

Topological Properties and Fixed Point Results for Modified Intuitionistic Generalized Fuzzy Metric Spaces

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Abstract. The principle originality inside the paper is to establish the concept of modified intuitionistic generalized fuzzy metric space and its fundamental topological properties. Moreover, we give the steps for proving some coupled coincidence point results for mapping with contractive condition in partial ordered modified intuitionistic generalized fuzzy metric space. Furthermore, an illustration is proved in the support of our main result.

1. INTRODUCTION

Numerous significant generalizations of metric space such as D^* -metric space, partial metric space, S -metric space, \mathcal{A} -metric space have been established by many eminent mathematicians in [1, 6, 21, 22], respectively. Mustafa and Sims [17] coined the notion of G -metric space and suggested as important generalizations of metric space.

Zadeh [29] is the first individual who was once credited with the conception of placing the thinking of fuzzy set for huge applications. George and Veeramani [10] modified the thought of a FM-space introduced by Kramosil and Michalek [14]. Moreover, Sun and Yang [28] gave the thinking of \mathcal{G} -fuzzy metric space.

The theory of intuitionistic fuzzy set was initiated by Atanassov [3] and Alaca et al. [2] characterized the idea of intuitionistic FM-space. Saadati et al. [20] introduced the MIFM-space, see also [4, 15].

Received: Nov. 4, 2024.

2020 *Mathematics Subject Classification.* 54H25, 47H10.

Key words and phrases. fuzzy Sets; compatible mappings; coupled coincidence point; MIGFM-space.

Definition 1.1. [20] Let X and Y be fuzzy sets such that $X(\vartheta, \eta, \tau) + Y(\vartheta, \eta, \tau) \leq 1$. The term $(M, E_{X,Y}, \mathcal{S})$ is called an MIFM-space if $M \neq \emptyset$, continuous τ -representable \mathcal{S} and $E_{X,Y}$ is a function $M \times M \times (0, \infty) \rightarrow \mathcal{H}^*$ satisfying the following conditions: for all $\vartheta, \eta, \sigma \in M, \tau, s > 0$

- (i) $E_{X,Y}(\vartheta, \eta, \tau) >_{\mathcal{H}^*} 0_{\mathcal{H}^*}$,
- (ii) $E_{X,Y}(\vartheta, \eta, \tau) = 1_{\mathcal{H}^*} \Leftrightarrow \vartheta = \eta$,
- (iii) $E_{X,Y}(\vartheta, \eta, \tau) = E_{X,Y}(\eta, \vartheta, \tau)$,
- (iv) $E_{X,Y}(\vartheta, \eta, \tau + s) \geq_{\mathcal{H}^*} \mathcal{S}(E_{X,Y}(\vartheta, \sigma, \tau), E_{X,Y}(\sigma, \eta, s))$,
- (v) $E_{X,Y}(\vartheta, \eta, \cdot) : (0, \infty) \rightarrow \mathcal{H}^*$ is continuous.

Here,

$$E_{X,Y}(\vartheta, \eta, \tau) = (X(\vartheta, \eta, \tau), Y(\vartheta, \eta, \tau)).$$

For more important related ideas and detailed concepts, we refer to [7–9, 11–13, 16, 18, 19, 24, 25]. Bhaskar and Lakshmikantham [5] formulated a very elementary and a fruitful concept of coupled fixed points and monotone property for mappings.

Definition 1.2. [5] Let (M, \leq) be a partially ordered set and $Y : M \times M \rightarrow M$ and $\Omega : M \rightarrow M$. We say Y has the mixed Ω -monotone property if Y is monotone Ω -non decreasing in its first argument and is monotone Ω -non increasing in its second argument, that is, for any $\vartheta, \eta \in M$

$$\vartheta_1, \vartheta_2 \in M, \Omega(\vartheta_1) \leq \Omega(\vartheta_2) \implies Y(\vartheta_1, \eta) \leq Y(\vartheta_2, \eta)$$

and

$$\eta_1, \eta_2 \in M, \Omega(\eta_1) \leq \Omega(\eta_2) \implies Y(\vartheta, \eta_1) \geq Y(\vartheta, \eta_2).$$

Refer to the recent article and the references therein for applications of partial order [23]

2. MAIN RESULTS

2.1. Modified Intuitionistic Generalized Fuzzy Metric Space. Here, the notion of the MIGFM-space is establish with supportive lemmas.

Definition 2.1. Let X and Y be fuzzy sets defined on $M \times M \times M \times (0, \infty)$ such that $X(\vartheta, \eta, \sigma, \tau) + Y(\vartheta, \eta, \sigma, \tau) \leq 1$. The triplet $(M, E_{X,Y}, \mathcal{S})$ is an MIGFM-space if M is non empty, continuous τ -representable \mathcal{S} and $E_{X,Y}$ is a function $M \times M \times M \times (0, \infty) \rightarrow \mathcal{H}^*$ hold the following conditions: for all $\vartheta, \eta, \sigma, \kappa \in M, \tau, s > 0$,

- (1) $E_{X,Y}(\vartheta, \eta, \sigma, \tau) >_{\mathcal{H}^*} 0_{\mathcal{H}^*}$,
- (2) $E_{X,Y}(\vartheta, \eta, \sigma, \tau) = 1_{\mathcal{H}^*} \Leftrightarrow \vartheta = \eta = \sigma$,
- (3) $E_{X,Y}(\vartheta, \eta, \sigma, \tau) = E_{X,Y}(P(\vartheta, \eta, \sigma), \tau)$, where P is permutation function,
- (4) $E_{X,Y}(\vartheta, \eta, \sigma, \tau) \leq_{\mathcal{H}^*} E_{X,Y}(\vartheta, \vartheta, \sigma, \tau)$,
- (5) $E_{X,Y}(\vartheta, \eta, \sigma, s + \tau) \geq_{\mathcal{H}^*} \mathcal{S}(E_{X,Y}(\vartheta, \kappa, \kappa, s), E_{X,Y}(\kappa, \eta, \sigma, \tau))$,
- (6) $E_{X,Y}(\vartheta, \eta, \sigma, \cdot) : (0, \infty) \rightarrow \mathcal{H}^*$ is continuous.

In this case, $E_{X,Y}$ is said to be MIGF-metric and

$$E_{X,Y}(\vartheta, \eta, \sigma, \tau) = \left(X(\vartheta, \eta, \sigma, \tau), Y(\vartheta, \eta, \sigma, \tau) \right).$$

Lemma 2.1. Let $(M, E_{X,Y}, \mathcal{S})$ be a MIGFM-space. Then for all $\vartheta, \eta, \sigma \in M$, $\tau > 0$, $E_{X,Y}(\vartheta, \eta, \sigma, \tau)$ is non-decreasing with respect to τ in $(\mathcal{H}^*, \leq_{\mathcal{H}^*})$.

Proof. As $\tau > 0$ and $\tau + s > \tau$, where $s > 0$, by letting $\kappa = \vartheta$ in the condition (v) of definition of MIGFM-space, one can have

$$E_{X,Y}(\vartheta, \eta, \sigma, \tau + s) \geq_{\mathcal{H}^*} \mathcal{S}(E_{X,Y}(\vartheta, \vartheta, \vartheta, s), E_{X,Y}(\vartheta, \eta, \sigma, \tau)) = E_{X,Y}(\vartheta, \eta, \sigma, \tau).$$

This implies that $(M, E_{X,Y}, \mathcal{S})$ is non-decreasing w.r.t τ in $(\mathcal{H}^*, \leq_{\mathcal{H}^*})$. □

Definition 2.2. Let $(M, E_{X,Y}, \mathcal{S})$ be a MIGFM-space. A sequence $\{\vartheta_a\}$ is said to converges to a point $\vartheta \in M$ if $E_{X,Y}(\vartheta_a, \vartheta_a, \vartheta, \tau) \rightarrow 1_{\mathcal{H}^*}$ whenever $a \rightarrow \infty$ for every $\tau > 0$.

Definition 2.3. A sequence $\{\vartheta_a\}$ in MIGFM-space $(M, E_{X,Y}, \mathcal{S})$ is called a Cauchy sequence if for $0 < \epsilon < 1$, $\tau > 0$, $\exists r_0 \in \mathbb{N}$ such that $E_{X,Y}(\vartheta_a, \vartheta_s, \vartheta, \tau) >_{\mathcal{H}^*} (\mathcal{R}_s(\epsilon), \epsilon)$ for each $a, s \geq r_0$, here \mathcal{R}_s is the standard negation on $([0, 1], \leq)$.

We know that \mathcal{S} is a continuous τ -representable on lattice \mathcal{H}^* as for every $\delta \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$, there is a $\alpha \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ such that

$$\mathcal{S}^{a-1} \{(\mathcal{R}(\alpha), \alpha), (\mathcal{R}(\alpha), \alpha), \dots, (\mathcal{R}(\alpha), \alpha)\} >_{\mathcal{H}^*} (\mathcal{R}(\delta), \delta).$$

Lemma 2.2. Let $(M, E_{X,Y}, \mathcal{S})$ be a MIGFM-space.

Define $\chi_{\alpha, E_{X,Y}} = \inf \{\tau > 0, E_{X,Y}(\vartheta, \eta, \sigma, \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha)\}$ for each $\alpha \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ and $\vartheta, \eta, \sigma \in M$. Then we have

(1) for any $\delta \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$, there exists $\alpha \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ such that

$$\begin{aligned} \chi_{\delta, E_{X,Y}}(\vartheta_1, \vartheta_2, \vartheta_a) &\leq \chi_{\alpha, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_2) + \chi_{\alpha, E_{X,Y}}(\vartheta_2, \vartheta_2, \vartheta_3) \\ &+ \dots + \chi_{\alpha, E_{X,Y}}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a) \end{aligned}$$

for any $\vartheta_1, \vartheta_2, \dots, \vartheta_a \in M$.

(2) The sequence $\{\vartheta_a\}$ is convergent in MIGFM-space $(M, E_{X,Y}, \mathcal{S})$ if and only if $\chi_{\alpha, E_{X,Y}}(\vartheta_a, \vartheta_a, \vartheta) \rightarrow 0$. Also the sequence $\{\vartheta_a\}$ is a Cauchy sequence if and only if it is Cauchy with $\chi_{\alpha, E_{X,Y}}$.

Proof. For first part, for every $\delta \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$, there is a element $\alpha \in \mathcal{H} / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ such that

$$\mathcal{S}^{a-1} \{(\mathcal{R}(\alpha), \alpha), (\mathcal{R}(\alpha), \alpha), \dots, (\mathcal{R}(\alpha), \alpha)\} >_{\mathcal{H}^*} (\mathcal{R}(\delta), \delta).$$

From definition of MIGFM-space, we have

$$\begin{aligned}
& E_{X,Y}(\vartheta_1, \vartheta_1, \vartheta_a, \chi_{\alpha, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_2) + \chi_{\alpha, E_{X,Y}}(\vartheta_2, \vartheta_2, \vartheta_3) + \cdots + \\
& \chi_{\alpha, E_{X,Y}}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a) + a\epsilon) \\
& \geq_{\mathcal{H}^*} \mathcal{S}^{a-1} \left(E_{X,Y}(\vartheta_1, \vartheta_1, \vartheta_2, \chi_{\alpha, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_2) + \epsilon), \right. \\
& E_{X,Y}(\vartheta_2, \vartheta_2, \vartheta_3, \chi_{\alpha, E_{X,Y}}(\vartheta_2, \vartheta_2, \vartheta_3) + \epsilon), \\
& \left. \cdots, E_{X,Y}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a, \chi_{\alpha, E_{X,Y}}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a) + \epsilon) \right) \\
& \geq_{\mathcal{H}^*} \mathcal{S}^{a-1} \{(\mathcal{R}(\alpha), \alpha), (\mathcal{R}(\alpha), \alpha), \cdots (\mathcal{R}(\alpha), \alpha)\} >_{\mathcal{H}^*} (\mathcal{R}(\delta), \delta),
\end{aligned}$$

this gives that

$$\begin{aligned}
\chi_{\delta, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_a) & \leq \chi_{\alpha, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_2) + \chi_{\alpha, E_{X,Y}}(\vartheta_2, \vartheta_2, \vartheta_3) \\
& + \cdots + \chi_{\alpha, E_{X,Y}}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a) + a\epsilon.
\end{aligned}$$

Since $\epsilon > 0$ is arbitrary, we get

$$\begin{aligned}
\chi_{\delta, E_{X,Y}}(\vartheta_1, \vartheta_2, \vartheta_a) & \leq \chi_{\alpha, E_{X,Y}}(\vartheta_1, \vartheta_1, \vartheta_2) + \chi_{\alpha, E_{X,Y}}(\vartheta_2, \vartheta_2, \vartheta_3) + \cdots + \\
& \chi_{\alpha, E_{X,Y}}(\vartheta_{a-1}, \vartheta_{a-1}, \vartheta_a).
\end{aligned}$$

For second part, we know that $E_{X,Y}$ is continuous and

$$\chi_{\alpha, E_{X,Y}}(\vartheta, \vartheta, \eta) = \inf \{ \tau > 0, E_{X,Y}(\vartheta, \vartheta, \eta, \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha) \}.$$

Hence we have $E_{X,Y}(\vartheta_a, \vartheta_a, \vartheta, \vartheta) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha)$. This gives that $E_{X,Y}(\vartheta_a, \vartheta_a, \vartheta) < \vartheta$ for every $\vartheta > 0$. \square

Lemma 2.3. Let $(M, E_{X,Y}, \mathcal{S})$ be a MIGFM-space. If

$E_{X,Y}(\vartheta_a, \vartheta_a, \vartheta_{a+1}, \tau) \geq_{\mathcal{H}^*} E_{X,Y}(\vartheta_0, \vartheta_0, \vartheta_1, \kappa^a \tau)$ for some $\kappa > 1$, $a \in \mathbb{N}$, then $\{\vartheta_a\}$ is a Cauchy sequence.

Proof. For every $\alpha \in H / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$, we have

$$\begin{aligned}
\chi_{\alpha, E_{X,Y}}(\vartheta_a, \vartheta_a, \vartheta_{a+1}, \tau) & = \inf \{ \tau > 0, E_{X,Y}(\vartheta_a, \vartheta_a, \vartheta_{a+1}, \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha) \} \\
& \leq \inf \{ \tau > 0, E_{X,Y}(\vartheta_0, \vartheta_0, \vartheta_1, \kappa^a \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha) \} \\
& = \inf \left\{ \frac{\tau}{\kappa^a} > 0, E_{X,Y}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha) \right\} \\
& = \frac{1}{\kappa^a} \inf \{ \tau > 0, E_{X,Y}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) >_{\mathcal{H}^*} (\mathcal{R}(\alpha), \alpha) \} \\
& = \frac{1}{\kappa^a} \chi_{\alpha, E_{X,Y}}(\vartheta_0, \vartheta_0, \vartheta_1, \tau).
\end{aligned}$$

By using Lemma 2.2, for any $\delta \in H \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$, there exists $\alpha \in H \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ such that

$$\begin{aligned} & \chi_{\delta, E_{X,Y}}(\vartheta_a, \vartheta_a, \vartheta_s) \\ & \leq \chi_{\alpha, E_{X,Y}}(\vartheta_a, \vartheta_a, \vartheta_{a+1}) + \chi_{\alpha, E_{X,Y}}(\vartheta_{a+1}, \vartheta_{a+1}, \vartheta_{a+2}) + \cdots + \chi_{\alpha, E_{X,Y}}(\vartheta_{s-1}, \vartheta_{s-1}, \vartheta_s) \\ & \leq \frac{1}{\kappa^a} \chi_{\alpha, E_{X,Y}}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) + \frac{1}{\kappa^{a+1}} \chi_{\alpha, E_{X,Y}}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) + \cdots + \\ & \quad \frac{1}{\kappa^{s-1}} \chi_{\alpha, E_{X,Y}}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) \\ & = \chi_{\alpha, E_{X,Y}}(\vartheta_0, \vartheta_0, \vartheta_1, \tau) \sum_{i=a}^{s-1} \frac{1}{\kappa^i} \rightarrow 0. \end{aligned}$$

This implies that sequence $\{\vartheta_a\}$ is a Cauchy sequence. □

Lemma 2.4. Let $(M, E_{X,Y}, \mathcal{S})$ be a MIGFM-space. Then $E_{X,Y}$ is continuous function on $M^3 \times (0, \infty)$.

Definition 2.4. The mappings $Y : M \times M \rightarrow M$ and $\Omega : M \rightarrow M$ are said to compatible mappings define on a MIGFM-space $(M, E_{X,Y}, \mathcal{S})$ if

$$\begin{aligned} E_{X,Y}(\Omega Y(\vartheta_a, \eta_a), \Omega Y(\vartheta_a, \eta_a), Y(\Omega \vartheta_a, \Omega \eta_a), \tau) &= 1_{\mathcal{H}^*}, \\ E_{X,Y}(\Omega Y(\eta_a, \vartheta_a), \Omega Y(\eta_a, \vartheta_a), Y(\Omega \eta_a, \Omega \vartheta_a), \tau) &= 1_{\mathcal{H}^*}, \end{aligned}$$

whenever $\{\vartheta_a\}$ and $\{\eta_a\}$ are sequences in M such that

$$\begin{aligned} \lim_{a \rightarrow \infty} \Omega(\vartheta_a) &= \lim_{a \rightarrow \infty} Y(\vartheta_a, \eta_a) = \vartheta, \\ \lim_{a \rightarrow \infty} \Omega(\eta_a) &= \lim_{a \rightarrow \infty} Y(\eta_a, \vartheta_a) = \eta \end{aligned}$$

for all $\vartheta, \eta \in M, \tau > 0$.

2.2. Coupled Coincident Point Results.

Theorem 2.1. Let $Y : M \times M \rightarrow M, \Omega : M \rightarrow M$ be functions on complete MIGFM-space $(M, E_{X,Y}, \mathcal{S})$ where (M, \leq) be a partially ordered set as

- (i) $Y(M \times M) \subset \Omega(M)$,
- (ii) Y has the mixed Ω -monotone property,
- (iii) $\forall \vartheta, \eta, u$ and $v \in M, \tau > 0$ as

$$E_{X,Y}(Y(\vartheta, \eta), Y(\vartheta, \eta), Y(u, v), \tau) \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\Omega \vartheta, \Omega \vartheta, \Omega u, \kappa \tau), \\ E_{X,Y}(\Omega \vartheta, \Omega \vartheta, Y(\vartheta, \eta), \kappa \tau), \\ E_{X,Y}(\Omega u, \Omega u, Y(u, v), \kappa \tau) \end{array} \right) \right\}$$

where $\kappa > 1, \phi : \mathcal{H}^* \rightarrow \mathcal{H}^*$ is continuous and $\phi(p) >_{\mathcal{H}^*} p$ for all $p \in \mathcal{H}^* / \{0_{\mathcal{H}^*}, 1_{\mathcal{H}^*}\}$ and also $\Omega(\vartheta) \leq \Omega(u), \Omega(\eta) \geq \Omega(v)$ or $\Omega(\vartheta) \geq \Omega(u), \Omega(\eta) \leq \Omega(v)$,

- (iv) Y, Ω satisfy compatibility property as well as continuity for Ω .

Also, suppose that

- (I) continuity for Y or

(II) M satisfy conditions as if non-decreasing sequence $\{\vartheta_a\}$ and $\{\eta_a\}$ as $\vartheta_a \rightarrow \vartheta, \eta_a \rightarrow \eta$ then $\vartheta_a \leq \vartheta, \eta_a \geq \eta \forall a \in \mathbb{N}$.

If $\exists \vartheta_0, \eta_0 \in M$ such that $\Omega(\vartheta_0) \leq Y(\vartheta_0, \eta_0)$ and $\Omega(\eta_0) \geq Y(\eta_0, \vartheta_0)$, then Y and Ω have a coupled coincidence point in M .

Proof. Let (ϑ_0, η_0) be a point in $M \times M$ as $\Omega(\vartheta_0) \leq Y(\vartheta_0, \eta_0)$ and $\Omega(\eta_0) \geq Y(\eta_0, \vartheta_0)$. By using condition (i), choose $\vartheta_1, \eta_1 \in M$ such that

$$Y(\vartheta_0, \eta_0) = \Omega(\vartheta_1), \quad Y(\eta_0, \vartheta_0) = \Omega(\eta_1). \quad (2.1)$$

Construct two sequences $\{\vartheta_a\}$ and $\{\eta_a\}$ in M such that

$$Y(\vartheta_a, \eta_a) = \Omega(\vartheta_{a+1}), \quad Y(\eta_a, \vartheta_a) = \Omega(\eta_{a+1}). \quad (2.2)$$

Now, with help of mathematical induction, we shall prove that

$$\Omega(\vartheta_a) \leq \Omega(\vartheta_{a+1}), \quad \Omega(\eta_a) \geq \Omega(\eta_{a+1}). \quad (2.3)$$

Let $a = 0$. Since

$$\Omega(\vartheta_0) \leq Y(\vartheta_0, \eta_0) \text{ and } \Omega(\eta_0) \geq Y(\eta_0, \vartheta_0).$$

Using (2.1), we have

$$\Omega(\vartheta_0) \leq \Omega(\vartheta_1) \text{ and } \Omega(\eta_0) \geq \Omega(\eta_1).$$

So, (2.3) satisfy for $a = 0$.

Since (2.3) holds for some fixed $s > 0$, one can see

$$\Omega(\vartheta_s) \leq \Omega(\vartheta_{s+1}), \quad \Omega(\eta_s) \geq \Omega(\eta_{s+1}).$$

As Y has the mixed Ω -monotone property and using (2.2), we have

$$\Omega(\vartheta_{a+1}) = Y(\vartheta_a, \eta_a) \leq Y(\vartheta_{a+1}, \eta_a), \quad \Omega(\eta_{a+1}) = Y(\eta_a, \vartheta_a) \geq Y(\eta_{a+1}, \vartheta_a). \quad (2.4)$$

Also

$$\Omega(\vartheta_{a+2}) = Y(\vartheta_{a+1}, \eta_{a+1}) \geq Y(\vartheta_{a+1}, \eta_a), \quad \Omega(\eta_{a+2}) = Y(\eta_{a+1}, \vartheta_{a+1}) \leq Y(\eta_{a+1}, \vartheta_a). \quad (2.5)$$

From (2.4) and (2.5), we get

$$\Omega(\vartheta_a) \leq \Omega(\vartheta_{a+1}), \quad \Omega(\eta_a) \geq \Omega(\eta_{a+1}). \quad (2.6)$$

Also, from (iii) and (2.2), we get

$$E_{X,Y}(Y(\vartheta_{a-1}, \eta_{a-1}), Y(\vartheta_{a-1}, \eta_{a-1}), Y(\vartheta_a, \eta_a), \tau) \\ \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau), \\ E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, Y(\vartheta_{a-1}, \eta_{a-1}), \kappa\tau), \\ E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, Y(\vartheta_a, \eta_a), \kappa\tau) \end{array} \right) \right\},$$

this implies,

$$E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \tau) \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau), \\ E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \kappa\tau) \end{array} \right) \right\}.$$

If $E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau) > E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \kappa\tau)$, then by the property $\phi(\tau) >_{\mathcal{H}^*} \tau$, we have

$$\begin{aligned} E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \tau) &\geq_{\mathcal{H}^*} \phi(E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \kappa\tau)) \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \kappa\tau), \end{aligned}$$

which is a contradiction (see Lemma 2.1).

If $E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau) < E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \kappa\tau)$, then

$$\begin{aligned} E_{X,Y}(\Omega\vartheta_a, \Omega\vartheta_a, \Omega\vartheta_{a+1}, \tau) &\geq_{\mathcal{H}^*} \phi(E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau)) \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\vartheta_{a-1}, \Omega\vartheta_{a-1}, \Omega\vartheta_a, \kappa\tau) \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\vartheta_{a-2}, \Omega\vartheta_{a-2}, \Omega\vartheta_{a-1}, \kappa^2\tau) \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\vartheta_{a-2}, \Omega\vartheta_{a-2}, \Omega\vartheta_{a-1}, \kappa^3\tau) \\ &\vdots \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\vartheta_0, \Omega\vartheta_0, \Omega\vartheta_1, \kappa^a\tau). \end{aligned}$$

From Lemma 2.4, we have $\{\Omega\vartheta_a\}$ is Cauchy sequence in M .

Taking $\vartheta = \eta_a, \eta = \vartheta_a, u = \eta_{a-1}, v = \vartheta_{a-1}$ in (iii), we get

$$\begin{aligned} &E_{X,Y}(Y(\eta_a, \vartheta_a), Y(\eta_a, \vartheta_a), Y(\eta_{a-1}, \vartheta_{a-1}), \tau) \\ &\geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a-1}, \kappa\tau), \\ E_{X,Y}(\Omega\eta_a, \Omega\eta_a, Y(\eta_a, \vartheta_a), \\ E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, Y(\eta_{a-1}, \vartheta_{a-1}), \kappa\tau) \end{array} \right) \right\} \end{aligned}$$

and

$$E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \tau) \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, \Omega(\eta_{a-1}), \kappa\tau), \\ E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega(\eta_{a+1}), \kappa\tau) \end{array} \right) \right\}.$$

If $E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, \Omega\eta_a, \kappa\tau) > E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \kappa\tau)$, then using the property $\phi(\tau) >_{\mathcal{H}^*} \tau$, we have

$$\begin{aligned} E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \tau) &\geq_{\mathcal{H}^*} \phi(E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \kappa\tau)) \\ &>_{\mathcal{H}^*} E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \kappa\tau), \end{aligned}$$

which is a contradiction (see Lemma 2.1).

If $E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, \Omega\eta_a, \tau) < E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \tau)$,

then

$$\begin{aligned}
 E_{X,Y}(\Omega\eta_a, \Omega\eta_a, \Omega\eta_{a+1}, \tau) &\geq_{\mathcal{H}^*} \phi(E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, \Omega\eta_a, \kappa\tau)) \\
 &>_{\mathcal{H}^*} E_{X,Y}(\Omega\eta_{a-1}, \Omega\eta_{a-1}, \Omega\eta_a, \kappa\tau) \\
 &>_{\mathcal{H}^*} E_{X,Y}(\Omega\eta_{a-2}, \Omega\eta_{a-2}, \Omega\eta_{a-1}, \kappa^2\tau) \\
 &>_{\mathcal{H}^*} E_{X,Y}(\Omega\eta_{a-3}, \Omega\eta_{a-3}, \Omega\eta_{a-2}, \kappa^3\tau) \\
 &\quad \vdots \\
 &>_{\mathcal{H}^*} E_{X,Y}(\Omega\eta_0, \Omega\eta_0, \Omega\eta_1, \kappa^a\tau).
 \end{aligned}$$

From Lemma 2.4, we have $\{\Omega\eta_a\}$ is a Cauchy sequence in M .

Since M is complete, there exist $\vartheta, \eta \in M$ such that

$$\begin{aligned}
 \lim_{a \rightarrow \infty} Y(\vartheta_a, \eta_a) &= \lim_{a \rightarrow \infty} \Omega(\vartheta_a) = \vartheta, \\
 \lim_{a \rightarrow \infty} Y(\eta_a, \vartheta_a) &= \lim_{a \rightarrow \infty} \Omega(\eta_a) = \eta.
 \end{aligned} \tag{2.7}$$

From condition (iv), we have as $a \rightarrow \infty$

$$\begin{aligned}
 E_{X,Y}(\Omega(Y(\vartheta_a, \eta_a)), \Omega(Y(\vartheta_a, \eta_a)), Y(\Omega(\vartheta_a), \Omega(\eta_a))) &= 1_{\mathcal{H}^*}, \\
 E_{X,Y}(\Omega(Y(\eta_a, \vartheta_a)), \Omega(Y(\eta_a, \vartheta_a)), Y(\Omega(\eta_a), \Omega(\vartheta_a))) &= 1_{\mathcal{H}^*}.
 \end{aligned} \tag{2.8}$$

Using continuity of Y and Ω in (2.8), we have

$$\begin{aligned}
 E_{X,Y}(\Omega(\vartheta), \Omega(\vartheta), Y(\vartheta, \eta), \tau) &= 1_{\mathcal{H}^*}, \\
 E_{X,Y}(\Omega(\eta), \Omega(\eta), Y(\eta, \vartheta), \tau) &= 1_{\mathcal{H}^*}.
 \end{aligned} \tag{2.9}$$

This implies $Y(\vartheta, \eta) = \Omega(\vartheta)$ and $Y(\eta, \vartheta) = \Omega(\eta)$.

Thus, established that Y and Ω have a coupled coincidence point in M .

Let (II) holds, then we have

$$\begin{aligned}
 \lim_{a \rightarrow \infty} E_{X,Y}(\Omega(Y(\vartheta_a, \eta_a)), \Omega(Y(\vartheta_a, \eta_a)), Y(\Omega(\vartheta_a), \Omega(\eta_a))) &= 1_{\mathcal{H}^*}, \\
 \lim_{a \rightarrow \infty} E_{X,Y}(\Omega(Y(\eta_a, \vartheta_a)), \Omega(Y(\eta_a, \vartheta_a)), Y(\Omega(\eta_a), \Omega(\vartheta_a))) &= 1_{\mathcal{H}^*}.
 \end{aligned} \tag{2.10}$$

Also, we have

$$\begin{aligned}
 \lim_{a \rightarrow \infty} \Omega(Y(\vartheta_a, \eta_a)) &= \lim_{a \rightarrow \infty} \Omega(\Omega\vartheta_a) = \Omega\vartheta, \\
 \lim_{a \rightarrow \infty} \Omega(Y(\eta_a, \vartheta_a)) &= \lim_{a \rightarrow \infty} \Omega(\Omega\eta_a) = \Omega\eta.
 \end{aligned} \tag{2.11}$$

By using definition of MIGFM-space and as $a \rightarrow \infty$, we get

$$\begin{aligned}
 E_{X,Y}(\Omega\vartheta, \Omega\eta, Y(\vartheta, \eta), \tau) &\geq_{\mathcal{H}^*} \mathcal{S} \left(\begin{array}{l} E_{X,Y}(\Omega(\Omega\vartheta_{a+1}), \Omega(\Omega\vartheta_{a+1}), Y(\vartheta, \eta), \kappa\eta), \\ E_{X,Y}(\Omega\vartheta, \Omega\vartheta, \Omega(\Omega\vartheta_{a-1}), \tau - \kappa\tau). \end{array} \right) \\
 &\geq_{\mathcal{H}^*} E_{X,Y}(\Omega(\Omega\vartheta_{a+1}), \Omega(\Omega\vartheta_{a+1}), Y(\vartheta, \eta), \kappa\tau).
 \end{aligned}$$

Using $\phi(\tau) >_{\mathcal{H}^*} \tau$, (3) and (2.8) in (2.10), we get

$$E_{X,Y}(\Omega\vartheta, \Omega\vartheta, Y(\vartheta, \eta), \tau) \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}((\Omega(\Omega\vartheta_a), \Omega(\Omega\vartheta_a), \Omega\vartheta, \tau)\kappa\tau), \\ E_{X,Y}(\Omega(\Omega\vartheta_a), \Omega(\Omega\vartheta_a), Y(\Omega\vartheta_a, \Omega\eta_a), \kappa\tau) \\ E_{X,Y}(\Omega\vartheta, \Omega\vartheta, Y(\vartheta, \eta), \kappa\tau) \end{array} \right) \right\} \\ >_{\mathcal{H}^*} \min \left(\begin{array}{l} E_{X,Y}((\Omega(\Omega\vartheta_a), \Omega(\Omega\vartheta_a), \Omega\vartheta, \tau)\kappa\tau), \\ E_{X,Y}(\Omega(\Omega\vartheta_a), \Omega(\Omega\vartheta_a), Y(\Omega\vartheta_a, \Omega\eta_a), \kappa\tau) \\ E_{X,Y}(\Omega\vartheta, \Omega\vartheta, Y(\vartheta, \eta), \kappa\tau) \end{array} \right).$$

By letting $a \rightarrow \infty$, we have

$$\lim_{a \rightarrow \infty} E_{X,Y}(\Omega\vartheta, \Omega\vartheta, Y(\vartheta, \eta), \tau) = 1_{\mathcal{H}^*}, \\ \lim_{a \rightarrow \infty} E_{X,Y}(\Omega\eta, \Omega\eta, Y(\eta, \vartheta), \tau) = 1_{\mathcal{H}^*}. \tag{2.12}$$

This implies that $Y(\vartheta, \eta) = \Omega(\vartheta)$. Similarly we get $Y(\eta, \vartheta) = \Omega(\eta)$.

Hence, Y and Ω have a coupled coincidence point in M . □

The following theorem is proved by letting $\Omega = I$ in Theorem 2.1.

Theorem 2.2. Let $Y : M \times M \rightarrow M$ is a function on complete MIGFM-space $(M, E_{X,Y}, \mathcal{S})$ where (M, \leq) be a partially ordered set as

- (1) mixed monotone property for Y ,
- (2) $\exists \kappa \in (0, 1)$ as

$$E_{X,Y}(Y(\vartheta, \eta), Y(\vartheta, \eta), Y(u, v), \tau) \geq_{\mathcal{H}^*} \phi \left\{ \min \left(\begin{array}{l} E_{X,Y}(\vartheta, \vartheta, u, \kappa\tau), \\ E_{X,Y}(\vartheta, \vartheta, Y(\vartheta, \eta), \kappa\tau) \\ E_{X,Y}(u, u, Y(u, v), \kappa\tau) \end{array} \right) \right\}$$

for all $\vartheta, \eta, u, v \in M, \tau > 0$ for which $\vartheta \leq u$ and $\eta \geq v$ or $\vartheta \geq u$ and $\eta \leq v$.

Moreover, if $\exists \vartheta_0 \eta_0 \in M$ as $\vartheta_0 \leq Y(\vartheta_0 \eta_0)$ and $\eta_0 \geq Y(\eta_0, \vartheta_0)$, then Y has a coupled fixed point in M while either one of the conditions (I) or (II) of Theorem 2.1 holds

An illustration is proved here for the support of Theorem 2.1.

Example 2.1. Let (M, \leq) be a partially ordered set with $M = [0, 1]$.

Define

$$\mathcal{S}(x, y) = (x_1 y_1, \min(x_2 + y_2, 1)).$$

for all $x = (x_1, x_2), y = (y_1, y_2)$ in \mathcal{H}^* .

Let $Y : M \times M \rightarrow M$ and $\Omega : M \rightarrow M$ are two mappings defined as

$$Y(\vartheta, \eta) = \begin{cases} \frac{\vartheta^4 - \eta^4}{4}, & \text{if } \vartheta \geq \eta \\ 0, & \text{if } \vartheta < \eta \end{cases}, \quad \Omega(\vartheta) = \vartheta^4.$$

This implies Y satisfies the definition of mixed Ω -monotone property. Let $E_{X,Y}$ is a mapping $M \times M \times M \times [0, \infty) \rightarrow \mathcal{H}^*$ defined as

$$E_{X,Y}(\vartheta_1, \vartheta_2, \vartheta_3, \tau) = \left(\frac{\tau}{\tau + G(\vartheta_1, \vartheta_2, \vartheta_3)}, \frac{G(\vartheta_1, \vartheta_2, \vartheta_3)}{\tau + G(\vartheta_1, \vartheta_2, \vartheta_3)} \right),$$

where $G(\vartheta_1, \vartheta_2, \vartheta_3)$ is G -metric space defined as

$$G(\vartheta_1, \vartheta_2, \vartheta_3) = |\vartheta_1 - \vartheta_2| + |\vartheta_2 - \vartheta_3| + |\vartheta_3 - \vartheta_1|$$

for all $\vartheta_1, \vartheta_2, \vartheta_3 \in M, \tau > 0$.

Then $(M, E_{X,Y}, \mathcal{S})$ is a complete MIGFM-space.

Assume the sequences $\{\vartheta_a\}$ and $\{\eta_a\}$ in M as

$$\begin{aligned} \lim_{a \rightarrow \infty} Y(\vartheta_a, \eta_a) &= \lim_{a \rightarrow \infty} \Omega(\vartheta_a) = 0 = v, \\ \lim_{a \rightarrow \infty} Y(\eta_a, \vartheta_a) &= \lim_{a \rightarrow \infty} \Omega(\eta_a) = 0 = v'. \end{aligned}$$

Define

$$Y(\vartheta_a, \eta_a) = \begin{cases} \frac{\vartheta_a^4 - \eta_a^4}{4}, & \text{if } \vartheta_a \geq \eta_a \\ 0, & \text{if } \vartheta_a < \eta_a \end{cases}, \quad \Omega(\vartheta_a) = \vartheta_a^4$$

and

$$Y(\eta_a, \vartheta_a) = \begin{cases} \frac{\eta_a^4 - \vartheta_a^4}{4}, & \text{if } \eta_a \geq \vartheta_a \\ 0, & \text{if } \eta_a < \vartheta_a \end{cases}, \quad \Omega(\eta_a) = \eta_a^4.$$

This gives that

$$\begin{aligned} \lim_{a \rightarrow \infty} E_{X,Y}(\Omega(Y(\vartheta_a, \eta_a)), \Omega(Y(\vartheta_a, \eta_a)), Y(\Omega(\vartheta_a, \eta_a))) &= 1_{\mathcal{H}^*}, \\ \lim_{a \rightarrow \infty} E_{X,Y}(\Omega(Y(\eta_a, \vartheta_a)), \Omega(Y(\eta_a, \vartheta_a)), Y(\Omega(\eta_a, \vartheta_a))) &= 1_{\mathcal{H}^*}. \end{aligned}$$

Thus $Y : M \times M \rightarrow M$ and $\Omega : M \rightarrow M$ are compatible mappings in M .

By Theorem 2.1, one can have $\Omega(\vartheta) \leq \Omega(p), \Omega(\eta) \geq \Omega(q)$.

This implies $\vartheta^4 \leq p^4, \eta^4 \geq q^4$.

If we consider $\vartheta \geq \eta, p \geq q$, then we have

$$\begin{aligned} E_{X,Y}(Y(\vartheta, \eta), Y(\vartheta, \eta), Y(p, q), \tau) &= \left\{ \frac{2\tau}{2\tau + |(\vartheta^4 - \eta^4) - (p^4 - q^4)|}, \frac{|(\vartheta^4 - \eta^4) - (p^4 - q^4)|}{2\tau + |(\vartheta^4 - \eta^4) - (p^4 - q^4)|} \right\} \\ &\geq_{\mathcal{H}^*} \left\{ \frac{2\tau}{2\tau + |\frac{3p^4 + q}{2}|}, \frac{|\frac{3p^4 + q}{2}|}{2\tau + |\frac{3p^4 + q}{2}|} \right\} \\ &= E_{X,Y}(\Omega p, \Omega p, Y(p, q), 2\tau) \\ &\geq_{\mathcal{H}^*} \min \left(\begin{array}{l} E_{X,Y}(\Omega \vartheta, \Omega \vartheta, \Omega p, 2\tau), \\ E_{X,Y}(\Omega \vartheta, \Omega \vartheta, Y(\vartheta, \eta), 2\tau), \\ E_{X,Y}(\Omega p, \Omega p, Y(p, q), 2\tau) \end{array} \right). \end{aligned}$$

If $\vartheta < \eta$, $p \geq q$, then we get

$$\begin{aligned} E_{X,Y}(Y(\vartheta, \eta), Y(\vartheta, \eta), Y(p, q), \tau) &= \frac{2\tau}{2\tau + |p^4 - q^4|} \\ &\geq_{\mathcal{H}^*} \left\{ \frac{2\tau}{2\tau + |p^4 - \vartheta^4|}, \frac{|p^4 - \vartheta^4|}{2\tau + |p^4 - \vartheta^4|} \right\} \\ &= E_{X,Y}(\Omega\vartheta, \Omega\vartheta, \Omega p, 2\tau) \\ &\geq_{\mathcal{H}^*} \min \left(\begin{array}{l} E_{X,Y}(\Omega\vartheta, \Omega\vartheta, \Omega p, 2\tau), \\ E_{X,Y}(\Omega\vartheta, \Omega\vartheta, Y(\vartheta, \eta), 2\tau), \\ E_{X,Y}(\Omega p, \Omega p, Y(p, q), 2\tau) \end{array} \right). \end{aligned}$$

If we let $\vartheta < \eta$, $p < q$, then we get condition (iii) of Theorem 2.1.

Hence, all conditions of Theorem 2.1 are satisfied with $\kappa = 2$.

So, (v, v') is the coupled coincidence point of Y and Ω .

Author's Contributions: All the authors contributed equally to prepare this paper.

Funding: This work was supported by Directorate of Research and Innovation, Walter Sisulu University, South Africa.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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