \cap V \neq \emptyset$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $F(z) \cap V \neq \emptyset$ for each $z \in U$;*

(ii) *lower faintly (τ_1, τ_2)-continuous if F has this property at every point of X.*

Theorem 3.2. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower faintly (τ_1, τ_2)-continuous;*

(2) *for each $x \in X$ and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^-(V)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq F^-(V)$;*

(3) *for each $x \in X$ and for each $(\sigma_1, \sigma_2)\theta$-closed set K of Y such that $x \in F^-(Y - K)$, there exists a $\tau_1\tau_2$-closed set H of X such that $x \in X - H$ and $F^+(K) \subseteq H$;*

(4) *$F^-(V)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(5) *$F^+(K)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*

(6) *$F^+(Y - V)$ is $\tau_1\tau_2$-closed in X for every $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(7) *$F^-(Y - K)$ is $\tau_1\tau_2$-open in X for every $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*

(8) *for each $x \in X$ and for each net (x_γ) which (τ_1, τ_2)-converges to x in X and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y such that $x \in F^-(V)$, the net (x_γ) is eventually in $F^-(V)$.*

Proof. The proof is similar to that of Theorem 3.1. □

Definition 3.4. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called faintly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called faintly (τ_1, τ_2)-continuous if f has this property at every point of X.*

Corollary 3.1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is faintly (τ_1, τ_2)-continuous;*

(2) *$f^{-1}(V)$ is $\tau_1\tau_2$-open in X for each $(\sigma_1, \sigma_2)\theta$-open set V of Y;*

(3) *$f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for each $(\sigma_1, \sigma_2)\theta$-closed set K of Y;*

(4) *for each $x \in X$ and for each $(\sigma_1, \sigma_2)\theta$-open set V of Y containing $f(x)$, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$.*

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Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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