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Some Separation Axioms via Soft Somewhat Open Sets

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Abstract. It is commonly known that some topological spaces include structures that may be used to expand abstract notions. somewhat open sets and soft sets is such sort of structures. We obtain several properties and symmetry of the soft somewhat-R_0 spaces and soft somewhat-R_1 spaces obtained. Furthermore, we present new theorems and results and investigated relation between this concepts and the other structures.

1. Introduction

Known as soft set theory, Molodtsov [1] proposed an alternative method in 1999 for handling partial information situations. This idea has been applied in a variety of contexts, including probability theory, game theory, theory of measurement, smoothness of functions, and Riemann integration. The fundamental idea of the theory of soft sets is the nature of parameter sets, which offers a broad framework for modeling data that is ambiguous. This basically advances the field of soft set theory in a little amount of time. Maji and colleagues [2] examined a (comprehensive) theoretical framework of soft set theory. They specifically established a few operators and operations that connect soft sets. Then, other mathematicians presented new forms of soft operators and operations and reformulated the operators and operations between soft sets provided in Maji et al.'s work; the reader is referred to [3] for a list of current contributions pertaining to soft operators and operations. Independent definitions of soft (generic) topology were provided in 2011 by Çağman et al. [4] and Shabir and Naz [5]. Nazmul and Samanta [6] provided a definition of soft continuity of functions in 2013. The literature thereafter started to publish a number of generalizations of soft continuity and soft openness of functions. soft semi-open functions [7], soft β-open functions [8], soft somewhere dense open [9], soft semi-continuous functions [7], soft β-continuous functions [8], and so on are examples of soft functions. Various types of relationships that belong and don't

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belong were examined in [5,10–12]. These relationships gave rise and symmetry to the multiplicity and diversity of soft topology concepts and ideas. Following this succinct introduction, we review several foundational ideas in Section 2. Subsequently, Section 3 is devoted to presenting the notion of soft somewhat open sets and soft somewhat-R_0 spaces. In Section 4 are investigate the concept of soft somewhat-R_1 spaces.

2. Preliminaries

This section presents some basic definitions and notations that will be used in the sequel. Henceforth, we mean by $\mathcal{X}$ an initial universe, γ a set of parameters and $2^{\mathcal{X}}$ the power set of $\mathcal{X}$.

Definition 2.1. *[1] Let $\Phi : \gamma \to 2^{\mathcal{X}}$ be a set-valued function form a parameters set γ to the power set of a nonempty set $\mathcal{X}$, then the pair (Φ, γ) said to be a soft set over $\mathcal{X}$, which defined as follows $(\Phi, \gamma) = \{(\alpha, \Phi(\alpha)) : \alpha \in \gamma$ and $\Phi(\alpha) \in 2^{\mathcal{X}\}}$ and we represented the soft set as $(\Phi, \gamma) = \Phi_\gamma$. Throughout this paper, $(\Phi, \gamma) = \Phi_\gamma$, $(\Omega, \gamma) = \Omega_\gamma$, $(F, \gamma) = F_\gamma$ and $(\Psi, \gamma) = \Psi_\gamma$ denote the soft sets over $\mathcal{X}$. We symbolized the family of all soft sets over $\mathcal{X}$ with parameters γ by $SS(\mathcal{X}_\gamma)$.*

Definition 2.2. (1) *[13] A soft set Φ_γ over $\mathcal{X}$ is called a soft element if $\Phi(\alpha) = \{x\}$ for all $\alpha \in \gamma$, where $x \in \mathcal{X}$. It is denoted by x_α.*

(2) *[14] The complement of Φ_γ is a soft set $\mathcal{X} \setminus \Phi_\gamma$ (or simply Φ^c_{γ},) where $\Phi^c : \gamma \to 2^{\mathcal{X}}$ is given by $\Phi^c(\alpha) = \mathcal{X} \setminus \Phi(\alpha)$ for all $\alpha \in \gamma$.*

(3) *[1] A soft set Φ_γ over $\mathcal{X}$ is called null if $\Phi(\alpha) = \emptyset$ for all $\alpha \in \gamma$.*

(4) *[1] A soft set Φ_γ over $\mathcal{X}$ is called absolute if $\Phi(\alpha) = \mathcal{X}$ for all $\alpha \in \gamma$.*

The null and absolute soft sets are respectively symbolized by $\emptyset_\gamma$ and $\mathcal{X}_\gamma$. Clearly $\emptyset^c_\gamma = \mathcal{X}_\gamma$ and $\mathcal{X}^c_\gamma = \emptyset_\gamma$.

Definition 2.3. *[5] A subfamily τ_γ of $SS(\mathcal{X}_\gamma)$ is called a soft topology on $\mathcal{X}$ if*

(1) *$\emptyset_\gamma$ and $\mathcal{X}_\gamma$ belong to τ_γ.*
(2) *finite intersection of a soft sets from τ_γ belongs to τ_γ.*
(3) *any union of a soft sets from τ_γ belongs to τ_γ.*

The element of τ_γ are called soft open sets, and their complements are called soft closed sets. A soft topological space $(\mathcal{X}_\gamma, \tau_\gamma)$ is denoted by ($\mathcal{STS}$).

Definition 2.4. *[5,15] A $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is called*

(1) *soft T_0-space if for each x_α, $y_\alpha \in \mathcal{X}_\gamma$ with $x_\alpha \neq y_\alpha$, there exist soft open sets (Φ, γ), (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$, $y_\alpha \notin (\Phi, \gamma)$ or $y_\alpha \in (\Psi, \gamma)$, $x_\alpha \notin (\Psi, \gamma)$.*
(2) *soft T_1-space if for each x_α, $y_\alpha \in \mathcal{X}_\gamma$ with $x_\alpha \neq y_\alpha$, there exist soft open sets (Φ, γ), (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$, $y_\alpha \notin (\Phi, \gamma)$ and $y_\alpha \in (\Psi, \gamma)$, $x_\alpha \notin (\Psi, \gamma)$.*
(3) *soft T_2-space if for each x_α, $y_\alpha \in \mathcal{X}_\gamma$ with $x_\alpha \neq y_\alpha$, there exist soft open sets (Φ, γ), (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$ and $y_\alpha \in (\Psi, \gamma)$ with $(\Phi, \gamma) \sqcap (\Psi, \gamma) = \emptyset_\gamma$.*

Definition 2.5. *[16] Let Δ be an arbitrary index set and $\Omega = \{(\Phi, \gamma)_\mu : \mu \in \Delta\}$ be a subfamily of $SS(\mathcal{X}_\gamma)$. Then:*

(1) *The union of all soft sets $(\Phi, \gamma)_\mu$ is the soft set (Ψ, γ), where $\Psi(e) = \sqcup_{\mu\in\Delta}(\Phi, \gamma)_\mu(e)$ for each $e \in \gamma$. We write $\sqcup_{\mu\in\Delta}(\Phi, \gamma)_\mu = (\Psi, \gamma)$.*
(2) *The intersection of all soft sets $(\Phi, \gamma)_\mu$ is the soft set (Ψ, γ), where $\Psi(e) = \sqcap_{\mu\in\Delta}(\Phi, \gamma)_\mu(e)$ for each $e \in \gamma$. We write $\sqcap_{\beta\in\Delta}(\Phi, \gamma)_\mu = (\Psi, \gamma)$.*

Now presents some basic definitions and notations of the concept of soft somewhat open sets and use it to establish some properties of soft somewhat-R_0 and soft somewhat-R_1 spaces

Definition 2.6. *[17] A subset (Φ, γ) of a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is said to be soft somewhat open (briefly soft sw-open) if either (Φ, γ) is null or $Int(\Phi, \gamma) \neq \emptyset_\gamma$.*

The complement of each soft sw-open set is called soft sw-closed. That is a soft set (Ψ, γ) is a soft sw-closed if $Cl(\Psi, \gamma) \neq \mathcal{X}_\gamma$ or $(\Psi, \gamma) = \mathcal{X}_\gamma$.

Proposition 2.1. *[17] Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a $\mathcal{STS}$, then*

(1) *A non-null set (Φ, γ) over $\mathcal{X}$ is a soft sw-open if and only if there is a soft open set (Ψ, γ) such that $\emptyset_\gamma \neq (\Psi, \gamma) \sqsubseteq (\Phi, \gamma)$.*
(2) *A proper soft set (Φ, γ) over $\mathcal{X}$ is a soft sw-closed if and only if there is a soft closed set (Ψ, γ) such that $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma) \neq \mathcal{X}_\gamma$.*
(3) *Every superset of a soft sw-open set is soft sw-open.*
(4) *Every subset of a soft sw-closed set is soft sw-closed.*
(5) *Any union of soft sw-open sets is soft sw-open.*
(6) *The intersection of two soft sw-open sets need not be soft sw-open.*

Definition 2.7. *Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be $\mathcal{STS}$ and $(\Phi, \gamma), (\Psi, \gamma) \in SS(\mathcal{X}_\gamma)$.*

(1) *The soft somewhat closure of (Φ, γ), symbolized by sw-$Cl_s(\Phi, \gamma)$, is the intersection of all soft somewhat closed supersets of (Φ, γ), i.e., sw-$Cl_s(\Phi, \gamma) = \sqcap\{(\Psi, \gamma) : (\Psi, \gamma)$ is soft somewhat closed and $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)\}$.*
(2) *The soft somewhat interior of (Φ, γ) is the set sw-$Int_s((\Phi, \gamma)) = \sqcup\{(\Psi, \gamma) : (\Psi, \gamma)$ is soft somewhat open and $(\Psi, \gamma) \sqsubseteq (\Phi, \gamma)\}$.*

3. Soft somewhat-R_0 spaces

Definition 3.1. *Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be soft topological space ($\mathcal{STS}$) and $(\Phi, \gamma) \in SS(\mathcal{X}_\gamma)$. Then the soft sw-kernel of (Φ, γ), denoted by sw-$K_s((\Phi, \gamma))$ is defined to be the set sw-$K_s((\Phi, \gamma)) = \sqcap\{(\Psi, \gamma) : (\Psi, \gamma)$ is a soft sw-open set and $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)\}$.*

Lemma 3.1. *Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be $\mathcal{STS}$ and $x_\alpha \in \mathcal{X}_\gamma$. Then, $y_\alpha \in$ sw-$K_s(\{x_\alpha\})$ if and only if $x_\alpha \in$ sw-$Cl_s(\{y_\alpha\})$.*

Proof. Suppose that $y_\alpha \notin sw\text{-}K_s(\{x_\alpha\})$. Then, there exists a soft sw-open set (Ψ, γ) such that $x_\alpha \in (\Psi, \gamma)$ and $y_\alpha \notin (\Psi, \gamma)$. Therefore, we have $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. The proof of the converse case can be done similarly. □

Lemma 3.2. *Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be $\mathcal{STS}$ and $x_\alpha \in \mathcal{X}_\gamma$. Then, $sw\text{-}K_s(\{(\Phi, \gamma)\}) = \{x_\alpha \in \mathcal{X}_\gamma : sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) \neq \emptyset_\gamma\}$.*

Proof. Let $x_\alpha \in sw\text{-}K_s(\{(\Phi, \gamma)\})$ and $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) = \emptyset_\gamma$. Hence $x_\alpha \notin [sw\text{-}Cl_s(\{x_\alpha\})]^c$ which is a soft sw-open set containing (Φ, γ). This is impossible, since $x_\alpha \in sw\text{-}K_s(\{(\Phi, \gamma)\})$. Consequently, $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) \neq \emptyset_\gamma$. Next, let $x_\alpha \in \mathcal{X}_\gamma$ such that $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) \neq \emptyset_\gamma$ and suppose that $x_\alpha \in sw\text{-}K_s(\{(\Phi, \gamma)\})$. Then, there exists a soft sw-open set (Ψ, γ) containing (Φ, γ) and $x_\alpha \notin (\Psi, \gamma)$. Let $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma)$. Hence, (Ψ, γ) is a soft sw-neighbourhood of y_α which does not contains x_α. By this contradiction $x_\alpha \in sw\text{-}K_s(\{(\Phi, \gamma)\})$ and hence $sw\text{-}K_s(\{(\Phi, \gamma)\}) = \{x_\alpha \in \mathcal{X}_\gamma : sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) \neq \emptyset_\gamma\}$. □

Definition 3.2. *An $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is said to be soft sw-R_0 space if every soft sw-open set contains the soft sw-closure of each of its singletons.*

Remark 3.1. *Since a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is soft sw-T_1 if and only if the singletons are soft sw-closed. So, it is clear that every soft sw-T_1 spaces is soft sw-R_0. But the converse is not true in general.*

Proposition 3.1. *For a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$, the following properties are equivalent:*

1. *$(\mathcal{X}_\gamma, \tau_\gamma)$ is soft sw-R_0-space;*
2. *For any soft sw-closed set (Φ, γ), $x_\alpha \notin (\Phi, \gamma)$ implies that $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)$ and $x_\alpha \notin (\Psi, \gamma)$ for some soft sw-open (Ψ, γ);*
3. *For any soft sw-closed set (Φ, γ), $x_\alpha \notin (\Phi, \gamma)$ implies that $(\Phi, \gamma) \sqcap sw\text{-}Cl_s(\{x_\alpha\}) = \emptyset_\gamma$;*
4. *For any distinct soft points x_α and y_α of $\mathcal{X}_\gamma$, either $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\})$ or $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$.*

Proof. (1) ⇒ (2): Let (Φ, γ) be a soft sw-closed and $x_\alpha \notin (\Phi, \gamma)$. Then, by (1) $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq \mathcal{X}_\gamma \setminus (\Phi, \gamma)$. Set $(\Psi, \gamma) = [sw\text{-}Cl_s(\{x_\alpha\})]^c$, then (Ψ, γ) is a soft sw-open, $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)$ and $x_\alpha \notin (\Psi, \gamma)$.
(2) ⇒ (3): Let (Φ, γ) be a soft sw-closed and $x_\alpha \notin (\Phi, \gamma)$. There exists (Ψ, γ) which is a soft sw-open, $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)$ and $x_\alpha \notin (\Psi, \gamma)$. Since (Ψ, γ) is a soft sw-open, $(\Psi, \gamma) \sqcap sw\text{-}Cl_s(\{x_\alpha\}) = \emptyset_\gamma$ and so $(\Phi, \gamma) \sqcap sw\text{-}Cl_s(\{x_\alpha\}) = \emptyset_\gamma$.
(3) ⇒ (4): Suppose that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$ for distinct soft points x_α and y_α. Then, there exists $z_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ such that $z_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. There exists a soft sw-open (Φ, γ) such that $y_\alpha \notin (\Phi, \gamma)$ and $z_\alpha \in (\Phi, \gamma)$, hence $x_\alpha \in (\Phi, \gamma)$. Therefore, we have $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. By (3) we obtain $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$.
(4) ⇒ (1): Let (Φ, γ) be a soft sw-closed set containing x_α. For any $y_\alpha \notin (\Phi, \gamma)$, $x_\alpha \neq y_\alpha$ and $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. Hence $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. By (4) $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$ for each

$y_\alpha \notin (\Phi,\gamma)$ and hence $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap \left(\sqcup_{y_\alpha\notin(\Phi,\gamma)} sw\text{-}Cl_s(\{y_\alpha\})\right) = \emptyset_\gamma$. On other hand, since (Φ,γ) be a soft sw-closed and $y_\alpha \notin (\Phi,\gamma)$ we have $sw\text{-}Cl_s(\{y_\alpha\}) \sqsubseteq [(\Phi,\gamma)]^c$ and hence $[(\Phi,\gamma)]^c = \left(\sqcup_{y_\alpha\notin(\Phi,\gamma)} sw\text{-}Cl_s(\{y_\alpha\})\right)$. Therefore, we obtain $[(\Phi,\gamma)]^c \sqcap sw\text{-}Cl_s(\{x_\alpha\}) = \emptyset_\gamma$ and $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$. Hence, $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft $sw\text{-}R_0$-space. □

Theorem 3.1. *An $\mathcal{STS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft $sw\text{-}R_0$-space if and only if for any $x_\alpha \neq y_\alpha$, $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$ implies that $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$.*

Proof. Suppose that $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft $sw\text{-}R_0$-space and $x_\alpha \neq y_\alpha$, such that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Then, there exists $z_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ such that $z_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. So, there exists a soft sw-open (Φ,γ) such that $y_\alpha \notin (\Phi,\gamma)$ and $z_\alpha \in (\Phi,\gamma)$, hence $x_\alpha \in (\Phi,\gamma)$. Therefore, we have $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. Thus, $x_\alpha \in [sw\text{-}Cl_s(\{y_\alpha\})]^c$ which is a soft sw-open and implies that $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq [sw\text{-}Cl_s(\{y_\alpha\})]^c$ and $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$.
Let (Φ,γ) be any soft sw-open set containing x_α. We show that $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$. Let $y_\alpha \notin (\Phi,\gamma)$ i.e., then $x_\alpha \neq y_\alpha$ and $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. This shows that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Hence, $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) = \emptyset_\gamma$. So, $y_\alpha \notin sw\text{-}Cl_s(\{x_\alpha\})$ and $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$. Therefore, $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft $sw\text{-}R_0$-space. □

Lemma 3.3. *The following statements are equivalent for any soft points x_α and y_α in $\mathcal{STS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$.*

(1) $sw\text{-}K_s(\{x_\alpha\}) \neq sw\text{-}K_s(\{y_\alpha\})$;
(2) $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$.

Proof. (1) ⇒ (2): Let $sw\text{-}K_s(\{x_\alpha\}) \neq sw\text{-}K_s(\{y_\alpha\})$. Then, there exists a soft point z_α such that $z_\alpha \in sw\text{-}K_s(\{x_\alpha\})$ and $z_\alpha \notin sw\text{-}K_s(\{y_\alpha\})$. It follows that $z_\alpha \in sw\text{-}K_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{z_\alpha\}) \neq \emptyset_\gamma$, and hence $x_\alpha \in sw\text{-}Cl_s(\{z_\alpha\})$. By $z_\alpha \notin sw\text{-}K_s(\{y_\alpha\})$, we have $\{y_\alpha\} \sqcap sw\text{-}Cl_s(\{z_\alpha\}) = \emptyset_\gamma$. Since $x_\alpha \in sw\text{-}Cl_s(\{z_\alpha\})$, $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq sw\text{-}Cl_s(\{z_\alpha\})$ and $\{y_\alpha\} \sqcap sw\text{-}Cl_s(\{z_\alpha\}) = \emptyset_\gamma$. Therefore, it follows that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$.
(2) ⇒ (1): Let $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Then there exists a soft point z_α such that $z_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ and $z_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. Then, there exists a soft sw-open set (Φ,γ) containing z_α and hence $x_\alpha \in (\Phi,\gamma)$ and $y_\alpha \notin (\Phi,\gamma)$. Therefore, $y_\alpha \notin sw\text{-}K_s(\{x_\alpha\})$ and thus $sw\text{-}K_s(\{x_\alpha\}) \neq sw\text{-}K_s(\{y_\alpha\})$. □

Theorem 3.2. *A $\mathcal{SITS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft $sw\text{-}R_0$-space if and only if for any pair of a soft points x_α and y_α, with $sw\text{-}K_s(\{x_\alpha\}) \neq sw\text{-}K_s(\{y_\alpha\})$ implies that $sw\text{-}K_s(\{x_\alpha\}) \sqcap sw\text{-}K_s(\{y_\alpha\}) = \emptyset_\gamma$.*

Proof. Let $(\mathcal{X}_\gamma,\tau_\gamma)$ be a soft $sw\text{-}R_0$-space. Thus, by Lemma 3.3 for any soft points x_α and y_α, if $sw\text{-}K_s(\{x_\alpha\}) \neq sw\text{-}K_s(\{y_\alpha\})$ implies that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Now we show $sw\text{-}K_s(\{x_\alpha\}) \sqcap sw\text{-}K_s(\{y_\alpha\}) = \emptyset_\gamma$. Suppose that $z_\alpha \in sw\text{-}K_s(\{x_\alpha\}) \sqcap sw\text{-}K_s(\{y_\alpha\})$. By $z_\alpha \in sw\text{-}K_s(\{x_\alpha\})$ and Lemma 3.1, it follows that $x_\alpha \in sw\text{-}Cl_s(\{z_\alpha\})$. Since $x_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ by Theorem 3.1, $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{z_\alpha\})$. Similarly, we have $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\}) = sw\text{-}Cl_s(\{z_\alpha\})$. This is a contradiction by Lemma 3.3. Therefore, we have $sw\text{-}K_s(\{x_\alpha\}) \sqcap sw\text{-}K_s(\{y_\alpha\}) = \emptyset_\gamma$.

Conversely, let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a $\mathcal{STS}$ such that for any pair of a soft points x_α and y_α, with sw-$K_s(\{x_\alpha\}) \neq sw$-$K_s(\{y_\alpha\})$ implies that sw-$K_s(\{x_\alpha\}) \sqcap sw$-$K_s(\{y_\alpha\}) = \emptyset_\gamma$. If sw-$Cl_s(\{x_\alpha\}) \neq sw$-$Cl_s(\{y_\alpha\})$, then by Lemma 3.3, sw-$K_s(\{x_\alpha\}) \neq sw$-$K_s(\{y_\alpha\})$. Hence, sw-$K_s(\{x_\alpha\}) \sqcap sw$-$K_s(\{y_\alpha\}) = \emptyset_\gamma$ which implies sw-$Cl_s(\{x_\alpha\}) \sqcap sw$-$Cl_s(\{y_\alpha\}) = \emptyset_\gamma$. Because $z_\alpha \in sw$-$Cl_s(\{x_\alpha\})$ implies that $x_\alpha \in sw$-$K_s(\{z_\alpha\})$ and therefore, sw-$K_s(\{x_\alpha\}) \sqcap sw$-$K_s(\{z_\alpha\}) \neq \emptyset_\gamma$. By hypothesis, we have sw-$K_s(\{x_\alpha\}) = sw$-$K_s(\{z_\alpha\})$. Then $z^\alpha \in sw$-$Cl_s(\{x_\alpha\}) \sqcap sw$-$Cl_s(\{y_\alpha\})$ implies that sw-$K_s(\{x_\alpha\}) = sw$-$K_s(\{z_\alpha\}) = sw$-$K_s(\{y_\alpha\})$. This is a contradiction. Therefore, sw-$Cl_s(\{x_\alpha\}) \sqcap sw$-$Cl_s(\{y_\alpha\}) = \emptyset_\gamma$. By Theorem 3.1, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space. □

Theorem 3.3. *For a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$, the following properties are equivalent:*

(1) *$(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space;*
(2) *For any non-null soft set (Φ, γ) and any non-null soft sw-open set (Ψ, γ) such that $(\Phi, \gamma) \sqcap (\Psi, \gamma) \neq \emptyset_\gamma$, there exist a soft sw-closed set (F, γ) such that $(\Phi, \gamma) \sqcap (F, \gamma) \neq \emptyset_\gamma$ and $(F, \gamma) \sqsubseteq (\Psi, \gamma)$;*
(3) *For any soft sw-open set (Ψ, γ), $(\Psi, \gamma) = \sqcup\{(F, \gamma) : (F, \gamma)$ is soft sw-closed set and $(F, \gamma) \sqsubseteq (\Psi, \gamma)\}$;*
(4) *For any soft sw-closed set (F, γ), $(F, \gamma) = \sqcap\{(\Psi, \gamma) : (\Psi, \gamma)$ is soft sw-open set and $(F, \gamma) \sqsubseteq (\Psi, \gamma)\}$;*
(5) *For any soft point $x_\alpha \in \mathcal{X}_\gamma$, sw-$Cl_s(\{x_\alpha\}) \sqsubseteq sw$-$K_s(\{x_\alpha\})$.*

Proof. (1) ⇒ (2): Let (Φ, γ) be non-null soft set and (Ψ, γ) be non-null soft sw-open set such that $(\Phi, \gamma) \sqcap (\Psi, \gamma) \neq \emptyset_\gamma$. There exists $x_\alpha \in (\Phi, \gamma) \sqcap (\Psi, \gamma)$. Since $x_\alpha \in (\Psi, \gamma)$ and $(\mathcal{X}_\gamma, \tau_\gamma, \mathbb{I}_\gamma)$ is soft sw-R_0-space, sw-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Psi, \gamma)$. Set $(F, \gamma) = sw$-$Cl_s(\{x_\alpha\})$, then (F, γ) is soft sw-closed set with $(F, \gamma) \sqsubseteq (\Psi, \gamma)$ and $(\Phi, \gamma) \sqcap (F, \gamma) \neq \emptyset_\gamma$.
(2) ⇒ (3): Let (Ψ, γ) be a soft sw-open set, then $\sqcup\{(F, \gamma) : (F, \gamma)$ is soft sw-closed set and $(F, \gamma) \sqsubseteq (\Psi, \gamma)\} \sqsubseteq (\Psi, \gamma)$. Let $x_\alpha \in (\Psi, \gamma)$. There exist a soft sw-closed set (F, γ) such that $x_\alpha \in (F, \gamma) \sqsubseteq (\Psi, \gamma)$. Therefore, we have $(F, \gamma) \sqsubseteq \sqcup\{(F, \gamma) : (F, \gamma)$ is soft sw-closed set and $(F, \gamma) \sqsubseteq (\Psi, \gamma)\}$. Hence, $(\Psi, \gamma) = \sqcup\{(F, \gamma) : (F, \gamma)$ is soft sw-closed set and $(F, \gamma) \sqsubseteq (\Psi, \gamma)\}$.
(3) ⇒ (4): It is clear.
(4) ⇒ (5): Let x_α be any soft point and $y_\alpha \notin sw$-$K_s(\{x_\alpha\})$. Then there exists a soft sw-open set (V, γ) such that $x_\alpha \in (V, \gamma)$ and $y_\alpha \notin (V, \gamma)$, hence sw-$Cl_s(\{y_\alpha\}) \sqcap (V, \gamma) = \emptyset_\gamma$. By (4) $\sqcap\{(\Psi, \gamma) : (\Psi, \gamma)$ is soft sw-open set and sw-$Cl_s(\{y_\alpha\}) \sqsubseteq (\Psi, \gamma)\} \sqcap (V, \gamma) = \emptyset_\gamma$ and $x_\alpha \notin (\Psi, \gamma)$. Therefore, sw-$Cl_s(\{x_\alpha\}) \sqcap (\Psi, \gamma) = \emptyset_\gamma$ and $y_\alpha \notin sw$-$Cl_s(\{x_\alpha\})$. Consequently, we obtain sw-$Cl_s(\{x_\alpha\}) \sqsubseteq sw$-$K_s(\{x_\alpha\})$.
(5) ⇒ (1): Let (Ψ, γ) be soft sw-open set such that $x_\alpha \in (\Psi, \gamma)$ and let $y_\alpha \in sw$-$K_s(\{x_\alpha\})$, then $x_\alpha \in sw$-$Cl_s(\{y_\alpha\})$ and $y_\alpha \in (\Psi, \gamma)$. This implies that sw-$K_s(\{x_\alpha\}) \sqsubseteq (\Psi, \gamma)$. Therefore, we obtain $x_\alpha \in sw$-$Cl_s(\{x_\alpha\}) \sqsubseteq sw$-$K_s(\{x_\alpha\}) \sqsubseteq (\Psi, \gamma)$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma, \mathbb{I}_\gamma)$ is a soft sw-R_0-space. □

Corollary 3.1. *For a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$, the following properties are equivalent:*

(1) *$(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space;*
(2) *For any soft point sw-$Cl_s(\{x_\alpha\}) = sw$-$K_s(\{x_\alpha\})$.*

Proof. (1) ⇒ (2): Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a soft sw-R_0-space. By Theorem 3.3 $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq sw\text{-}K_s(\{x_\alpha\})$ for all $x_\alpha \in \mathcal{X}_\gamma$. Let $y_\alpha \in sw\text{-}K_s(\{x_\alpha\})$, then $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$, so by Theorem 3.1, $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\})$. Therefore, $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ and hence $sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq sw\text{-}Cl_s(\{x_\alpha\})$. This show that $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}K_s(\{x_\alpha\})$.
(2) ⇒ (1): This is obvious by Theorem 3.3. □

Corollary 3.2. *For any soft point x_α in a soft sw-R_0-space, if $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}K_s(\{x_\alpha\}) = \{x_\alpha\}$. Then, $sw\text{-}K_s(\{x_\alpha\}) = \{x_\alpha\}$.*

Proof. This is obvious by item (5) of Theorem 3.3. □

Theorem 3.4. *For a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$, the following properties are equivalent:*

(1) *$(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space;*
(2) *$x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$ if and only if $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ for any soft points $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$.*

Proof. (1) ⇒ (2): Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a soft sw-R_0-space. Let $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$ and (Ψ, γ) be soft sw-open set such that $y_\alpha \in (\Psi, \gamma)$. By hypothesis, $x_\alpha \in (\Psi, \gamma)$. Therefore, every soft sw-open set containing y_α contains x_α. Hence $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$.
(2) ⇒ (1): Let (Φ, γ) be soft sw-open set such that $x_\alpha \in (\Phi, \gamma)$. If $y_\alpha \notin (\Phi, \gamma)$, then $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$ and hence $y_\alpha \notin sw\text{-}Cl_s(\{x_\alpha\})$. This implies that $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space. □

Theorem 3.5. *For a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$, the following properties are equivalent:*

(1) *$(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space;*
(2) *If (Φ, γ) is soft sw-closed set, then $sw\text{-}K_s((\Phi, \gamma)) = (\Phi, \gamma)$;*
(3) *If (Φ, γ) is soft sw-closed set and $x_\alpha \in (\Phi, \gamma)$, then $sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)$;*
(4) *For $x_\alpha \in \mathcal{X}_\gamma$, then $sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq sw\text{-}Cl_s(\{x_\alpha\})$.*

Proof. (1) ⇒ (2): Let (Φ, γ) is soft sw-closed set and $x_\alpha \notin (\Phi, \gamma)$. Thus, $(\Phi, \gamma)^c$ is soft sw-open and $x_\alpha \in (\Phi, \gamma)^c$. Since $(\mathcal{X}_\gamma, \tau_\gamma, \mathbb{I}_\gamma)$ is a soft sw-R_0-space, $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)^c$ and $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma) = \emptyset_\gamma$ and by Lemma 3.2, $x_\alpha \notin sw\text{-}K_s((\Phi, \gamma))$. Therefore, $sw\text{-}K_s((\Phi, \gamma)) = (\Phi, \gamma)$.
(2) ⇒ (3): In general if $(\Phi, \gamma) \sqsubseteq (\Psi, \gamma)$ implies that $sw\text{-}K_s((\Phi, \gamma)) \sqsubseteq sw\text{-}K_s((\Psi, \gamma))$. Therefore, if follows that by (2) $sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq sw\text{-}K_s((\Phi, \gamma)) = (\Phi, \gamma)$.
(3) ⇒ (4): Since $x_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ and $sw\text{-}Cl_s(\{x_\alpha\})$ is sw-closed, by (3) $sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq sw\text{-}Cl_s(\{x_\alpha\})$.
(4) ⇒ (1): Let $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$. Then by Lemma 3.1, $y_\alpha \in sw\text{-}K_s(\{x_\alpha\})$. Since $x_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ and $sw\text{-}Cl_s(\{x_\alpha\})$ is sw-closed, by (4) we obtain $y_\alpha \in sw\text{-}K_s(\{x_\alpha\}) \sqsubseteq sw\text{-}Cl_s(\{x_\alpha\})$. Therefore, $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$ implies that $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$. The converse is similar and by Theorem 3.4, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0-space. □

Definition 3.3. *[] Let $\mathcal{F} \sqsubseteq SS(\mathcal{X}_\gamma)$ be non-null subset, then $\mathcal{F}$ is called a soft filter base on $\mathcal{X}_\gamma$ if*

(1) $\emptyset_\gamma \notin \mathcal{F}$;

(2) *for all* $(\Phi,\gamma),(\Psi,\gamma) \in \mathcal{F}$, *there exists* $(H,\gamma) \in \mathcal{F}$ *such that* $(H,\gamma) \sqsubseteq (\Phi,\gamma) \sqcap (\Psi,\gamma)$.

Definition 3.4. *A soft filter base* $\mathcal{F}$ *is called sw-convergent to a soft point* x_α, *if for any soft sw-open set* (Φ,γ) *containing* x_α, *there exists* $(\Psi,\gamma) \in \mathcal{F}$ *such that* $(\Psi,\gamma) \sqsubseteq (\Phi,\gamma)$.

Lemma 3.4. *Let* $(\mathcal{X}_\gamma,\tau_\gamma)$ *be a* $\mathcal{STS}$ *and let* x_α *and* y_α *be two soft points such that every soft net* $\{x_\alpha^d : d \in D\}$ *over* $\mathcal{X}_\gamma$, *sw-convergent to* y_α *is sw-convergent to* x_α, *where D is a directed set. Then* $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$.

Proof. Suppose that $x_\alpha^d = y_\alpha$ for each $d \in D$. Then $\{x_\alpha^d : d \in D\}$ is a soft net in $sw\text{-}Cl_s(\{y_\alpha\})$. Since $\{x_\alpha^d : d \in D\}$ sw-convergent to y_α, then $\{x_\alpha^d : d \in D\}$ sw-convergent to x_α and this implies that $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$. □

Theorem 3.6. *For a* $\mathcal{STS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$, *the following properties are equivalent:*

(1) $(\mathcal{X}_\gamma,\tau_\gamma)$ *is a soft sw-*R_0*-space;*

(2) *If* x_α *and* y_α *be two soft points, then* $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$ *if and only if every soft net* $\{x_\alpha^d : d \in D\}$ *over* $\mathcal{X}_\gamma$, *sw-convergent to* y_α *is sw-convergent to* x_α.

Proof. (1) ⇒ (2): If x_α and y_α be two soft points such that $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$. Suppose that $\{x_\alpha^d : d \in D\}$ be a soft net such that $\{x_\alpha^d : d \in D\}$ sw-convergent to y_α. Since $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$, by Theorem 3.1, we have $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\})$, Therefore, $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$. This means that $\{x_\alpha^d : d \in D\}$ sw-convergent to x_α. Conversely, let x_α and y_α be two soft points such that every soft net $\{x_\alpha^d : d \in D\}$ sw-convergent to y_α is sw-convergent to x_α. Then, $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$ by Lemma 3.4. By Theorem 3.1 we have $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\})$. Therefore, $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$.

(2) ⇒ (1): Assume that x_α and y_α be two soft points such that $sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\}) \neq \emptyset_\gamma$. Let $z_\alpha \in sw\text{-}Cl_s(\{x_\alpha\}) \sqcap sw\text{-}Cl_s(\{y_\alpha\})$. So, there exists a soft net such that $\{x_\alpha^d : d \in D\}$ in $sw\text{-}Cl_s(\{x_\alpha\})$ such that $\{x_\alpha^d : d \in D\}$ sw-convergent to z_α. Since $z_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$, then $\{x_\alpha^d : d \in D\}$ sw-convergent to y_α. It follows that $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$. By the same way we obtain $x_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})$. Therefore, $sw\text{-}Cl_s(\{x_\alpha\}) = sw\text{-}Cl_s(\{y_\alpha\})$ and by Theorem 3.1 we have $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft sw-R_0-space. □

4. Soft somewhat-R_1 spaces

Definition 4.1. *A* $\mathcal{STS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$ *is said to be soft sw-*R_1 *space if any two soft points* x_α *and* y_α *with* $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$, *there exist two disjoint soft sw-open sets* (Φ,γ) *and* (Ψ,γ) *such that* $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$ *and* $sw\text{-}Cl_s(\{y_\alpha\}) \sqsubseteq (\Psi,\gamma)$.

Proposition 4.1. *If A* $\mathcal{STS}$ $(\mathcal{X}_\gamma,\tau_\gamma)$ *is a soft sw-*R_1 *space, then it is a soft sw-*R_0.

Proof. Let (Φ,γ) be a soft sw-open set containing x_α. If $y_\alpha \notin (\Phi,\gamma)$, then since $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$, $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Hence, there exists a soft sw-open set (Ψ,γ) such that $sw\text{-}Cl_s(\{y_\alpha\}) \sqsubseteq (\Psi,\gamma)$ and $x_\alpha \notin (\Psi,\gamma)$, which implies that $y_\alpha \notin sw\text{-}Cl_s(\{x_\alpha\})$. Thus $sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$. Therefore, $(\mathcal{X}_\gamma,\tau_\gamma)$ is a soft sw-R_0 space. □

Theorem 4.1. *A $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is soft sw-R_1 space if and only if for any soft points x_α and y_α, such that sw-$K_s(\{x_\alpha\}) \neq$ sw-$K_s(\{y_\alpha\})$, there exist two disjoint soft sw-open sets (Φ, γ) and (Ψ, γ) such that sw-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)$ and sw-$Cl_s(\{y_\alpha\}) \sqsubseteq (\Psi, \gamma)$.*

Proof. It follows that from Lemma 3.3. □

Theorem 4.2. *The following properties are equivalent for a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$:*

(1) *a soft sw-T_2-space;*
(2) *a soft sw-R_1 and a soft sw-T_1-space;*
(3) *a soft sw-R_1 and a soft sw-T_0-space.*

Proof. (1) ⇒ (2): Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-T_2-space, then it is a soft *sw*-T_1-space. If for any soft points x_α and y_α, such that *sw*-$Cl_s(\{x_\alpha\}) \neq$ *sw*-$Cl_s(\{y_\alpha\})$, then $x_\alpha \neq y_\alpha$, then there exist two disjoint soft *sw*-open sets (Φ, γ) and (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$ and $y_\alpha \in (\Psi, \gamma)$ and *sw*-$Cl_s(\{x_\alpha\}) = \{x_\alpha\} \sqsubseteq (\Phi, \gamma)$ and *sw*-$Cl_s(\{y_\alpha\}) = \{y_\alpha\} \sqsubseteq (\Psi, \gamma)$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-R_1-space.
(2) ⇒ (3): Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-T_1-space, then it is a soft *sw*-T_0-space.
(3) ⇒ (1): Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-R_1-space, then it is a soft *sw*-R_0-space. Let x_α and y_α be disjoint soft points. Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-T_0-space, there exist a soft *sw*-open sets (Φ, γ) such that $x_\alpha \in (\Phi, \gamma)$ and $y_\alpha \notin (\Phi, \gamma)$, we have $x_\alpha \in$ *sw*-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)$ and $y_\alpha \in (\Phi, \gamma)^c \sqsubseteq [sw\text{-}Cl_s(\{x_\alpha\})]^c$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-T_1-space. Since *sw*-$Cl_s(\{x_\alpha\}) = \{x_\alpha\} \neq \{y_\alpha\} =$ *sw*-$Cl_s(\{y_\alpha\})$. Then, there exist two disjoint soft *sw*-open sets (Φ, γ) and (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$ and $y_\alpha \in (\Psi, \gamma)$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft *sw*-T_2-space.

□

A soft point x_α of a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is a *sw*-θ-accumulation soft point of a soft subset (Φ, γ) if for each soft *sw*-open (Ψ, γ) containing x_α, *sw*-$Cl_s((\Psi, \gamma)) \sqcap (\Phi, \gamma) \neq \emptyset_\gamma$. The soft set *sw*-$Cl_s((\Phi, \gamma))$ of all *sw*-θ-accumulation soft points of (Φ, γ) is called the soft *sw*-θ-closure of (Φ, γ). The soft set (Φ, γ) is said to be soft *sw*-θ-closed if *sw*-$Cl_s((\Phi, \gamma)) = (\Phi, \gamma)$. Complement of a soft *sw*-θ-closed set is said to be a soft *sw*-θ-open. It is clear that *sw*-$Cl_s((\Phi, \gamma)) \sqsubseteq$ *sw*-$\theta Cl_s((\Phi, \gamma))$.

Lemma 4.1. *Let x_α and y_α be any soft points in a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$. Then $y_\alpha \in$ sw-$\theta Cl_s(\{x_\alpha\})$ if and only if $x_\alpha \in$ sw-$\theta Cl_s(\{y_\alpha\})$.*

Proof. Let $y_\alpha \notin$ *sw*-$\theta Cl_s(\{x_\alpha\})$. This implies that there exists a soft *sw*-open set (Φ, γ) such that $y_\alpha \in (\Phi, \gamma)$ and *sw*-$Cl_s((\Phi, \gamma)) \sqcap \{x_\alpha\} = \emptyset_\gamma$. Also, $[sw\text{-}Cl_s((\Phi, \gamma))]^c$ is a soft *sw*-open set containing x_α which means that $x_\alpha \notin$ *sw*-$\theta Cl_s(\{y_\alpha\})$. □

Theorem 4.3. *A $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space if and only if sw-$Cl_s(\{x_\alpha\}) =$ sw-$\theta Cl_s(\{x_\alpha\})$, for any soft point x_α.*

Proof. Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a soft *sw*-R_1 space. Suppose *sw*-$Cl_s(\{x_\alpha\}) \neq$ *sw*-$\theta Cl_s(\{x_\alpha\})$. Then there exists $y_\alpha \in$ *sw*-$\theta Cl_s(\{x_\alpha\}) \setminus$ *sw*-$Cl_s(\{x_\alpha\})$. Then there exists a soft *sw*-open set (Φ, γ) containing

y_α such that sw-$Cl_s((\Phi,\gamma)) \sqcap \{x_\alpha\} \neq \emptyset_\gamma$ but $(\Phi,\gamma) \sqcap \{x_\alpha\} = \emptyset_\gamma$. Thus sw-$Cl_s(\{y_\alpha\}) \sqsubseteq (\Phi,\gamma)$, sw-$Cl_s(\{x_\alpha\}) \sqcap (\Phi,\gamma) = \emptyset_\gamma$. Hence, sw-$Cl_s(\{x_\alpha\}) \neq sw$-$Cl_s(\{y_\alpha\})$. Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space, there exists two disjoint soft sw-open sets (Υ,γ), (Ω,γ) such that sw-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Upsilon,\gamma)$ and sw-$Cl_s(\{y_\alpha\}) \sqsubseteq (\Omega,\gamma)$. Therefore, $(\Upsilon,\gamma)^c$ is soft sw-closed set at y_α which does not contain x_α. Thus $y_\alpha \notin sw$-$\theta Cl_s(\{x_\alpha\})$. This is a contradiction and hence sw-$Cl_s(\{x_\alpha\}) = sw$-$\theta Cl_s(\{x_\alpha\})$.

Let sw-$Cl_s(\{x_\alpha\}) = sw$-$\theta Cl_s(\{x_\alpha\})$, for any soft point x_α. Now, we show $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0 space. Let (Φ,γ) be soft sw-open set such that $x_\alpha \in (\Phi,\gamma)$ and $y_\alpha \notin (\Phi,\gamma)$. Since sw-$Cl_s(\{y_\alpha\}) = sw$-$\theta Cl_s(\{y_\alpha\}) \sqsubseteq (\Phi,\gamma)^c$, we have $x_\alpha \notin sw$-$\theta Cl_s(\{y_\alpha\})$ and by 4.1, $y_\alpha \notin sw$-$Cl_s(\{x_\alpha\}) = sw$-$\theta Cl_s(\{x_\alpha\})$. It follows that sw-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$. Therefore, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_0 space. For to show $\mathcal{X}_\gamma$ is a soft sw-R_1 space, let $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$ with sw-$Cl_s(\{x_\alpha\}) \neq sw$-$Cl_s(\{y_\alpha\})$, then $x_\alpha \neq y_\alpha$. Since sw-$Cl_s(\{x_\alpha\}) = sw$-$\theta Cl_s(\{x_\alpha\})$ for all soft point x_α that is $y_\alpha \notin sw$-$\theta Cl_s(\{x_\alpha\})$ and there exists soft sw-open set (Φ,γ) containing y_α such that $x_\alpha \notin sw$-$Cl_s((\Phi,\gamma))$. Therefore, we obtain $y_\alpha \in (\Phi,\gamma)$, $x_\alpha \in [sw\text{-}Cl_s((\Phi,\gamma))]^c$ and $(\Phi,\gamma) \sqcap [sw\text{-}Cl_s((\Phi,\gamma))]^c = \emptyset_\gamma$. This shows that $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-T_2 space. It follows from Theorem 4.2 that $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space. □

Theorem 4.4. *A $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space if and only if for each soft sw-open set (Φ,γ) and each $x_\alpha \in (\Phi,\gamma)$, sw-$\theta Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$.*

Proof. Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a soft sw-R_1 space and (Φ,γ) be a soft sw-open set with $x_\alpha \in (\Phi,\gamma)$. Let $y_\alpha \in (\Phi,\gamma)^c$. Since $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space, by Theorem 4.3 sw-$Cl_s(\{y_\alpha\}) = sw$-$\theta Cl_s(\{y_\alpha\}) \sqsubseteq (\Phi,\gamma)^c$. Hence, we have that $x_\alpha \notin sw$-$\theta Cl_s(\{y_\alpha\})$ and by Lemma 4.1, $y_\alpha \notin sw$-$\theta Cl_s(\{x_\alpha\})$. It follows that sw-$\theta Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$.

Suppose that $(\mathcal{X}_\gamma, \tau_\gamma)$ is not a soft sw-R_1 space, by Theorem 4.3, sw-$Cl_s(\{x_\alpha\}) \neq sw$-$\theta Cl_s(\{x_\alpha\})$, then there exists $y_\alpha \in sw$-$\theta Cl_s(\{x_\alpha\}) \setminus sw$-$Cl_s(\{x_\alpha\})$. Then, there exists a soft sw-open set (Φ,γ) which containing y_α such that sw-$Cl_s((\Phi,\gamma)) \sqcap \{x_\alpha\} \neq \emptyset_\gamma$ but $(\Phi,\gamma) \sqcap \{x_\alpha\} = \emptyset_\gamma$. Then, sw-$\theta Cl_s(\{y_\alpha\}) \sqsubseteq (\Phi,\gamma)$ and sw-$\theta Cl_s(\{y_\alpha\}) \sqcap \{x_\alpha\} = \emptyset_\gamma$. Hence $x_\alpha \notin sw$-$\theta Cl_s(\{y_\alpha\})$. Thus, $y_\alpha \notin sw$-$\theta Cl_s(\{x_\alpha\})$. Which is a contradiction, we obtain $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space. □

Theorem 4.5. *The following properties are equivalent for a $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$:*

(1) *$(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-R_1 space.*
(2) *for each $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$ one of the following holds:*
   (a) *if (Φ,γ) be a soft sw-open, then $x_\alpha \in (\Phi,\gamma)$ if and only if $y_\alpha \in (\Phi,\gamma)$.*
   (b) *there exist disjoint soft sw-open sets (Φ,γ) and (Ψ,γ) such that $x_\alpha \in (\Phi,\gamma)$ and $y_\alpha \in (\Psi,\gamma)$.*
(3) *if $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$ such that sw-$Cl_s(\{x_\alpha\}) \neq sw$-$Cl_s(\{y_\alpha\})$, then there exist two soft sw-closed sets (Υ,γ) and (Ω,γ) such that $x_\alpha \in (\Upsilon,\gamma)$, $y_\alpha \in (\Omega,\gamma)$, $y_\alpha \notin (\Upsilon,\gamma)$, $x_\alpha \notin (\Omega,\gamma)$, and $\mathcal{X}_\gamma = (\Upsilon,\gamma) \sqcup (\Omega,\gamma)$.*

Proof. (1) ⇒ (2): Let $(\mathcal{X}_\gamma, \tau_\gamma)$ be a soft sw-R_1 space and $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$. Then, there are two cases:

- Case (a): sw-$Cl_s(\{x_\alpha\}) = sw$-$Cl_s(\{y_\alpha\})$ and (Φ,γ) be a soft sw-open, then $x_\alpha \in (\Phi,\gamma)$ implies that $y_\alpha \in sw$-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$ and $y_\alpha \in (\Phi,\gamma)$ implies that $x_\alpha \in sw$-$Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi,\gamma)$.

- Case (b): $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Then, there exist disjoint soft sw-open sets (Φ, γ) and (Ψ, γ) such that $x_\alpha \in sw\text{-}Cl_s(\{x_\alpha\}) \sqsubseteq (\Phi, \gamma)$ and $y_\alpha \in sw\text{-}Cl_s(\{y_\alpha\})(\Psi, \gamma)$.

(2) $\Rightarrow$ (3): Let $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$ such that $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$. Then, $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$ or $y_\alpha \notin sw\text{-}Cl_s(\{x_\alpha\})$, say $x_\alpha \notin sw\text{-}Cl_s(\{y_\alpha\})$. So, there exist a soft sw-open set (U, γ) such that $x_\alpha \in (U, \gamma)$ and $y_\alpha \notin (U, \gamma)$, which implies by case (b) there exist disjoint soft sw-open sets (Φ, γ) and (Ψ, γ) such that $x_\alpha \in (\Phi, \gamma)$ and $y_\alpha \in (\Psi, \gamma)$. Then, let $(\Upsilon, \gamma) = (\Psi, \gamma)^c$ and $(\Omega, \gamma) = (\Phi, \gamma)^c$ are soft sw-closed sets such that $x_\alpha \in (\Upsilon, \gamma)$, $y_\alpha \in (\Omega, \gamma)$, $y_\alpha \notin (\Upsilon, \gamma)$, $x_\alpha \notin (\Omega, \gamma)$, and $\mathcal{X}_\gamma = (\Upsilon, \gamma) \sqcup (\Omega, \gamma)$.
(3) $\Rightarrow$ (1): Suppose $(\mathcal{X}_\gamma, \tau_\gamma)$ is not soft $sw\text{-}R_0$ space then there exist a soft sw-open set (Φ, γ) and $x_\alpha \in (\Phi, \gamma)$ such that $sw\text{-}Cl_s(\{x_\alpha\}) \not\sqsubseteq (\Phi, \gamma)$. Let $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\}) \sqcap (\Phi, \gamma)^c$. Then, $sw\text{-}Cl_s(\{x_\alpha\}) \neq sw\text{-}Cl_s(\{y_\alpha\})$ and hence there exist two soft sw-closed sets (Υ, γ) and (Ω, γ) such that $x_\alpha \in (\Upsilon, \gamma)$, $y_\alpha \in (\Omega, \gamma)$, $y_\alpha \notin (\Upsilon, \gamma)$, $x_\alpha \notin (\Omega, \gamma)$, and $\mathcal{X}_\gamma = (\Upsilon, \gamma) \sqcup (\Omega, \gamma)$. Then, $y_\alpha \in (\Upsilon, \gamma)^c$ which is soft sw-open set and $x_\alpha \notin (\Upsilon, \gamma)^c$, which is a contradiction by $y_\alpha \in sw\text{-}Cl_s(\{x_\alpha\})$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is soft $sw\text{-}R_0$ space. Let $a_\alpha, b_\alpha \in \mathcal{X}_\gamma$ such that $sw\text{-}Cl_s(\{a_\alpha\}) \neq sw\text{-}Cl_s(\{b_\alpha\})$. Then, there exist two soft sw-closed sets (U, γ) and (V, γ) such that $a_\alpha \in (U, \gamma)$, $b_\alpha \in (V, \gamma)$, $b_\alpha \notin (U, \gamma)$, $a_\alpha \notin (V, \gamma)$, and $\mathcal{X}_\gamma = (U, \gamma) \sqcup (V, \gamma)$. Thus, $a_\alpha \in (V, \gamma)^c$ and $b_\alpha \in (U, \gamma)^c$ which are disjoint a soft sw-open sets implies that $sw\text{-}Cl_s(\{a_\alpha\}) \in (V, \gamma)^c$ and $sw\text{-}Cl_s(\{b_\alpha\}) \in (U, \gamma)^c$. Hence, $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft $sw\text{-}R_1$ space. □

Theorem 4.6. *A $\mathcal{STS}$ $(\mathcal{X}_\gamma, \tau_\gamma)$ is a soft sw-T_2 space if and only if for $x_\alpha, y_\alpha \in \mathcal{X}_\gamma$ such that $x_\alpha \neq y_\alpha$, there exist a soft sw-closed sets (Υ, γ) and (Ω, γ) such that $x_\alpha \in (\Upsilon, \gamma)$, $y_\alpha \in (\Omega, \gamma)$, $y_\alpha \notin (\Upsilon, \gamma)$, $x_\alpha \notin (\Omega, \gamma)$, and $\mathcal{X}_\gamma = (\Upsilon, \gamma) \sqcup (\Omega, \gamma)$.*

Proof. The proof follows from Theorems 4.2 and 4.5. □

5. Conclusion

One of the main and most significant branches of mathematics is topology, which establishes numerous connections between mathematical models and other branches of science. The soft set theory, developed by Molodtsov and readily applicable to a wide range of situations including uncertainties from social life, has garnered significant attention from scientists in recent times. We have kept up our investigation of the characteristics of soft topological spaces in the current work. We have created some intriguing features and introduced new axioms of soft somewhat separation We will continue researching the characteristics of soft somewhat closed sets and soft somewhat open sets, including their genetic aspects, in our next work. Then go over a few theorems on the equivalency of soft, slightly distinct spaces. The somewhat topological qualities established in this study may be generalized in the fuzzy soft sets and will be helpful in the fuzzy systems, as the authors proposed topological structures on fuzzy soft sets [19–21]. Since there are compact linkages between information systems and soft sets [22–25], we may enhance these types of connections by utilizing the insights gleaned from research on soft topological space. We anticipate that the research presented in this paper will assist and encourage further investigations into soft topology, enabling the development of a broad framework for practical applications.

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References

[1] D. Molodtsov, Soft Set Theory–First Results, Comp. Math. Appl. 37 (1999), 19–31. https://doi.org/10.1016/S0898-1221(99)00056-5.

[2] P.K. Maji, R. Biswas, A.R. Roy, Soft Set Theory, Comp. Math. Appl. 45 (2003), 555–562. https://doi.org/10.1016/S0898-1221(03)00016-6.

[3] T.M. Al-Shami, M.E. El-Shafei, T-Soft Equality Relation, Turk. J. Math. 44 (2020), 1427–1441. https://doi.org/10.3906/mat-2005-117.

[4] N. Cagman, S. Karatas, S. Enginoglu, Soft Topology, Comp. Math. Appl. 62 (2011), 351–358. https://doi.org/10.1016/j.camwa.2011.05.016.

[5] M. Shabir, M. Naz, On Soft Topological Spaces, Comp. Math. Appl. 61 (2011), 1786–1799. https://doi.org/10.1016/j.camwa.2011.02.006.

[6] S.K. Nazmul, S.K. Samanta, Neighbourhood Properties of Soft Topological Spaces, Ann. Fuzzy Math. Inf. 6 (2013), 1–15.

[7] J. Mahanta, P.K. Das, On Soft Topological Space via Semiopen and Semiclosed Soft Sets, Kyungpook Math. J. 54 (2014), 221–236. https://doi.org/10.5666/KMJ.2014.54.2.221.

[8] Y. Yumak, A.K. Kaymakci, Soft β-Open Sets and their Applications, J. New Theory 4 (2015), 80–89.

[9] T. Al-Shami, I. Alshammari, B. Asaad, Soft Maps via Soft Somewhere Dense Sets, Filomat 34 (2020), 3429–3440. https://doi.org/10.2298/FIL2010429A.

[10] M. E. El-Shafei, M. Abo-Elhamayel and T. M. Al-shami, Partial soft separation axioms and soft compact spaces, Filomat 32(13) (2018), 4755–4771. https://doi.org/10.2298/FIL1813755E.

[11] A. Al-Omari, W. Alqurashi, Hyperconnectedness and Resolvability of Soft Ideal Topological Spaces, Mathematics 11 (2023), 4697. https://doi.org/10.3390/math11224697.

[12] A. Al-Omari, Soft Topology in Ideal Topological Spaces, Hacettepe J. Math. Stat. 48 (2018), 1277–1285. https://doi.org/10.15672/HJMS.2018.557.

[13] A. A. Allam, Ramadhan A. Mohammed, Tahir H. Ismail, A New Approach to Soft Belonging, Ann. Fuzzy Math. Inf. 13 (2017), 145–152. https://doi.org/10.30948/AFMI.2017.13.1.145.

[14] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On Some New Operations in Soft Set Theory, Comp. Math. Appl. 57 (2009), 1547–1553. https://doi.org/10.1016/j.camwa.2008.11.009.

[15] S. Bayramov, C.G. Aras, A New Approach to Separability and Compactness in Soft Topological Spaces, TWMS J. Pure Appl. Math. 9 (2018), 82–93.

[16] I. Zorlutuna, M. Akdag, W.K. Min, S. Atmaca, Remarks on Soft Topological Spaces, Ann. Fuzzy Math. Inf. 3 (2012), 171–185.

[17] Z.A. Ameen, B.A. Asaad, T.M. Al-shami, Soft Somewhat Continuous and Soft Somewhat Open Functions, TWMS J. Appl. Eng. Math. 13 (2023), 792–806.

[18] T.M. Al-shami, Soft Somewhat Open Sets: Soft Separation Axioms and Medical Application to Nutrition, Comp. Appl. Math. 41 (2022), 216. https://doi.org/10.1007/s40314-022-01919-x.

[19] B. Tanay, M.B. Kandemir, Topological Structure of Fuzzy Soft Sets, Comp. Math. Appl. 61 (2011), 2952–2957. https://doi.org/10.1016/j.camwa.2011.03.056.

[20] J.K. Syed, L.A. Aziz, Fuzzy Inventory Model without Shortages Using Signed Distance Method, Appl. Math. Inf. Scie. 1 (2007), 203–209.

[21] J. Xiao, M. Tong, Q. Fan, S. Xiao, Generalization of Belief and Plausibility Functions to Fuzzy Sets. Appl. Math. Inf. Sci. 6 (2012), 697–703.

[22] Z. Xiao, L. Chen, B. Zhong, S. Ye, Recognition for Soft Information Based on the Theory of Soft Sets, in: Proceedings of 2005 International Conference on Services Systems and Services Management, 2005, IEEE, Chongquing, China, 2005: pp. 1104–1106. https://doi.org/10.1109/ICSSSM.2005.1500166.

[23] D. Pei, D. Miao, From Soft Sets to Information Systems, in: 2005 IEEE International Conference on Granular Computing, IEEE, Beijing, China, 2005: pp. 617–621. https://doi.org/10.1109/GRC.2005.1547365.

[24] A. Al-Omari, W. Alqurashi, Connectedness of Soft-Ideal Topological Spaces, Symmetry 16 (2024), 143. https://doi.org/10.3390/sym16020143.

[25] S. Al Ghour, A. Al-Omari, T. Noiri, On Homogeneity and Homogeneity Components in Generalized Topological Spaces, Filomat 27 (2013), 1097–1105. https://doi.org/10.2298/FIL1306097A.