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The Influence of Quarantine and Uprooting Control Measures in Reducing Rice Tungro Disease through a Mathematical Model

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ABSTRACT. Tungro virus is one of the most important diseases that affect the rice plant, as it is known as cancer because of the severe damage it causes both in quantity and quality in production. This disease is transmitted by the green leafhoppers (Nephotettix virescens), which are the most responsible vector for the disease's transmission. In this paper, we consider a mathematical model that describes the transmission dynamics of vector-borne rice tungro disease (RTD), which represents the predator-prey interaction between insect vectors and biological agents. Moreover, we incorporated two control efforts to formulate the optimal control model (OCM) in order to examine the best strategy for reducing the infection of RTD. The description of the two implementing controls is quarantine control (u_1) such as uprooting and burning infected plants and chemical control (u_2) such as using insecticides, respectively. The Hamiltonian and necessary optimality conditions (NOCs) are presented based on Pontryagin's maximum principle (PMP). We show numerical simulations in some figures by using the forward-backward sweep method (FBSM) to investigate the suggested control strategies. The results demonstrate that each integrated strategy can reduce infection transmission, but the combination of the two controls is the best strategy for the others.

1. Introduction

Rice is the main crop in Asia, especially in China, it is playing an important role in Chinese life and most of them are depending on rice as a staple food with high productivity and nutritive

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value. Therefore, the government of Chinese increased the funding more than 2.5 times compared with maize and used a genetic improvement to increase [1]. But rice is susceptible to infection by more than forty types of diseases. Crop diseases reduce production so we must predict disease in the early stage to protect more crops [2]. In traditional ways to identify disease, we can use the visual symptoms caused by pathogens or identification of pathogens in the laboratory [3]. In our paper we focus on the tungro disease, which has the potential to do more damage, there are two types of tungro viruses rice tungro Bacilliform Virus (RTBV) and rice tungro spherical virus (RTSV), these viruses are transmitted by the green leafhoppers. The most common control strategies are by spraying insecticides which means it will reduce the number of green leafhoppers vector.

Mathematical modelling (MM) plays an important role in studying, analyzing and controlling the spread of diseases to protect susceptible plants population from infection and reduce the number of infected vectors population based on ordinary differential equations to describe infectious diseases [4,5]. The aim of MM is to provide an answer for all phenomena [6] Moreover, it is concerned with learning significant recommendations to achieve the objective of generating a model of a problem situation that clears the behavior of variables involved and how they tie in the phenomena [7]. The application of mathematics is evident in almost every aspect that we could think of when we budget our monthly income, manage our time, and even in the infrastructures we see outside so some countries like the Philippines put mathematics at the top of their education system [8]. Using the MM, we can discuss the dynamical model of tungro and the effectiveness of insecticide [9]. There are some papers in the literature that have discussed tungro disease (see e.g., [10-16]).

Optimal control theory (OCT) determines state trajectories and controls at a period of time in the presence of constraints represented in a dynamic system to maximize or minimize the objective function. The importance of OCT is to translate problems into mathematical models involving some suggested controls to be optimal control problems (OCP) and through it can understand the spreading mechanism of epidemic diseases to predict the future behavior of the transmission by using some control strategies to reduce the spread of these diseases [17]. The OCP is applied to many real-life problems in different fields for example, in [18] authors proposed a model of pine wilt disease with three optimal controls vaccination, cutting of infected trees and spraying with insecticide to reduce the number of infected trees.

In the system of breast cancer model [19], the aim is to control the spreading of cancer by using two controls chemotherapy and a ketogenic diet which have an effective role to control this disease. In [20], the model of cholera has been considered, the aim of this research is to control the spreading of cholera by using three controls vaccination, therapeutic treatment (including hydration therapy, antibiotics, etc.), and water sanitation. In [21], the system of COVID-19 has presented and divided the population into susceptible, exposed, asymptomatic, symptomatic, hospitalized, and vaccinated then they add one control which is non-pharmaceutical interventions.

Our contribution to this research work is to modify RTD mathematical model and formulate OCP by adding two control efforts namely quarantine control (u_1) such as uprooting and burning of infected plants and chemical control (u_2) such as the use of insecticides. Moreover, we improve the FBSM scheme to solve NOCs numerically to present some different control strategies that reduce the transmission dynamic of tungro virus to protect the rice plant from this disease. The simulation of each strategy is displayed graphically by some figures.

We organized this work as follows: Section 2 describes the mathematical model of RTD. In Section 3, we formulate the OCP based on the mathematical model of RTD, and we present the Hamiltonian and NOCs by using Pontryagin's maximum principle [22]. In Section 4, the numerical simulation results for three strategies are illustrated to show the best effective strategy. In Section 5, we present our brief conclusion.

2. A Mathematical Model of RTD

Here, we develop the mathematical model for RTD in [4] by considering predators, quarantine, implementing roguing mechanisms (uprooting and burning of infected plants) and insecticide spraying. So, in constructing the model of the spread of RTD, there are some assumptions used, including:

1) The rate of replanting of rice plants Λ_R and leafhoppers vector birth Λ_T is constant.
2) The population of rice plants is divided into two classes, susceptible rice plants (R_S), infected rice plants (R_I).
3) The insect vector population is split into two classes, susceptible (T_S) and infected (T_I).
4) The transmission of disease can happen if the susceptible rice plants interact with infected insect vector and the susceptible insect vector interact with infected rice plants.

5) The infected plants and insect vectors cannot recover and P is biological agents as predators of vectors.

The schematic compartment diagram of the tungro disease transmission in rice plants flows linked with vector and predator-prey in Figure (1) and other variables and parameters used can be seen in Table (1).

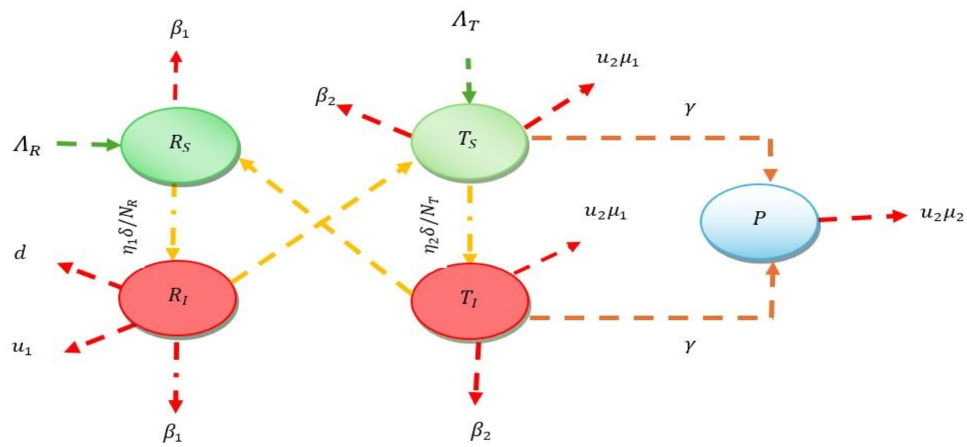

Figure 1. The diagram for RTD transmission dynamic.

Parameter	Definition	Value	Refs
Λ_R	The rate of replanting of rice plants	8	[4]
Λ_T	The rate of leafhoppers vector birth	10	[4]
η_1	The transmission rate from T_I to R_S	0.15	[4]
η_2	The transmission rate from R_I to T_S	0.25	[4]
N_R	The total population of rice plant	400	[4]
N_T	The total population of the green leafhopper insects	300	[4]
N_p	The total population of the biological agent	50	[4]
d	The per capita death rate of the host	0.025	[4]
δ	The suction rate of the vector	0.8	[4]
β_1	The death rate of N_R	0.05	[4]
β_2	The death rate of N_T	0.1	[4]
μ_1	The mortality ratio of vectors is caused by insecticide	4	[4]
μ_2	The death ratio of predators is caused by insecticide	4	[4]
γ	The rate of predation	0 & 0.02	[4]
r	The recruitment rate of biological agent	1	[4]

Table 1. The definition and values for variables and parameters of RTD model.

The system of differential equations that describe RTD mathematical model can be considered as the following:

$$\frac{dR_S}{\mathrm{d}t} = \Lambda_R - \frac{\eta_1\delta}{N_R}R_S T_I - \beta_1 R_S,$$
$$\frac{\mathrm{d}R_I}{\mathrm{d}t} = \frac{\eta_1\delta}{N_R}R_S T_I - (\beta_1 + d + u_1)R_I\ ,$$
$$\frac{\mathrm{d}T_S}{\mathrm{d}t} = \Lambda_T - \frac{\eta_2\delta}{N_T}T_S R_I - (\beta_2 + \mu_1 u_2)T_s - \gamma P T_S \tag{2.1}$$
$$\frac{\mathrm{d}T_I}{\mathrm{d}t} = \frac{\eta_2\delta}{N_T}T_S R_I - (\beta_2 + \mu_1 u_2)T_I - \gamma P T_I,$$
$$\frac{dP}{dt} = rP\left(1 - \frac{P}{N_P}\right) + \gamma P(T_S + T_I) - u_2\mu_2 P,$$

with the following initial conditions [3]:

$$R_S(0) = 225,\ \ T_S(0) = 200,\ \ \ R_I(0) = 175,\ \ \ T_I(0) = 100,\ \ P(0) = 5, \tag{2.2}$$

where γ is the prediction rate of biological agents on susceptible and infected plants, u_1 is the quarantine measures and u_2 is insecticides measures on both the vector and the biological agents.

3. Optimal Control Problem

In this section, we aim to minimize the number of populations of infected plants R_I and infected insects T_I, to reduce the infection of tungro disease. So, to minimize R_I and T_I, we implement two control measures quarantine (implementing roguing mechanism) and insecticide spraying, respectively, and formulate the following OCP.

By considering the state system (2.1) with the following set of admissible control functions

$$\Omega = \{(u_1(\cdot), u_2(\cdot))\ : 0 \le u_1(t), u_2(t) \le 1, \forall\, t \in [0, t_f]\},$$

the cost function is given by

$$J(u_1(\cdot), u_2(\cdot)) = \min_{\Omega} \int_0^{t_f} \left[A_1 R_I + A_2 T_I + \frac{1}{2}[B_1 u_1^2 + B_2 u_2^2]\right] dt \tag{3.1}$$

where A_1 and A_2 describes the balancing constant coefficients of infected rice plants and leafhopper vector population, while B_1 and B_2 are weight coefficients for each control costs measure.

Therefore, following PMP [19] the Hamiltonian $H(R_S, R_I, T_R, T_I, P, \lambda_i, u_1, u_2)$ is defined by

$H(R_S,\ R_I,\ T_R,\ T_I,\ P,\ \lambda,\ u_1,\ u_2)\ = A_1R_I + A_2T_I + \frac{1}{2}[B_1u_1^2 + B_2u_2^2]\ +\ \lambda_1\left[\Lambda_R - \frac{\eta_1\delta}{N_R}R_sT_I - \beta_1R_s\right] + \lambda_2\left[\frac{\eta_1\delta}{N_R}R_sT_I - (\beta_1 + d + u_1)R_I\right] + \lambda_3\left[\Lambda_T - \frac{\eta_2\delta}{N_T}T_sR_I - (\beta_2 + \mu_1u_2)T_s - \gamma PT_s\right]\ +\ \lambda_4\left[\frac{\eta_2\delta}{N_T}T_sR_I - (\beta_2 + \mu_1u_2)T_I - \gamma PT_I\right]\ +\ \lambda_5\left[rP\left(1 - \frac{P}{N_P}\right) + \gamma P(T_S + T_I) - u_2\mu_2P\right]$ (3.2)

where $\lambda_1(t),\ \lambda_2(t),\ \lambda_3(t),\ \lambda_4(t),\ \lambda_5(t)$ are the co-state functions associated with state functions. We clarify the NOCs for our OCP in the following theorem.

Theorem 3.1. Suppose that $R_S^*(\cdot), R_I^*(\cdot), T_R^*(\cdot), T_I^*(\cdot)$ and $P^*(\cdot)$ are optimal solutions for the system (2.1) subject to initial conditions (2.2) with optimal controls $u_1^*(\cdot)$ and $u_2^*(\cdot)$ on the interval $[0, t_f]$ that minimizes the cost function (3.1) through the control set Ω. Then, there exist co-state functions $\lambda_1(\cdot), \lambda_2(\cdot), \lambda_3(\cdot), \lambda_4(\cdot)$ and $\lambda_5(\cdot)$ satisfying,

$$
\begin{aligned}
\frac{d\lambda_1}{dt} &= \frac{\eta_1\delta}{N_R}\ [\lambda_1 - \lambda_2]\ T_I^* + \lambda_1\ \beta_1\ ,\\
\frac{d\lambda_2}{dt} &= -A_1 + \lambda_2(\beta_1 + d + u_1^*) + \frac{\eta_2\ \delta}{N_T}[\lambda_3 - \lambda_4]\ T_S^*\ ,\\
\frac{d\lambda_3}{dt} &= \frac{\eta_2\ \delta}{N_T}\ [\lambda_3 - \lambda_4]\ R_I^* + \lambda_3(\beta_2 + \mu_1\ u_2^* + \gamma\ P^*) - \lambda_5\ \gamma\ P^*\ , \qquad (3.3)\\
\frac{d\lambda_4}{dt} &= -A_2 + \frac{\eta_1\delta}{N_R}\ [\lambda_1 - \lambda_2]\ R_S^* + \lambda_4\ (\beta_2 + \mu_1\ u_2^* + \gamma\ P^*) - \lambda_5\ \gamma\ P^*,\\
\frac{d\lambda_5}{dt} &= \lambda_3\ \gamma\ T_5^* + \lambda_4\ \gamma\ T_I^* + \lambda_5\left[-r + \frac{2\ r\ P^*}{N_P} - \gamma\ (T_S^* + T_I^*) + u_2^*\ \mu_2\right],
\end{aligned}
$$

with transversality conditions $\lambda_1(t_f) = \lambda_2(t_f) = \lambda_3(t_f) = \lambda_4(t_f) = \lambda_5(t_f)$ =0. Furthermore,

$$u_1^* = max\ \{min\ [\frac{\lambda_2\ R_I^*}{B_1}, 1], 0\},$$

$$u_2^* = max\ \{min\ [\frac{\mu_1\lambda_3\ T_S^* + \mu_1\lambda_4\ T_I^* + \lambda_5\ \mu_2^*\ P^*}{B_2}, 1], 0\}. \qquad (3.4)$$

Proof. Based on PMP, we differentiate the Hamiltonian $H(R_S^*,\ R_I^*,\ T_R^*,\ T_I^*,\ P^*,\ \lambda_i,\ u_1^*,\ u_2^*)$ in Equation (3.2) with respect to state variables in order to obtain the co-state variables as follows

$$
\begin{aligned}
\frac{d\lambda_1}{\mathrm{d}t} &= -\frac{\partial H}{\partial R_S} = \frac{\eta_1\delta}{N_R}\ [\lambda_1 - \lambda_2]\ T_I^* + \lambda_1\ \beta_1,\\
\frac{d\lambda_2}{\mathrm{d}t} &= -\frac{\partial H}{\partial R_I} = -A_1 + \lambda_2(\beta_1 + d + u_1^*) + \frac{\eta_2\ \delta}{N_T}[\lambda_3 - \lambda_4]\ T_S^*,\\
\frac{d\lambda_3}{\mathrm{d}t} &= -\frac{\partial H}{\partial T_S} = \frac{\eta_2\ \delta}{N_T}\ [\lambda_3 - \lambda_4]\ R_I^* + \lambda_3(\beta_2 + \mu_1\ u_2^* + \gamma\ P^*) - \lambda_5\ \gamma\ P^*, \qquad (3.5)\\
\frac{d\lambda_4}{\mathrm{d}t} &= -\frac{\partial H}{\partial T_I} = -A_2 + \frac{\eta_1\delta}{N_R}\ [\lambda_1 - \lambda_2]\ R_S^* + \lambda_4\ (\beta_2 + \mu_1\ u_2^* + \gamma\ P^*) - \lambda_5\ \gamma\ P^*,\\
\frac{d\lambda_5}{\mathrm{d}t} &= -\frac{\partial H}{\partial P} = \lambda_3\ \gamma\ T_5^* + \lambda_4\ \gamma\ T_I^* + \lambda_5\left[-r + \frac{2\ r\ P^*}{N_P} - \gamma\ (T_S^* + T_I^*) + u_2^*\ \mu_2\right],
\end{aligned}
$$

with the transversality conditions $\lambda_i(t_f) = 0,\ i = 1,\ \ldots.,\ 5$. Furthermore, we differentiate the Hamiltonian with respect to control variables and using the following optimality condition:

$$\frac{\partial H(R_S^*,\ R_I^*,\ T_R^*,\ T_I^*,\ P^*,\ \lambda_i,\ u_1^*,\ u_2^*)}{\partial u_1^*} = \frac{\partial H(R_S^*,\ R_I^*,\ T_R^*,\ T_I^*,\ P^*,\ \lambda_i,\ u_1^*,\ u_2^*)}{\partial u_2^*} = 0$$

Therefore, we have

$0 = B_1\, u_1^* - \lambda_2\, R_I^*,$

$0 = B_2\, u_2^* - \mu_1 \lambda_3\, T_S^* - \mu_1 \lambda_4\, T_I^* - \lambda_5\, \mu_2^*\, P^*.$

By using the compactness condition for controls, we get Equation (3.4).

4. Numerical Results

In this section, the effects of quarantine and chemical controls are studied numerically and presented in different optimal control strategies to find the best strategy for reducing the infection of RTD. We obtain the results by solving the optimality system of state and co-state equations by FBSM. The state equations are solved by starting with the initial value for the controls and using the forward Runge-Kutta scheme. Then, the co-state equations are solved using the backward Runge-Kutta scheme with transversality conditions. We get the solution of state and co-state equations by updating the controls until the consecutive iteration is close enough to each other. Then, we present some strategies based on the intervention of suggested controls in order to reduce the number of infected rice plants. In this simulation, the population with a control effect is labelled with a blue line and without control with a red line. The balanced and weight coefficients in the cost function are $\boldsymbol{A_1 = A_2 = 10}$ and $\boldsymbol{B_1 = B_2 = 20}$, respectively. The parameter values used are stated in Table 1. We investigate three optimal control strategies that are presented as follows:

Strategy 1: in this strategy, we activated the quarantine control measure (i.e., $\boldsymbol{u_1 \neq 0}$) and stopped the chemical control measure (i.e., $\boldsymbol{u_2 = 0}$). It is clear from Fig. 2, the population of infected plants decreases rapidly when quarantine control measures are implemented. While in Fig. 3, The control profile for this strategy is displayed in Fig. 4. This figure shows that at the beginning of the period, the maximum efforts will be made for these control measures. Then, approximately after 6 days of the period, the measures using this control will gradually decrease until it reaches their lowest value at approximately 40 days of the proposed period.

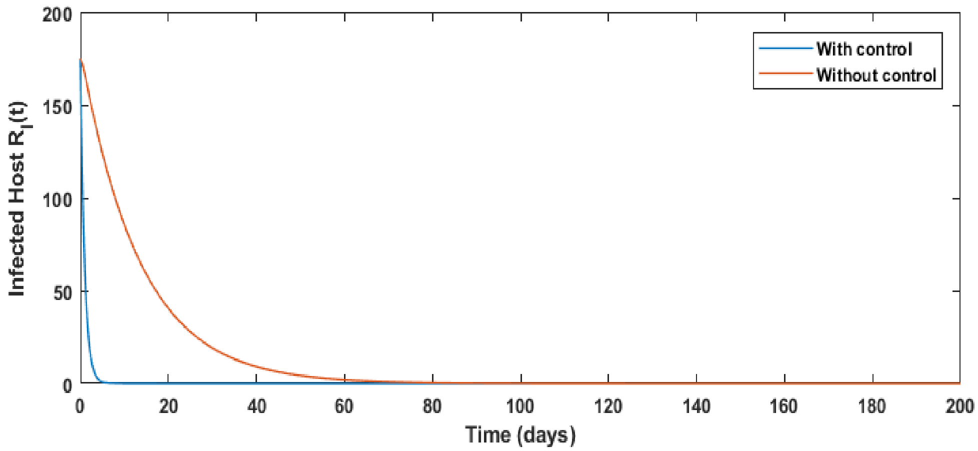

Figure 2. The population of Infected plants with and without control.

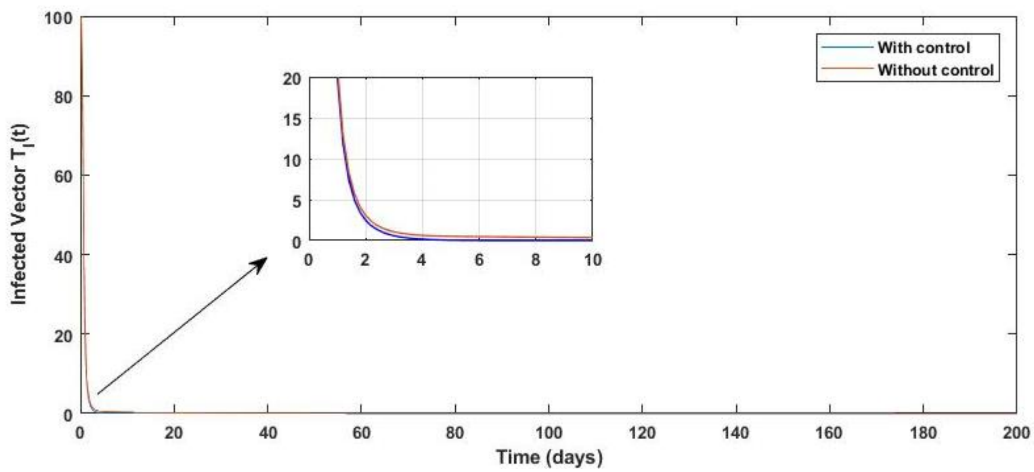

Figure 3. The population of Infected vectors with and without control.

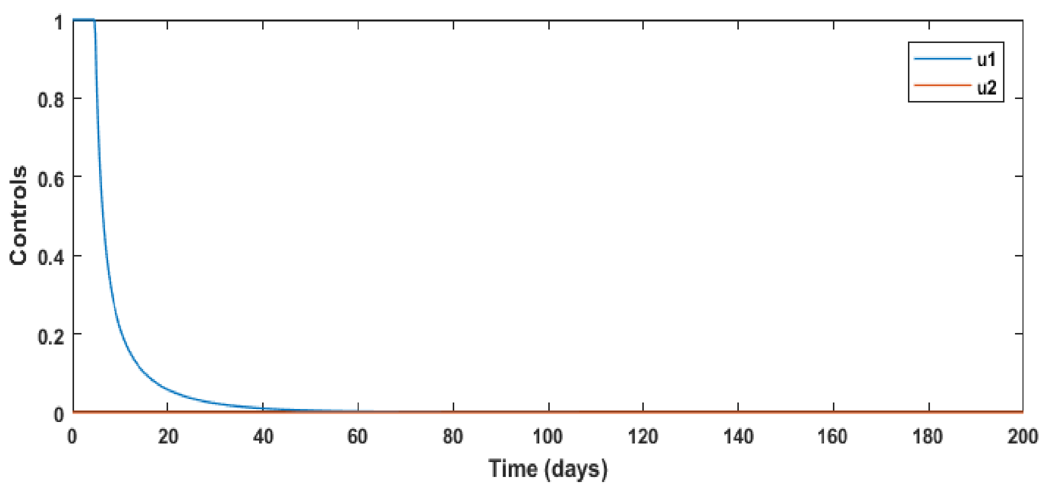

Figure 4. Control profile.

Strategy 2: in this strategy, we focus on the effect of chemical control (i.e., $\boldsymbol{u_2} \neq \mathbf{0}$) and neglected the effect of quarantine control measure (i.e., $\boldsymbol{u_1} = \mathbf{0}$). The simulation of this strategy is presented in Figs. 5-7, where we observe in Fig. 5, the population of infected plants decreases slightly compared to Fig. 2 and there is a bit of variation between with and without control. While the effect of this control appears clearly on the vector because insects are sprayed with some chemical pesticides, this is what appears in Fig. 6. Also, the population of infected vectors decreases more compared to infected vectors in Fig. 3. The control profile is shown in Fig. 7, where it is clear that the control remains at the maximum value during the initial periods until approximately 45 days. Then, the control used gradually decreases.

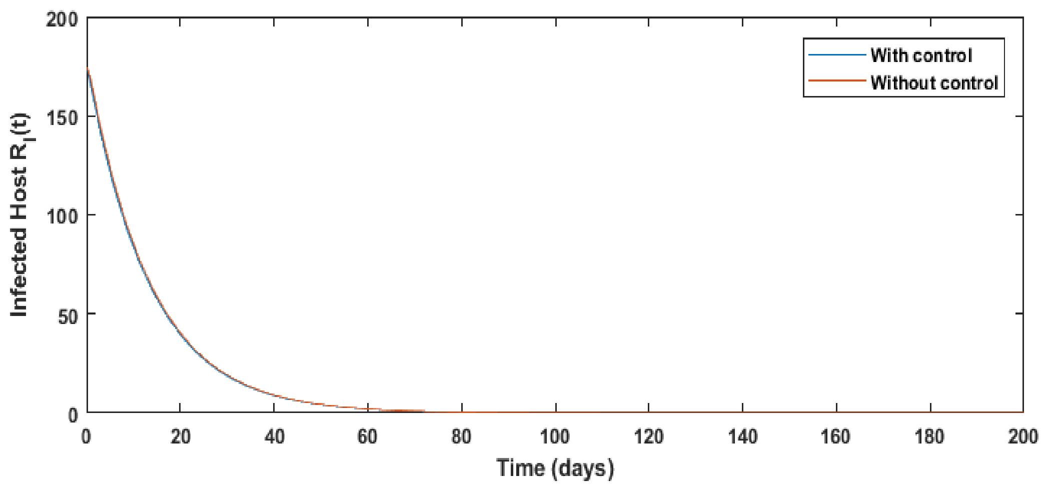

Figure 5. The population of Infected plants with and without control.

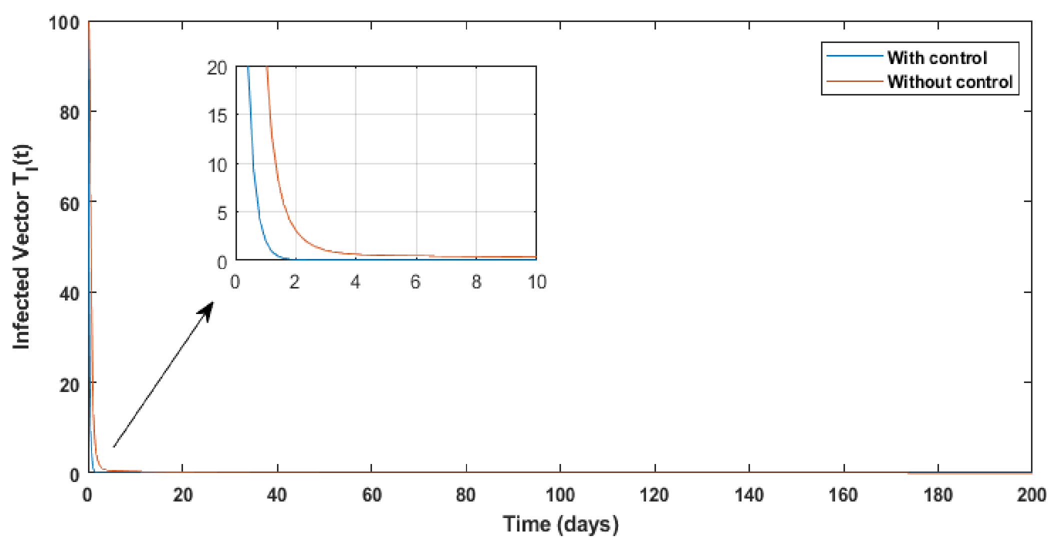

Figure 6. The population of Infected vectors with and without control.

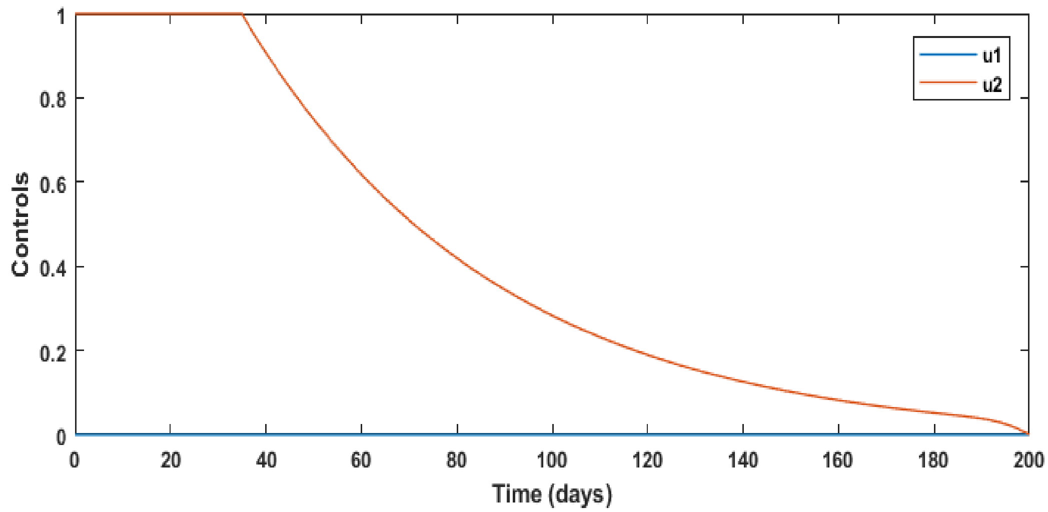

Figure 7. Control profile.

Strategy 3: in this strategy, we combine all the available controls (i.e., $\boldsymbol{u_1} \neq \mathbf{0}$ and $\boldsymbol{u_2} \neq \mathbf{0}$). Here you will find a noticeable decrease in the numbers of infected plants and infected insects, as shown in Fig. 8 and Fig. 9, respectively. We also note from the profile of the controls shown in Fig. 10 that the proposed controls remain at their highest value at the beginning of the period and then gradually decrease until they reach their lowest value at 40 days of the proposed period, and this means that the disease is eliminated in the shortest possible time. Furthermore, compared to the previous two strategies, we find that in the case of applying all controls (i.e., Strategy 3), it is the best case to eliminate the disease as soon as possible.

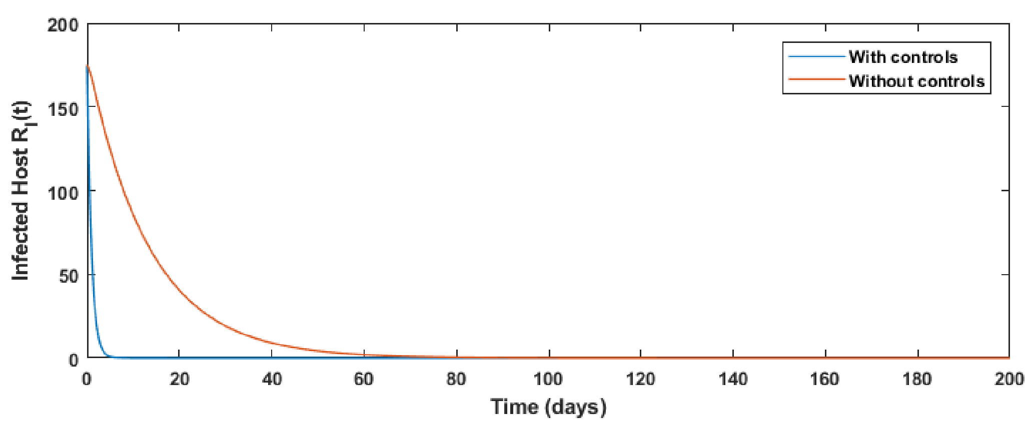

Figure 8. The population of Infected plants with and without control.

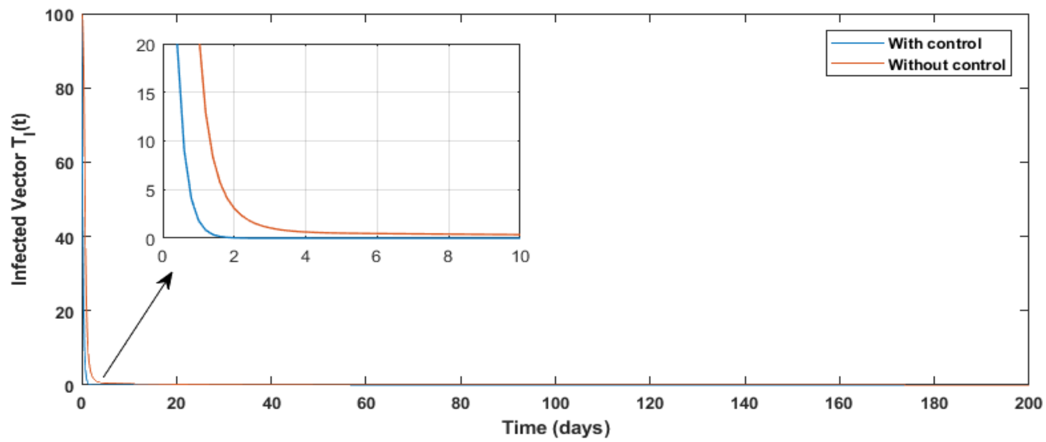

Figure 9. The population of Infected vectors with and without control.

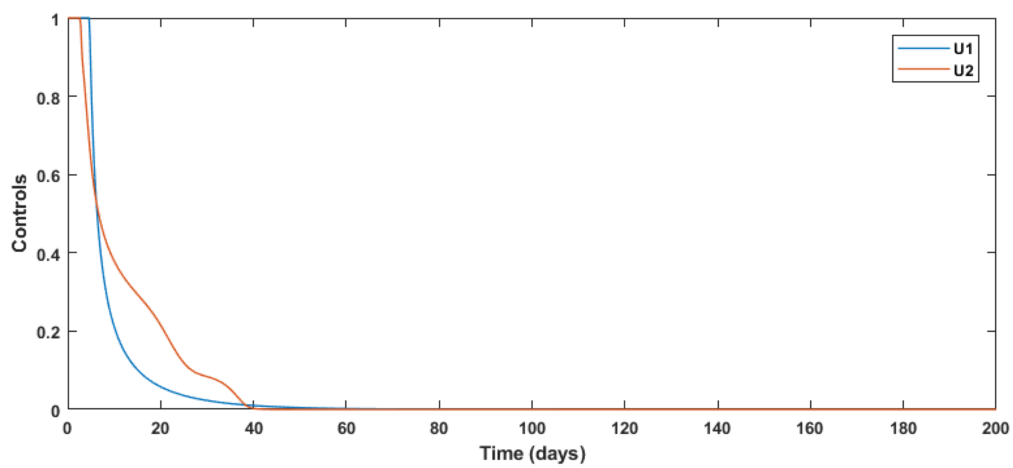

Figure 10. Control profile

5. Conclusions

In this work, A mathematical model for the spread of tungro disease of rice has been discussed. The two control efforts namely quarantine control ($\boldsymbol{u_1}$) such as uprooting and burning infected plants and chemical control ($\boldsymbol{u_2}$) such as using insecticides have been suggested. Moreover, we formulated an OCP and proved their characterizations based on PMP and presented the solutions for the OCP with different control strategies to find the best strategies for controlling tungro in the rice plant population. A numerical simulation for the suggested problem has been presented in some figures. From this simulation, we found that the most effective strategy for reducing the infection of rice and eliminating the infected host and vector in the shortest possible time is to use all available controls at the same time (i.e., Strategy 3), which gives the best results compared with the other strategies.

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References

[1] S. Phadikar, J. Sil, A.K. Das, Rice Diseases Classification Using Feature Selection and Rule Generation Techniques, Comput. Electron. Agric. 90 (2013), 76–85. https://doi.org/10.1016/j.compag.2012.11.001.

[2] J.G.A. Barbedo, A Review on the Main Challenges in Automatic Plant Disease Identification Based on Visible Range Images, Biosyst. Eng. 144 (2016), 52–60. https://doi.org/10.1016/j.biosystemseng.2016.01.017.

[3] W. Suryaningrat, N. Anggriani, A.K. Supriatna, N. Istifadah, The Optimal Control of Rice Tungro Disease with Insecticide and Biological Agent, AIP Conf. Proc. 2264 (2020), 040002. https://doi.org/10.1063/5.0023569.

[4] H.M. Ali, I.G. Ameen, Save the Pine Forests of Wilt Disease Using a Fractional Optimal Control Strategy, Chaos, Solitons Fractals 132 (2020), 109554. https://doi.org/10.1016/j.chaos.2019.109554.

[5] A.L. Jenner, Applications of Mathematical Modelling in Oncolytic Virotherapy and Immunotherapy, Bull. Aust. Math. Soc. 101 (2020), 522–524. https://doi.org/10.1017/S0004972720000283.

[6] R.P. Rangel, M.D.L.G. Magaña, R.U. Azpeitia, E. Nesterova, Mathematical Modeling in Problem Situations of Daily Life, J. Educ. Hum. Dev. 5 (2016), 62-76. https://doi.org/10.15640/jehd.v5n1a7.

[7] M.L.K.P. Santos, R.R. Belecina, R.V. Diaz, Mathematical Modeling: Effects on Problem Solving Performance and Math Anxiety of Students, Int. Lett. Soc. Humanist. Sci. 65 (2015), 103-115.

[8] R. Amelia, N. Anggriani, N. Istifadah, A.K. Supriatna, Stability Analysis for Yellow Virus Disease Mathematical Model of Red Chili Plants, J. Phys.: Conf. Ser. 1722 (2021), 012043. https://doi.org/10.1088/1742-6596/1722/1/012043.

[9] R. Amelia, N. Anggriani, A.K. Supriatna, N. Istifadah, Mathematical Model for Analyzing the Dynamics of Tungro Virus Disease in Rice: A Systematic Literature Review, Mathematics 10 (2022), 2944. https://doi.org/10.3390/math10162944.

[10] W. Suryaningrat, N. Anggriani, A.K. Supriatna, Mathematical analysis and numerical simulation of spatial-temporal model for rice tungro disease spread, Commun. Math. Biol. Neurosci. 2022 (2022), 44. https://doi.org/10.28919/cmbn/7160.

[11] A. Maryati, N. Anggriani, E. Carnia, Stability Analysis of Tungro Disease Spread Model in Rice Plant Using Matrix Method, Barekeng 16 (2022), 217–228. https://doi.org/10.30598/barekengvol16iss1pp215-226.

[12] R. Amelia, N. Anggriani, A.K. Supriatna, N. Istifadah, Analysis and Optimal Control of the Tungro Virus Disease Spread Model in Rice Plants by Considering the Characteristics of the Virus, Roguing, and Pesticides, Mathematics 11 (2023), 1151. https://doi.org/10.3390/math11051151.

[13] N. Anggriani, N. Istifadah, E. Carnia, et al. The Effect of Insecticide in Tungro Disease Transmission Model with Vegetative and Generative Phase, Commun. Math. Biol. Neurosci. 2024 (2024), 41. https://doi.org/10.28919/cmbn/8433.

[14] S. Lertnaweephorn, U.W. Humphries, A. Khan, Stability Analysis and Optimal Control for Leaf Brown Spot Disease of Rice, AIMS Math. 8 (2023), 9602–9623. https://doi.org/10.3934/math.2023485.

[15] N. Anggriani, M. Yusuf, A.K. Supriatna, The Effect of Insecticide on the Vector of Rice Tungro Disease: Insight from a Mathematical Model, Information, 20 (2017), 6197-6206.

[16] S. Lenhart, J.T. Workman, Optimal Control Applied to Biological Models, Chapman and Hall/CRC, (2007).

[17] M.A. Khan, K. Ali, E. Bonyah, et al. Mathematical Modeling and Stability Analysis of Pine Wilt Disease with Optimal Control, Sci. Rep. 7 (2017), 3115. https://doi.org/10.1038/s41598-017-03179-w.

[18] S.I. Oke, M.B. Matadi, S.S. Xulu, Optimal Control Analysis of a Mathematical Model for Breast Cancer, Math. Comput. Appl. 23 (2018), 21. https://doi.org/10.3390/mca23020021.

[19] C. Modnak, Optimal Control Modeling and Simulation, with Application to Cholera Dynamics, Old Dominion University, (2013).

[20] T.A. Perkins, G. España, Optimal Control of the COVID-19 Pandemic with Non-Pharmaceutical Interventions, Bull. Math. Biol. 82 (2020), 118. https://doi.org/10.1007/s11538-020-00795-y.

[21] L.S. Pontryagin, Mathematical Theory of Optimal Processes, CRC Press, (1987).