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Restarted Shooting Method Applied to Jeffery-Hamel Flow Problem

Nawal Alzaid*, **Kholoud Alzahrani, Huda Bakodah**

Department of Mathematics and Statistics, Faculty of Science, University of Jeddah, Jeddah, Saudi Arabia

**Corresponding author:* naalzaid@uj.edu.sa

Abstract. In the current manuscript, an efficient numerical method based on coupling the restarted decomposition method and the shooting method has been implemented to tackle the nonlinear differential equation that describes blood flow in human arteries. A complete outline of the coupled method has been provided and further utilized on the generalized Jeffery-Hamel blood flow problem. In addition, the acquired computational results are contrasted with the numerical results of other computational approaches. Lastly, the efficacy of the devised numerical method is confirmed by the maximum error remainder and is reported through comparison tables and figures.

1. Introduction

Ordinary differential equations (ODEs) have been well-acknowledged in modeling different physical scenarios, including population growth, Newton's cooling, radioactive decay, and the various biological processes among others. In particular, the flow of biological fluids in the human arteries, veins, and capillaries is very important to human health when safely circulated; besides, among the hindrances that oppose the safe movement of blood in human bodies is the stenosis of arteries, as characterized by biomedical experts to cause high blood pressure as a result of "fats in artery lumen and the fibrous concentration tissue which keeps in the interior side and restricts the normal movement of arterial blood, in other words, decreasing of interior angle of arteries causes stenosis of arteries" ([1]). In this regard, mathematics plays a significant role -through the window of mathematical modeling – in modeling the safe movement of biological fluids in human bodies through the use of ODEs and partial differential equations (PDEs). Notably, we make mention of the famous mathematical model for the movement of blood in human bodies by Jeffery–Hamel ([2], [3]), which greatly helped investigators to affirm their already established experimental and theoretical findings. Certainly, the Jeffery–Hamel flow problem was modeled using nonlinear

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ODE, just like many other mathematical models that have the presence of nonlinearity in the underlying assumptions. Most scientific problems such as Jeffery–Hamel flow and other fluid mechanic problems are inherently in form of nonlinearity. Except a limited number of these problems, most of them do not have exact solution. Therefore, these nonlinear equations should be solved using other methods. Therefore, many mathematicians have over the years introduced dissimilar computational approaches for the solution of both the initial-value problems (IVPs) and BVPs, such as the He's homotopy perturbation technique ([4], [5]), Zhou's transformation approach ([6]), and the Adomian decomposition method (ADM) ([7]) to mention a few. Indeed, ADM is among the few pertinent numerical methods that have been productively used to efficiently tackle various nonlinear models ([8], [9]). Further, ADM has undergone quite a lot of modifications in the past and recent times to improve its rapidness, efficiency and computational time among others; in this regard, we make mention the Restarted Adomian decomposition method (RADM) by Babolian et al. ([10]) that enhanced the classical ADM; read also the work of Sadeghi et al. ([11]) that deployed the RADM on the class of nonlinear Volterra integral equations. However, the current manuscript introduces an efficient numerical method based on coupling the RADM and the shooting method to solve the Jeffery–Hamel flow problem. Besides, this approach starts by transforming the governing BVP into two appropriate IVPs, and thereafter, solves the resulting IVPs recurrently via the application of RADM, which is easier to execute than the two methods individually. Additionally, the paper is composed in the following pattern: Section 2 gives the governing equations for the Jeffery-Hamel blood flow problem. Section 3 outlines the RADM procedure. Section 4 delineates the proposed coupling between the shooting method and the RADM to solve the Jeffery–Hamel flow problem. What is more, Section 5 gives the numerical application; while Section 6 gives some concluding annotations.

2. Geometry of the problem

The flow of arterial blood was described by Jeffrey and Hamel ([2, 3]) using the nonlinear Magneto-Hydro-Dynamic (MHD) equation ([12–14]). Therefore, making consideration of a conducting incompressible viscous fluid that flows from a source /sink continuously in a two-dimensional channel (walls) positioned in planes at an angle of 2δ; see Figure (1) for the schematic plan of this flow. Moreover, as the velocity depends on the radial and azimuthal axes r and θ, it is then further assumed that the velocity exists along the radial direction upon which the governing continuity-Navier-Stokes equation in cylindrical system take the following form

$$\frac{\rho}{r}\frac{\partial}{\partial r}\left(ru(r,\theta)\right)=0, \tag{2.1}$$

$$u(r,\theta)\frac{\partial u(r,\theta)}{\partial r}=-\frac{1}{\rho}\frac{\partial P}{\partial r}+\nu\left[\frac{\partial^2 u(r,\theta)}{\partial r^2}+\frac{1}{r}\frac{\partial u(r,\theta)}{\partial r}+\frac{1}{r^2}\frac{\partial^2 u(r,\theta)}{\partial \theta^2}-\frac{u(r,\theta)}{r^2}\right], \tag{2.2}$$

$$-\frac{1}{\rho r}\frac{\partial P}{\partial \theta}+\frac{2\nu}{r^2}\frac{\partial u(r,\theta)}{\partial \theta}=0, \tag{2.3}$$

where $u(r,\theta)$ is the velocity of the flow in the radial r and azimuthal θ variables, ρ is the density of the fluid, ν is the kinematic viscosity's coefficient, while P is the pressure. In addition, the following boundary data are imposed:

- at the channel's centerline

$$\frac{\partial u(r,\theta)}{\partial \theta} = 0, \tag{2.4}$$

- on the channel's wall

$$u(r,\theta) = 0. \tag{2.5}$$

Furthermore, when the cross-sectional area of the governing artery is assumed to be constant, – as in the case of a cylindrical coordinate system – the model equations in (2.1)-(2.3) without the impact of viscoelasticity recast to a nonlinear ODE after integrating (2.1) with respect to r as follows

$$y(\theta) = ru(r,\theta). \tag{2.6}$$

Next, let us adopt the following fresh function $y(x)$

$$y(x) = \frac{y(\theta)}{y_{\max}}, \quad x = \frac{\theta}{\delta}, \tag{2.7}$$

then, we express the function $u(r,\theta)$ and all its related derivatives in terms of the fresh function $y(x)$ as follows

$$\frac{\partial u(r,\theta)}{\partial r} = -\frac{y_{\max} y(x)}{r^2}, \tag{2.8}$$

$$\frac{\partial^2 u(r,\theta)}{\partial r^2} = \frac{2y_{\max} y(x)}{r^3}, \tag{2.9}$$

$$\frac{\partial u(r,\theta)}{\partial \theta} = \frac{y_{\max} y'(x)}{r\delta}, \tag{2.10}$$

$$\frac{\partial^2 u(r,\theta)}{\partial \theta^2} = \frac{y_{\max} y''(x)}{r\delta^2}. \tag{2.11}$$

In addition, one obtains the following related pressure upon integrating (2.3) with respect to θ

$$P = \frac{2\nu}{r}\rho u(r,\theta). \tag{2.12}$$

Next, on finding the derivative of P above in r through the use of (2.7), one obtains

$$\frac{1}{\rho}\frac{\partial P}{\partial r} = -\frac{4\nu y_{\max} y(x)}{r^3}, \tag{2.13}$$

upon which when (2.8)-(2.13) are substituted into (2.3) gives

$$-\frac{(y_{\max})^2 y^2(x)}{r^3} = \frac{4\nu y_{\max} y(x)}{r^3} + \nu\left[\frac{2y_{\max} y(x)}{r^3} - \frac{y_{\max} y(x)}{r^3} + \frac{y_{\max} y''(x)}{r^3\delta^2} - \frac{y_{\max} y(x)}{r^3}\right],$$

or simply

$$y_{\max} y^2(x) + 4\nu y_{\max} y(x) + \nu\frac{y_{\max} y''(x)}{\delta^2} = 0. \tag{2.14}$$

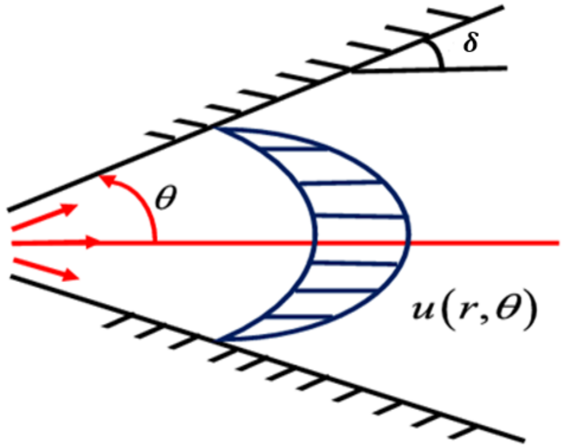

Figure 1. Schematic diagram of the problem.

Now, differentiating (2.14) with respect to x, one arrives at the governing reduced model for the flow of arterial blood by Jeffrey and Hamel as follows [2,3]

$$y'''(x) + 2\delta Re y(x)y'(x) + 4\delta^2 y'(x) = 0, \tag{2.15}$$

while at the same time, according to the relations (2.4)-(2.7) the related boundary data take the following expression

$$y(0) = 1, \quad y'(0) = 0, \quad y(1) = 0. \tag{2.16}$$

Moreover, from (2.15), Re and δ denote the Reynolds number and the angle between the examining inclined plates, respectively. In fact, the Reynolds number is exclusively expressed as follows

$$Re \equiv \frac{y_{\max}\delta}{\nu} = \frac{U_{\max} r\delta}{\nu} \quad \begin{pmatrix} \text{Convergent Channel: } \delta < 0,\, U_{\max} < 0 \\ \text{Divergent Channel: } \delta > 0,\, U_{\max} > 0 \end{pmatrix}, \tag{2.17}$$

where $U_{\max}$ in the latter equation is the maximum velocity at the center of the considering channel, that is, at $r = 0$.

3. Description of the restarted Adomian method

The classical ADM gives the resulting solution as a fast converging infinite series, which is why one may find its relevance in solving different functional equations, including delay, integral and integro-differential equations to mention a few ([15,16]). Therefore, while considering the IVP for a nonlinear universal third-order ODE, we present the ADM procedure as follows

$$y''' + Ry + Ny = g(x), \tag{3.1}$$

with the following prescribed initial data

$$y(a) = \alpha, \quad y'(a) = \lambda, \quad y''(a) = t. \tag{3.2}$$

More so, from (3.1), Ry and Ny are the linear and the nonlinear differential operators having the order less than 3, respectively, while $g(x)$ is a forcing term. Additionally, the constants α, λ, and t in (3.2) are prescribed real constants. Therefore, we further make consideration to the differential operator from (3.1) as follows

$$L(.) = \frac{d^3y}{dx^3}(.), \tag{3.3}$$

together with its inversion operator L^{-1} as follows

$$L^{-1}(.) = \int_a^x \int_a^x \int_a^x (.)\, dx\, dx\, dx. \tag{3.4}$$

Now, through the application of the classical ADM procedure, the solution $y(x)$ and the nonlinear operator Ny are thus expressed via infinite series representations as follows

$$y(x) = \sum_{m=0}^{\infty} y_m(x), \tag{3.5}$$

and

$$Ny = \sum_{m=0}^{\infty} A_m, \tag{3.6}$$

where the A_m's are the Adomian polynomials of $y_0, y_1, y_2, \ldots, y_m$ that are computed using

$$A_m = \frac{1}{m!}\frac{d^m}{d\lambda^m}\left[N\left(\sum_{i=0}^{m} \lambda^i y_i\right)\right]_{\lambda=0}, \quad m = 0, 1, 2, \ldots \quad .$$

Next, upon applying L^{-1} earlier defined in (3.4) into (3.1), we reveals

$$y(x) = \varphi(x) + L^{-1}(g(x)) - L^{-1}(Ry(x)) - L^{-1}(N(y(x))). \tag{3.7}$$

with the function $\varphi(x)$ representing the combined terms initiating from integrating y''' in (3.1) and from using the given conditions in (3.2), upon which $L\varphi(x) = 0$. Furthermore, substitution of (3.5)-(3.6) into (3.7) admits the following convergent components as

$$\begin{aligned} y_0 &= \varphi(x) + L^{-1}(g(x)), \\ y_1 &= -L^{-1}(Ry_0) - L^{-1}(A_0), \\ &\vdots \\ y_m &= -L^{-1}(Ry_{m-1}) - L^{-1}(A_{m-1}). \end{aligned} \tag{3.8}$$

However, with regard to the modification by Babolian et al. ([17]) that initialized the RADM, we can modify (3.7) by adding a term to both sides of the equation. Let G be the proper term, which is determined next; then from (3.7) one gets

$$y(x) + G = \varphi(x) + L^{-1}(g(x)) - L^{-1}(Ry(x)) - L^{-1}(N(y(x))) + G. \tag{3.9}$$

Next, upon utilizing Wazwaz's modification of the classical ADM ([18]), one thus obtains the resulting recurrent scheme for (3.9) as follows

$$\begin{aligned} y_0 &= G, \\ y_1 &= \varphi(x) + L^{-1}(g(x)) - L^{-1}(Ry_0) - L^{-1}(A_0) - G, \\ y_2 &= -L^{-1}(Ry_1) - L^{-1}(A_1), \\ &\vdots \\ y_m &= -L^{-1}(Ry_{m-1}) - L^{-1}(A_{m-1}). \end{aligned} \tag{3.10}$$

Algorithm of RADM Here, we write down an implementable algorithm for the computational implementation of RADM by first choosing some small natural numbers m, n.

Step I: Apply the ADM procedure on (3.1) and compute $y_0, y_1, \ldots, y_m$.

Set $w^1 = y_0 + y_1 + \cdots + y_m$.

Step II: For $i = 2 : n$, do

$$\begin{aligned} G &= w^{i-1}, \\ y_0 &= G, \\ y_1 &= \varphi(x) + L^{-1}(g(x)) - L^{-1}(Ry_0) - L^{-1}(A_0) - G, \\ y_2 &= -L^{-1}(Ry_1) - L^{-1}(A_1), \\ &\vdots \\ y_m &= -L^{-1}(Ry_{m-1}) - L^{-1}(A_{m-1}). \end{aligned}$$

Set $w^i = y_0 + y_1 + \cdots + y_m$.

End.

Markedly, the RADM algorithm will be applied in n steps; and in every step, m terms of the classical ADM with updated y_0 are obtained. It should equally be noticed that the polynomials $A_0, \ldots, A_{m-1}$ are used in each step; whereas for the classical ADM, mn terms are obtained, that is, using $A_0, \ldots, A_{mn-1}$.

4. Restarted shooting method

In order to address the study problem, we shall elucidate the method by which the Restarted shooting method (RSM) resolves third-order nonlinear problems in this section. Consider the following third-order nonlinear two-point BVP

$$y''' = f(x, y, y', y''), \quad x \in [a, b], \tag{4.1}$$

together with the following two-point boundary data

$$y(a) = \alpha, \quad y'(a) = \lambda, \quad y(b) = \beta. \tag{4.2}$$

This method relies on using the shooting method ([19]) and converting the governing BVP into the following third-order equation

$$y''' = f(x, y, y', y''), \quad x \in [a, b], \tag{4.3}$$

with the specific initial conditions

$$y(a) = \alpha, \quad y'(a) = \lambda, \quad y''(a) = t. \tag{4.4}$$

After that, we will use the RSM directly to treat the IVP in equations (4.3) - (4.4) by choosing the parameters $t = t_k$ in a manner to ensure that

$$\lim_{k \to \infty} y(b, t_k) = y(b) = \beta,$$

where $y(x, t_k)$ presents the solution to the IVP given in equations (4.3) - (4.4) with $t = t_k$, while the function $y(x)$ represents the solution to the BVP in equations (4.1) - (4.2). Therefore, the expected solution to the resulting first IVP is required in a sequence form after constraining initial guess $t_0 = \frac{\beta - \alpha}{b - a}$. Then, we make use of Newton's method to find the value of t_1 as follows

$$t_1 = t_0 - \frac{y(b, t_0) - \beta}{\frac{dy}{dt}(b, t_0)}. \tag{4.5}$$

So, on determining the value of $\frac{dy}{dt}(b, t_0)$, the IVP in equations (4.3) - (4.4) are scaled to depend on x and t variables as in the following scaled-IVP

$$y'''(x, t) = f(x, y(x, t), y'(x, t), y''(x, t)), \tag{4.6}$$

with the following initial data

$$y(a, t) = \alpha, \quad y'(a, t) = \lambda, \quad y''(a, t) = t. \tag{4.7}$$

Next, on finding the partial derivative of equation (4.6) in t, we further let $z(x, t) = \frac{\partial y}{\partial t}(x, t)$. Then, the scaled-IVP in equations (4.6) - (4.7) is thus simplified as

$$z''' = \frac{\partial f}{\partial y}(x, y, y', y'')z(x, t) + \frac{\partial f}{\partial y'}(x, y, y', y'')z'(x, t) + \frac{\partial f}{\partial y''}(x, y, y', y'')z''(x, t), \tag{4.8}$$

for $a \leq x \leq b$, with the simplified initial data as follows

$$z(a) = 0, \quad z'(a) = 0, \quad z''(a) = 1. \tag{4.9}$$

Finally, we will solve the simplified-IVP in equations (4.8) - (4.9) at t_k using the RSM directly, that gives $\frac{dy}{dt}(b, t_0)$. Also, to determine the complete sequence, the guess points t_k for $k = 2, 3, \ldots$ together with nonlinear function $y(b, t) - \beta = 0$ are thus found via the application of the Secant iterative method as follows

$$t_k = t_{k-1} - \frac{(y(b, t_{k-1}) - \beta)(t_{k-2} - t_{k-1})}{y(b, t_{k-2}) - y(b, t_{k-1})}, \quad k = 2, 3, \ldots \quad .$$

Remarkably, the computational procedure in the proposed scheme will be terminated upon satisfying the following termination condition

$$|y(b,t_k)-\beta| \le \text{tolerance}.$$

5. Numerical results and discussion

The current section makes use of the proposed RSM procedure to solve the Jeffrey-Hamel problem, featuring the two-point third-order nonlinear ODE for arterial blood flow earlier expressed in (2.15)-(2.16). Firstly, we consider the following two IVPs

$$y'''(x) = -2\delta\, Re\, y(x)\, y'(x) - 4\delta^2\, y'(x), \quad y(0)=1, \quad y'(0)=0, \quad y''(0)=t_k, \tag{5.1}$$

and

$$z'''(x) = -2\delta\, Re\, y'(x)\, z(x) - 2\delta(Re\, y(x)+2\delta)\, z'(x), \quad z(0)=0, \quad z'(0)=0, \quad z''(0)=1. \tag{5.2}$$

Then, when employing the RADM on equations (5.1) - (5.2), guesses are chosen $n=2$ and $m=5$, the recursive relations in the first step are obtained as follows

$$\begin{cases} y_0 = 1 + \dfrac{x^2}{2}\, t_k, \\ y_{m+1} = -2\delta\, Re\, L^{-1}(A_m) - 4\delta^2\, L^{-1}(y'_m), \quad m \ge 0, \end{cases}$$

and

$$\begin{cases} z_0 = \dfrac{x^2}{2}, \\ z_{m+1} = -2\delta\, Re\, L^{-1}(y'_m z_m) - 2\delta L^{-1}\left((Re\, y_m + 2\delta) z'_m\right), \quad m \ge 0, \end{cases}$$

where A_m in the above schemes denotes the Adomian polynomials corresponding to the nonlinear term $y\,y'$, and by calculating the components $y_0, y_1, \ldots, y_5$ and $z_0, z_1, \ldots, z_5$, the approximate solutions of equations (5.1) - (5.2) in the first step are obtained in a series form as follows

$$y(x) = G_1 = \sum_{m=0}^{5} y_m(x),$$

and

$$z(x) = G_2 = \sum_{m=0}^{5} z_m(x).$$

Next, the recursive relations in the second step are obtained as follows

$$\begin{cases} y_0 = G_1, \\ y_1 = 1 + \dfrac{x^2}{2}\, t_k - 2\delta\, Re\, L^{-1}(A_0) - 4\delta^2\, L^{-1}(y'_0) - G_1, \\ y_{m+1} = -2\delta\, Re\, L^{-1}(A_m) - 4\delta^2\, L^{-1}(y'_m), \quad m \ge 1, \end{cases}$$

and

$$\begin{cases} z_0 = G_2, \\ z_1 = \dfrac{x^2}{2} - 2\delta\, Re\, L^{-1}(y_0' z_0) - 2\delta L^{-1}\left((Re\, y_0 + 2\delta) z_0'\right) - G_2, \\ z_{m+1} = -2\delta\, Re\, L^{-1}(y_m' z_m) - 2\delta L^{-1}\left((Re\, y_m + 2\delta) z_m'\right), \quad m \geq 1. \end{cases}$$

Consequently, the solution of equation (5.1) is obtained in a series form as follows:

$$y(x) = \sum_{m=0}^{5} y_m = y_0 + y_1 + \cdots + y_5.$$

Therefore, when using 6 iterations, than $y(x, t_k)$ represents the solution to the Jeffrey-Hamel arterial blood flow nonlinear MHD problem (2.15)-(2.16) with $t = t_k$; see Tables 1 and 2 for the simulated numerical results while considering cases 1-4 of different δ and Re values.

Table 1. The absolute error for RSM.

x	**Case 1** $\delta = 0.0524, Re = 110$	**Case 2** $\delta = -0.0873, Re = 80$	**Case 3** $\delta = 0.1309, Re = 50$	**Case 4** $\delta = 0.0873, Re = 50$
0.0	0	0	0	0
0.2	1.3×10^{-6}	9.3×10^{-13}	4.4×10^{-6}	7.1×10^{-8}
0.4	4.7×10^{-6}	4.3×10^{-12}	1.6×10^{-5}	2.6×10^{-7}
0.6	9.4×10^{-6}	1.2×10^{-11}	3.1×10^{-5}	5.3×10^{-7}
0.8	1.5×10^{-5}	2.6×10^{-11}	4.8×10^{-5}	8.6×10^{-7}
1.0	2.2×10^{-5}	4.9×10^{-11}	7.1×10^{-5}	1.3×10^{-6}

Table 2. Comparison between different methods.

Numerical methods	**Maximum error**			
	Case 1	**Case 2**	**Case 3**	**Case 4**
RSM	2.2×10^{-5}	4.9×10^{-11}	7.1×10^{-5}	1.3×10^{-6}
EDSM [19]	1.9×10^{-3}	8.9×10^{-10}	2.1×10^{-2}	2.5×10^{-5}
HPM [1]	7.6×10^{-2}	1.4×10^{-2}	1.3×10^{-1}	2.4×10^{-2}
PSO [12]	1.7×10^{-4}	8.6×10^{-4}	2.5×10^{-4}	5.1×10^{-5}

The Jeffery–Hamel problem for the flow of blood in the human system is solved by RSM for cases 1-4 of different δ and Re values; some different real values for the angle between the examining inclined plates and the Reynolds number, respectively. Further, in Table 1, we estimated the error in $y(x, t_5)$ using

$$\mathrm{ERR} = |y(x, t_5) - y(x, t_4)|,$$

because of the fact that the present problem does not have a known closed-form solution. Equally, from Table 2, we can see that RSM is the most competent technique for solving the governing model in comparison with the methods used in [1, 12, 19]. Again, we portray the approximate solutions of the RSM for the four different cases in Figure 2. Figure 3 also shows that the value of δ is varied for four distinct scenarios, namely 0.03, 0.10, 0.13, and 0.20, while the value of Re is kept at 50. The values of Re were varied for four distinct scenarios (20, 40, 60, and 80, respectively) while the value of $\delta = 0.03$ remained fixed in Figure 4. Notably, it is observed from Figures 2-4 that an increase x results in a decrease in the velocity of the arterial blood flow in the human body; that is, both have inverse relation. Moreover, this trend is noted across all cases of different δ and Re values. In addition, changing the product of δ and Re leads to a change in the MHD Jeffery–Hamel blood flow that is, increase in product value of δ and Re causes the flow to decrease i.e., they have inverse relation.

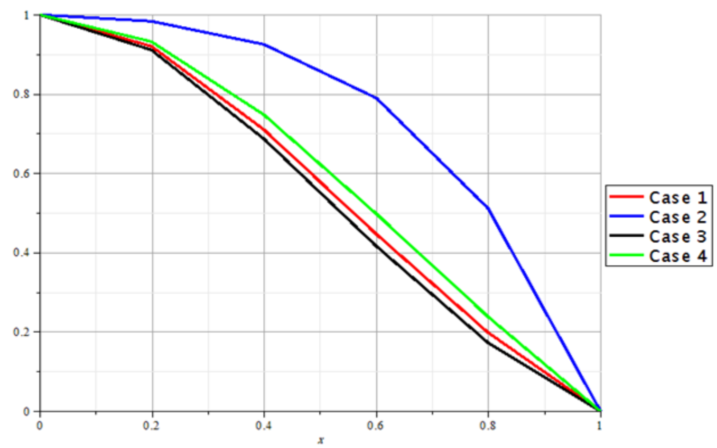

FIGURE 2. Graphical comparison, depicting approximate solution of RSM for the four cases.

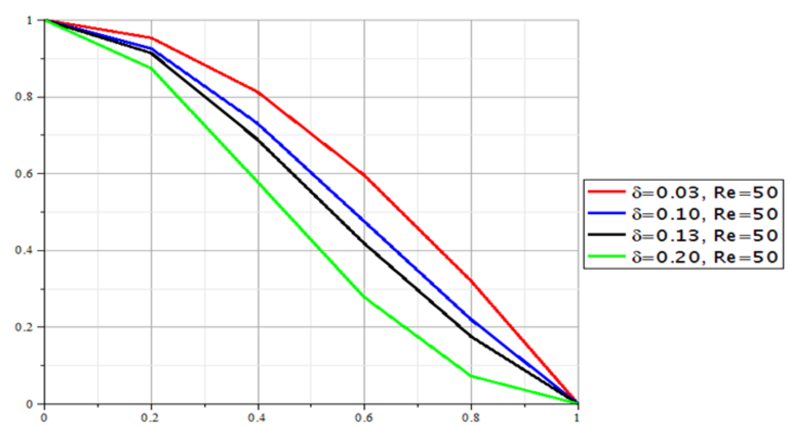

FIGURE 3. Graphical comparison, depicting approximate solution of RSM for the different values of δ when $Re = 50$.

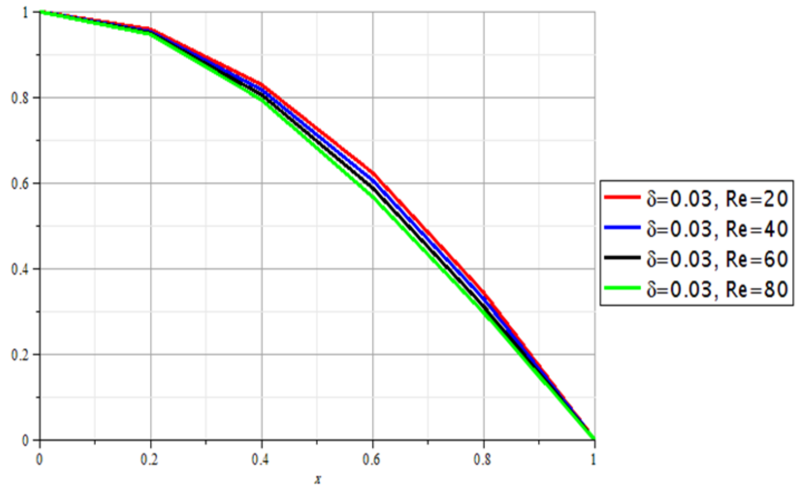

Figure 4. Graphical comparison, depicting approximate solution of RSM for the different values of Re when $\delta = 0.03$.

6. Conclusion

In conclusion, this paper presents a computational method to effectively address a class of third-order nonlinear ODEs that have two-point boundary data. Specifically, we used the modified RADM in conjunction with the iterative shooting method to devise a highly effective strategy called the RSM. The RSM as a reliable computational method has then been demonstrated on the Jeffery-Hamel problem for the movement of arterial blood with a huge success. Besides, the study beseeched a particular transformation method to recast the governing two-dimensional flow equations to a condensed third-order nonlinear one-dimensional ODE. Subsequently, the problem was subjected to RSM testing for various values of Re and δ by using Maple Software. Following the method's implementation, it was discovered that the current approach outperforms previous approaches identified in the literature [1, 12, 19]. Lastly, we have supported the findings of the present study with some comparison tables and figures – demonstrating the usefulness of the devised technique. In addition, the devised technique can be applied to diverse models of real-life applications; certainly, the future undertaking would look at the possibilities of tackling higher-order nonlinear ODEs using the proposed method, as well as accelerating the rapidness of the scheme's convergence by incorporating enhanced algorithms.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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