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Magneto Dynamic Stability of Bounded Cylindrical Streaming Fluid Jet Pervaded by Toroidal Magnetic Field

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ABSTRACT. The hydromangetic stability of bounded fluid jet under the influence of the electromagnetic (with toroidal varying field) force has been developed. A general dispersion relation valid for all modes of perturbation is derived. The geometric factor q which is the radii ratio of the tenuous-fluid regions plays an important role for stabilizing the model. The axial and transvers magnetic fields interior and exterior the fluid jet are stabilizing. The magnetic fields decrease the streaming destabilizing domains and at the same time give a sort or rigidity to the fluid molecules. For any value of the applying magnetic field strength, the instability character of the streaming model could be suppressed and dispersed. These results are confirmed numerically upon using computer programs.

1. Introduction

The classical stability analysis of a full jet has been extensively studied by Rayleigh [1], Nayfeh [2] and Chandrasekhar [3]. The last author studied the hydromagnetic stability of a fluid jet pervaded by a constant magnetic field for axisymmetric perturbation. Radwan *et al*. ([4],[5]) developed the MHD stability of that model for all axisymmetric and non-axisymmetric modes subject to electromagnetic forces and pervaded by constant magnetic field. See also Hide [6] . Khurana [7] presents some theoretical research on the propagation of Rossby-MHD waves in

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homogeneous media and inertial-MHD waves over rigid boundaries. Radwan [8] investigated Stability of a streaming magnetized fluid jet penetrated internally by a toroidal varying magnetic field. Radwan et al. [9] investigated the MHD stability of a fluid jet with surface tension that was pierced by a toroidal changing magnetic field in all axisymmetric and non-axisymmetric perturbation modes. Sakuraba [10] studied a linear analysis of thermally generated magneto-convection, with a focus on convection within the Earth's core. Radwan and Hasan [11] investigated the self-gravitating instability of a fluid cylinder surrounded by a magnetic field and supplied with surface tension. Elazab et al. [12] studied the magnetohydrodynamic stability criterion of a self-gravitating streaming fluid cylinder with integrated capillary effects. Hasan [13] investigated the stability of an oscillating streaming self-gravitating dielectric incompressible fluid cylinder surrounded by a tenuous medium with minimal motion and a transversely shifting electric field for all axisymmetric and non-axisymmetric perturbation modes. Abdeen and Hasan [14] studied the magnetohydro-dynamic stability of a fluid jet pervaded by a transversely changing magnetic field while its surrounding tenuous medium is pierced by a uniform magnetic field.

Hasan et al. [15] investigated the magnetohydrodynamic stability criterion of a self-gravitating streaming fluid cylinder under the combined influence of self-gravitating, magnetic, and capillary forces. Barakat [16] investigated the magnetohydrodynamic (MHD) stability of an oscillating fluid in the presence of a longitudinal magnetic field. See (Raphaldini and Raupp, [17]). Barakat et al. [18] investigated the self-gravitating stability of a fluid cylinder embedded in a confined liquid containing a magnetic field for all symmetric and asymmetric perturbation modes. Hendy and Amin [19] investigated the influence of magnetodynamic stability on the bounded annular cylinder (tar) penetrated by an unstable magnetic field.

Hendy [20] investigated the effects of non-axisymmetric self-gravitating instability on a capillary incompressible bounded cylindrical hollow jet. See also Wright et al., [21]. Hasan et al. [22] explained magnetohydrodynamic stability in a uniform cylinder of an incompressible inviscid fluid due to self-gravitation, magnetic field, and capillary forces. Medvedev et al. [23] investigated the ideal magnetohydrodynamic properties of axisymmetric balance magnetoplasma formations of three rings conveying current dependent on plasma pressure. The electrically conductive fluid instability under the effect of a transversal magnetic field created by two parallel plates examined by Hussain et al. [24]. Barakat [25] calculated the self-gravitating stability of a fluid cylinder immersed in a confined liquid with a magnetic field for all symmetric and asymmetric perturbation modes. Recently, Elazab et al. [26] investigated Magnetohydro-dynamic Stability of Self-gravitating Streaming Fluid Cylinder. The purpose of the present work is to investigate the magnetodynamic stability of cylindrical streaming fluid jet pervaded internally by toroidal varying magnetic field and surrounded by bounded medium with uniform magnetic field. The advantages here are that model is realistic and that there is no singular solution any more in the region surrounding the fluid jet. The conclusion would be validated both theoretically and numerically for all (non)-axisymmetric forms of perturbations.

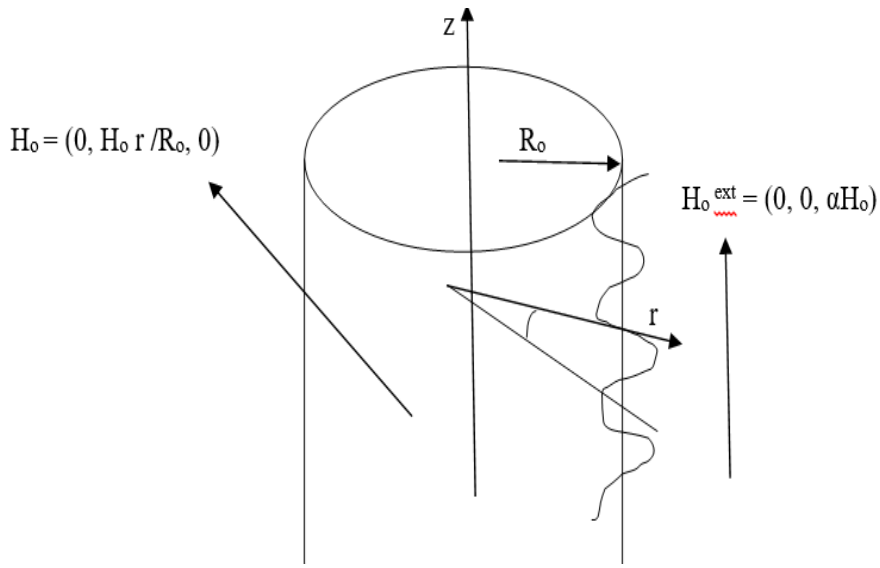

Fig. 1 Sketch for MHD fluid jet

2. Basic State

We consider a uniform infinite cylinder of (radius R_0) incompressible and inviscid fluid. In the initial unperturbed state the fluid is assumed to be streaming uniformally with the velocity $\underline{u}_0 = (0,0,U)$ and pervaded by the magnetic field,

$$\underline{H}_0^{\text{int}} = \left(0, \frac{\beta H_0 r}{R_0}, \gamma H_0\right).$$

The tenuous medium surrounding the fluid cylinder is penetrated by

$$\underline{H}_0^{ext} = (0,0,\alpha H_0),$$

where α, β and γ are constants and H_0 is the intensity of the magnetic field in the exterior region of the cylinder as $\alpha = 1$. The components of $\underline{u}_0$, $\underline{H}_0$ and $\underline{H}_0^{ext}$ are considered along the utilizing cylindrical polar coordinates (r, ϕ, z) system with the *z*-axis coinciding with the axis of the cylinder.

The magnetohydrodynamic basic equations are given by

$$\rho\left(\frac{\partial u}{\partial t}+(\underline{u}\cdot\nabla)\underline{u}\right)=-\nabla P+\mu\left(curl\,\underline{H}\wedge\underline{H}\right) \tag{2.1}$$

$$\nabla\cdot\underline{u}=0 \tag{2.2}$$

$$\frac{\partial H}{\partial t}=curl\left(\underline{u}\wedge\underline{H}\right) \tag{2.3}$$

$$\nabla\cdot\underline{H}=0 \tag{2.4}$$

In the surrounding bounded medium

$$\nabla\wedge\underline{H}^{ext}=0 \tag{2.5}$$

$$\nabla\cdot\underline{H}^{ext}=0 \tag{2.6}$$

Here ρ, $\underline{u}$ and P are the fluid mass density, velocity vector and kinetic pressure; $\underline{H}$ and $\underline{H}^{ext}$ are the magnetic field intensities inside and outside the fluid jet, μ the magnetic field permeability coefficient.

The initial state with $\underline{u}_0=0$ is studied upon using the basic equations (2.1)-(2.6). Consequently, the distribution of the pressure is given by

$$P_0+\frac{\mu}{2}\left(\underline{H}_0\cdot\underline{H}_0\right)=const. \tag{2.7}$$

By applying the balance of the pressure at $r=R_0$, we get

$$P_0=\frac{\mu}{2}H_0^2\left(\alpha^2-\frac{\beta^2r^2}{R^2}-\gamma^2\right) \tag{2.8}$$

from which we see that P_0 is variable.

3. Perturbation Analysis

Linearization of equations (2.1)-(2.6), is accomplished by substituting the expansions

$$Q(r,\phi,z,t)=Q_0(r)+\varepsilon(t)Q_1(r,\phi,z)\,,\qquad |Q_1|<Q_0 \tag{3.1}$$

and retaining first-order terms only in the fluctuating variables Q_1, where Q stands for p, $\underline{u}$, $\underline{H}$ and $\underline{H}^{ext}$. The amplitude $\varepsilon(t)$ of the perturbation at time t is given by

$$\varepsilon(t)=\varepsilon_0\exp(\sigma t) \tag{3.2}$$

where $\varepsilon_0(=\varepsilon \text{ at } t=0)$ is the initial amplitude and σ (complex) is the growth rate. In view of the expansion (3.1) and based on the linear perturbation technique, the perturbed radial distance of the fluid cylinder is given by

$$r = R_0 + \varepsilon(t) R_1, \tag{3.3}$$

with

$$R_1 = R_0 \exp[i(kz + m\phi)] \tag{3.4}$$

Here R_1 is the elevation of the surface wave measured from the unperturbed position, k (real) is the longitudinal wavenumber and m (integer) is the transverse wavenumber.

By an appeal to the expansions (3.1), the linearized equations are given by

$$\rho\left(\frac{\partial \underline{u}_1}{\partial t} + (\underline{u}_0 \cdot \nabla)\underline{u}_1\right) = -\nabla P_1 + \mu\left[-\nabla(\underline{H}_0 \cdot \underline{H}_1) + (\underline{H}_0 \cdot \nabla)\underline{H}_1 + (\underline{H}_1 \cdot \nabla)\underline{H}_0\right] \tag{3.5}$$

$$\nabla \cdot \underline{u}_1 = 0 \tag{3.6}$$

$$\nabla \cdot \underline{H}_1 = 0 \tag{3.7}$$

$$\frac{\partial \underline{H}_1}{\partial t} = (\underline{H}_0 \cdot \nabla)\underline{u}_1 - (\underline{u}_0 \cdot \nabla)\underline{H}_1 - (\underline{u}_1 \cdot \nabla)\underline{H}_0 \tag{3.8}$$

$$\nabla \cdot \underline{H}_1^{ext} = 0 \tag{3.9}$$

$$\nabla \wedge \underline{H}_1^{ext} = 0 \tag{3.10}$$

By combining equations (3.5)-(3.8) and taking into account the space dependence, we get

$$\nabla^2 \Pi_1 = 0 \tag{3.11}$$

where $\rho\Pi_1 = P_1 + \mu (\underline{H}_0 \cdot \underline{H}_1)$ is the total magnetohydrodynamic pressure.

Also upon combining equations (3.9) and (3.10), we get

$$\underline{H}_1^{ext} = -\nabla \psi_1^{ext} \tag{3.12}$$

with

$$\nabla^2 \psi_1^{ext} = 0 \tag{3.13}$$

where ψ_1^{ext} is, some scalar function, the magnetic potential.

From the viewpoint of the linearized perturbation technique and the expansions (3.1)-(3.4), we may write

$$Q_1(r, \phi, z; t) = Q_1^*(r) \exp(i(kz + m\phi) + \sigma t) \tag{3.14}$$

Consequently, the non-singular solutions of the second order differential equations (3.11) and (3.13) are given by

$$\Pi_1 = A I_m(kr) \exp[i(kz + m\phi) + \sigma t] \tag{3.15}$$

$$\psi_1^{ext} = (B K_m(kr) + C I_m(kr)) \exp[i(kz + m\phi) + \sigma t] \tag{3.16}$$

Here $I_m(kr)$ and $K_m(kr)$ are the modified Bessel functions of the first and second kind of the order *m* while *A,B* and *C* are constants of integration.

The latters could be identified upon applying appropriate boundary conditions.

Under the present circumstances, these boundary conditions are given as follows:

1- The normal component of the velocity $\underline{u}$ must be compatible with the velocity of the fluid-tenuous medium interface equation (3.3) at $r = R_0$. This condition gives

$$A = \frac{-\left[(\sigma + iUk)^2 + \Omega_A^2\right]R_0^2}{xI'_m(x)} \tag{3.17}$$

where $x(= kR_0)$ is a dimensionless longitudinal wavenumber.

2- The normal component of the exterior magnetic field $\underline{H}^{ext}$ vanishes at $r = qR_0$.

3- The normal component of the magnetic field $\underline{H}$ must be continuous across the interface equation (3.3) at $r = R_0$.

Consequently the last two conditions give

$$C = \frac{-BK'_m(qkR_0)}{I'_m(qkR_0)} \tag{3.18}$$

$$B = \frac{-iR_0\alpha H_0 I'_m(qkR)}{\left[K'_m(qkR_0)\,I'_m(kR_0) - I'_m(qkR_0)K'_m(kR_0)\right]} \tag{3.19}$$

4- The jump of the total MHD pressure must be continuous across the interface equation (3.3) at $r = R_0$. This condition reads

$$P_1 + R_1\frac{\partial P_0}{\partial r} + \left\langle \frac{\mu}{2}(\underline{H}\cdot\underline{H})_1 \right\rangle = 0, \quad \text{at} \quad r = R_0 \tag{3.20}$$

Note that, the jump of magnetic field is given by

$$\left\langle \frac{\mu}{2}(\underline{H}\cdot\underline{H})_1 \right\rangle = \mu\left[(H_0\cdot H_1) - (H_0\cdot H_1)^{ext}\right] + \frac{\mu}{2}R_1\frac{\partial}{\partial r}\left[(\underline{H}_0\cdot\underline{H}_0) - (\underline{H}_0\cdot\underline{H}_0)^{ext}\right] \tag{3.21}$$

By utilizing the compatibility condition (3.20), the following relation is obtained

$$(\sigma + iUk)^2 = \frac{\mu H_0^2}{\rho R_0^2}\left[-(m\beta + \gamma x)^2 + \alpha^2 x^2\frac{I'_m(x)L_y^m}{I_m(x)L_{x,y}^m}\right] \tag{3.22}$$

where

$$L_y^m = I'_m(y)K_m(x) - I_m(x)K'_m(y) \tag{3.23}$$

$$L_y^m = I'_m(y)K'_m(x) - I'_m(x)K'_m(y) \tag{3.24}$$

with

$$y = qx \tag{3.25}$$

is a dimensionless longitudinal wavenumber due to the existence of the bounded tenuous medium.

4. Stability Discussions

Equation (3.22) is the desired eigenvalue relation of magnetized fluid cylinder, pervaded by toroidal varying magnetic field, surrounded by bounded tenuous magnetized medium. It related the growth rate σ with the magnetic fields parameters α, β and γ, the intensity of the magnetic field H_0 in the tenuous medium as $(\alpha = 1)$, the fluid density ρ, the radius R_0 of the fluid cylinder, the wavenumbers *x, y* and *m*, the modified Bessel functions I_m and K_m (with different arguments) and with their derivatives. Since equation (3.22) is a general dispersion relation, we may recover some reported works as limiting cases from it upon assuming some postulates.

If we assume that $U = 0$ and $q \to \infty$, we have a stationary liquid cylinder surrounded by unbounded tenuous medium of negligible inertia. Its dispersion relation is given from equation (3.22) by

$$\sigma^2 = \frac{-\mu H_0^2}{\rho R_0^2}\left[(m\beta + \gamma x)^2 - \alpha^2 x^2 \frac{I'_m(x)K_m(x)}{K'_m(x)I_m(x)}\right] \tag{4.1}$$

where use has been made of the limit

$$\lim_{y\to\infty} I'_m(y) \to \infty \tag{4.2}$$

$$\lim_{y\to\infty} K'_m(y) \to 0 \tag{4.3}$$

If we assume that $U = 0$, $\beta = 0$ and $q \to \infty$, we have a stationary fluid cylinder surrounded by infinite tenuous medium pervaded by uniform magnetic field interior and exterior the fluid cylinder. In such case the dispersion relation is given by

$$\sigma^2 = \frac{-\mu H_0^2}{\rho R_0^2} x^2 \left[\gamma^2 - \alpha^2 \frac{I'_m(x)K_m(x)}{K'_m(x)I_m(x)}\right] \tag{4.4}$$

If we assume that $U = 0$, $\gamma = 0$ and $q \to \infty$, in this case we have, initially, non-streaming liquid cylinder pervaded by transverse varying magnetic field while it is surrounded by unbounded medium penetrated by axial magnetic field.

If we assume that the present model is not streaming in the unperturbed state, the dispersion relation (3.22) becomes

$$\sigma^2 = \frac{\mu H_0^2}{\rho R_0^2}\left[-(m\beta+\gamma x)^2 + \frac{I'_m(x)}{I_m(x)}\left(x^2\alpha^2\,\frac{(K'_m(y)I_m(x)-K_m(x)I'_m(y))}{K'_m(y)I'_m(x)-K'_m(x)I'_m(y)}\right)\right] \tag{4.5}$$

If the liquid cylinder is non-conducting but streaming in the initial state while the surrounding bounded medium is conducting, then the dispersion relation is simply given by

$$\frac{(\sigma+ikU)^2}{\mu H_0^2/(\rho R_0^2)} = x^2\alpha^2\,\frac{I'_m(x)}{I_m(x)}\frac{(K'_m(y)I_m(x)-K_m(x)I'_m(y))}{(K'_m(y)I'_m(x)-K'_m(x)I'_m(y))} \tag{4.6}$$

Now, in order to discuss the stability states of the problem under consideration, we have to study some characteristic behavior of the modified Bessel functions.

Consider the recurrence relations

$$2I'_m(x) = I_{m-1}(x) + I_{m+1}(x) \tag{4.7}$$

$$2K'_m(x) = -K_{m-1}(x) - K_{m+1}(x) \tag{4.8}$$

It is well known (cf. Abramowitz and Stegun [27] for non-zero real value of *x*, that $I_m(x)$ is positive definite and monotonic increasing while $K_m(x)$ is monotonic decreasing but never negative. Therefore, on using the recurrence relations (4.7) and (4.8) we may show, for $x \neq 0$, that

$$I'_m(x) > 0, \tag{4. 9}$$

$$K'_m(x) < 0, \tag{4.10}$$

In addition, one has to mention here, since $1 < q < \infty$, that

$$y > x \tag{4.11}$$

So that

$$I_m(y) > I_m(x) \tag{4.12}$$

but

$$K_m(x) > K_m(y) \tag{4.13}$$

Based on these inequalities, we may show, for non-zero real values of *x* and *y*, that

$$L_y^m = I'_m(y)K_m(x) + I_m(x)|\,K'_m(y)| = +\upsilon e \tag{4.14}$$

$$L_{x,y}^m = -I'_m(y)|\,K'_m(x)| + I'_m(y)|\,K'_m(x)| = -\upsilon e \tag{4.15}$$

Therefore, for $x \neq 0$ and $y \neq 0$, the quantity Q_m :

$$Q_m = \frac{I'_m(x) L_y^m}{I'_m(x) L_{x,y}^m} < 0 \tag{4.16}$$

is negative for all axisymmetric mode $m = 0$ and non-axisymmetric modes $m \neq 0$ of perturbation.

Now, let us returning to the general dispersion relation (30).

The effect of the toroidal magnetic field interior the fluid cylinder is represented by the term $-(m\beta + x\gamma)^2$ following $\frac{\mu H_0^2}{\rho R_0^2}$ in the relation (3.22). It is a quadratic quantity with negative sign, so it is negative quantity. Therefore, the interior toroidal magnetic field has strong stabilizing influence. The influence of the longitudinal magnetic field pervaded in the bounded tenuous medium is represented the quantity $\alpha^2 x^2 Q_m$ following $\frac{\mu H_0^2}{\rho R_0^2}$ in the general dispersion relation (3.22). it, for $\alpha \neq 0$, $x \neq 0$, is negative for all $m = 0$ and $m \geq 1$ modes of perturbation. Therefore, the axial magnetic field penetrated in the tenuous region has strong stabilizing influence. The latter is true not only in the axisymmetric perturbation mode $m = 0$ but also in those of non-axisymmetric $m \geq 1$. The streaming has a destabilizing effect and valid for all short and long wavelengths.

Based on the foregone analytical discussions we may deduce the following results.

1- The axial magnetic fields interior and exterior the fluid cylinder are stabilizing for all $m \geq 0$, $x \neq 0$ and $y \neq 0$.

2- The transverse magnetic field interior the fluid cylinder is stabilizing for all modes $m = 0$ and $m \geq 1$ of perturbations for all non-zero real values of *x* and *y*.

3- The streaming of the fluid is destabilizing. The latter effect is independent of the kind of perturbation and whether the wavelength is short or long.

We conclude that the present streaming model is destabilizing or stabilizing according to restrictions. This may be clearer via the numerical analysis, where we can easily identify the stable domains and those of instability and their characteristics.

5. Numerical Analysis

In order to verify the analytical results and determine exactly where the stable and unstable domains are, the dispersion relation (3.22) is formulated in the non-dimension form

$$\left(\sigma^* - U^*\right)^2 = -(m\beta + \gamma x)^2 + \alpha^2 x^2 \frac{I'_m(x) L_y^m}{I_m(x) L_{x,y}^m} \tag{5.1}$$

With

$$\sigma^* = \frac{\sigma}{\sqrt{\dfrac{\mu H_0^2}{\rho R_0^2}}}, \tag{5.2}$$

$$U^* = \frac{-ikU}{\sqrt{\dfrac{\mu H_0^2}{\rho R_0^2}}}, \tag{5.3}$$

The relation (5.1) has been computed in the computer for $m = 0$. It is remarkable that the transverse magnetic field has no any influence on the stability of the present model in such case. Different values for α, γ and q are given while x is assumed to be regular values $0 < x \le 5$ where $y = qx$. See figs. (2)- (5).

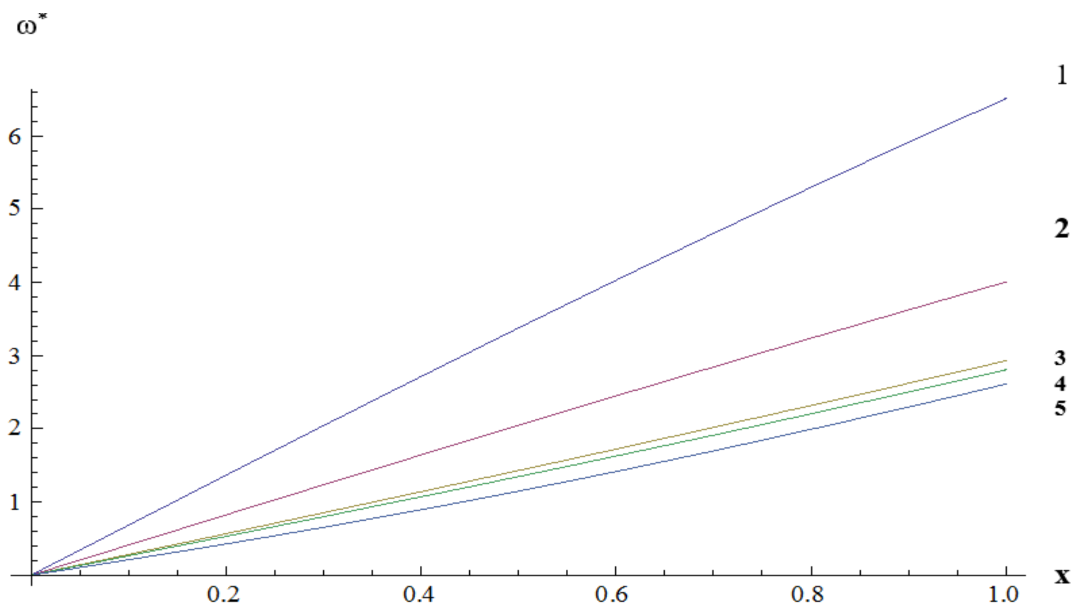

Fig. (2) Magneto hydrodynamic stable domains for different values of (q, α, γ) with

$$\omega^* = \omega\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2},\quad U^* = -iUk\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2} = 0$$

$1-(q,\alpha,\gamma) = (1.1,3,2)$ 2- $(q,\alpha,\gamma) = (1.1,3,4)$ 3- $(q,\alpha,\gamma) = (1.1,4,5)$

4- $(q,\alpha,\gamma) = (1.1,5,8)$ 5- $(q,\alpha,\gamma) = (1.1,6,9)$

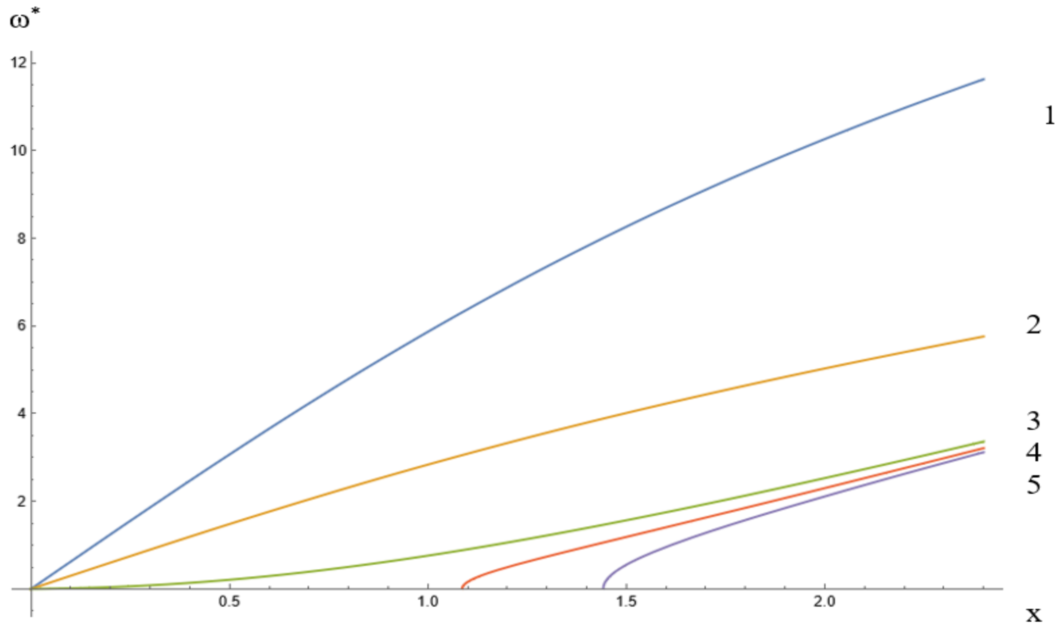

Fig. (3) Magneto hydrodynamic stable domains for different values of (q,α,γ) with

$$\omega^* = \omega\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2}, \quad U^* = -iUk\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2} = 0$$

$1-(q,\alpha,\gamma)=(1.1,3,2)$ 2- $(q,\alpha,\gamma)=(1.3,3,2)$ 3- $(q,\alpha,\gamma)=(1.8,3,2)$

4- $(q,\alpha,\gamma)=(2,3,2)$ 5- $(q,\alpha,\gamma)=(5,3,2)$

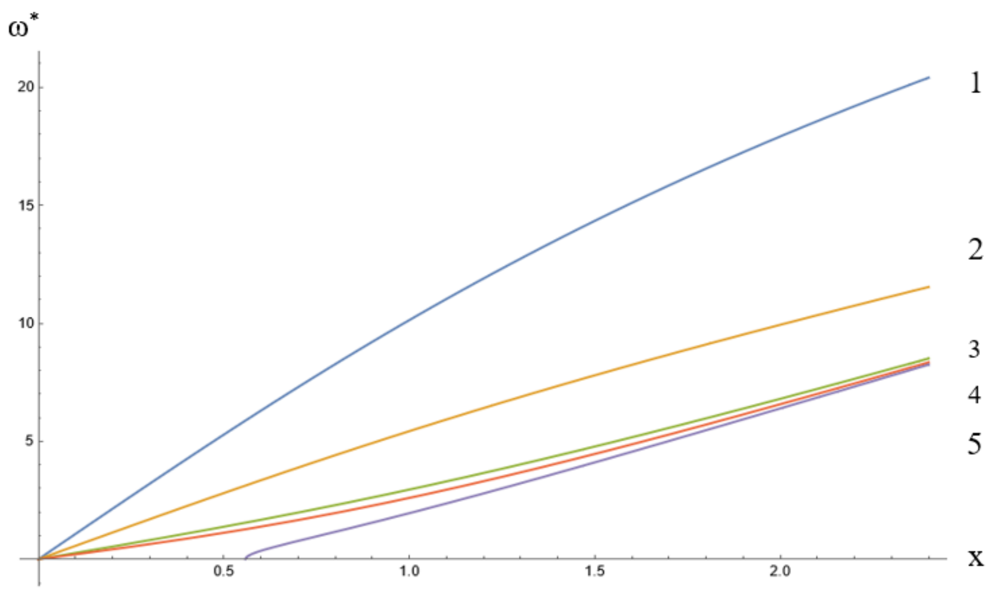

Fig. (4) Magneto hydrodynamic stable domains for different values of (q,α,γ) with

$$\omega^* = \omega\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2}, \quad U^* = -iUk\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2} = 0$$

$1-(q,\alpha,\gamma)=(1.1,5,2)$ 2- $(q,\alpha,\gamma)=(1.3,5,2)$ 3- $(q,\alpha,\gamma)=(1.8,5,2)$

4- $(q,\alpha,\gamma)=(2,5,2)$ 5- $(q,\alpha,\gamma)=(5,5,2)$

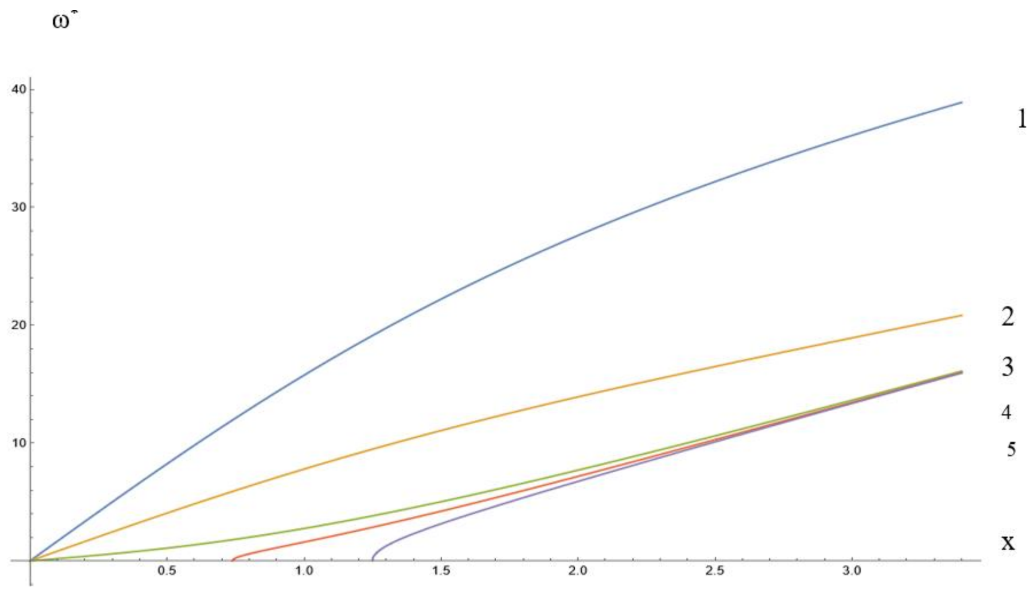

Fig. (5) Magneto hydrodynamic stable domains for different values of (q,α,γ) with

$$\omega^{*}=\omega\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2},\ U^{*}=-iUk\left(\frac{\mu H_0^2}{\rho R_0^2}\right)^{-1/2}=0$$

$1-(q,\alpha,\gamma)=(1.1,8,5)$ 2- $(q,\alpha,\gamma)=(1.3,8,5)$ 3- $(q,\alpha,\gamma)=(1.8,8,5)$

4- $(q,\alpha,\gamma)=(2,8,5)$ 5- $(q,\alpha,\gamma)=(5,8,5)$

6. Conclusion

Based on the numerical results presented, we can conclude the following. For the same value of U, the unstable domains decrease as h increases. This suggests that the magnetic field has a stabilizing impact on both short and long wavelengths. For the same h value, the unstable domains increase as U values increase.

1- The current model's stability is unaffected by the transverse magnetic field. This indicates that the streaming is highly disruptive.
2- The model described in this work is both successful and intelligent in simulating the effect of longitudinal wave number, magnetic field, and fluid stream velocity on growth rate.
3- The provided model is quite efficient in directly forecasting the stable and unstable domains for the growth rate for different parameters.

4- The model can generate a large number of outcomes for each modification in magnetic field and fluid stream velocity in a single run, which takes only a few minutes and accepts mistake.
5- The model was able to comprehend the stability behavior of a streaming jet field and became capable of providing growth rates for various parameters without having to solve such a problem again using analytic or any other approach.

Final, the streaming has a destabilizing effect in both hydrodynamic and MHD forms. The axial magnetic field that pierced the mantle jet stabilizes all non-axisymmetric perturbation modes. It is discovered that when the magnetic field strength is so high that the Alfven magnetic wave velocity is significantly greater than the streaming velocity, the magnetic field's stabilizing influence predominates and overcomes the capillary and streaming destabilizing forces, and stability occurs.

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