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CONVERGENCE TO COMMON FIXED POINT FOR NEARLY ASYMPTOTICALLY NONEXPANSIVE MAPPINGS IN BANACH SPACES

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Abstract. The purpose of this paper is to study modified S-iteration process to converge to common fixed point for two nearly asymptotically nonexpansive mappings in the framework of Banach spaces. Also we establish some strong convergence theorems and a weak convergence theorem for said mappings and iteration scheme under appropriate conditions.

1. Introduction

Let C be a nonempty subset of a Banach space E and $T: C \to C$ a nonlinear mapping. We denote the set of all fixed points of T by $F(T)$. The set of common fixed points of two mappings S and T will be denoted by $F = F(S) \cap F(T)$. The mapping T is said to be Lipschitzian [1, 16] if for each $n \in \mathbb{N}$, there exists a constant $k_n > 0$ such that

$$\|T^n x - T^n y\| \leq k_n \|x - y\|$$

for all $x,\, y \in C$.

A Lipschitzian mapping T is said to be uniformly k-Lipschitzian if $k_n = k$ for all $n \in \mathbb{N}$ and asymptotically nonexpansive [4] if $k_n \geq 1$ for all $n \in \mathbb{N}$ with $\lim_{n\to\infty} k_n = 1$.

It is easy to observe that every nonexpansive mapping T (i.e., $\|Tx - Ty\| \leq \|x - y\|$ for all $x,\, y \in C$) is asymptotically nonexpansive with constant sequence $\{1\}$ and every asymptotically nonexpansive mapping is uniformly k-Lipschitzian with $k = \sup_{n\in\mathbb{N}} k_n$.

The asymptotic fixed point theory has a fundamental role in nonlinear functional analysis (see, [2]). The theory has been studied by many authors (see, e.g., [6], [7], [10], [12], [21]) for various classes of nonlinear mappings (e.g., Lipschitzian, uniformly k-Lipschitzian and non-Lipschitzian mappings). A branch of this theory related to asymptotically nonexpansive mappings has been developed by many authors (see, e.g., [4], [5], [9], [11], [12], [14], [15], [17]-[19]) in Banach spaces with

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suitable geometrical structure.

Fix a sequence $\{a_n\} \subset [0, \infty)$ with $\lim_{n\to\infty} a_n = 0$, then according to Agarwal et al. [1], T is said to be nearly Lipschitzian with respect to $\{a_n\}$ if for each $n \in \mathbb{N}$, there exist constants $k_n \geq 0$ such that $\|T^n x - T^n y\| \leq k_n(\|x - y\| + a_n)$ for all $x, y \in C$. The infimum of constants k_n for which the above inequality holds is denoted by $\eta(T^n)$ and is called nearly Lipschitz constant.

A nearly Lipschitzian mapping T with sequence $\{a_n, \eta(T^n)\}$ is said to be nearly asymptotically nonexpansive if $\eta(T^n) \geq 1$ for all $n \in \mathbb{N}$ and $\lim_{n\to\infty} \eta(T^n) = 1$ and nearly uniformly k-Lipschitzian if $\eta(T^n) \leq k$ for all $n \in \mathbb{N}$.

In 2007, Agarwal et al. [1] introduced the following iteration process:

$$\begin{aligned} x_1 &= x \in C, \\ x_{n+1} &= (1 - \alpha_n)T^n x_n + \alpha_n T^n y_n, \\ y_n &= (1 - \beta_n)x_n + \beta_n T^n x_n, \ n \geq 1 \end{aligned} \tag{1.1}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $(0, 1)$. They showed that this process converge at a rate same as that of Picard iteration and faster than Mann for contractions and also they established some weak convergence theorems using suitable conditions in the framework of uniformly convex Banach space.

We modify iteration scheme (1.1) for two nonlinear mappings.

Let C be a nonempty subset of a Banach space E and $S, T \colon C \to C$ be two nearly asymptotically nonexpansive mappings. For given $x_1 = x \in C$, the iterative sequence $\{x_n\}$ defined as follows:

$$\begin{aligned} x_1 &= x \in C, \\ x_{n+1} &= (1 - \alpha_n)T^n x_n + \alpha_n S^n y_n, \\ y_n &= (1 - \beta_n)x_n + \beta_n T^n x_n, \ n \geq 1 \end{aligned} \tag{1.2}$$

where $\{\alpha_n\}$ and $\{\beta_n\}$ are sequences in $(0, 1)$. The iteration scheme (1.2) is called modified S-iteration scheme for two nonlinear mappings.

If we put $S = T$, then iteration scheme (1.2) reduces to S-iteration scheme (1.1).

The aim of this paper is to establish some strong convergence theorems and a weak convergence theorem of modified S-iteration scheme (1.2) for two nearly asymptotically nonexpansive mappings in the framework of Banach spaces.

2. Preliminaries

For the sake of convenience, we restate the following concepts.

A mapping $T \colon C \to C$ is said to be demiclosed at zero, if for any sequence $\{x_n\}$ in C, the condition x_n converges weakly to $x \in C$ and Tx_n converges strongly to 0 imply $Tx = 0$.

A mapping $T\colon C \to C$ is said to be semi-compact [3] if for any bounded sequence $\{x_n\}$ in C such that $\|x_n - Tx_n\| \to 0$ as $n \to \infty$, then there exists a subsequence $\{x_{n_k}\} \subset \{x_n\}$ such that $x_{n_k} \to x^* \in C$ strongly.

We say that a Banach space E satisfies the *Opial's condition* [13] if for each sequence $\{x_n\}$ in E weakly convergent to a point x and for all $y \neq x$

$$\liminf_{n\to\infty} \|x_n - x\| < \liminf_{n\to\infty} \|x_n - y\|.$$

The examples of Banach spaces which satisfy the Opial's condition are Hilbert spaces and all $L^p[0, 2\pi]$ with $1 < p \neq 2$ fail to satisfy Opial's condition [13].

Now, we state the following useful lemma to prove our main results.

Lemma 2.1. (See [20]) Let $\{\alpha_n\}_{n=1}^{\infty}$, $\{\beta_n\}_{n=1}^{\infty}$ and $\{r_n\}_{n=1}^{\infty}$ be sequences of nonnegative numbers satisfying the inequality

$$\alpha_{n+1} \leq (1+\beta_n)\alpha_n + r_n, \quad \forall n \geq 1.$$

If $\sum_{n=1}^{\infty} \beta_n < \infty$ and $\sum_{n=1}^{\infty} r_n < \infty$, then $\lim_{n\to\infty} \alpha_n$ exists.

3. **Main Results**

In this section, we prove some strong convergence theorems and a weak convergence theorem for two nearly asymptotically nonexpansive mappings in the framework of Banach spaces.

Theorem 3.1. Let E be a Banach space and C be a nonempty closed convex subset of E. Let $S, T\colon C \to C$ be two nearly asymptotically nonexpansive mappings with sequences $\{a'_n, \eta(S^n)\}$, $\{a''_n, \eta(T^n)\}$ and $F = F(S) \cap F(T) \neq \emptyset$ is closed such that $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} \Big(\eta(S^n)\eta(T^n) - 1\Big) < \infty$. Let $\{x_n\}$ be the modified S-iteration scheme defined by (1.2). Then $\{x_n\}$ converges to a common fixed point of the mappings S and T if and only if $\liminf_{n\to\infty} d(x_n, F) = 0$.

Proof. The necessity is obvious. Thus we only prove the sufficiency. Let $q \in F$. For the sake of convenience, set

$$A_n x = (1-\beta_n)x + \beta_n T^n x$$

and

$$G_n x = (1-\alpha_n)T^n x + \alpha_n S^n A_n x.$$

Then $y_n = A_n x_n$ and $x_{n+1} = G_n x_n$. Moreover, it is clear that q is a fixed point of G_n for all n. Let $\eta = \sup_{n\in\mathbb{N}} \eta(S^n) \vee \sup_{n\in\mathbb{N}} \eta(T^n)$ and $a_n = \max\{a'_n, a''_n\}$ for all n.

Consider

$$\begin{aligned} \|A_nx - A_ny\| &= \|((1-\beta_n)x + \beta_nT^nx) - ((1-\beta_n)y + \beta_nT^ny)\| \\ &= \|(1-\beta_n)(x-y) + \beta_n(T^nx - T^ny)\| \\ &\leq (1-\beta_n)\|x-y\| + \beta_n\eta(T^n)(\|x-y\| + a''_n) \\ &\leq (1-\beta_n)\|x-y\| + \beta_n\eta(T^n)\|x-y\| + \beta_na_n\eta(T^n) \\ &\leq (1-\beta_n)\eta(T^n)\|x-y\| + \beta_n\eta(T^n)\|x-y\| \\ &\quad +\beta_na_n\eta(T^n) \\ &\leq \eta(T^n)\|x-y\| + a_n\eta(T^n). \qquad (3.1) \end{aligned}$$

Choosing $x = x_n$ and $y = q$, we get

$$\|y_n - q\| \leq \eta(T^n)\|x_n - q\| + a_n\eta(T^n). \qquad (3.2)$$

Now, consider

$$\begin{aligned} \|G_nx - G_ny\| &= \|((1-\alpha_n)T^nx + \alpha_nS^nA_nx) - ((1-\alpha_n)T^ny + \alpha_nS^nA_ny)\| \\ &= \|(1-\alpha_n)(T^nx - T^ny) + \alpha_n(S^nA_nx - S^nA_ny)\| \\ &\leq (1-\alpha_n)\eta(T^n)(\|x-y\| + a''_n) + \alpha_n\eta(S^n)(\|A_nx - A_ny\| + a'_n) \\ &\leq (1-\alpha_n)\eta(T^n)(\|x-y\| + a_n) + \alpha_n\eta(S^n)(\|A_nx - A_ny\| + a_n) \\ &\leq (1-\alpha_n)\eta(T^n)\|x-y\| + \alpha_n\eta(S^n)\|A_nx - A_ny\| \\ &\quad +(1-\alpha_n)a_n\eta(T^n) + \alpha_na_n\eta(S^n). \qquad (3.3) \end{aligned}$$

Now using (3.1) in (3.3), we get

$$\begin{aligned} \|G_nx - G_ny\| &\leq (1-\alpha_n)\eta(T^n)\|x-y\| + \alpha_n\eta(S^n)[\eta(T^n)\|x-y\| \\ &\quad +a_n\eta(T^n)] + (1-\alpha_n)a_n\eta(T^n) + \alpha_na_n\eta(S^n) \\ &\leq (1-\alpha_n)\eta(T^n)\eta(S^n)\|x-y\| + \alpha_n\eta(T^n)\eta(S^n)\|x-y\| \\ &\quad +(1-\alpha_n+2\alpha_n)a_n\eta(T^n)\eta(S^n) \\ &\leq \eta(T^n)\eta(S^n)\|x-y\| + 2a_n\eta(T^n)\eta(S^n) \\ &\leq \Big[1 + \Big(\eta(T^n)\eta(S^n) - 1\Big)\Big]\|x-y\| + 2a_n\eta^2 \\ &= (1+P_n)\|x-y\| + Q_n, \qquad (3.4) \end{aligned}$$

where $P_n = \Big(\eta(T^n)\eta(S^n) - 1\Big)$ and $Q_n = 2a_n\eta^2$. Since by hypothesis $\sum_{n=1}^{\infty}\Big(\eta(S^n)\eta(T^n) - 1\Big) < \infty$ and $\sum_{n=1}^{\infty} a_n < \infty$. It follows that $\sum_{n=1}^{\infty} P_n < \infty$ and $\sum_{n=1}^{\infty} Q_n < \infty$.

Choosing $x = x_n$ and $y = q$ in (3.4), we get

$$\|x_{n+1} - q\| = \|G_nx_n - q\| \leq (1+P_n)\|x_n - q\| + Q_n. \qquad (3.5)$$

Applying Lemma 2.1 in (3.5), we have $\lim_{n\to\infty}\|x_n - q\|$ exists.

Next, we shall prove that $\{x_n\}$ is a Cauchy sequence. Since $1+x \le e^x$ for $x \ge 0$, therefore, for any $m, n \ge 1$ and for given $q \in F$, from (3.5), we have

$$\begin{aligned}
\|x_{n+m} - q\| &\le (1 + P_{n+m-1})\|x_{n+m-1} - q\| + Q_{n+m-1} \\
&\le e^{P_{n+m-1}}\|x_{n+m-1} - q\| + Q_{n+m-1} \\
&\le e^{P_{n+m-1}}[e^{P_{n+m-2}}\|x_{n+m-2} - q\| + Q_{n+m-2}] + Q_{n+m-1} \\
&\le e^{(P_{n+m-1}+P_{n+m-2})}\|x_{n+m-2} - q\| \\
&\quad + e^{(P_{n+m-1}+P_{n+m-2})}[Q_{n+m-2} + Q_{n+m-1}] \\
&\le \dots \\
&\le e^{\left(\sum_{k=n}^{n+m-1} P_k\right)}\|x_n - q\| + e^{\left(\sum_{k=n}^{n+m-1} P_k\right)}\sum_{k=n}^{n+m-1} Q_k \\
&\le e^{\left(\sum_{n=1}^{\infty} P_n\right)}\|x_n - q\| + e^{\left(\sum_{n=1}^{\infty} P_n\right)}\sum_{k=n}^{n+m-1} Q_k \\
&= K\,\|x_n - q\| + K\sum_{k=n}^{n+m-1} Q_k \qquad (3.6)
\end{aligned}$$

where $K = e^{\left(\sum_{n=1}^{\infty} P_n\right)} < \infty$. Since

$$\lim_{n\to\infty} d(x_n, F) = 0, \qquad \sum_{n=1}^{\infty} Q_n < \infty \qquad (3.7)$$

for any given $\varepsilon > 0$, there exists a positive integer n_1 such that

$$d(x_n, F) < \frac{\varepsilon}{4(K+1)}, \qquad \sum_{k=n}^{n+m-1} Q_k < \frac{\varepsilon}{2K} \quad \forall\, n \ge n_1. \qquad (3.8)$$

Hence, there exists $q_1 \in F$ such that

$$\|x_n - q_1\| < \frac{\varepsilon}{2(K+1)} \quad \forall\, n \ge n_1. \qquad (3.9)$$

Consequently, for any $n \ge n_1$ and $m \ge 1$, from (3.6), we have

$$\begin{aligned}
\|x_{n+m} - x_n\| &\le \|x_{n+m} - q_1\| + \|x_n - q_1\| \\
&\le K\,\|x_n - q_1\| + K\sum_{k=n}^{n+m-1} Q_k + \|x_n - q_1\| \\
&\le (K+1)\|x_n - q_1\| + K\sum_{k=n}^{n+m-1} Q_k \\
&< (K+1)\,\frac{\varepsilon}{2(K+1)} + K\,\frac{\varepsilon}{2K} = \varepsilon. \qquad (3.10)
\end{aligned}$$

This implies that $\{x_n\}$ is a Cauchy sequence in E and so is convergent since E is complete. Assume that $\lim_{n\to\infty} x_n = q^*$. Since C is closed, therefore $q^* \in C$. Next, we show that $q^* \in F$. Now $\lim_{n\to\infty} d(x_n, F) = 0$ gives that $d(q^*, F) = 0$. Since F is closed, $q^* \in F$. Thus $\{x_n\}$ converges strongly to a common fixed point of the mappings S and T. This completes the proof.

Theorem 3.2. Let E be a Banach space and C be a nonempty closed convex subset of E. Let $S, T\colon C \to C$ be two nearly asymptotically nonexpansive mappings with sequences $\{a'_n, \eta(S^n)\}$, $\{a''_n, \eta(T^n)\}$ and $F = F(S) \cap F(T) \neq \emptyset$ is closed such that $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} \Big(\eta(S^n)\eta(T^n) - 1\Big) < \infty$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[\delta, 1-\delta]$ for some $\delta \in (0,1)$. Let $\{x_n\}$ be the modified S-iteration scheme defined by (1.2). If either S is semi-compact and $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ or T is semi-compact and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$, then the sequence $\{x_n\}$ converge strongly to a point of F.

proof. Suppose that T is semi-compact and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. Then there exists a subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $x_{n_j} \to q \in C$. Also, we have $\lim_{j\to\infty} \|x_{n_j} - Tx_{n_j}\| = 0$ and we make use of the fact that every nearly asymptotically nonexpansive mapping is nearly k-Lipschitzian. Hence, we have

$$\begin{aligned} \|q - Tq\| &\leq \|q - x_{n_j}\| + \|x_{n_j} - Tx_{n_j}\| + \|Tx_{n_j} - Tq\| \\ &\leq (1+k)\|q - x_{n_j}\| + \|x_{n_j} - Tx_{n_j}\| \to 0. \end{aligned}$$

Thus $q \in F$. By (3.5),

$$\|x_{n+1} - q\| \leq (1 + P_n)\|x_n - q\| + Q_n.$$

Since by hypothesis $\sum_{n=1}^{\infty} P_n < \infty$ and $\sum_{n=1}^{\infty} Q_n < \infty$, by Lemma 2.2, $\lim_{n\to\infty} \|x_n - q\|$ exists and $x_{n_j} \to q \in F$ gives that $x_n \to q \in F$. This shows that $\{x_n\}$ converges strongly to a point of F. This completes the proof.

As an application of Theorem 3.1, we establish another strong convergence result as follows.

Theorem 3.3. Let E be a Banach space and C be a nonempty closed convex subset of E. Let $S, T\colon C \to C$ be two nearly asymptotically nonexpansive mappings with sequences $\{a'_n, \eta(S^n)\}$, $\{a''_n, \eta(T^n)\}$ and $F = F(S) \cap F(T) \neq \emptyset$ is closed such that $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} \Big(\eta(S^n)\eta(T^n) - 1\Big) < \infty$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[\delta, 1-\delta]$ for some $\delta \in (0,1)$. Let $\{x_n\}$ be the modified S-iteration scheme defined by (1.2). If S and T satisfy the following conditions:

(i) $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$.

(ii) There exists a constant $A > 0$ such that

$$\Big[a_1\|x_n - Sx_n\| + a_2\|x_n - Tx_n\|\Big] \geq A\, d(x_n, F)$$

where a_1 and a_2 are two non-negative real numbers such that $a_1 + a_2 = 1$.

Then the sequence $\{x_n\}$ converge strongly to a point of F.

proof. From conditions (i) and (ii), we have $\lim_{n\to\infty} d(x_n, F) = 0$, it follows as in the proof of Theorem 3.1, that $\{x_n\}$ must converge strongly to a point of F. This completes the proof.

Theorem 3.4. Let E be a Banach space satisfying Opial's condition and C be a nonempty closed convex subset of E. Let $S, T\colon C \to C$ be two nearly asymptotically nonexpansive mappings with sequences $\{a'_n, \eta(S^n)\}$, $\{a''_n, \eta(T^n)\}$ and $F = F(S) \cap F(T) \neq \emptyset$ is closed such that $\sum_{n=1}^{\infty} a_n < \infty$ and $\sum_{n=1}^{\infty} \Big(\eta(S^n)\eta(T^n) - 1\Big) < \infty$. Let $\{\alpha_n\}$ and $\{\beta_n\}$ be sequences in $[\delta, 1-\delta]$ for some $\delta \in (0,1)$. Let $\{x_n\}$ be the modified S-iteration scheme defined by (1.2). Suppose that S and T have a common fixed point, $I - S$ and $I - T$ are demiclosed at zero and $\{x_n\}$ is an approximating common fixed point sequence for S and T, that is, $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and $\lim_{n\to\infty} \|x_n - Tx_n\| = 0$. Then $\{x_n\}$ converges weakly to a common fixed point of S and T.

Proof: Let q be a common fixed point of S and T. Then $\lim_{n\to\infty} \|x_n - q\|$ exists as proved in Theorem 3.1. We prove that $\{x_n\}$ has a unique weak subsequential limit in $F = F(S) \cap F(T)$. For, let u and v be weak limits of the subsequences $\{x_{n_i}\}$ and $\{x_{n_j}\}$ of $\{x_n\}$, respectively. By hypothesis of the theorem, we know that $\lim_{n\to\infty} \|x_n - Sx_n\| = 0$ and $I - S$ is demiclosed at zero, therefore we obtain $Su = u$. Similarly, $Tu = u$. Thus $u \in F = F(S) \cap F(T)$. Again in the same fashion, we can prove that $v \in F = F(S) \cap F(T)$. Next, we prove the uniqueness. To this end, if u and v are distinct then by Opial's condition,

$$\begin{aligned}
\lim_{n\to\infty} \|x_n - u\| &= \lim_{n_i\to\infty} \|x_{n_i} - u\| \\
&< \lim_{n_i\to\infty} \|x_{n_i} - v\| \\
&= \lim_{n\to\infty} \|x_n - v\| \\
&= \lim_{n_j\to\infty} \|x_{n_j} - v\| \\
&< \lim_{n_j\to\infty} \|x_{n_i} - u\| \\
&= \lim_{n\to\infty} \|x_n - u\|.
\end{aligned}$$

This is a contradiction. Hence $u = v \in F$. Thus $\{x_n\}$ converges weakly to a common fixed point of the mappings S and T. This completes the proof.

Remark 3.1. Our results extend and generalize the corresponding results of [8], [14], [15], [17], [18], [20] and many others from the existing literature to the case of modified S-iteration scheme and more general class of nonexpansive and asymptotically nonexpansive mappings considered in this paper.

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