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# Control and Function Projective Synchronization of 3D Chaotic System

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**Abstract.** In this paper, we study a linear feedback control to dampen the chaotic behavior of a 3D dynamic system. Based on the Routh-Hurwitz criterion, the conditions are determined to achieve control. Furthermore, function projective synchronization (FPS) between two identical 3D chaotic systems is demonstrated. The proof of asymptotic stability of solutions for the error dynamical system depends on the negative eigenvalues of the system. Additionally, numerical simulations are utilized to demonstrate the impact and effectiveness of the proposed methods.

## 1. Introduction

Chaotic dynamical systems have attracted the attention of many researchers due to their sensitive dependence on initial conditions and their unpredictable behavior, as well as their applications in many disciplines such as engineering, biology, information processing, and secure communication [1]. In the past few decades, many control mechanisms have been successfully used to regulate chaotic behavior, such as linear and nonlinear feedback control, active control, and adaptive control ( [2], [3], [4], [5], [6], [7], [8], [9]). Recently, following the seminal work by Pecora and Carroll [10] on chaotic synchronization for identical chaotic systems with differing initial conditions, numerous synchronization methods have been developed effectively utilized to manage chaos, such as projective synchronization, anti synchronization, complete synchronization, generalized synchronization, and modified projective synchronization ( [11], [12], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22], [23], [24]).

The above-mentioned projective synchronization method involves synchronizing its drive and response systems with a constant scaling factor. A new scheme called function projective synchronization (FPS), introduced by Chen *et al* [25], enables the synchronization of dynamical states

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with a scaling function factor. By setting the scaling function as unity or constant, one can achieve complete synchronization or projective synchronization, making FPS a more inclusive definition of projective synchronization ( [26], [27], [28]).

The object of this paper is to suppress the chaos of 3D chaotic dynamic system and to study the function projective synchronization (FPS) of two identical 3D chaotic systems with predefined parameters.

The subsequent sections of this paper are structured as follows: Section 2 displays an overview of the 3D chaotic system and its basic dynamic properties. Section 3 demonstrates the linear feedback control of the 3D chaotic system. Section 4 delves into the function projective synchronization of two identical 3D chaotic systems. Section 5 provides numerical results to illustrate the efficacy of the proposed schemes. The last section includes the conclusion.

## 2. Description of system

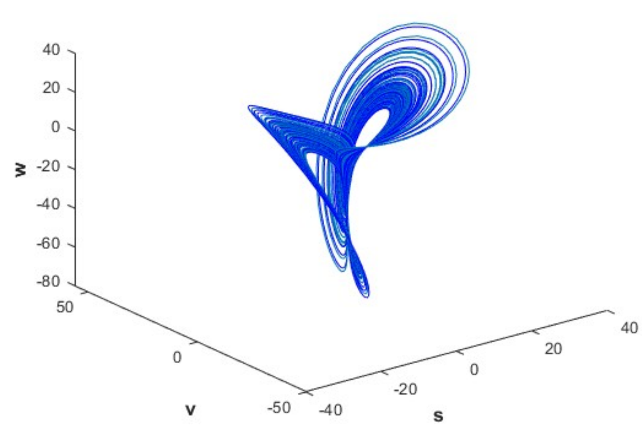
The $3D$ chaotic dynamical system ( [29], [30]) can be shown as follows:

$$\begin{cases} \dot{s} = \alpha(v - s), \\ \dot{v} = -\gamma v - sw, \\ \dot{w} = -\beta + sv. \end{cases} \tag{2.1}$$

where $\alpha, \beta$ and $\gamma$ are positive real parameters. This system will be chaotic at $\alpha = 10, \beta = 100$, and $\gamma = 10.3$, the chaotic attractor is shown in FIGURE 1.
At previous parameter values, the system possesses two equilibrium points, which are:
$P_1 = (\sqrt{\beta}, \sqrt{\beta}, -\gamma)$ and $P_2 = (-\sqrt{\beta}, -\sqrt{\beta}, -\gamma)$.
The divergence of system (2.1) is found as follows:

$$\nabla.F = \frac{\partial \dot{s}}{\partial s} + \frac{\partial \dot{v}}{\partial v} + \frac{\partial \dot{w}}{\partial w} = -\alpha - \gamma = -20.3 < 0.$$

Therefore, the (2.1) is the dissipative system.



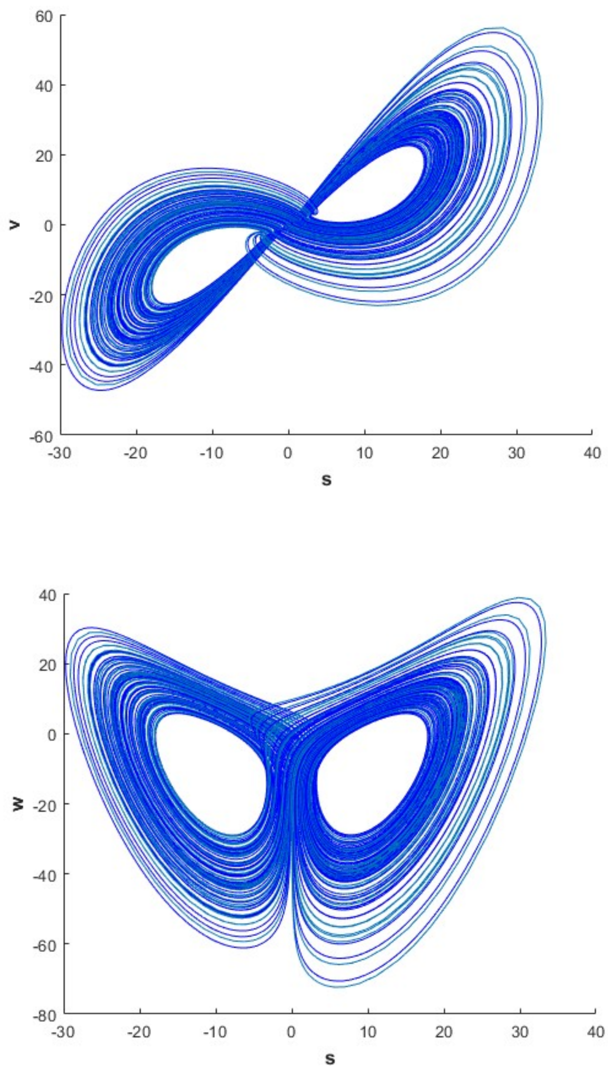



Figure 1. The 3D chaotic system at $\alpha = 10, \beta = 100$, and $\gamma = 10.3$

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## 3. Controlling 3D chaotic system

This study aims to direct the trajectory of the 3D chaotic system towards a specific unstable equilibrium point.

3.1. **First.** To direct the chaotic trajectory towards the unstable equilibrium point $P_1 = (s_1, v_1, w_1) = (\sqrt{\beta}, \sqrt{\beta}, -\gamma)$, we employ traditional feedback control.

Let system (2.1) be controlled on the form:

$$\begin{cases} \dot{s} = \alpha(v - s) - k_{11}(s - s_1), \\ \dot{v} = -\gamma v - sw - k_{12}(v - v_1), \\ \dot{w} = -\beta + sv - k_{13}(w - w_1). \end{cases} \tag{3.1}$$

The controlled system (3.1) has one equilibrium point $P_1 = (s_1, v_1, w_1)$. Linearization of the system is performed around this specific equilibrium point. Then the linearized system can be expressed as follows:

$$\begin{cases} \dot{S} = -(\alpha + k_{11})S + \alpha V, \\ \dot{V} = -w_1 S - (\gamma + k_{12})V - s_1 W, \\ \dot{W} = v_1 S + s_1 V - k_{13} W. \end{cases} \tag{3.2}$$

where $P_1 = (s_1, v_1, w_1) = (\sqrt{\beta}, \sqrt{\beta}, -\gamma)$, then:

$$\begin{cases} \dot{S} = -(\alpha + k_{11})S + \alpha V, \\ \dot{V} = \gamma S - (\gamma + k_{12})V - \sqrt{\beta} W, \\ \dot{W} = \sqrt{\beta} S + \sqrt{\beta} V - k_{13} W. \end{cases} \tag{3.3}$$

The previous system (3.3) possesses a fixed point $(0, 0, 0)$ that is analogues to the fixed point $(s_1, v_1, w_1)$ of the controlled system (3.1).

**Lemma 3.1.** *The asymptotic stability of the zero solution in the linearized system (3.3) is shown under the conditions that the gained matrix satisfies $k_{12} = k_{13} = 0$ and $k_{11} > 0$.*

*Proof.* The verification of this lemma is contingent upon the criteria of Routh-Hurwitz. The Jacobian matrix corresponding to system (3.3) is provided by:

$$J = \begin{pmatrix} -\alpha - k_{11} & \alpha & 0 \\ \gamma & -\gamma & -\sqrt{\beta} \\ \sqrt{\beta} & \sqrt{\beta} & 0 \end{pmatrix},$$

then the characteristic equation of the Jacobian matrix is:

$$\lambda^3 + (\alpha + \gamma + k_{11})\lambda^2 + (\beta + \gamma k_{11})\lambda + (\beta k_{11} + 2\alpha\beta) = 0, \tag{3.4}$$

where

$$a_1 = \alpha + \gamma + k_{11},$$
$$a_2 = \beta + \gamma k_{11},$$
$$a_3 = \beta k_{11} + 2\alpha\beta.$$

The Routh-Hurwitz criterion has been met, which indicates that the eigenvalues possess negative real components. Consequently, the zero solution of the system (3.3) is deemed asymptotically stable. □

3.2. **Second.** To direct the chaotic trajectory towards the unstable equilibrium point $P_2 = (s_2, v_2, w_2) = (-\sqrt{\beta}, -\sqrt{\beta}, -\gamma)$, we employ traditional feedback control.

Let system (2.1) be controlled on the form:

$$\begin{cases} \dot{s} = \alpha(v - s) - k_{21}(s - s_2), \\ \dot{v} = -\gamma v - sw - k_{22}(v - v_2), \\ \dot{w} = -\beta + sv - k_{23}(w - w_2). \end{cases} \tag{3.5}$$

The controlled system (3.5) has one equilibrium point $P_2 = (s_2, v_2, w_2)$. Linearization of the system is performed around this specific equilibrium point. Then the linearized system can be expressed as follows:

$$\begin{cases} \dot{S} = -(\alpha + k_{21})S + \alpha V, \\ \dot{V} = -w_2 S - (\gamma + k_{22})V - s_2 W, \\ \dot{W} = v_2 S + s_2 V - k_{23} W, \end{cases} \tag{3.6}$$

where $P_2 = (s_2, v_2, w_2) = (-\sqrt{\beta}, -\sqrt{\beta}, -\gamma)$, then:

$$\begin{cases} \dot{S} = -(\alpha + k_{21})S + \alpha V, \\ \dot{V} = \gamma S - (\gamma + k_{22})V + \sqrt{\beta} W, \\ \dot{W} = -\sqrt{\beta} S - \sqrt{\beta} V - k_{23} W. \end{cases} \tag{3.7}$$

The previous system (3.7) possesses a fixed point $(0, 0, 0)$ that is analogues to the fixed point $(s_2, v_2, w_2)$ of the controlled system (3.5).

**Lemma 3.2.** *The asymptotic stability of the zero solution in the linearized system (3.7) is shown under the conditions that the gained matrix satisfies $k_{22} = k_{23} = 0$ and $k_{21} > 0$.*

*Proof.* The verification of this lemma is contingent upon the criteria of Routh-Hurwitz. The Jacobian matrix corresponding to system (3.7) is provided by:

$$J = \begin{pmatrix} -\alpha - k_{21} & \alpha & 0 \\ \gamma & -\gamma & \sqrt{\beta} \\ -\sqrt{\beta} & -\sqrt{\beta} & 0 \end{pmatrix},$$

then the characteristic equation of the Jacobian matrix is:

$$\lambda^3 + (\alpha + \gamma + k_{21})\lambda^2 + (\beta + \gamma k_{21})\lambda + (\beta k_{21} + 2\alpha\beta) = 0, \tag{3.8}$$

where

$$a_1 = \alpha + \gamma + k_{21},$$
$$a_2 = \beta + \gamma k_{21},$$
$$a_3 = \beta k_{21} + 2\alpha\beta.$$

The Routh-Hurwitz criterion has been met, which indicates that the eigenvalues possess negative real components. Consequently, the zero solution of the system (3.7) is deemed asymptotically stable. □

## 4. Function projective synchronization of 3D chaotic dynamical system

In the synchronization process, there is a drive system and a response system, which can be written in the following formula, respectively:

$$\dot{S} = F(S), \tag{4.1}$$

$$\dot{V} = G(V) + U(t, S, V), \tag{4.2}$$

where $S = (s_1, s_2, \ldots, s_n)^T, V = (v_1, v_2, \ldots, v_n)^T \in \mathbb{R}^n$ are the state vectors of the systems (4.1) and (4.2), $F, G : \mathbb{R}^n \to \mathbb{R}^n$ are differentiable vector functions, $U(t, S, V)$ is a controller function which will be designed later.

**Remark 4.1.** *The concept of function projective synchronization (FPS) for the drive system (4.1) and the response system (4.2) involves the existence of a vector function such that*

$$\lim_{t\to+\infty} \|e(t)\| = \lim_{t\to+\infty} \|V - \Lambda(s)S\| = 0, \tag{4.3}$$

*where $e(t)$ is called the error vector, $\Lambda(S) = diag\{h_1(S),\ h_2(S), \ldots, h_n(S)\}$ such that $h_i(S)(i = 1, 2, \ldots, n)$ are continuous differentiable functions and $h_i(S) \neq 0$ for all $t$, $\|.\|$ represents a vector norm induced by the matrix norm.*

This study will involve examining the FPS of a 3D system (2.1) with predetermined parameters and establishing a controller function for the FPS of the drive and response systems. The goal is to devise a controller that allows the response system to mimic the behavior of the drive system and ultimately achieve optimal performance.

The 3D system as a drive system is given by:

$$\begin{cases} \dot{s}_1 = \alpha(v_1 - s_1), \\ \dot{v}_1 = -\gamma v_1 - s_1 w_1, \\ \dot{w}_1 = -\beta + s_1 v_1, \end{cases} \tag{4.4}$$

and the 3D system as a response system can be written as:

$$\begin{cases} \dot{s}_2 = \alpha(v_2 - s_2) + u_1, \\ \dot{v}_2 = -\gamma v_2 - s_2 w_2 + u_2, \\ \dot{w}_2 = -\beta + s_2 v_2 + u_3, \end{cases} \tag{4.5}$$

where $u_1, u_2$ and $u_3$ are the nonlinear controllers. Based on the FPS scheme outlined previously, we can select the scaling function matrix without loss of generality as: $\Lambda(S) = diag\{h_{11}s_1 + h_{12},\ h_{21}v_1 + h_{22},\ h_{31}w_1 + h_{32}\}$ where $h_{ij}(i = 1, 2, 3,\ j = 1, 2)$ are constant numbers.

The error vectors are shown as follows:

$$\begin{cases} e_s = s_2 - (h_{11}s_1 + h_{12})s_1, \\ e_v = v_2 - (h_{21}v_1 + h_{22})v_1, \\ e_w = w_2 - (h_{31}w_1 + h_{32})w_1. \end{cases} \tag{4.6}$$

Then the error dynamical system is given by:

$$\begin{cases} \dot{e}_s = -\alpha e_s + \alpha v_2 - 2h_{11}\alpha s_1 v_1 + \alpha h_{11} s_1^2 - \alpha h_{12} v_1 + u_1, \\ \dot{e}_v = -\gamma e_v - s_2 w_2 + h_{21}\gamma v_1^2 + 2h_{21} s_1 v_1 w_1 + h_{22} s_1 w_1 + u_2, \\ \dot{e}_w = -\beta + s_2 v_2 + 2\beta h_{31} w_1 - 2h_{31} s_1 v_1 w_1 + h_{32}\beta - s_1 v_1 h_{32} + u_3. \end{cases} \tag{4.7}$$

Therefore, we can choose the controller functions as:

$$\begin{cases} u_1 = -\alpha v_2 + 2h_{11}\alpha s_1 v_1 - \alpha h_{11} s_1^2 + \alpha h_{12} v_1, \\ u_2 = s_2 w_2 - h_{21}\gamma v_1^2 - 2h_{21} s_1 v_1 w_1 - h_{22} s_1 w_1, \\ u_3 = \beta - s_2 v_2 - 2\beta h_{31} w_1 + 2h_{31} s_1 v_1 w_1 - h_{32}\beta + s_1 v_1 h_{32} - 7w_2 + 7h_{31} w_1^2 + 7h_{32} w_1. \end{cases} \tag{4.8}$$

Hence, the error dynamical system is:

$$\begin{cases} \dot{e}_s = -\alpha e_s, \\ \dot{e}_v = -\gamma e_v, \\ \dot{e}_w = -7e_w. \end{cases} \tag{4.9}$$

The presence of three negative eigenvalues $(-\alpha, -\gamma,\ and\ -7)$ in the closed loop system (4.9) signifies that the error states $e_x, e_y$, and $e_z$ tend towards zero as time t approaches infinity. Consequently, the identical 3D chaotic systems achieved FPS.

## 5. Numerical simulations

5.1. **Controlling for 3D chaotic dynamical system.** This section includes numerical simulation outcomes aimed at validating previous analytical results where $\alpha = 10, \beta = 100$, and $\gamma = 10.3$. FIGURE 2. demonstrates the trajectory of the controlled system (3.1) tends to the unstable equilibrium point $P_1 = (s_1, v_1, w_1) = (\sqrt{\beta}, \sqrt{\beta}, -\gamma)$ of the uncontrolled 3D chaotic system (2.1). FIGURE 3. demonstrates the trajectory of the controlled system (3.5) tends to the unstable equilibrium point $P_2 = (s_2, v_2, w_2) = (-\sqrt{\beta}, -\sqrt{\beta}, -\gamma)$ of the uncontrolled 3D chaotic system (2.1).

5.2. **Synchronization for 3D chaotic dynamical system.** This section includes numerical simulation outcomes aimed at validating previous analytical results where $\alpha = 10, \beta = 100$, and $\gamma = 10.3$.

In all simulations, the initial values of the drive and the response systems are selected as: $(s_1(0), v_1(0), w_1(0))^T = (5, -5, 3)$ and $(s_2(0), v_2(0), w_2(0))^T = (18, -1, 14)$. By choosing the scaling function factor as $h_1 = 3s + 3$, $h_2 = 3v + 3$, $h_3 = 3w + 3$, FIGURE 4. displays the function projective synchronization (FPS) for two identical 3D chaotic systems. When we take the scaling factor as: $h_1 = 2$, $h_2 = -3$, $h_3 = 1$, FIGURE 5. displays the modified projective synchronization (MPS) for two identical 3D chaotic systems. Furthermore, we simplify the scaling factor as $h_1 = h_2 = h_3 = 2.5$, FIGURE 6. shows the projective synchronization (PS) for two identical 3D chaotic systems. Moreover, in FIGURE 7. the anti-synchronization for two 3D chaotic systems appears when the scaling factors are $h_i = 1 (i = 1, 2, 3)$. Finally, the complete synchronization for two 3D chaotic systems is shown in FIGURE 8. when the scaling factors are taken as $h_i = 1 (i = 1, 2, 3)$.

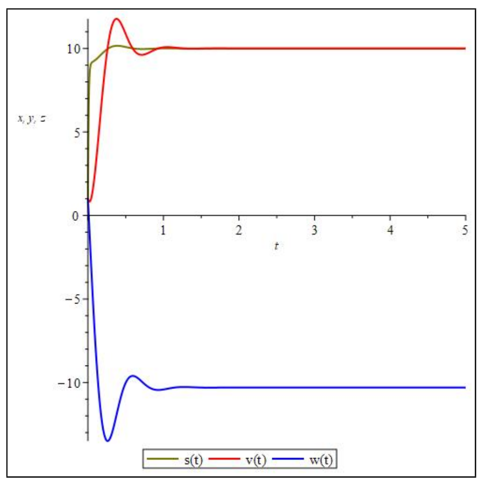


FIGURE 2. The trajectories of system (3.1) tends to $P_1$.

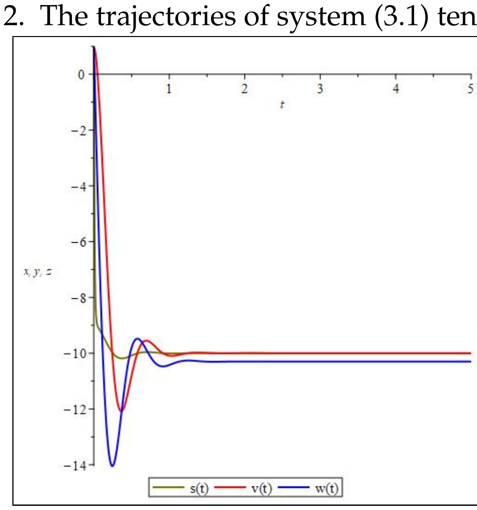


FIGURE 3. The trajectories of system (3.5) tends to $P_2$.

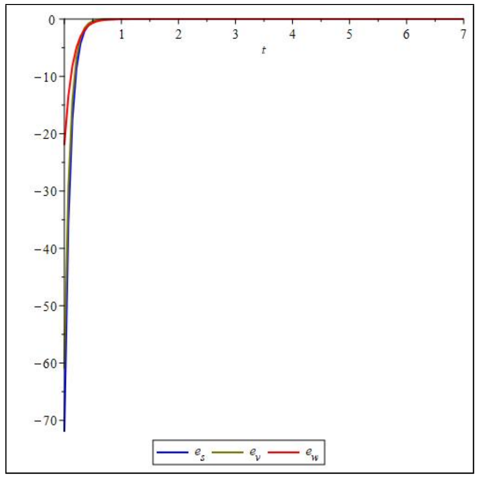


FIGURE 4. The error vectors $e_s, e_v$ and $e_w$ head to zero for achieving FPS.

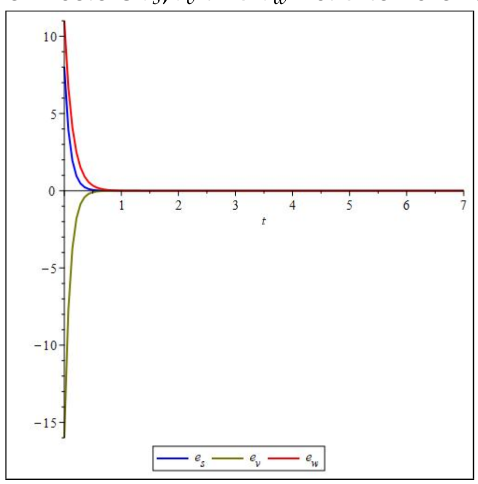


FIGURE 5. The error vectors $e_s, e_v$ and $e_w$ head to zero for achieving MPS.

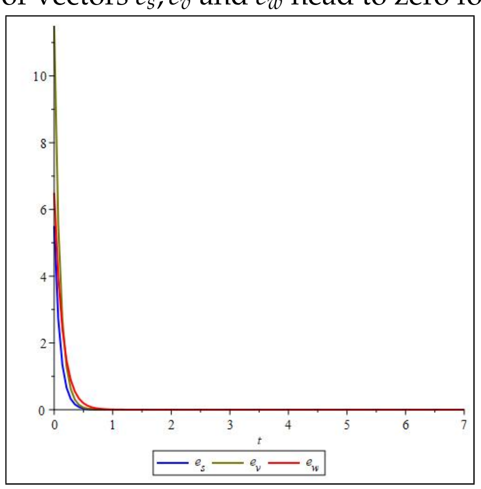


FIGURE 6. The error vectors $e_s, e_v$ and $e_w$ head to zero for achieving PS.

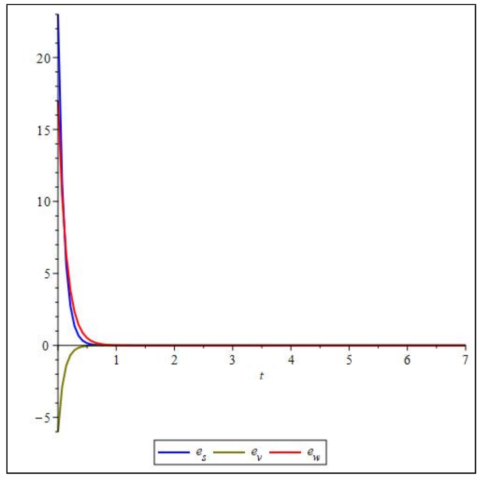

FIGURE 7. The error vectors $e_s, e_v$ and $e_w$ head to zero for achieving anti synchronization.

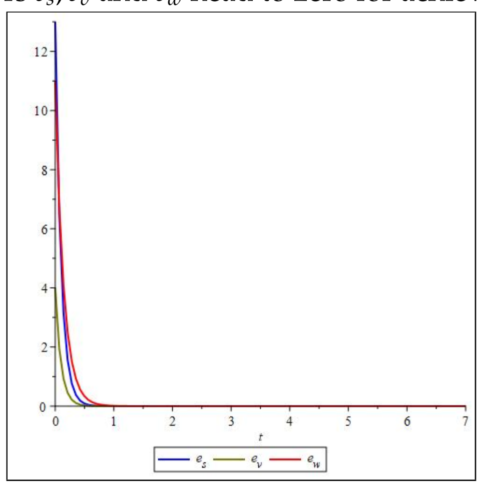

FIGURE 8. The error vectors $e_s, e_v$ and $e_w$ head to zero for achieving complete synchronization.

## 6. CONCLUSION

In this paper, the 3D chaotic dynamical system and its fundamental properties are presented, along with the linear feedback control method to suppress the chaos of this system. The controlling conditions were determined using the Routh-Hurwitz criterion, and the function projective synchronization has been utilized to synchronize two identical 3D chaotic systems with known parameters. Through the application of Lyapunov stability theory, the sufficient condition for function projective synchronization is derived. Furthermore, numerical simulations are performed to demonstrate the effectiveness of the obtained results.

**Conflicts of Interest:** The authors declare that there are no conflicts of interest regarding the publication of this paper.

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