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Upper and Lower Weakly Quasi (τ_1, τ_2)-Continuous Multifunctions

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Abstract. This paper presents new classes of multifunctions called upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations and some properties concerning upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions are considered.

1. Introduction

In 1961, Marcus [41] introduced the concept of quasi continuous functions. Popa [48] introduced and investigated the notion of almost quasi continuous functions. Neubrunnovaá [42] showed that quasi continuity is equivalent to semi-continuity due to Levine [39]. Popa and Stan [49] introduced and studied the notion of weakly quasi continuous functions. Weak quasi continuity is implied by quasi continuity and weak continuity [40] which are independent of each other. It is shown in [45] that weak quasi continuity is equivalent to weak semi-continuity due to Arya and Bhamini [1] and Kar and Bhattacharyya [32]. In [18], the present authors studied some properties of (Λ, sp)-open sets and (Λ, sp)-closed sets. Viriyapong and Boonpok [61] investigated several characterizations of (Λ, sp)-continuous functions by utilizing the notions of (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [31] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [30] introduced and studied the notion of almost $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous

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functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise almost M-continuous functions, (τ_1, τ_2)-continuous functions, almost (τ_1, τ_2)-continuous functions, almost nearly (τ_1, τ_2)-continuous functions and weakly (τ_1, τ_2)-continuous functions were presented in [55], [56], [3], [50], [9], [8], [15], [22], [27], [28], [4], [5], [36] and [7], respectively. Srisarakham et al. [54] introduced and investigated the notion of quasi $\theta(\tau_1, \tau_2)$-continuous functions. Kong-ied et al. [37] introduced and studied the concept of almost quasi (τ_1, τ_2)-continuous functions. Chiangpradit et al. [29] introduced and investigated the notion of weakly quasi (τ_1, τ_2)-continuous functions.

In 1975, Bânzara and Crivăţ [2] introduced and studied the concept of quasi continuous multifunctions. Popa and Noiri [46] introduced the concept of almost quasi continuous multifunctions and investigated some characterizations of such multifunctions. The notion of weakly quasi continuous multifunctions was introduced and investigated by the present authors [44]. Several characterizations of weakly quasi continuous multifunctions have been obtained in [46]. Popa and Noiri [47] introduced and studied the concepts of upper and lower θ-quasi continuous multifunctions. Noiri and Popa [43] investigated some characterizations of upper and lower θ-quasi continuous multifunctions. Laprom et al. [38] introduced and investigated the concept of $\beta(\tau_1, \tau_2)$-continuous multifunctions. Moreover, some characterizations of $(\tau_1, \tau_2)\alpha$-continuous multifunctions, $(\tau_1, \tau_2)\delta$-semicontinuous multifunctions, almost weakly (τ_1, τ_2)-continuous multifunctions, $\star$-continuous multifunctions, $\beta(\star)$-continuous multifunctions, weakly quasi (Λ, sp)-continuous multifunctions, α-$\star$-continuous multifunctions, almost α-$\star$-continuous multifunctions, almost quasi $\star$-continuous multifunctions, weakly α-$\star$-continuous multifunctions, $s\beta(\star)$-continuous multifunctions, weakly $s\beta(\star)$-continuous multifunctions, $\theta(\star)$-quasi continuous multifunctions, almost $\iota^\star$-continuous multifunctions, weakly (Λ, sp)-continuous multifunctions, $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\alpha(\Lambda, sp)$-continuous multifunctions, almost $\beta(\Lambda, sp)$-continuous multifunctions, (τ_1, τ_2)-continuous multifunctions, almost (τ_1, τ_2)-continuous multifunctions, weakly (τ_1, τ_2)-continuous multifunctions, weakly quasi (τ_1, τ_2)-continuous multifunctions and s-$(\tau_1, \tau_2)p$-continuous multifunctions were established in [62], [23], [20], [25], [19], [60], [6], [14], [24], [13], [11], [12], [17], [21], [10], [34], [16], [58], [53], [35], [57], [51] and [59], respectively. Khampakdee et al. [33] introduced and investigated the concept of c-(τ_1, τ_2)-continuous multifunctions. Pue-on et al. [52] introduced and studied the notion of almost quasi (τ_1, τ_2)-continuous multifunctions. In this paper, we introduce the concepts of upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions. We also investigate some characterizations of upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [26] if $A = \tau_1\text{-Cl}(\tau_2\text{-Cl}(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [26] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [26] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 2.1. [26] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) *$A \subseteq \tau_1\tau_2\text{-}Cl(A)$ and $\tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Cl(A)) = \tau_1\tau_2\text{-}Cl(A)$.*
(2) *If $A \subseteq B$, then $\tau_1\tau_2\text{-}Cl(A) \subseteq \tau_1\tau_2\text{-}Cl(B)$.*
(3) *$\tau_1\tau_2\text{-}Cl(A)$ is $\tau_1\tau_2$-closed.*
(4) *A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2\text{-}Cl(A)$.*
(5) *$\tau_1\tau_2\text{-}Cl(X - A) = X - \tau_1\tau_2\text{-}Int(A)$.*

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [62] (resp. $(\tau_1, \tau_2)s$-*open* [23], $(\tau_1, \tau_2)p$-*open* [23], $(\tau_1, \tau_2)\beta$-*open* [23]) if $A = \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$ (resp. $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(A))$, $A \subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A))$, $A \subseteq \tau_1\tau_2\text{-Cl}(\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed*, $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)s$-closed sets of X containing A is called the $(\tau_1, \tau_2)s$-*closure* [23] of A and is denoted by (τ_1, τ_2)-sCl(A). The union of all $(\tau_1, \tau_2)s$-open sets of X contained in A is called the $(\tau_1, \tau_2)s$-*interior* [23] of A and is denoted by (τ_1, τ_2)-sInt(A).

Lemma 2.2. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *$(\tau_1, \tau_2)\text{-}sCl(A) = \tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(A)) \cup A$* [20];
(2) *$(\tau_1, \tau_2)\text{-}sInt(A) = \tau_1\tau_2\text{-}Cl(\tau_1\tau_2\text{-}Int(A)) \cap A$.*

Let A be a subset of a bitopological space (X, τ_1, τ_2). A point $x \in X$ is called a $(\tau_1, \tau_2)\theta$-*cluster point* [62] of A if $\tau_1\tau_2\text{-Cl}(U) \cap A \neq \emptyset$ for every $\tau_1\tau_2$-open set U containing x. The set of all $(\tau_1, \tau_2)\theta$-cluster points of A is called the $(\tau_1, \tau_2)\theta$-*closure* [62] of A and is denoted by $(\tau_1, \tau_2)\theta$-Cl(A). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)\theta$-*closed* [62] if $(\tau_1, \tau_2)\theta\text{-Cl}(A) = A$. The complement of a $(\tau_1, \tau_2)\theta$-closed set is said to be $(\tau_1, \tau_2)\theta$-*open*. The union of all $(\tau_1, \tau_2)\theta$-open sets of X contained in A is called the $(\tau_1, \tau_2)\theta$-*interior* [62] of A and is denoted by $(\tau_1, \tau_2)\theta$-Int(A).

Lemma 2.3. [62] *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *If A is $\tau_1\tau_2$-open in X, then $\tau_1\tau_2\text{-}Cl(A) = (\tau_1, \tau_2)\theta\text{-}Cl(A)$.*

(2) $(\tau_1, \tau_2)\theta$-$Cl(A)$ *is* $\tau_1\tau_2$*-closed in* X.

By a multifunction $F : X \to Y$, we mean a point-to-set correspondence from X into Y, and we always assume that $F(x) \neq \emptyset$ for all $x \in X$. For a multifunction $F : X \to Y$, we shall denote the upper and lower inverse of a set B of Y by $F^+(B)$ and $F^-(B)$, respectively, that is, $F^+(B) = \{x \in X \mid F(x) \subseteq B\}$ and

$$F^-(B) = \{x \in X \mid F(x) \cap B \neq \emptyset\}.$$

In particular, $F^-(y) = \{x \in X \mid y \in F(x)\}$ for each point $y \in Y$. For each $A \subseteq X$, $F(A) = \cup_{x \in A} F(x)$.

3. Upper and lower weakly quasi (τ_1, τ_2)-continuous multifunctions

In this section, we introduce the concepts of upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions. Moreover, several characterizations of upper weakly quasi (τ_1, τ_2)-continuous multifunctions and lower weakly quasi (τ_1, τ_2)-continuous multifunctions are discussed.

Definition 3.1. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be upper weakly quasi* (τ_1, τ_2)*-continuous at a point* $x \in X$ *if for each* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \subseteq V$ *and each* $\tau_1\tau_2$*-open set* U *of* X *containing* x*, there exists a nonempty* $\tau_1\tau_2$*-open set* G *such that* $G \subseteq U$ *and* $F(G) \subseteq \sigma_1\sigma_2$-$Cl(V)$. *A multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ *is said to be upper weakly quasi* (τ_1, τ_2)*-continuous if* F *is upper weakly quasi* (τ_1, τ_2)*-continuous at each point* x *of* X.

Theorem 3.1. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper weakly quasi* (τ_1, τ_2)*-continuous;*
(2) *for each* $x \in X$ *and every* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \subseteq V$*, there exists a* $(\tau_1, \tau_2)s$*-open set* U *of* X *containing* x *such that* $F(U) \subseteq \sigma_1\sigma_2$-$Cl(V)$;
(3) $\tau_1\tau_2$-$Int(\tau_1\tau_2$-$Cl(F^-(\sigma_1\sigma_2$-$Int(K)))) \subseteq F^-(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y;
(4) $F^+(V) \subseteq (\tau_1, \tau_2)$-$sInt(F^+(\sigma_1\sigma_2$-$Cl(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y;
(5) (τ_1, τ_2)-$sCl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2$-$Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y.

Proof. (1) ⇒ (2): Let $\mathscr{U}(x)$ the family of all $\tau_1\tau_2$-open sets of X containing x. Let V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \subseteq V$. For each $H \in \mathscr{U}(x)$, there exists a nonempty $\tau_1\tau_2$-open set G_H such that $G_H \subseteq H$ and $F(G_H) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Let $W = \cup\{G_H \mid H \in \mathscr{U}(x)\}$. Then, W is $\tau_1\tau_2$-open in X, $x \in \tau_1\tau_2$-Cl(W) and $F(W) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Put $U = W \cup \{x\}$, then $W \subseteq U \subseteq \tau_1\tau_2$-Cl$(W)$. Thus, U is a $(\tau_1, \tau_2)s$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$.

(2) ⇒ (4): Let V be any $\sigma_1\sigma_2$-open set of Y and $x \in F^+(V)$. Then, $F(x) \subseteq V$ and there exists a $(\tau_1, \tau_2)s$-open set U of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. Thus, $x \in U \subseteq (\tau_1, \tau_2)$-sInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$ and so $F^+(V) \subseteq (\tau_1, \tau_2)$-sInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$.

(4) ⇒ (5): Let V be any $\sigma_1\sigma_2$-open set of Y. Then by (4), we have

$$\begin{aligned} X - F^-(\sigma_1\sigma_2\text{-Cl}(V)) &= F^+(Y - \sigma_1\sigma_2\text{-Cl}(V)) \\ &\subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V)))) \\ &= (\tau_1,\tau_2)\text{-sInt}(F^+(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq (\tau_1,\tau_2)\text{-sInt}(F^+(Y - V)) \\ &= (\tau_1,\tau_2)\text{-sInt}(X - F^-(V)) \\ &= X - (\tau_1,\tau_2)\text{-sCl}(F^-(V)) \end{aligned}$$

and hence (τ_1,τ_2)-sCl$(F^-(V)) \subseteq F^-(\sigma_1\sigma_2$-Cl$(V))$.

(5) ⇒ (3): Let K be any $\sigma_1\sigma_2$-closed set of Y. By (5) and Lemma 2.2, we have

$$\begin{aligned} \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(K)))) &\subseteq (\tau_1,\tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(K)) \\ &= F^-(K). \end{aligned}$$

(3) ⇒ (4): Let V be any $\sigma_1\sigma_2$-open set of Y. By (3) and Lemma 2.2,

$$\begin{aligned} X - (\tau_1,\tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V))) &= (\tau_1,\tau_2)\text{-sCl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &= F^-(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^-(Y - V) \\ &= X - F^+(V) \end{aligned}$$

and hence $F^+(V) \subseteq (\tau_1,\tau_2)$-sInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$.

(4) ⇒ (1): Let $x \in X$ and V be any $\sigma_1\sigma_2$-open set of Y such that $F(x) \subseteq V$. By (4), we have $F^+(V) \subseteq (\tau_1,\tau_2)$-sInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$. Put $U = (\tau_1,\tau_2)$-sInt$(F^+(\sigma_1\sigma_2$-Cl$(V)))$, then U is $(\tau_1,\tau_2)s$-open set of X containing x such that $F(U) \subseteq \sigma_1\sigma_2$-Cl$(V)$. This shows that F is upper weakly quasi (τ_1,τ_2)-continuous. □

Definition 3.2. *A multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be lower weakly quasi (τ_1,τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y such that $F(x) \cap V \neq \emptyset$ and each $\tau_1\tau_2$-open set U of X containing x, there exists a nonempty $\tau_1\tau_2$-open set G such that $G \subseteq U$ and $\sigma_1\sigma_2$-Cl$(V) \cap F(z) \neq \emptyset$ for each $z \in G$. A multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is said to be lower weakly quasi (τ_1,τ_2)-continuous if F is lower weakly quasi (τ_1,τ_2)-continuous at each point x of X.*

Theorem 3.2. *For a multifunction $F : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) *F is lower weakly quasi (τ_1,τ_2)-continuous;*

(2) *for each* $x \in X$ *and every* $\sigma_1\sigma_2$*-open set* V *of* Y *such that* $F(x) \cap V \neq \emptyset$*, there exists a* $(\tau_1, \tau_2)s$*-open set* U *of* X *containing* x *such that* $\sigma_1\sigma_2\text{-}Cl(V) \cap F(z) \neq \emptyset$ *for every* $z \in U$*;*

(3) $\tau_1\tau_2\text{-}Int(\tau_1\tau_2\text{-}Cl(F^+(\sigma_1\sigma_2\text{-}Int(K)))) \subseteq F^+(K)$ *for every* $\sigma_1\sigma_2$*-closed set* K *of* Y*;*

(4) $F^-(V) \subseteq (\tau_1, \tau_2)\text{-}sInt(F^-(\sigma_1\sigma_2\text{-}Cl(V)))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(5) $(\tau_1, \tau_2)\text{-}sCl(F^+(V)) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*.*

Proof. The proof is similar to that of Theorem 3.1. □

Theorem 3.3. *For a multifunction* $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$*, the following properties are equivalent:*

(1) F *is upper weakly quasi* (τ_1, τ_2)*-continuous;*

(2) $(\tau_1, \tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int((\sigma_1, \sigma_2)\theta\text{-}Cl(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(3) $(\tau_1, \tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ *for every subset* B *of* Y*;*

(4) $(\tau_1, \tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every* $\sigma_1\sigma_2$*-open set* V *of* Y*;*

(5) $(\tau_1, \tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every* $(\sigma_1, \sigma_2)p$*-open set* V *of* Y*;*

(6) $(\tau_1, \tau_2)\text{-}sCl(F^-(\sigma_1\sigma_2\text{-}Int(K))) \subseteq F^-(K)$ *for every* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*.*

Proof. (1) ⇒ (2): Let B be any subset of Y. Since $(\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ is $\sigma_1\sigma_2$-closed in Y, by Theorem 3.1,

$$\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B))))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-Cl}(B))$$

and by Lemma 2.2, we have

$$(\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}((\sigma_1, \sigma_2)\theta\text{-Cl}(B)))) \subseteq F^-((\sigma_1, \sigma_2)\theta\text{-Cl}(B)).$$

(2) ⇒ (3): This is obvious since $\sigma_1\sigma_2\text{-Cl}(B) \subseteq (\sigma_1, \sigma_2)\theta\text{-Cl}(B)$ for every subset B of Y.

(3) ⇒ (4): This is obvious since $\sigma_1\sigma_2\text{-Cl}(V) = (\sigma_1, \sigma_2)\theta\text{-Cl}(V)$ for every $\sigma_1\sigma_2$-open set V of Y.

(4) ⇒ (5): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Then, we have $V \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ and $\sigma_1\sigma_2\text{-Cl}(V) = \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Now, put $G = \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$, then G is $\sigma_1\sigma_2$-open in Y and $\sigma_1\sigma_2\text{-Cl}(G) = \sigma_1\sigma_2\text{-Cl}(V)$. Thus by (4), $(\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V))$.

(5) ⇒ (6): Let K be any $(\sigma_1, \sigma_2)r$-closed set of Y. Since $\sigma_1\sigma_2\text{-Int}(K)$ is $(\sigma_1, \sigma_2)p$-open in Y, by (5), we have

$$\begin{aligned}(\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(K))) &= (\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= F^-(K).\end{aligned}$$

(6) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed in Y and by (6),

$$\begin{aligned}(\tau_1, \tau_2)\text{-sCl}(F^-(V)) &\subseteq (\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).\end{aligned}$$

It follows from Theorem 3.1 that F is upper weakly quasi (τ_1, τ_2)-continuous. □

Theorem 3.4. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower weakly quasi (τ_1, τ_2)-continuous;*
(2) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int((\sigma_1, \sigma_2)\theta\text{-}Cl(B)))) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ *for every subset B of Y;*
(3) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(B)))) \subseteq F^+((\sigma_1, \sigma_2)\theta\text{-}Cl(B))$ *for every subset B of Y;*
(4) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $\sigma_1\sigma_2$-open set V of Y;*
(5) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y;*
(6) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(K))) \subseteq F^+(K)$ *for every $(\sigma_1, \sigma_2)r$-closed set K of Y.*

Proof. The proof is similar to that of Theorem 3.3. □

Theorem 3.5. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper weakly quasi (τ_1, τ_2)-continuous;*
(2) (τ_1, τ_2)-$sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;*
(3) (τ_1, τ_2)-$sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)s$-open set V of Y;*
(4) (τ_1, τ_2)-$sCl(F^-(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y.*

Proof. (1) ⇒ (2): Let V be any $(\sigma_1, \sigma_2)\beta$-open set of Y. Then, we have $V \subseteq \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ and hence $\sigma_1\sigma_2\text{-Cl}(V) = \sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Since $\sigma_1\sigma_2\text{-Cl}(V)$ is a $(\sigma_1, \sigma_2)r$-closed set, by Theorem 3.3, $(\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V))$.

(2) ⇒ (3): This is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) ⇒ (4): For any $(\sigma_1, \sigma_2)p$-open set V of Y, $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)r$-closed and $\sigma_1\sigma_2\text{-Cl}(V)$ is $(\sigma_1, \sigma_2)s$-open in Y.

(4) ⇒ (1): Let V be any $\sigma_1\sigma_2$-open set of Y. Then, V is $(\sigma_1, \sigma_2)p$-open in Y. By (4), we have

$$(\tau_1, \tau_2)\text{-sCl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).$$

It follows from Theorem 3.3 that F is upper weakly quasi (τ_1, τ_2)-continuous. □

Theorem 3.6. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower weakly quasi (τ_1, τ_2)-continuous;*
(2) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)\beta$-open set V of Y;*
(3) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)s$-open set V of Y;*
(4) (τ_1, τ_2)-$sCl(F^+(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))) \subseteq F^+(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y.*

Proof. The proof is similar to that of Theorem 3.5. □

Theorem 3.7. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is upper weakly quasi (τ_1, τ_2)-continuous;*
(2) $\tau_1\tau_2$-$Int(\tau_1\tau_2\text{-}Cl(F^-(V))) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y;*
(3) (τ_1, τ_2)-$sCl(F^-(V)) \subseteq F^-(\sigma_1\sigma_2\text{-}Cl(V))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y;*
(4) $F^+(V) \subseteq (\tau_1, \tau_2)$-$sInt(F^+(\sigma_1\sigma_2\text{-}Cl(V)))$ *for every $(\sigma_1, \sigma_2)p$-open set V of Y.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Since F is upper weakly quasi (τ_1, τ_2)-continuous, by Lemma 2.2 and Theorem 3.3,

$$\begin{aligned}\tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(V))) &\subseteq \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).\end{aligned}$$

(2) $\Rightarrow$ (3): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. By (2) and Lemma 2.2, we have

$$\begin{aligned}(\tau_1, \tau_2)\text{-sCl}(F^-(V)) &= F^-(V) \cup \tau_1\tau_2\text{-Int}(\tau_1\tau_2\text{-Cl}(F^-(V))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(V)).\end{aligned}$$

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y. Then by (3), we have

$$\begin{aligned}X - (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V))) &= (\tau_1, \tau_2)\text{-sCl}(X - F^+(\sigma_1\sigma_2\text{-Cl}(V))) \\ &= (\tau_1, \tau_2)\text{-sCl}(F^-(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq F^-(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Cl}(V))) \\ &= X - F^+(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq X - F^+(V)\end{aligned}$$

and hence $F^+(V) \subseteq (\tau_1, \tau_2)\text{-sInt}(F^+(\sigma_1\sigma_2\text{-Cl}(V)))$.

(4) $\Rightarrow$ (1): Since every $\sigma_1\sigma_2$-open set is $(\sigma_1, \sigma_2)p$-open, this follows from Theorem 3.1. □

Theorem 3.8. *For a multifunction $F : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *F is lower weakly quasi (τ_1, τ_2)-continuous;*
(2) *$\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(F^+(V))) \subseteq F^+(\sigma_1\sigma_2$-Cl$(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;*
(3) *(τ_1, τ_2)-sCl$(F^+(V)) \subseteq F^+(\sigma_1\sigma_2$-Cl$(V))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y;*
(4) *$F^-(V) \subseteq (\tau_1, \tau_2)$-sInt$(F^-(\sigma_1\sigma_2$-Cl$(V)))$ for every $(\sigma_1, \sigma_2)p$-open set V of Y.*

Proof. The proof is similar to that of Theorem 3.7. □

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