Int. J. Anal. Appl. (2025), 23:14

International Journal of Analysis and Applications

Almost Nearly (τ_1, τ_2)-Continuous Functions

Butsakorn Kong-ied[1], **Areeyuth Sama-Ae**[2], **Chawalit Boonpok**[1,*]

[1] *Mathematics and Applied Mathematics Research Unit, Department of Mathematics, Faculty of Science, Mahasarakham University, Maha Sarakham, 44150, Thailand*

[2] *Department of Mathematics and Computer Science, Faculty of Science and Technology, Prince of Songkla University, Pattani Campus, Pattani, 94000, Thailand*

**Corresponding author:* chawalit.b@msu.ac.th

Abstract. This paper presents a new class of functions called almost nearly (τ_1, τ_2)-continuous functions. Furthermore, several characterizations and some properties concerning almost nearly (τ_1, τ_2)-continuous functions are discussed.

1. Introduction

The field of the mathematical science which goes under the name of topology is concerned with all questions directly or indirectly related to continuity. Stronger and weaker forms of open sets in topological spaces such as semi-open sets, preopen sets, α-open sets and β-open sets play an important role in the research of generalizations of continuity. By utilizing these sets several authors introduced and studied various types of generalizations of continuity for functions. In [8], the authors studied some properties of (Λ, sp)-open sets. Viriyapong and Boonpok [35] investigated some characterizations of (Λ, sp)-continuous functions by using (Λ, sp)-open sets and (Λ, sp)-closed sets. Dungthaisong et al. [17] introduced and studied the concept of $g_{(m,n)}$-continuous functions. Duangphui et al. [16] introduced and investigated the notion of $(\mu, \mu')^{(m,n)}$-continuous functions. Furthermore, several characterizations of almost (Λ, p)-continuous functions, strongly $\theta(\Lambda, p)$-continuous functions, almost strongly $\theta(\Lambda, p)$-continuous functions, $\theta(\Lambda, p)$-continuous functions, weakly (Λ, b)-continuous functions, $\theta(\star)$-precontinuous functions, $\star$-continuous functions, θ-$\mathscr{I}$-continuous functions, almost (g, m)-continuous functions, pairwise M-continuous functions were presented in [29], [31], [1], [26], [5], [6], [7], [9], [12], [13], respectively. Singal and

Received: Nov. 4, 2024.

2020 *Mathematics Subject Classification.* 54C08, 54E55.

Key words and phrases. $\tau_1\tau_2$-open set; $\mathscr{N}(\tau_1, \tau_2)$-closed set; almost nearly (τ_1, τ_2)-continuous function.

https://doi.org/10.28924/2291-8639-23-2025-14

ISSN: 2291-8639

Singal [28] introduced the concept of almost continuous functions as a generalization of continuity. Popa [24] defined almost quasi-continuous functions as a generalization of almost continuity and quasi-continuity [21]. Munshi and Bassan [22] studied the notion of almost semi-continuous functions. Maheshwari et al. [19] introduced the concept of almost feebly continuous functions as a generalization of almost continuity. Malghan and Hanchinamani [20] introduced the concept of N-continuous functions. Noiri and Ergun [23] investigated some characterizations of N-continuous functions. In [4], the present authors introduced and investigated the concept of (τ_1, τ_2)-continuous functions. Furthermore, several characterizations of almost (τ_1, τ_2)-continuous functions, weakly (τ_1, τ_2)-continuous functions, almost quasi (τ_1, τ_2)-continuous functions and weakly quasi (τ_1, τ_2)-continuous functions were established in [2], [3], [18] and [14], respectively. In this paper, we introduce the concept of almost nearly (τ_1, τ_2)-continuous tifunctions. We also investigate several characterizations of almost nearly (τ_1, τ_2)-continuous functions.

2. Preliminaries

Throughout the present paper, spaces (X, τ_1, τ_2) and (Y, σ_1, σ_2) (or simply X and Y) always mean bitopological spaces on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a bitopological space (X, τ_1, τ_2). The closure of A and the interior of A with respect to τ_i are denoted by τ_i-Cl(A) and τ_i-Int(A), respectively, for $i = 1, 2$. A subset A of a bitopological space (X, τ_1, τ_2) is called $\tau_1\tau_2$-*closed* [11] if $A = \tau_1$-Cl$(\tau_2$-Cl$(A))$. The complement of a $\tau_1\tau_2$-closed set is called $\tau_1\tau_2$-*open*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $\tau_1\tau_2$-closed sets of X containing A is called the $\tau_1\tau_2$-*closure* [11] of A and is denoted by $\tau_1\tau_2$-Cl(A). The union of all $\tau_1\tau_2$-open sets of X contained in A is called the $\tau_1\tau_2$-*interior* [11] of A and is denoted by $\tau_1\tau_2$-Int(A).

Lemma 2.1. [11] *Let A and B be subsets of a bitopological space (X, τ_1, τ_2). For the $\tau_1\tau_2$-closure, the following properties hold:*

(1) *$A \subseteq \tau_1\tau_2$-$Cl(A)$ and $\tau_1\tau_2$-$Cl(\tau_1\tau_2$-$Cl(A)) = \tau_1\tau_2$-$Cl(A)$.*
(2) *If $A \subseteq B$, then $\tau_1\tau_2$-$Cl(A) \subseteq \tau_1\tau_2$-$Cl(B)$.*
(3) *$\tau_1\tau_2$-$Cl(A)$ is $\tau_1\tau_2$-closed.*
(4) *A is $\tau_1\tau_2$-closed if and only if $A = \tau_1\tau_2$-$Cl(A)$.*
(5) *$\tau_1\tau_2$-$Cl(X - A) = X - \tau_1\tau_2$-$Int(A)$.*

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $(\tau_1, \tau_2)r$-*open* [36] (resp. $(\tau_1, \tau_2)s$-*open* [10], $(\tau_1, \tau_2)p$-*open* [10], $(\tau_1, \tau_2)\beta$-*open* [10]) if $A = \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$ (resp. $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A))$, $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A))$, $A \subseteq \tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(A)))$). The complement of a $(\tau_1, \tau_2)r$-open (resp. $(\tau_1, \tau_2)s$-open, $(\tau_1, \tau_2)p$-open, $(\tau_1, \tau_2)\beta$-open) set is called $(\tau_1, \tau_2)r$-*closed* (resp. $(\tau_1, \tau_2)s$-*closed*, $(\tau_1, \tau_2)p$-*closed*, $(\tau_1, \tau_2)\beta$-*closed*). A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\alpha(\tau_1, \tau_2)$-*open* [34] if $A \subseteq \tau_1\tau_2$-Int$(\tau_1\tau_2$-Cl$(\tau_1\tau_2$-Int$(A)))$. The complement of an $\alpha(\tau_1, \tau_2)$-open set is said to be $\alpha(\tau_1, \tau_2)$-*closed*. Let A be a subset of a bitopological space (X, τ_1, τ_2). The intersection of all $(\tau_1, \tau_2)p$-closed (resp. $(\tau_1, \tau_2)s$-closed, $\alpha(\tau_1, \tau_2)$-closed) sets of X

containing A is called the (τ_1, τ_2)*p-closure* [32] (resp. (τ_1, τ_2)*s-closure* [36], $\alpha(\tau_1, \tau_2)$*-closure* [33]) of A and is denoted by (τ_1, τ_2)-pCl(A) (resp. (τ_1, τ_2)-sCl(A), $\alpha(\tau_1, \tau_2)$-Cl(A)). The union of all $(\tau_1, \tau_2)p$-open (resp. $(\tau_1, \tau_2)s$-open, $\alpha(\tau_1, \tau_2)$-open) sets of X contained in A is called the (τ_1, τ_2)*p-interior* [32] (resp. (τ_1, τ_2)*s-interior* [36], $\alpha(\tau_1, \tau_2)$*-interior* [33]) of A and is denoted by (τ_1, τ_2)-pInt(A) (resp. (τ_1, τ_2)-sInt(A), $\alpha(\tau_1, \tau_2)$-Int(A)).

Lemma 2.2. [32] *For subsets A and B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *$A \subseteq (\tau_1, \tau_2)$-$pCl(A)$ and (τ_1, τ_2)-$pCl((\tau_1, \tau_2)$-$pCl(A)) = (\tau_1, \tau_2)$-$pCl(A)$.*
(2) *If $A \subseteq B$, then (τ_1, τ_2)-$pCl(A) \subseteq (\tau_1, \tau_2)$-$pCl(B)$.*
(3) *(τ_1, τ_2)-$pCl(A)$ is $(\tau_1, \tau_2)p$-closed.*
(4) *A is $(\tau_1, \tau_2)p$-closed if and only if $A = (\tau_1, \tau_2)$-$pCl(A)$.*
(5) *(τ_1, τ_2)-$pCl(X - A) = X - (\tau_1, \tau_2)$-$pInt(A)$.*
(6) *$x \in (\tau_1, \tau_2)$-$pCl(A)$ if and only if $A \cap U \neq \emptyset$ for every $(\tau_1, \tau_2)p$-open set U of X containing x.*

Lemma 2.3. [33] *For subsets A and B of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *$A \subseteq \alpha(\tau_1, \tau_2)$-$Cl(A)$ and $\alpha(\tau_1, \tau_2)$-$Cl(\alpha(\tau_1, \tau_2)$-$Cl(A)) = \alpha(\tau_1, \tau_2)$-$Cl(A)$.*
(2) *If $A \subseteq B$, then $\alpha(\tau_1, \tau_2)$-$Cl(A) \subseteq \alpha(\tau_1, \tau_2)$-$Cl(B)$.*
(3) *$\alpha(\tau_1, \tau_2)$-$Cl(A)$ is $\alpha(\tau_1, \tau_2)$-closed.*
(4) *A is $\alpha(\tau_1, \tau_2)$-closed if and only if $A = \alpha(\tau_1, \tau_2)$-$Cl(A)$.*
(5) *$\alpha(\tau_1, \tau_2)$-$Cl(X - A) = X - \alpha(\tau_1, \tau_2)$-$Int(A)$.*

Lemma 2.4. *For a subset A of a bitopological space (X, τ_1, τ_2), the following properties hold:*

(1) *(τ_1, τ_2)-$sCl(A) = \tau_1\tau_2$-$Int(\tau_1\tau_2$-$Cl(A)) \cup A$* [10];
(2) *(τ_1, τ_2)-$sInt(A) = \tau_1\tau_2$-$Cl(\tau_1\tau_2$-$Int(A)) \cap A$* [27].

A subset A of a bitopological space (X, τ_1, τ_2) is said to be $\mathscr{N}(\tau_1, \tau_2)$*-closed* [30] if every cover of A by $(\tau_1, \tau_2)r$-open sets of X has a finite subcover.

Lemma 2.5. [15] *Let (X, τ_1, τ_2) be a bitopological space. If V is a $\tau_1\tau_2$-open set of X having $\mathscr{N}(\tau_1, \tau_2)$-closed complement, then $\tau_1\tau_2$-$Int(\tau_1\tau_2$-$Cl(V))$ is a $(\tau_1, \tau_2)r$-open set having $\mathscr{N}(\tau_1, \tau_2)$-closed complement.*

Lemma 2.6. [15] *Let (X, τ_1, τ_2) be a bitopological space. If V is a $(\tau_1, \tau_2)p$-open set of X having $\mathscr{N}(\tau_1, \tau_2)$-closed complement, then $\tau_1\tau_2$-$Int(\tau_1\tau_2$-$Cl(V))$ is a $(\tau_1, \tau_2)r$-open set having $\mathscr{N}(\tau_1, \tau_2)$-closed complement.*

3. Almost nearly (τ_1, τ_2)-continuous functions

In this section, we introduce the notion of almost nearly (τ_1, τ_2)-continuous functions. Furthermore, several characterizations of almost nearly (τ_1, τ_2)-continuous functions are discussed.

Definition 3.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous at a point $x \in X$ if for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V))$. A function*

$f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is said to be almost nearly (τ_1, τ_2)-continuous if f has this property at each point x of X.

Theorem 3.1. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1, τ_2)-continuous at $x \in X$;*

(2) *$x \in \tau_1\tau_2$-Int$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$ for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(3) *$x \in \tau_1\tau_2$-Int$(f^{-1}((\sigma_1, \sigma_2)$-sCl$(V)))$ for each $\sigma_1\sigma_2$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(4) *$x \in \tau_1\tau_2$-Int$(f^{-1}(V))$ for each $(\sigma_1, \sigma_2)r$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(5) *for each $(\sigma_1, \sigma_2)r$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement, there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq V$.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. By (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $f(U) \subseteq \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$. Thus, we have $x \in U \subseteq f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)))$ and hence $x \in \tau_1\tau_2$-Int$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$.

(2) $\Rightarrow$ (3): This follows from Lemma 2.4.

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. It follows from Lemma 2.4 that $V = \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V)) = (\sigma_1, \sigma_2)$-sCl$(V)$.

(4) $\Rightarrow$ (5): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (4), $x \in \tau_1\tau_2$-Cl$(f^{-1}(V))$ and therefore there exists a $\tau_1\tau_2$-open set U of X such that $x \in U \subseteq f^{-1}(V)$; hence $f(U) \subseteq V$.

(5) $\Rightarrow$ (1): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By Lemma 2.5, $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Thus by (5), there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$f(U) \subseteq \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)).$$

This shows that f is almost nearly (τ_1, τ_2)-continuous. □

Theorem 3.2. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1, τ_2)-continuous;*

(2) *$f^{-1}(V) \subseteq \tau_1\tau_2$-Int$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$ for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement;*

(3) *$\tau_1\tau_2$-Cl$(f^{-1}(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(K)))) \subseteq f^{-1}(K)$ for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y;*

(4) *$\tau_1\tau_2$-Cl$(f^{-1}(\sigma_1\sigma_2$-Cl$(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(B))))) \subseteq f^{-1}(\sigma_1\sigma_2$-Cl$(B))$ for every every subset B of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;*

(5) $f^{-1}(\sigma_1\sigma_2\text{-}Int(B)) \subseteq \tau_1\tau_2\text{-}Int(f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(\sigma_1\sigma_2\text{-}Int(B)))))$ *for every every subset* B *of* Y *such that* $Y - \sigma_1\sigma_2\text{-}Int(B)$ *is* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed;*

(6) $f^{-1}(V)$ *is* $\tau_1\tau_2$*-open in* X *for each* $(\sigma_1, \sigma_2)r$*-open set* V *of* Y *having* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed complement;*

(7) $f^{-1}(K)$ *is* $\tau_1\tau_2$*-closed in* X *for every* $\mathscr{N}(\sigma_1, \sigma_2)$*-closed and* $(\sigma_1, \sigma_2)r$*-closed set* K *of* Y*.*

Proof. (1) $\Rightarrow$ (2): Let V be any $\sigma_1\sigma_2$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement and $x \in f^{-1}(V)$. Then, $f(x) \in V$. By Theorem 3.1, we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ and hence $f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$.

(2) $\Rightarrow$ (3): Let K be any $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $\sigma_1\sigma_2$-closed set K of Y. Then, $Y - K$ is a $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (2), we have

$$\begin{aligned} X - f^{-1}(K) &= f^{-1}(Y - K) \\ &\subseteq \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - K)))) \\ &= \tau_1\tau_2\text{-Int}(X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \\ &= X - \tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))). \end{aligned}$$

Thus, $\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))) \subseteq f^{-1}(K)$.

(3) $\Rightarrow$ (4): Let B be any subset of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2\text{-Cl}(B)$ is a $\sigma_1\sigma_2$-closed $\mathscr{N}(\sigma_1, \sigma_2)$-closed set of Y and by (3),

$$\tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(B))))) \subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(B)).$$

(4) $\Rightarrow$ (5): Let B be any subset of Y such that $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed. Since $Y - \sigma_1\sigma_2\text{-Int}(B)$ is $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed, by (4) we have

$$\begin{aligned} f^{-1}(\sigma_1\sigma_2\text{-Int}(B)) &= X - f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(B)) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - B)) \\ &\subseteq X - \tau_1\tau_2\text{-Cl}(f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y - B))))) \\ &= X - \tau_1\tau_2\text{-Cl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))) \\ &= \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(B))))). \end{aligned}$$

(5) $\Rightarrow$ (6): Let V be any $(\sigma_1, \sigma_2)r$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, we have $Y - \sigma_1\sigma_2\text{-Int}(V)$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and by (5), $f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(V))$. Thus, $f^{-1}(V)$ is $\tau_1\tau_2$-open in X.

(6) $\Rightarrow$ (7): Let K be any $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $(\sigma_1, \sigma_2)r$-closed set of Y. Then, $Y - K$ is a $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (6), we have $f^{-1}(Y - K) = X - f^{-1}(K)$ is $\tau_1\tau_2$-open in X and hence $f^{-1}(K)$ is $\tau_1\tau_2$-closed in X.

(7) $\Rightarrow$ (1): Let $x \in X$ and V be any $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $(\sigma_1, \sigma_2)r$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed. By (7), $f^{-1}(Y - V) = X - f^{-1}(V)$ is $\tau_1\tau_2$-closed in X. Thus, $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Then, there exists a $\tau_1\tau_2$-open

set U of X containing x such that $f(U) \subseteq V$. It follows from Theorem 3.1 that f is almost nearly (τ_1, τ_2)-continuous at x. This shows that f is almost nearly (τ_1, τ_2)-continuous. □

Corollary 3.1. *A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is almost nearly (τ_1, τ_2)-continuous if $f^{-1}(K)$ is $\tau_1\tau_2$-closed in X for every $\mathscr{N}(\sigma_1, \sigma_2)$-closed set K of Y.*

Proof. Let V be any $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - V$ is $\mathscr{N}(\sigma_1, \sigma_2)$-closed and $(\sigma_1, \sigma_2)r$-closed. By the hypothesis, $X - f^{-1}(V) = f^{-1}(Y - V)$ is $\tau_1\tau_2$-closed in X and hence $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. It follows from Theorem 3.2 that f is almost nearly (τ_1, τ_2)-continuous. □

Theorem 3.3. *For a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1, τ_2)-continuous;*
(2) *$\tau_1\tau_2$-$Cl(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$-$Cl(V))$ for every every $(\sigma_1, \sigma_2)\beta$-open set V of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;*
(3) *$\tau_1\tau_2$-$Cl(f^{-1}(V)) \subseteq f^{-1}(\sigma_1\sigma_2$-$Cl(V))$ for every every $(\sigma_1, \sigma_2)s$-open set V of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure;*
(4) *$f^{-1}(V) \subseteq \tau_1\tau_2$-$Int(f^{-1}(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V))))$ for every every $(\sigma_1, \sigma_2)p$-open set V of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement.*

Proof. (1) $\Rightarrow$ (2): Let V be any $(\sigma_1, \sigma_2)\beta$-open set of Y having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. Then, $\sigma_1\sigma_2$-Cl(V) is $(\sigma_1, \sigma_2)r$-closed in Y. Since f is almost nearly (τ_1, τ_2)-continuous, by Theorem 3.2 we have $f^{-1}(\sigma_1\sigma_2$-Cl$(V))$ is $\tau_1\tau_2$-closed in X. Thus, $\tau_1\tau_2$-Cl$(f^{-1}(V)) \subseteq \tau_1\tau_2$-Cl$(f^{-1}(\sigma_1\sigma_2$-Cl$(V))) = f^{-1}(\sigma_1\sigma_2$-Cl$(V))$.

(2) $\Rightarrow$ (3): The proof is obvious since every $(\sigma_1, \sigma_2)s$-open set is $(\sigma_1, \sigma_2)\beta$-open.

(3) $\Rightarrow$ (4): Let V be any $(\sigma_1, \sigma_2)p$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then by Lemma 2.6, $\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1, \sigma_2)r$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, $Y - \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1, \sigma_2)r$-closed and $\mathscr{N}(\sigma_1, \sigma_2)$-closed set. Therefore, $Y - \sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))$ is a $(\sigma_1, \sigma_2)s$-open set having the $\mathscr{N}(\sigma_1, \sigma_2)$-closed $\sigma_1\sigma_2$-closure. By (3), we have

$$\begin{aligned} X - \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) &= \tau_1\tau_2\text{-Cl}(f^{-1}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &\subseteq f^{-1}(\sigma_1\sigma_2\text{-Cl}(Y - \sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \\ &\subseteq X - f^{-1}(V) \end{aligned}$$

and hence $f^{-1}(V) \subseteq \tau_1\tau_2$-Int$(f^{-1}(\sigma_1\sigma_2$-Int$(\sigma_1\sigma_2$-Cl$(V))))$.

(4) $\Rightarrow$ (1): Let V be any $(\sigma_1, \sigma_2)r$-open set of Y having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. Then, V is a $(\sigma_1, \sigma_2)p$-open set having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement. By (4), we have

$$f^{-1}(V) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))) = \tau_1\tau_2\text{-Int}(f^{-1}(V))$$

and hence $f^{-1}(V)$ is $\tau_1\tau_2$-open in X. Thus by Theorem 3.2, f is almost nearly (τ_1,τ_2)-continuous. □

Lemma 3.1. [15] *For a bitopological space (X,τ_1,τ_2), the following properties hold:*

(1) *$\alpha(\tau_1,\tau_2)$-$Cl(U) = \tau_1\tau_2$-$Cl(U)$ for every $(\tau_1,\tau_2)\beta$-open set U of X;*
(2) *(τ_1,τ_2)-$pCl(U) = \tau_1\tau_2$-$Cl(U)$ for every $(\tau_1,\tau_2)s$-open set U of X.*

Corollary 3.2. *For a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1,τ_2)-continuous;*
(2) *$\tau_1\tau_2$-$Cl(f^{-1}(V)) \subseteq f^{-1}(\alpha(\sigma_1,\sigma_2)$-$Cl(V))$ for every every $(\sigma_1,\sigma_2)\beta$-open set V of Y having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure;*
(3) *$\tau_1\tau_2$-$Cl(f^{-1}(V)) \subseteq f^{-1}((\sigma_1,\sigma_2)$-$pCl(V))$ for every every $(\sigma_1,\sigma_2)s$-open set V of Y having the $\mathscr{N}(\sigma_1,\sigma_2)$-closed $\sigma_1\sigma_2$-closure.*

Theorem 3.4. *For a function $f : (X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$, the following properties are equivalent:*

(1) *f is almost nearly (τ_1,τ_2)-continuous;*
(2) *for each $x \in X$ and for every $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set K of Y such that $x \in f^{-1}(Y-K)$, there exists a $\tau_1\tau_2$-closed set H of X such that $x \in X-H$ and $f^{-1}(\sigma_1\sigma_2$-$Cl(\sigma_1\sigma_2$-$Int(K))) \subseteq H$;*
(3) *$f^{-1}(\sigma_1\sigma_2$-$Int(\sigma_1\sigma_2$-$Cl(V)))$ is $\tau_1\tau_2$-open in X for every $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement;*
(4) *$f^{-1}(\sigma_1\sigma_2$-$Cl(\sigma_1\sigma_2$-$Int(K)))$ is $\tau_1\tau_2$-closed in X for every $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set K of Y.*

Proof. (1) ⇒ (2): Let $x \in X$ and K be any $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set of Y such that

$$x \in f^{-1}(Y-K).$$

Then, $Y-K$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Since f is almost nearly (τ_1,τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that

$$U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y-K))) = X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))).$$

It is clear that $H = X-U$ is $\tau_1\tau_2$-closed in X and $f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \subseteq H$.

(2) ⇒ (1): The proof is similar to the proof (1) ⇒ (2).

(1) ⇒ (3): Let V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement and

$$x \in f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))).$$

Then, we have $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))$ is a $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. Thus by (1), there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Since U is $\tau_1\tau_2$-open, we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))))$ and hence

$$f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V))) \subseteq \tau_1\tau_2\text{-Int}(f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))).$$

Thus, $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ is $\tau_1\tau_2$-open in X.

(3) $\Rightarrow$ (1): The proof is clear.

(3) $\Rightarrow$ (4): Let K be any $\sigma_1\sigma_2$-closed and $\mathscr{N}(\sigma_1,\sigma_2)$-closed set of Y. Then, $Y-K$ is a $\sigma_1\sigma_2$-open set having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement. By (3), $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y-K)))$ is $\tau_1\tau_2$-open in X. Since $\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y-K)) = Y-\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))$, it follows that

$$\begin{aligned} f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(Y-K))) &= f^{-1}(Y-\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))) \\ &= X - f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K))). \end{aligned}$$

Thus, $f^{-1}(\sigma_1\sigma_2\text{-Cl}(\sigma_1\sigma_2\text{-Int}(K)))$ is $\tau_1\tau_2$-closed in X.

(4) $\Rightarrow$ (3): It can be obtained similarly as (3) $\Rightarrow$ (4). □

Recall that a net (x_γ) in a topological space (X,τ) is said to be *eventually* in the set $U \subseteq X$ if there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$.

Definition 3.2. [25] *A sequence (x_n) is called (τ_1,τ_2)-converge to a point x if for every $\tau_1\tau_2$-open set V containing x, there exists an index γ_0 such that for all $\gamma \geq \gamma_0$, $x_\gamma \in V$.*

Theorem 3.5. *A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is almost nearly (τ_1,τ_2)-continuous if and only if for each $x \in X$ and for each net (x_γ) which (τ_1,τ_2)-converges to x in X and for each $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement such that $x \in f^{-1}(V)$, the net (x_γ) is eventually in $f^{-1}(\sigma_1\sigma_2\text{-}Int(\sigma_1\sigma_2\text{-}Cl(V)))$.*

Proof. Let (x_γ) be a net which (τ_1,τ_2)-converges to x in X and V be any $\sigma_1\sigma_2$-open set of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement such that $x \in f^{-1}(V)$. Since f is almost nearly (τ_1,τ_2)-continuous, there exists a $\tau_1\tau_2$-open set U of X containing x such that $U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. Since (x_γ) (τ_1,τ_2)-converges to x, it follows that there exists an index $\gamma_0 \in \nabla$ such that $x_\gamma \in U$ for all $\gamma \geq \gamma_0$. Therefore,

$$x_\gamma \in U \subseteq f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$$

for all $\gamma \geq \gamma_0$. Thus, the net (x_γ) is eventually in $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$.

Conversely, suppose that f is not almost nearly (τ_1,τ_2)-continuous. Then, there exists a point x of X and a $\sigma_1\sigma_2$-open set V of Y having $\mathscr{N}(\sigma_1,\sigma_2)$-closed complement with $x \in f^{-1}(V)$ such that

$$U \not\subseteq f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$$

for each $\tau_1\tau_2$-open set U of X containing x. Let $x_U \in U$ and $x_U \notin f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$ for each $\tau_1\tau_2$-open set U of X containing x. Then, for each $\tau_1\tau_2$-neighbourhood net (x_U), (x_U) (τ_1,τ_2)-converges to x, but (x_U) is not eventually in $f^{-1}(\sigma_1\sigma_2\text{-Int}(\sigma_1\sigma_2\text{-Cl}(V)))$. This is a contradiction. Thus, f is almost nearly (τ_1,τ_2)-continuous. □

The $\tau_1\tau_2$-*frontier* [4] of a subset A of a bitopological space (X,τ_1,τ_2), denoted by $\tau_1\tau_2\text{-fr}(A)$, is defined by $\tau_1\tau_2\text{-fr}(A) = \tau_1\tau_2\text{-Cl}(A) \cap \tau_1\tau_2\text{-Cl}(X-A) = \tau_1\tau_2\text{-Cl}(A) - \tau_1\tau_2\text{-Int}(A)$.

Theorem 3.6. *The set of all points x of X at which a function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is not almost nearly (τ_1, τ_2)-continuous is identical with the union of the $\tau_1\tau_2$-frontier of the inverse images of $(\sigma_1, \sigma_2)r$-open sets containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement.*

Proof. Let x be a point of X at which f is not almost nearly (τ_1, τ_2)-continuous. Then, by Theorem 3.1 there exists a $(\sigma_1, \sigma_2)r$-open set V of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $U \cap (X - f^{-1}(V)) \neq \emptyset$ for every $\tau_1\tau_2$-open set U of X containing x. Thus, $x \in \tau_1\tau_2\text{-Cl}(X - f^{-1}(V))$. On the other hand, we have $x \in f^{-1}(V) \subseteq \tau_1\tau_2\text{-Cl}(f^{-1}(V))$ and hence $x \in \tau_1\tau_2\text{-fr}(f^{-1}(V))$.

Conversely, suppose that V is a $(\sigma_1, \sigma_2)r$-open set of Y containing $f(x)$ and having $\mathscr{N}(\sigma_1, \sigma_2)$-closed complement such that $x \in \tau_1\tau_2\text{-fr}(f^{-1}(V))$. If f is almost nearly (τ_1, τ_2)-continuous at $x \in X$, then by Theorem 3.1, we have $x \in \tau_1\tau_2\text{-Int}(f^{-1}(V))$. This is a contradiction and hence f is not almost nearly (τ_1, τ_2)-continuous at x. □

Acknowledgements: This research project was financially supported by Mahasarakham University.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] C. Boonpok, J. Khampakdee, Almost Strong $\theta(\Lambda, p)$-Continuity for Functions, Eur. J. Pure Appl. Math. 17 (2024), 300–309. https://doi.org/10.29020/nybg.ejpam.v17i1.4975.

[2] C. Boonpok, P. Pue-On, Characterizations of Almost (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 33. https://doi.org/10.28924/2291-8639-22-2024-33.

[3] C. Boonpok, C. Klanarong, On Weakly (τ_1, τ_2)-Continuous Functions, Eur. J. Pure Appl. Math. 17 (2024), 416–425. https://doi.org/10.29020/nybg.ejpam.v17i1.4976.

[4] C. Boonpok, N. Srisarakham, (τ_1, τ_2)-Continuity for Functions, Asia Pac. J. Math. 11 (2024), 21. https://doi.org/10.28924/APJM/11-21.

[5] C. Boonpok, N. Srisarakham, Weak Forms of (Λ, b)-Open Sets and Weak (Λ, b)-Continuity, Eur. J. Pure Appl. Math. 16 (2023), 29–43. https://doi.org/10.29020/nybg.ejpam.v16i1.4571.

[6] C. Boonpok, $\theta(\star)$-Precontinuity, Mathematica, 65 (2023), 31–42. https://doi.org/10.24193/mathcluj.2023.1.04.

[7] C. Boonpok, on Some Spaces via Topological Ideals, Open Math. 21 (2023), 20230118. https://doi.org/10.1515/math-2023-0118.

[8] C. Boonpok, J. Khampakdee, (Λ, sp)-Open Sets in Topological Spaces, Eur. J. Pure Appl. Math. 15 (2022), 572–588. https://doi.org/10.29020/nybg.ejpam.v15i2.4276.

[9] C. Boonpok, on Characterizations of $\star$-Hyperconnected Ideal Topological Spaces, J. Math. 2020 (2020), 9387601. https://doi.org/10.1155/2020/9387601.

[10] C. Boonpok, $(\tau_1, \tau_2)\delta$-Semicontinuous Multifunctions, Heliyon, 6 (2020), e05367. https://doi.org/10.1016/j.heliyon.2020.e05367.

[11] C. Boonpok, C. Viriyapong, M. Thongmoon, on Upper and Lower (τ_1, τ_2)-Precontinuous Multifunctions, J. Math. Computer Sci. 18 (2018), 282–293. https://doi.org/10.22436/jmcs.018.03.04.

[12] C. Boonpok, Almost (G, m)-Continuous Functions, Int. J. Math. Anal. 4 (2010), 1957–1964.

[13] C. Boonpok, M-Continuous Functions in Biminimal Structure Spaces, Far East J. Math. Sci. 43 (2010), 41–58.

[14] M. Chiangpradit, S. Sompong, C. Boonpok, Weakly Quasi (τ_1, τ_2)-Continuous Functions, Int. J. Anal. Appl. 22 (2024), 125. https://doi.org/10.28924/2291-8639-22-2024-125.

[15] N. Chutiman, A. Sama-Ae, C. Boonpok, Almost Nearly (τ_1, τ_2)-Continuity for Multifunctions, (Accepted).

[16] T. Duangphui, C. Boonpok and C. Viriyapong, Continuous Functions on Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1165–1174.

[17] W. Dungthaisong, C. Boonpok, C. Viriyapong, Generalized Closed Sets in Bigeneralized Topological Spaces, Int. J. Math. Anal. 5 (2011), 1175–1184.

[18] B. Kong-ied, S. Sompong, C. Boonpok, Almost Quasi (τ_1, τ_2)-Continuous Functions, Asia Pac. J. Math. 11 (2024), 64. https://doi.org/10.28924/APJM/11-64.

[19] S.N. Maheshwari, G.I. Chae, P.C. Jain, Almost Feebly Continuous Functions, Ulsan Inst. Tech. Rep. 13 (1982), 195–197.

[20] S.R. Malghan, V.V. Hanchinamani, N-Continuous Functions, Ann. Soc. Sci. Bruxelles, 98 (1984), 69–79.

[21] S. Marcus, Sur les Fonctions Quasicontinues au Sens de S. Kempisty, Colloq. Math. 8 (1961), 47–53. https://eudml.org/doc/210887.

[22] B.M. Munshi, D.S. Bassan, Almost Semi-Continuous Mappings, Math. Student, 49 (1981), 239–248.

[23] T. Noiri, N. Ergun, Notes on N-Continuous Functions, Res. Rep. Yatsushiro Coll. Tech. 11 (1989), 65–68.

[24] V. Popa, on a Decomposition of Quasicontinuity in Topological Spaces, Stud. Cerc. Mat. 30 (1978), 31–35.

[25] P. Pue-on, A. Sama-Ae, C. Boonpok, Upper and Lower Faint (τ_1, τ_2)-Continuity, Int. J. Anal. Appl. 22 (2024), 169. https://doi.org/10.28924/2291-8639-22-2024-169.

[26] P. Pue-on, C. Boonpok, $\theta(\Lambda, p)$-Continuity for Functions, Int. J. Math. Comput. Sci. 19 (2024), 491–495.

[27] P. Pue-on, S. Sompong, C. Boonpok, Almost Quasi (τ_1, τ_2)-Continuity for Multifunctions, Int. J. Anal. Appl. 22 (2024), 97. https://doi.org/10.28924/2291-8639-22-2024-97.

[28] M.K. Singal, A.R. Singal, Almost Continuous Mappings, Yokohama J. Math. 16 (1968), 63–73.

[29] N. Srisarakham, C. Boonpok, Almost (Λ, p)-Continuous Functions, Int. J. Math. Comput. Sci. 18 (2023), 255–259.

[30] M. Thongmoon, A. Sama-Ae, C. Boonpok, Upper and Lower Near (τ_1, τ_2)-Continuity, (Accepted).

[31] M. Thongmoon, C. Boonpok, Strongly $\theta(\Lambda, p)$-Continuous Functions, Int. J. Math. Comput. Sci. 19 (2024), 475–479.

[32] N. Viriyapong, S. Sompong, C. Boonpok, Slightly $(\tau_1, \tau_2)p$-Continuous Multifunctions, Int. J. Anal. Appl. 22 (2024), 152. https://doi.org/10.28924/2291-8639-22-2024-152.

[33] C. Viriyapong, S. Sompong, C. Boonpok, Upper and Lower Slight $\alpha(\tau_1, \tau_2)$-Continuity, Eur. J. Pure Appl. Math. 17 (2024), 2142–2154. https://doi.org/10.29020/nybg.ejpam.v17i3.5321.

[34] N. Viriyapong, S. Sompong, C. Boonpok, (τ_1, τ_2)-Extremal Disconnectedness in Bitopological Spaces, Int. J. Math. Comput. Sci. 19 (2024), 855–860.

[35] C. Viriyapong, C. Boonpok, (Λ, sp)-Continuous Functions, WSEAS Trans. Math. 21 (2022), 380–385. https://doi.org/10.37394/23206.2022.21.45.

[36] C. Viriyapong, C. Boonpok, $(\tau_1, \tau_2)\alpha$-Continuity for Multifunctions, J. Math. 2020 (2020), 6285763. https://doi.org/10.1155/2020/6285763.