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Modelling Extreme Rainfall using Extended Generalized Extreme Value Distribution

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Abstract. This study assesses the performance of extended generalized extreme value (GEV) distribution based on Kumaraswamy generalized extreme value (KumGEV) distribution using the maximum likelihood estimates on extreme rainfall data obtained from a weather station in Phitsanulok province, a total of 408 months during January 1987 to December 2021. The findings indicate that the KumGEV distribution provides a better fit than the traditional GEV distribution, with estimated parameters $\mu = 41.4966$ ($SE = 0.6015$), $\sigma = 8.9467$ ($SE = 0.0797$), $\xi = -0.0502$ ($SE = 0.0308$), $a = 0.0310$ ($SE = 0.0060$), and $b = 0.2738$ ($SE = 0.0155$). Additionally, the analysis of return levels derived from both GEV and KumGEV distributions shows an upward trend over return periods of 10, 20, 50, and 100 years, highlighting significant changes in rainfall patterns over time.

1. Introduction

The natural disasters due to extreme rainfall have been nowadays occurring more frequently with greater strength, causing increasingly prominent and destructive impact on landscape. These severe weather occurrences inflict substantial damage on agriculture and ecosystems, wreak havoc on infrastructure, and disrupt daily human activities, often resulting in injuries and loss of lives. The assessment of climate risks is increasingly concerned with understanding the variability and intensity of such extreme events, as well as emphasizing their critical impact on our environment and society. One of the fundamental approaches to avoid human and material damage is to acquire a thorough understanding of climate change mechanisms. This involves conducting an in-depth

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study of the spatial and temporal distribution of climatic variables such as rainfall and temperature. Such knowledge enables forecasting disaster occurrences and promptly implementing appropriate measures to mitigate negative consequences and reduce the impact of unforeseen events [4, 9, 12–19].

Extreme value analysis (EVA) is a statistical method focused on predicting rare and high impact events of specific data sets or assessing the risk of such events, including excessive rainfall, floods, extreme weather conditions, or severe storms. This technique is widely utilized across various fields, such as engineering, communications, finance, risk management, insurance, economics, material sciences, hydrology, and meteorology. EVA employs historicaldata to estimate the parameters of extreme value (EV) distributions, often called limiting distributions for the largest or smallest of independent, identically distributed variables. Numerous studies have demonstrated its practical application, highlighting its effectiveness in real-world data analysis [1, 2, 7, 10, 11].

Two primary parametric methods are commonly used to model the extremes of a random process: the block-maxima approach and the threshold exceedance approach. The block-maxima method divides observations into continuous, non-overlapping segments with equally sized blocks and identifies the highest value in each block. The maxima from these blocks are modeled using a Generalized Extreme Value (GEV) distribution. In contrast, the threshold exceedance method involves selecting observations that exceed a pre-defined threshold, focusing on data points that are either smaller or larger than the threshold. Parameter estimation is not just a technical procedure in analyzing probability distributions but a crucial aspect that can significantly impact the results and conclusions of our analyses. This is evident from the extensive literature on the subject, which underscores the importance of accurate parameter estimation. The GEV distribution, a combination of the Gumbel, Fréchet, and Weibull distributions (types I, II, and III extreme value distributions), is a basis model due to its flexible, continuous shape parameters. This research emphasizes on the GEV distribution as a baseline to explore new classes of distributions that might provide a more accurate representation of extreme event data. The development of GEV distribution often requires more flexibility to model extreme values across various domains. In 2017, Eljabri et al. [8] introduced an enhanced version called the Kumaraswamy GEV (KumGEV) distribution. They explored its mathematical properties in more detail, the maximum likelihood method for parameter estimation and the observed information matrix. The benefit of this new model is demonstrated through its application to real-world data.

This study aims to compare the GEV distribution with the KumGEV distribution. The KumGEV distribution's return level was also calculated using the Delta method to account for variability in dataset. Next, we apply both GEV and KumGEV distributions to fit the rainfall data, obtained from the weather station 378201-Phitsanulok in Phitsanulok province, Thailand, from 1987 to 2021, followed by a goodness-of-fit test. Finally, a conclusion is drawn in the last section.

2. Extreme Value Theory

In this section, we will present the generalized extreme value (GEV) and the Kumaraswamy Generalized extreme value (KumGEV) distribution, which includes the cumulative distribution function, probability density function, quantile function, and return level.

A random variable X follows the GEV distribution, $G(x;\mu,\sigma,\xi)$ if its cumulative distribution function (CDF) is given by

$$G(x;\mu,\sigma,\xi) = \begin{cases} \exp\{-[1+\xi(x-\mu)/\sigma]^{-1/\xi}\}, & \xi \neq 0, \\ \exp\{-\exp[-(x-\mu)/\sigma]\}, & \xi \to 0, \end{cases} \tag{2.1}$$

which is defined in the set $\{x : 1+\xi(x-\mu)/\sigma > 0\}$, where $\mu \in \mathbb{R}$ is a location parameter, $\sigma > 0$ is a scale parameter and $\xi \in \mathbb{R}$ is a shape parameter. The condition for a distribution possessing any of the extreme value distributions is given as follows: $\xi = 0$ for Gumbel distribution, $\xi > 0$ for Fréchet distribution and $\xi < 0$ for Weibull distribution. The corresponding probability density function (PDF) of GEV distribution, $g(x;\mu,\sigma,\xi)$, is then obtained as

$$g(x;\mu,\sigma,\xi) = \begin{cases} \sigma^{-1}[1+\xi(x-\mu)/\sigma]^{-(1/\xi)-1}\exp\{-[1+\xi(x-\mu)/\sigma]^{-1/\xi}\}, & \xi \neq 0, \\ \sigma^{-1}\exp[-(x-\mu)/\sigma]\exp\{-\exp[-(x-\mu)/\sigma]\}, & \xi \to 0. \end{cases} \tag{2.2}$$

Plots of the GEV distribution probability density function for specific values of parameters are displayed in Figure 1.

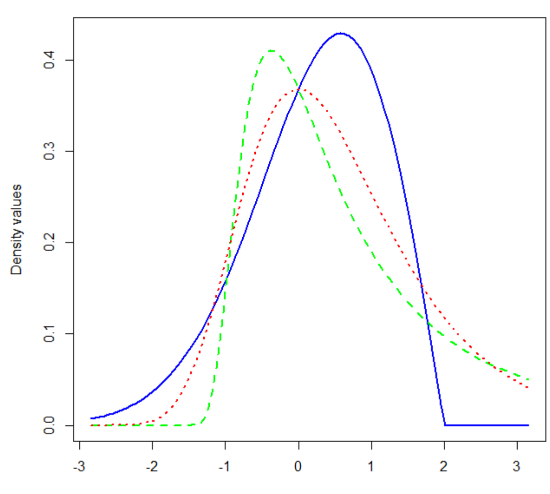

Figure 1. Generalized extreme value distributions for Fréchet: $\xi = 1/2$ (blue), Gumbel: $\xi = 0$ (red) and Weibull: $\xi = -1/2$ (green).

The estimates of extreme quantiles z_u^G of the GEV distribution, known as return level, are then obtained by inverting $G(x;\mu,\sigma,\xi)$,

$$z_u^G = \begin{cases} \mu + \dfrac{\sigma}{\xi}\Big\{[-\log(u)^{-\xi} - 1]\Big\}, & \xi \neq 0, \\ \mu - \sigma \log[-\log(u)], & \xi \to 0, \end{cases} \tag{2.3}$$

where $u \in [0,1]$, $u = 1 - T^{-1}$, and T is the number of recurrent times.

2.1. **The Kumaraswamy Generalized Extreme Value Distribution.** In the framework of the Kumaraswamy generalized extreme value (KumGEV) distribution presented by Eljabri et al. [8] in 2017, the integration of the GEV distribution in (2.1) with the concepts developed by Cordeiro [5] leads to the definition of the Kumaraswamy-G (Kw-G) family. Specifically, the Kw-G Cumulative Distribution Function (CDF) family is characterized as follows. If $G(x)$ represents the CDF of a random variable X, then the CDF of the Kw-G family is derived from this baseline function.

$$H_{a,b}(x) = 1 - [1 - G^a(x)]^b, \tag{2.4}$$

where $a > 0$ and $b > 0$ are two additional shape parameters to the exponentiated G distribution, which has skewed characteristics with a heavy tail. The probability density function (PDF) corresponding to (2.4) is

$$h_{a,b}(x) = abg(x)G^{a-1}(x)[1 - G^a(x)]^{b-1}, \tag{2.5}$$

where $g(x) = dG(x)/dx$.

The cumulative distribution function and probability density function of the KumGEV distribution are then given by

$$F(x) = 1 - \{1 - \exp(-ax)^b\}, \tag{2.6}$$

and

$$f(x) = \sigma^{-1} abx^{1+\xi} \exp(-ax)\{1 - \exp(-ax)^{b-1}\}, \tag{2.7}$$

respectively, where a and b are positive shape parameters. Equation (2.6) and (2.7) can be reduced to some sub-models for specific values of a and b. Plots of the KumGEV probability density function for specific values of parameters are displayed in Figure 2.

Special cases of the KumGEV: Let X follow the KumGEV distribution with parameters (μ,σ,ξ,a,b). The baseline GEV distribution is obtained as a special case when $a = 1$ and $b = 1$.

If X is a random variable with probability density function (2.7),
we write $X \sim \text{KumGEV}(\mu,\sigma,\xi,a,b)$. The KumGEV quantile function is obtained by inverting (2.6)

$$x = Q(x) = F^{-1}(z) = \mu + \frac{\sigma}{\xi}\left\{\left[-\frac{1}{a}\log\{1 - (1-z)^{1/b}\}\right]^{-\xi}\right\}. \tag{2.8}$$

Thus, one can generate KumGEV variates from (2.8) by $X = Q(Z)$, where Z is a uniform variate on the unit interval (0, 1).

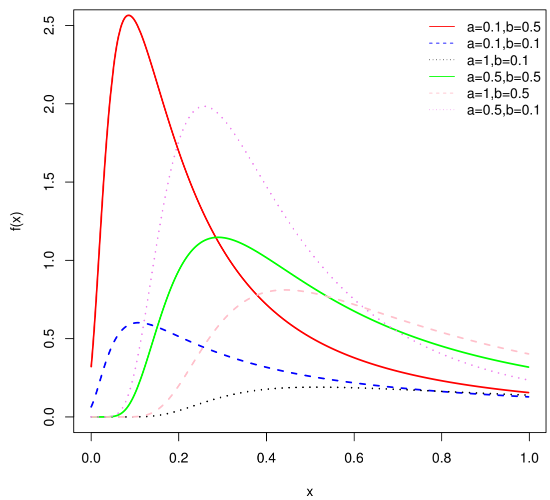

FIGURE 2. Plots of the KumGEV probability density function as $\mu = 0.5$, $\sigma = 0.3$, and $\xi = 0.5$.

2.2. **The return level of KumGEV distribution.** Next, we propose the KumGEV concept to include the return level R_T analysis, which predicts the data return period, as described below.

A return level, also known as a recurrence interval, is a statistical measure used to estimate the likelihood of an event exceeding a specific value, such as maximum level of rainfall or river discharge, to recur over a long period, This measure makes use of historical data and is often derived in risk analysis. More specifically, the return period is the inverse of the probability of exceeding a certain level in any given year which reflects the expected waiting time or the average number of years between occurrences of such an event.

Assuming that the event components are independently distributed, the probability that an exceeding event will occur for the first time t in years is $p(1-p)^{t-1}$, $t = 1, 2, \ldots$, which is the geometric probability mass function whose mean is equal to $T = 1/p$, when the yearly exceedance probability $p = P(X \geq R_T)$ is assumed to remain constant throughout the future years of interest. The survival probability can estimate the probability of exceeding R_T, $1 - F(R_T)$, the return period then being equal to $1/P(X \geq R_T)$. Thus, for a given return period T, the corresponding return level can be obtained as follows:

$$R_T = F^{-1}(1 - 1/T), \tag{2.9}$$

which yields

$$R_T = \mu + \frac{\sigma}{\xi}\left\{\left[-\frac{1}{a}\log\{1-(1-t)^{1/b}\}\right]^{-\xi}\right\}, \tag{2.10}$$

where $t = 1 - T^{-1}$.

The confidence interval of the return level for KumGEV distribution is performed using the Delta method as $Var(R_T) \approx \nabla R_T^t V \nabla R_T$, where V is a covariance matrix of $(\mu, \sigma, \xi, a, b)^t$ and

$$\nabla R_T^t = [\frac{\partial R_T}{\partial \mu}, \frac{\partial R_T}{\partial \sigma}, \frac{\partial R_T}{\partial \xi}, \frac{\partial R_T}{\partial a}, \frac{\partial R_T}{\partial b}],$$

where

$$\begin{aligned}
\frac{\partial R_T}{\partial \mu} =& 1, \\
\frac{\partial R_T}{\partial \sigma} =& \frac{1}{\xi}((-a^{-1}\log(1-[1-t]^{\frac{1}{b}}))^{-\xi} - 1), \\
\frac{\partial R_T}{\partial \xi} =& -\sigma\xi^{-1}(-a^{-1}\log(1-[1-t]^{\frac{1}{b}}))^{-\xi}\log(-a^{-1}\log(1-[1-t]^{\frac{1}{b}})) \\
& -\sigma\xi^{-2}(-a^{-1}\log(1-[1-t]^{\frac{1}{b}}))^{-\xi} + \sigma\xi^{-2}, \\
\frac{\partial R_T}{\partial a} =& -\sigma a^{-1}(-a\log(1-[1-t]^{\frac{1}{b}}))^{-\xi}, \\
\frac{\partial R_T}{\partial b} =& \sigma a b^{-2}(-a\log(1-[1-t]^{\frac{1}{b}}))^{-\xi-1}[1-t]^{\frac{1}{b}}(1-[1-t]^{\frac{1}{b}})^{-1}\log[1-t].
\end{aligned}$$

3. Data and Research Methodology

This study applies the GEV distributions and KumGEV distributions to the highest monthly rainfall data in Phitsanulok province, Thailand, from 1987 to 2021. Data are collected by weather station 378201-Phitsanulok with the courtesy of the Thailand Meteorological Department [6], covering an area of 10,815.8 square kilometers (6,759,909 rai) or 6.4% of the northern area accounting for 2.1% of the country's total area. The rainy season starts around May and ends around October, with an average rainfall of 1,375 millimeters per year. The temperature varies in the range of 19°C to 37°C and very rarely below 15°C or above 39°C. A total of 368 observations, the maximum rainfall is 167.10 *mm* with a mean of 38.81 *mm* and a standard deviation of 29.30 *mm*. For parameter estimation in GEV and KumGEV distributions, we utilize the maximum likelihood method accompanying the Newton-Raphson procedure.

In testing the unit root test of data using the Augmented Dickey-Fuller (ADF) test, it is a test to consider the extreme values under a stationary or nonstationary extreme because if the data is not constant, it will cause problems in spurious regression to avoid data with unstable means and variances at different times using the unit root test by ADF test. The results of the data stability test are shown in Table 1.

Table 1 shows the statistical description of the data. The p-value of the Augmented Dickey-Fuller (ADF) test is 0.01, which is the stationarity of rainfall data.

3.1. **Goodness-of-fit tests.** Next, we use formal goodness-of-fit tests to determine which distribution fits the data best. We will employ the Cramér-von Mises (W^*) and Anderson-Darling (A^*)

TABLE 1. Summary statistics of the monthly highest rainfall data in Phitsanulok province from 1987 - 2021

N	Maximum	Mean	Standard deviation	Coefficient of skewness	ADF test
368	167.1	38.8133	29.3043	0.8584	$-13.828(0.01)$

statistics, as described by Chen and Balakrishnan ([3]). We considered the W^* and A^* statistics to test the validity of the null hypothesis H_0 against the alternative hypothesis H_1 as stated below:

H_0: the data follow the specified distribution,
H_1: the data do not follow the specified distribution.

In general, smaller values of these statistics indicate a better fit of the distribution to the data. Let $F(x;\theta)$ be a CDF, where the form of F is known, but θ is unknown. To obtain the statistics W^* and A^*, we can proceed as follows:

(1) Calculation of transformed values:
For each data point x_i (sorted in ascending order), calculate $v_i = F(x_i;\hat{\theta})$.
(2) Standard normal transformation:
Transform v_i to y_i using the inverse of the standard normal CDF, $\Phi^{-1}(v_i)$.
(3) Normalization:
Calculate the normalized values u_i using:

$$u_i = \Phi\left(\frac{y_i - \bar{y}}{s_y}\right)$$

where $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$ and $s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$.
(4) Calculation of W^2 and A^2 statistics:
Compute W^2:

$$W^2 = \sum_{i=1}^{n}\left(u_i - \frac{2i-1}{2n}\right)^2 + \frac{1}{12n}$$

Compute A^2:

$$A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}\left[(2i-1)\log(u_i) + (2n+1-2i)\log(1-u_i)\right]$$

(5) Adjustments for final statistics:
Adjust W^2 and A^2 to obtain W^* and A^*:

$$W^* = W^2\left(1 + \frac{0.5}{n}\right)$$

$$A^* = A^2\left(1 + \frac{0.75}{n} + \frac{2.25}{n^2}\right)$$

The fit of the KumGEV distribution is compared with several other competitive models namely Basic GEV, Weibull, Exponentiated Weibull, Gamma, and Inverse Gamma distribution with the following CDF and PDFs.

Weibull distribution (Weibull)

$$F(x) = 1 - \exp -(x/\sigma)^c, \quad x > 0, c, \sigma > 0 \tag{3.1}$$

and

$$f(x) = \frac{c}{\sigma}\left(\frac{x}{\sigma}\right)^{c-1} \exp -(x/\sigma)^c, \quad x > 0, c, \sigma > 0 \tag{3.2}$$

Exponentiated Weibull distribution (EW)

$$F(x) = [1 - \exp -(x/\sigma)^c]^k, \quad x > 0, c, k, \sigma > 0 \tag{3.3}$$

and

$$f(x) = ck\sigma^c x^{c-1} \exp -(\sigma x)^c [1 - \exp -(x/\sigma)^c]^{k-1}, \quad x > 0, c, k, \sigma > 0 \tag{3.4}$$

Gamma distribution (Gamma)

$$F(x) = \frac{1}{\Gamma(c)\sigma^c}\gamma\left(c, \frac{x}{\sigma}\right), \quad x > 0, c, \sigma > 0 \tag{3.5}$$

and

$$f(x) = \frac{1}{\Gamma(c)\sigma^c} x^{c-1} \exp -(x/\sigma), \quad x > 0, c, \sigma > 0 \tag{3.6}$$

Inverse Gamma distribution (IG)

$$F(x) = \frac{\Gamma\left(c, \frac{x}{\sigma}\right)}{\Gamma(c)}, \quad x > 0, c, \sigma > 0 \tag{3.7}$$

and

$$f(x) = \frac{1}{\Gamma(c)\sigma^c} x^{-c-1} \exp -(\sigma/x), \quad x > 0, c, \sigma > 0 \tag{3.8}$$

The values of W^* and A^* for GEV, KumGEV, Weibull, Exponentiated Weibull, Gamma, and Inverse Gamma distribution are given in Table 2 for goodness-of-fit tests on rainfall data. Based on the values of W^* and A^*, the KumGEV distribution shows the p-values of these statistics, providing the best fit compared to the other distributions.

3.2. **Parameter and return level estimation.** The GEV and KumGEV distribution results in Table 2 show the Goodness-of-fit tests, which best fit the extreme rainfall data in Phitsanulok province, are based on the Block Maxima approach. Therefore, we illustrate the results of parameter estimates in the GEV and KumGEV distributions in Table 3.

In Table 3, The GEV parameters are estimated using maximum likelihood estimation. The estimated results are (μ, σ, ξ) = (24.9578, 22.5914, 0.0324), with standard errors (1.3935, 1.0627, 0.0515). The approximate 95% confidence intervals (CI) for the parameters are thus (22.22, 27.69)

Table 2. Goodness-of-fit tests with Cramér-von Mises and Anderson- Darling statistics

Distribution	W^*	p-value	A^*	p-value
GEV	0.2561	0.1808	2.1826	0.0731
KumGEV	0.2229	0.2271	1.9123	0.1027
Weibull	0.9613	0.0030	6.8196	0.0004
EW	0.0011	0.0000	2.0859	0.0000
Gamma	0.0040	0.0000	8.2211	0.0000
IG	0.0349	0.0000	61.9210	0.0000

Table 3. Maximum likelihood estimates, standard errors and 95% CI of all parameters in GEV and KumGEV distribution

Distribution	Parameter	Estimate	Standard error	95% CI
GEV	μ	24.9578	1.3935	(22.22, 27.69)
	σ	22.5914	1.0627	(20.51, 24.67)
	ξ	0.0324	0.0515	$(-0.07, 0.13)$
KumGEV	μ	41.4966	0.6015	(40.32, 42.68)
	σ	8.9467	0.0797	(8.79, 9.10)
	ξ	-0.0502	0.0308	$(-0.11, 0.01)$
	a	0.0310	0.0060	(0.02, 0.04)
	b	0.2738	0.0155	(0.24, 0.30)

for μ, (20.51, 24.67) for σ, and $(-0.07, 0.13)$ for ξ. Since the confidence intervals of ξ contain 0, the Gumbel distribution is the optimal model for the GEV class.

Similarly, the KumGEV parameters are estimated using the maximum likelihood method. The estimated results, as shown in Table 3, are (μ, σ, ξ, a, b) = (41.4966, 8.9467,−0.0502, 0.0310, 0.2738), with standard errors (0.6015, 0.0797, 0.0308, 0.0060, 0.0155). The approximate 95% CI for the parameters are thus (40.32, 42.68) for μ, (8.79, 9.10) for σ, $(-0.11, 0.01)$ for ξ, (0.02, 0.04) for a, and (0.24, 0.30) for b.

Table 4 shows the return level estimates at different return periods (T) of 10, 20, 50, and 100 years, based on two different distributions: GEV and KumGEV. As we can see, the estimates of return levels of the GEV distribution show an increasing trend in the estimated return levels with increasing return periods. For example, the estimated return levels for 10 years and 100 years are 77.6981 *mm* and 137.0483 *mm*, respectively. Consequently, the lower and upper limits for CIs also increase with the return period.

Similarly, the KumGEV distribution shows an increasing trend in the estimated return levels with increasing return periods. For example, the estimated return levels for 10 years and 100 years are 80.6145 *mm* and 128.4912 *mm*, respectively. Consequently, the lower and upper limits for CIs also increase with the return period.

Table 4. Return level estimates (*mm*) at selected return periods (T) for the GEV and KumGEV distributions

Distribution	T	Return Level	95% CI
GEV	10	77.6981	(71.29,84.11)
	20	95.4000	(85.54,105.26)
	50	118.937	(102.24,135.64)
	100	137.0483	(113.46,160.64)
KumGEV	10	80.6145	(69.79, 81.76)
	20	97.2075	(85.71, 99.30)
	50	116.1407	(106.74, 121.61)
	100	128.4912	(122.85, 138.44)

As we can see, the two distributions indicate similar patterns of increased return level estimates as return periods rise. However, the estimated values and corresponding CIs obtained from the two distributions are quite different, more precise CIs with KumGEV, which could be attributed to the different characteristics of the distributions and the specific data used in the analysis. Therefore, it is essential to carefully select the distribution appropriate for a particular data set to obtain reliable results for practical use.

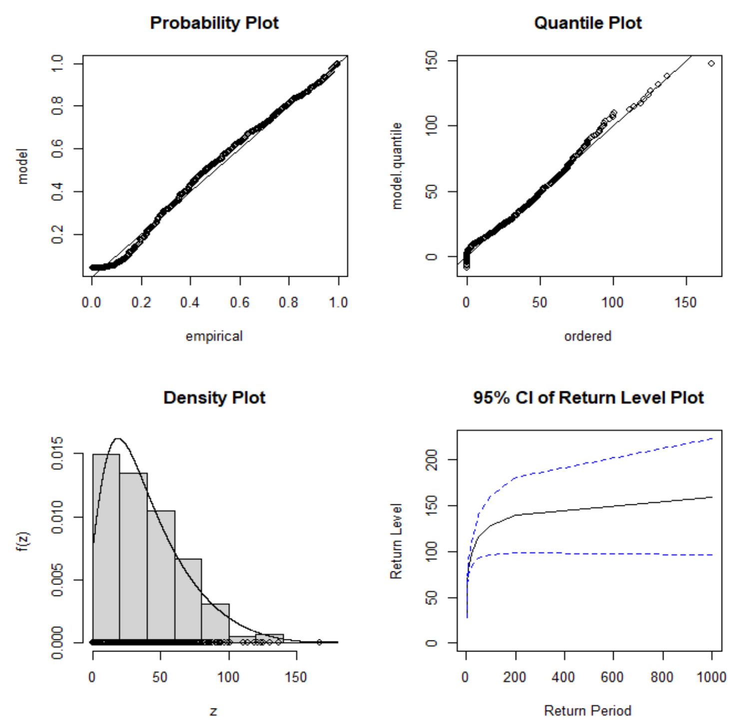

Figure 3. Diagnostic plots for KumGEV distribution, applying to rainfall data in Phitsanulok province, Thailand.

Figure 3 displays the diagnostic plots for the KumGEV distributions, respectively. Most probability and quantile plot points lie close to the unit diagonal. This implies that the KumGEV distribution functions provide good fits as consistent with the Cramér-von Mises (W^*) and Anderson-Darling (A^*) statistics.

4. Conclusion

For research methodology, we describe the maximum likelihood estimation of all parameters and return levels of GEV and KumGEV distribution and their confidence intervals. Due to the complexity of likelihood functions, the Newton-Raphson procedure is then implemented to find the estimated values of all parameters. In addition, the KumGEV distributions are applied to rainfall data and yield more favorable results than the GEV distribution based on the goodness-of-fit tests: Cramér-von Mises and Anderson-Darling statistics. We hope this generalization will attract applications in statistics, mathematics, biology, the environment, engineering, and other areas.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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