

Neutrosophic Run Test for Randomness in Imprecise Data with Application

Muhammad Aslam^{1*}, Eid Sadun Alotaibi²

¹Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

²Department of Mathematics and statistics, AlKhurmah University College, Taif University, P.O. Box11099, Taif 21944, Saudi Arabia

*Corresponding author: aslam_ravian@hotmail.com

ABSTRACT. The statistical tests have been applied under the assumption that the observations in the sample should be random. The existing run test for randomness is applied when no uncertainty is presented. In practice, when implementing the test, the uncertainty is always present and should be evaluated in implementing the statistical tests. In this paper, the modification of the existing run test for randomness is presented using the idea of neutrosophic statistics. The proposed neutrosophic run test for randomness will be applied when the decision-makers are uncertain about the level of significance, observations, and sample size. The application of the proposed test will be given using the data of women with HIV/AIDS. From HIV/AIDS Data, it is concluded that the presence of indeterminacy may affect the decision about the null hypothesis.

1. Introduction

Mostly, the statistical tests are applied under the assumption that the data obtained from the sample should be random. Therefore, before implementing the statistical test, the randomness in the sample is checked using the run test. The run test is easy to apply and interpret. The run test is applied to test the null hypothesis that the data is random vs. the alternative hypothesis that the data is departed from the randomness. The median of the data is calculated and +signs

Received Dec. 15, 2024

2020 Mathematics Subject Classification. 62F03.

Key words and phrases. run test; classical statistics; neutrosophy; simulation; HIV/AIDS Data

are assigned when the observation is larger than the median and -signs are assigned when the observations are less than the median. The run scope is calculated based on the sequence of signs and compared with the tabulated values. The null hypothesis is accepted if the calculated runs are within the tabulated values, otherwise, the alternative hypothesis is accepted [1] presented the method and application of the run test. [2] discussed the applications of the run test. [3] worked on multivariate run test. [4] applied the run test using the idea of the length of runs. More application of the test can be seen in [5].

The statistical tests have been applied in the medical field for testing the medical hypothesis and forecasting purposes. [6] presented the modeling for HIV data. [7] applied the run test using the data obtained from the clinical survey. [8] applied the test for DNA data. [9] presented the statistical analysis for HIV data. [10] applied the statistical analysis of HIV data collected from Nigeria. [11] worked on the randomness of the data obtained from the clinical trials. More applications of the statistical tests in the field of medicine can be seen in [12], [13], [14], [15] and [16].

The fuzzy logic gets important when the data in hand is vague and the classical tests cannot be applied for the decision-making. As mentioned by [17] mentioned that “statistical data are frequently not precise numbers but more or less non-precise also called fuzzy. Measurements of continuous variables are always fuzzy to a certain degree”. By analyzing the vague data, the fuzzy-based tests give only information about the measure of truth and the measure of indeterminacy. The applications of the statistical tests/analysis using the fuzzy logic can be seen in [18], [19], [20], [21], [22], [23], [24], [25], [26], [27].

The fuzzy-logic ignores the information about the measure of indeterminacy while analyzing the vague data. The neutrosophic logic that is called the generalization of fuzzy logic was introduced by [28]. The neutrosophic logic can be applied for any type of set and gives information about the measure of uncertainty/indeterminacy when analyzing the interval or vague data. More details with the application of the neutrosophic logic can be seen in [29] and [30]. The neutrosophic logic was used by [31] to extend the theory of classical statistics. Classical statistics has no able to deal with the vague information that can be expected when the process is complex or parameters under study are indeterminate. [32] and [33] worked on the methods to analyze the neutrosophic data. [34], [35] and [36] proposed some statistical tests using the

idea of neutrosophy. Aslam [37] worked on the negative binomial distribution and related algorithm using the neutrosophic statistics. Aslam [38] proposed the Bayes estimator using neutrosophic statistics and presented the application and extensive simulation study.

The run tests available in the literature cannot be applied when the data is vague and have uncertain, imprecise, and indeterminate observations. By exploring the literature and the best of our knowledge, there is no work on run test for randomness using the neutrosophic statistics. In this paper, we will introduce the neutrosophic run test for randomness when uncertainty is found in the observations, level of significance, and sample size. The application of the proposed test will be given on the data of women with HIV/AIDS data. It is expected that the proposed test will be effective to be applied when uncertainty is present in the observations, level of significance, and the sample size. It is expected that the proposed test will be powerful test than the existing run test under classical statistics.

2. Methodology

The existing test under classical statistics is applied for testing the randomness in the sample when the decision-makers are uncertain about the sample size, level of significance (α) and observations in the data. But in practice, when applying the statistical tests, the uncertainty is always presented that is ignored in implementing the statistical tests. In this section, we will introduce the modification of the existing run test for testing the randomness in the sample using the idea of neutrosophy. Suppose that $n_N = n_L + n_U I_{n_N}; I_{n_N} \in [I_{n_L}, I_{n_U}]$ be neutrosophic random sample having lower value n_L , indeterminate part $n_U I_{n_N}$, and the measure of indeterminacy $I_{n_N} \in [I_{n_L}, I_{n_U}]$. Note that when $I_{n_U} = 0$, $n_N = n_L$. Suppose that $K_N = K_L + K_U I_{K_N}; I_{K_N} \in [I_{K_L}, I_{K_U}]$ present the neutrosophic value of the number of runs has a lower value K_L , indeterminate part $K_U I_{K_N}$, and the measure of indeterminacy $I_{K_N} \in [I_{K_L}, I_{K_U}]$. Note that a succession of the values with +sign or -sign is called a run. The operational process of the proposed test is given as: all observations larger than the median of the data are assigned +sign and all observations are less than the median is assigned -sign. In the case of the odd number of observations, the median observation is ignored in counting $K_N \in [K_L, K_U]$. The proposed test will be implemented as follows.

Case-I: when $n_N \in [n_L, n_U] > 30$

When $n_N \in [n_L, n_U] > 30$, the test statistic $K_N \in [K_L, K_U]$ is compared with neutrosophic normal distribution having the following mean

$$\mu_N = \mu_L + \mu_U I_{\mu_N}; I_{\mu_N} \in [I_{\mu_L}, I_{\mu_U}] \quad (1)$$

where $\mu_N = (n_N + 1)$; $n_N \in [n_L, n_U]$, μ_L is the lower value of the interval, $\mu_U I_{\mu_N}$ is the upper value of the interval, and $I_{\mu_N} \in [I_{\mu_L}, I_{\mu_U}]$ is the indeterminacy. The neutrosophic variance of the statistic is given by

$$\sigma_N^2 = \sigma_L^2 + \sigma_U^2 I_{\sigma_N^2}; I_{\sigma_N^2} \in [I_{\sigma_L^2}, I_{\sigma_U^2}] \quad (2)$$

where $\sigma_N^2 = 1/2 (2n_N - 2)/(2n_N - 1)$; $n_N \in [n_L, n_U]$, σ_L^2 is the lower value of the interval, $\sigma_U^2 I_{\sigma_N^2}$ is the upper value of the interval, and $I_{\sigma_N^2} \in [I_{\sigma_L^2}, I_{\sigma_U^2}]$ is the indeterminacy.

Case-II: when $n_N \in [n_L, n_U] < 30$

When $n_N \in [n_L, n_U] < 30$, the values of the test statistic $K_N \in [K_L, K_U]$ is compared with the critical value at α , see [1]. The null hypothesis (H_0) is rejected if $K_N \in [K_L, K_U]$ is greater than the tabulated value, otherwise the alternative hypothesis (H_1) is accepted. The proposed test can be implement using the following steps.

Step-1: State H_0 : the observations in the sample are random vs. the alternative hypothesis H_1 : observations in the sample are departures from randomness.

Step-2: State the level of significance (α) and select the critical values from [1].

Step-3: Count the number of runs $K_N \in [K_L, K_U]$

Step-4: Accept H_0 if $K_N \in [K_L, K_U]$ is within the acceptance region.

The operational processes of the proposed test for case-I and case-II are shown in Figures 1-2.

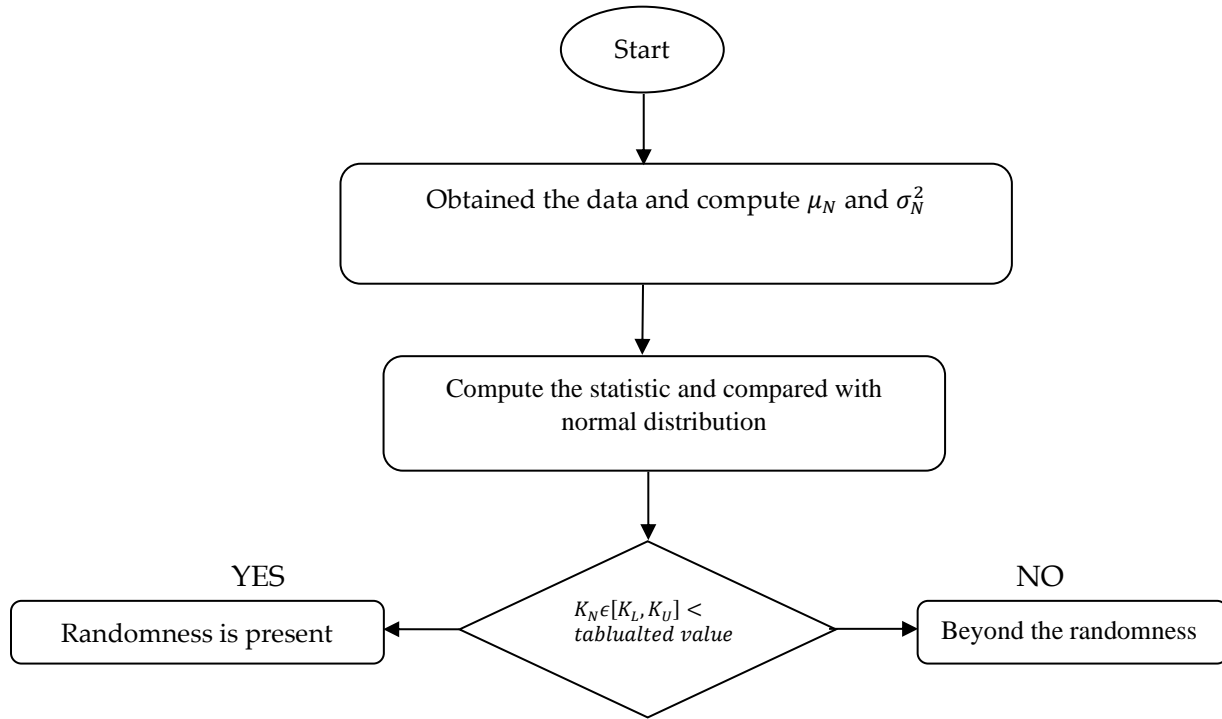


Figure 1: The operational procedure of the proposed run test for case-I

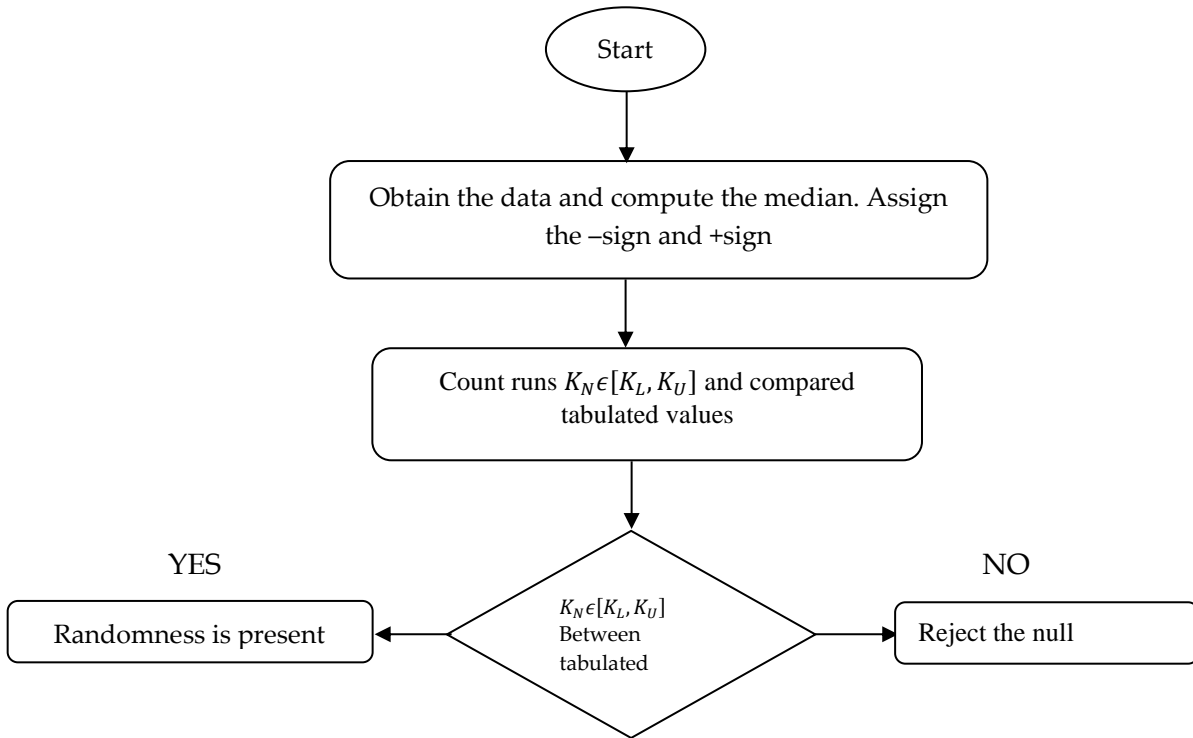


Figure 2: The operational procedure of the proposed run test for case-II

3. Application using HIV/AIDS Data

This section addresses the application of the proposed neutrosophic run test using HIV/AIDS Data. [39] conducted a survey about women with HIV/AIDS. The information about the women having HIV/AIDS is collected from the interview method. The main questions were about the age of women with HIV/AIDS, age at marriage of women with HIV/AIDS, education of women with HIV/AIDS, and years living with HIV/AIDS. The data of the first variable was retrieved from [39] and reported in Table 1. To implement the proposed test, the values for the second variable are generated using simulation. Table 1 shows the different sample sizes for each variable. The decision-makers are interested to test whether the observations belonging to each variable is random or not. For testing the randomness of HIV/AIDS data, it is assumed that the decision-makers are uncertain about the size of the sample. The measure of uncertainty in the sample is $I_{n_N}=20\%$. They decided to use the proposed test to see the randomness in HIV/AIDS data. Suppose that $n_N \in [8,10]$ and level of significance $\alpha=2\%$. As, $n_N \in [n_L, n_U] < 30$, using case-II, the analysis of the proposed runs test is shown in Table 2. Table 2 presents the values of $K_N \in [K_L, K_U]$ for age of women with HIV/AIDS, age at marriage of women with HIV/AIDS, education of women with HIV/AIDS, and years living with HIV/AIDS. The proposed test using the data of women with HIV/AIDS is stated as follows

Step-1: State H_0 : the observations in the age of women with HIV/AIDS, age at marriage of women with HIV/AIDS, education of women with HIV/AIDS, and years living with HIV/AIDS are random vs. the alternative hypothesis H_1 : observations in the age of women with HIV/AIDS, age at marriage of women with HIV/AIDS, education of women with HIV/AIDS, and years living with HIV/AIDS are a departure from randomness.

Step-2: State the level of significance ($\alpha = 0.02$) and select the critical values from [1].

Step-3: Count the number of runs as shown in Table 2

Step-4: Accept H_0 if $K_N \in [K_L, K_U]$ is within the acceptance region.

The decision about H_0 for all variables is shown in Table 2. From Table 2, it can be seen that under indeterminacy, the randomness is found in the age of women with HIV/AIDS, age at marriage of women with HIV/AIDS, and years living with HIV/AIDS. On the other hand, the observations in the education of women with HIV/AIDS are departed from the randomness.

The neutrosophic forms of the statistic $K_N \in [K_L, K_U]$ along with the measure of indeterminacy are shown in Table 3. From Table 3, it can be seen that the indeterminacy is found in the age of women with HIV/AIDS, age at marriage of women with HIV/AIDS. The operational process for HIV/AIDS is shown in Figure 3.

Table 1: The data of women with HIV/AIDS

Variables	HIV/AIDS Data									
Age (year)	46	22	40	32	22	24	35	25		
Age (year)	46	22	40	32	22	24	35	25	38	38
Age at Marriage	15	12	18	15	17	18	17	21		
Age at Marriage	15	12	18	15	17	18	17	21	20	13
Education	8	2	6	5	5	5	7	5		
Education	8	2	6	5	5	5	7	5	8	5
Years living with HIV/AIDS	8	0.0547	2	2	3	1	2	3		
Years living with HIV/AIDS	8	0.0547	2	2	3	1	2	3	3.5	7

Table 2: Analysis of women with HIV/AIDS

Variables	$n_N \in [n_L, n_U]$	$K_N \in [K_L, K_U]$	Decision about H_0
Age (year)	[8,10]	[6,7]	Do not reject
Age at Marriage	[8,10]	[4,5]	Do not reject
Education	[8,10]	[3,3]	reject
Years living with HIV/AIDS	[8,10]	[5,5]	Do not reject

Table 3: the neutrosophic forms using the HIV/AIDS data

Variables	$n_N \in [n_L, n_U]$	$K_N \in [K_L, K_U]$	$K_N = K_L + K_U I_{K_N}$
Age (year)	[8,10]	[6,7]	$6 + 7I_{K_N}; I_{K_N} \in [0,0.14]$
Age at Marriage	[8,10]	[4,5]	$4 + 5I_{K_N}; I_{K_N} \in [0,0.20]$
Education	[8,10]	[3,3]	$3 + 3I_{K_N}; I_{K_N} \in [0,0]$
Years living with HIV/AIDS	[8,10]	[5,5]	$5 + 5I_{K_N}; I_{K_N} \in [0,0]$

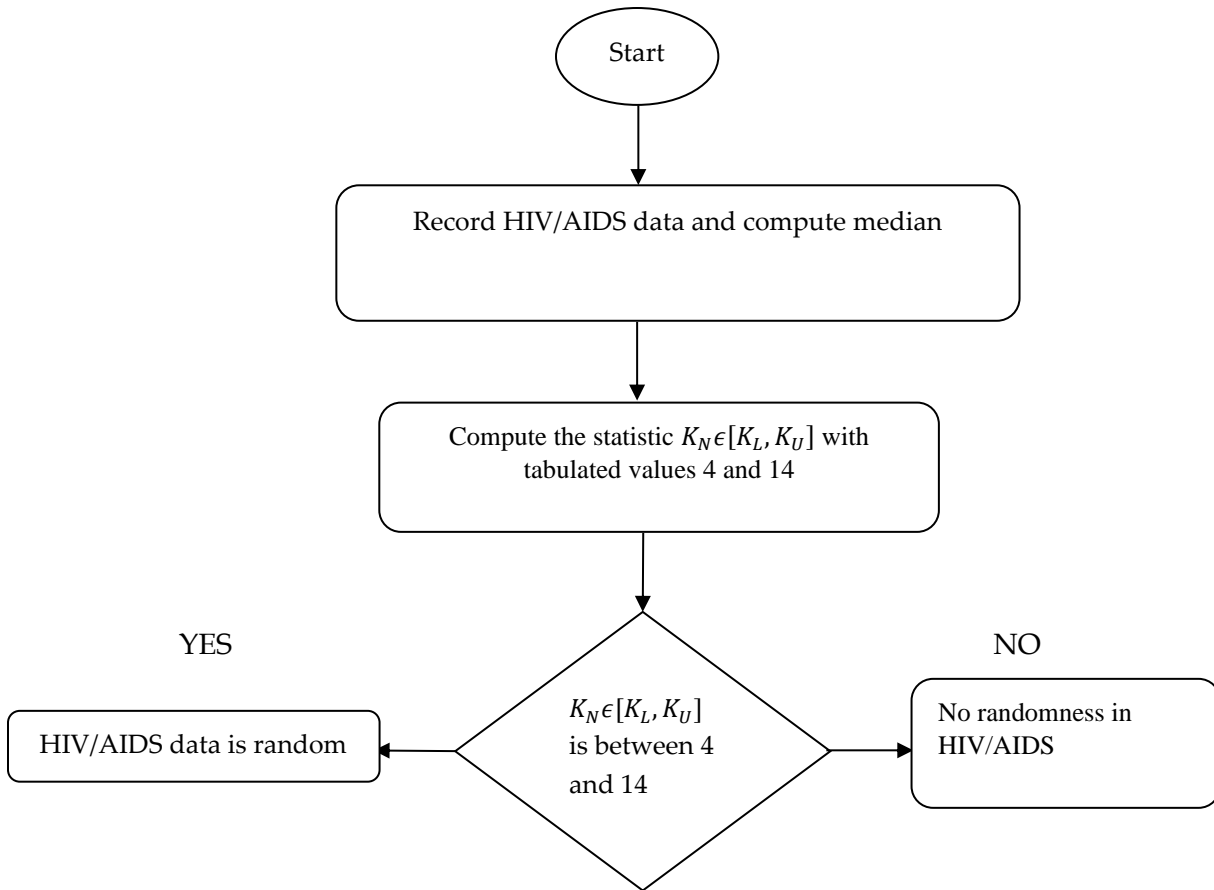


Figure 3: The operational process for HIV/AIDS

4. Simulation Study

To see whether the increase in the measure of indeterminacy/uncertainty $I_{K_N} \in [I_{K_L}, I_{K_U}]$ causes the change in $K_N \in [K_L, K_U]$, a simulation study is performed for age (year) and age at Marriage of women with HIV/AIDS. The values of $I_{K_N} \in [I_{K_L}, I_{K_U}]$ are chosen from $[0,0]$ to $[0,1]$ for both variables under study. A simulation is performed using the neutrosophic forms of two variables and the values of $K_N \in [K_L, K_U]$ are placed in Tables 4-5. Table 4 depicts the values of the statistic $K_N \in [K_L, K_U]$ for age of women with HIV/AIDS and Table 5 depicts the values of the statistic $K_N \in [K_L, K_U]$ for age at marriage of women with HIV/AIDS. From Table 4, it can be noted that the values of $K_N \in [K_L, K_U]$ increase as the values of $I_{K_N} \in [I_{K_L}, I_{K_U}]$ increase from $[0,0]$ to $[0,1]$. For example, when $I_{K_N} \in [0,0]$, the values of $K_N \in [6,6]$. On the other hand, when $I_{K_N} \in [0,1]$, the values

of $K_N \in [6,13]$. For $\alpha=0.02$, the tabulated values are 4 and 14, it can be read that as the values of $I_{K_N} \in [I_{K_L}, I_{K_U}]$ increase, the values of $K_N \in [K_L, K_U]$ become close to the upper critical value. The decision about H_0 is also shown in Table 4. Table 5, it can be noted that the values of $K_N \in [K_L, K_U]$ increase as the values of $I_{K_N} \in [I_{K_L}, I_{K_U}]$ increase from $[0,0]$ to $[0,1]$. For example, when $I_{K_N} \in [0,0]$, the values of $K_N \in [4,4]$. On the other hand, when $I_{K_N} \in [0,1]$, the values of $K_N \in [4,9]$. For $\alpha=0.02$, the tabulated values are 4 and 14, it can be read that as the values of $I_{K_N} \in [I_{K_L}, I_{K_U}]$ increase, the values of $K_N \in [K_L, K_U]$ become close to the upper critical value. The decision about H_0 is also shown in Table 4.

Table 4: Age (year)

I_{K_N}	$K_N \in [K_L, K_U]$	Decision about H_0	I_{K_N}	$K_N \in [K_L, K_U]$	Decision about H_0
[0,0]	[6,6]	Do not reject H_0	[0,0.18]	[6,7]	Do not reject H_0
[0,0.01]	[6,6]	Do not reject H_0	[0,0.2]	[6,7]	Do not reject H_0
[0,0.02]	[6,6]	Do not reject H_0	[0,0.3]	[6,8]	Do not reject H_0
[0,0.04]	[6,6]	Do not reject H_0	[0,0.4]	[6,9]	Do not reject H_0
[0,0.06]	[6,6]	Do not reject H_0	[0,0.5]	[6,10]	Do not reject H_0
[0,0.08]	[6,7]	Do not reject H_0	[0,0.6]	[6,10]	Do not reject H_0
[0,0.10]	[6,7]	Do not reject H_0	[0,0.7]	[6,11]	Do not reject H_0
[0,0.12]	[6,7]	Do not reject H_0	[0,0.8]	[6,12]	Do not reject H_0
[0,0.14]	[6,7]	Do not reject H_0	[0,0.9]	[6,12]	Do not reject H_0
[0,0.16]	[6,7]	Do not reject H_0	[0,1]	[6,13]	Do not reject H_0

Table 5: Age at Marriage

I_{K_N}	$K_N \in [K_L, K_U]$	Decision about H_0	I_{K_N}	$K_N \in [K_L, K_U]$	Decision about H_0
[0,0]	[4,4]	Do not reject H_0	[0,0.18]	[4,5]	Do not reject H_0
[0,0.01]	[4,4]	Do not reject H_0	[0,0.2]	[4,5]	Do not reject H_0
[0,0.02]	[4,4]	Do not reject H_0	[0,0.3]	[4,6]	Do not reject H_0
[0,0.04]	[4,4]	Do not reject H_0	[0,0.4]	[4,6]	Do not reject H_0
[0,0.06]	[4,4]	Do not reject H_0	[0,0.5]	[4,7]	Do not reject H_0
[0,0.08]	[4,4]	Do not reject H_0	[0,0.6]	[4,7]	Do not reject H_0
[0,0.10]	[4,5]	Do not reject H_0	[0,0.7]	[4,8]	Do not reject H_0
[0,0.12]	[4,5]	Do not reject H_0	[0,0.8]	[4,8]	Do not reject H_0
[0,0.14]	[4,5]	Do not reject H_0	[0,0.9]	[4,9]	Do not reject H_0
[0,0.16]	[4,5]	Do not reject H_0	[0,1]	[4,9]	Do not reject H_0

5. Advantages

Several existing tests are the special case of the proposed neutrosophic run test. The proposed test reduces to the existing run test under classical statistics if no uncertainty is found in the sample size, level of significance, and observations. The proposed test is also an extension of the run test under interval-statistics and run test under interval-based analysis. In this section, the efficiency of the proposed tests will be compared with the three runs tests in terms of information, flexibility and adequacy. The neutrosophic forms of age (year) and age at Marriage of women with HIV/AIDS are given as: $K_N = 6 + 7I_{K_N} \in [0, 0.14]$, and $K_N = 4 + 5I_{K_N} \in [0, 0.20]$, respectively. Using the neutrosophic results of age (year) and age at Marriage of women with HIV/AIDS, it can be seen that the values of the statistic $K_N \in [K_L, K_U]$ ranges from 6 to 7 and 4 to 5, respectively. The first values 6 and 4, respectively show the values of statistic under classical statistics. It can be seen that the proposed test gives the results in intervals rather than the exact values. Therefore, the proposed test is flexible than the existing run test under classical statistics. By comparing the results of the proposed test with the run test under fuzzy logic, it can be seen that the proposed test gives the information about three measures that the probability of acceptance that 0.98 (a measure of truth) level of significance that is 0.02 (a measure of falseness) and the measures of indeterminacy which are 0.14 and 0.20, respectively. Therefore, the proposed test is also efficient than the run test under fuzzy logic which gives information about only two measures. The proposed test is found to be efficient than the test under interval-statistics. The interval statistics only utilizes the information within the interval and is unable to give information about the measure of indeterminacy. In nutshell, the proposed test is informative, flexible and adequate than the existing three-run tests.

6. Comparisons in Power of Test

Now the efficiency of the proposed test will be compared with the existing run test under classical statistics in terms of the power of the test. As mentioned earlier, the probability of rejecting the null hypotheses when it is true is denoted by α and the probability of accepting the null hypothesis when it is false is called type-II and denoted by β . The power of the test is denoted by $1 - \beta$. The power of both tests for various values of α is shown in Table 6. The power curves of both tests are shown in Table 6 and curves are shown in Figure 4. From Table 6 and Figure 4, it is quite clear that the proposed test gives the higher values of the power of the test as

compared to the existing run test under classical statistics. For example, when $\alpha=0.02$, the power of the proposed test can be expected from 0.62 to 0.64 and the power of the test from the existing test is 0.60. From the study, it is concluded that the proposed test is efficient than the existing run test in terms of the power of the test.

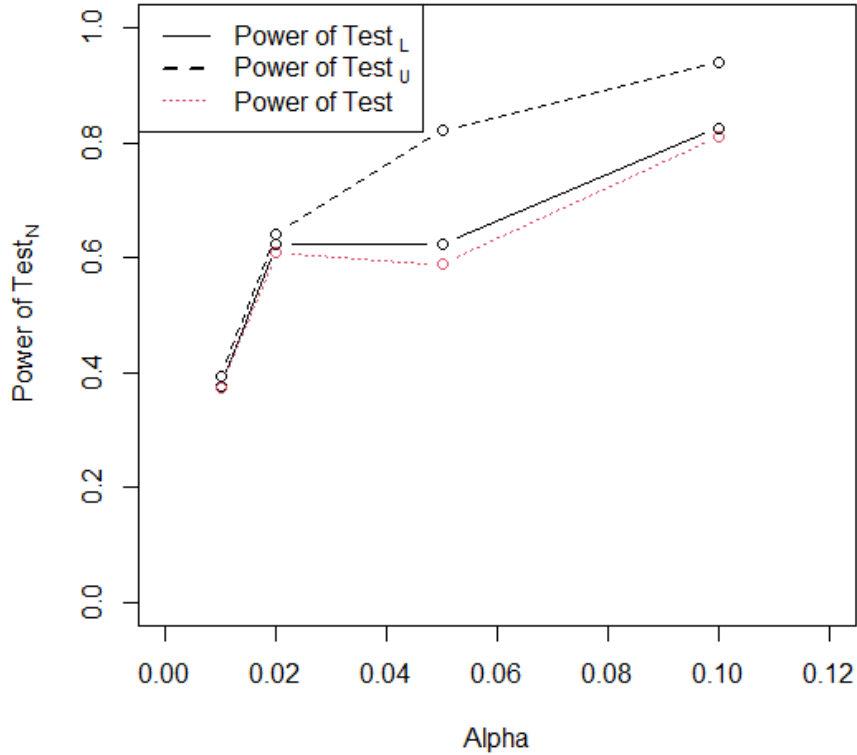


Figure 4: power curves of two tests

Table 6: the values of power of the tests

α	Power of the proposed test	Power of the existing test
0.1	[0.826,0.94]	0.811
0.05	[0.624,0.822]	0.588
0.02	[0.624,0.64]	0.608
0.01	[0.376,0.394]	0.372

7. Concluding Remarks

In this paper, the run test using the idea of neutrosophic logic was presented and applied using data obtained from women with HIV/AIDS. The proposed test was the generalization of the run test under classical statistics. When all parameters in the test are uncertain, the proposed test reduces to the existing test. Using the HIV/AIDS Data, it is concluded that the presence of indeterminacy affects the results of the tests. Based on the simulation and comparative studies, it is concluded that the proposed test is informative, flexible to be applied in uncertainty. The proposed test using big data can be extended for future research. The development of software for the proposed test is also a fruitful area of future research.

Acknowledgements: The authors are deeply thankful to the editor and reviewers for their valuable suggestions to improve the quality and presentation of the paper. The authors would like to acknowledge the Deanship of Graduate Studies and Scientific Research, Taif University for funding this work.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

- [1] G.K. Kanji, *100 Statistical Tests*, Sage, 2006.
- [2] M. Dakhilalian, E.M. Jazi, M.J. Taghiyar, Analysis of Randomness of Runs and Its Application for Statistical Tests, *Int. J. Comput. Sci. Netw. Secur.* 9 (2009), 83-90.
- [3] D. Paindaveine, On Multivariate Runs Tests for Randomness, *J. Amer. Stat. Assoc.* 104 (2009), 1525-1538. <https://doi.org/10.1198/jasa.2009.tm09047>.
- [4] A. Doğanaksoy, F. Sulak, M. Uğuz, O. Şeker, Z. Akcengiz, New Statistical Randomness Tests Based on Length of Runs, *Math. Probl. Eng.* 2015 (2015), 626408. <https://doi.org/10.1155/2015/626408>.
- [5] H. Haramoto, Automation of Statistical Tests on Randomness to Obtain Clearer Conclusion, in: P. L'Ecuyer, A.B. Owen (Eds.), *Monte Carlo and Quasi-Monte Carlo Methods 2008*, Springer, Berlin, Heidelberg, 2009: pp. 411-421. https://doi.org/10.1007/978-3-642-04107-5_26.
- [6] D. Petros Kelkile, Statistical Analysis of Adult HIV/AIDS Patients and Modelling of AIDS Disease Progression, *Sci. J. Appl. Math. Stat.* 4 (2016), 189. <https://doi.org/10.11648/j.sjams.20160405.12>.

- [7] M.A. Bujang, F.E. Sapri, An Application of the Runs Test to Test for Randomness of Observations Obtained from a Clinical Survey in an Ordered Population, *Malays. J. Med. Sci.* 25 (2018), 146–151. <https://doi.org/10.21315/mjms2018.25.4.15>.
- [8] L.C. Meiser, J. Koch, P.L. Antkowiak, et al. DNA Synthesis for True Random Number Generation, *Nature Commun.* 11 (2020), 5869. <https://doi.org/10.1038/s41467-020-19757-y>.
- [9] D. Dargasso Danna, Statistical Analysis on Correlates of HIV Testing Outcomes Based on Ethiopia Demographic and Health Survey (EDHS) 2011 Data, *Sci. J. Public Health* 6 (2018), 120. <https://doi.org/10.11648/j.sjph.20180605.11>.
- [10] P.I. Adamu, P.E. Oguntunde, H.I. Okagbue, O.O. Agboola, On the Epidemiology and Statistical Analysis of HIV/AIDS Patients in the Insurgency Affected States of Nigeria, *Open Access Maced. J. Med. Sci.* 6 (2018), 1315–1321. <https://doi.org/10.3889/oamjms.2018.229>.
- [11] V.W. Berger, L.J. Bour, K. Carter, et al. A Roadmap to Using Randomization in Clinical Trials, *BMC Med. Res. Methodol.* 21 (2021), 168. <https://doi.org/10.1186/s12874-021-01303-z>.
- [12] E. García-Berthou, C. Alcaraz, Incongruence between Test Statistics and P Values in Medical Papers, *BMC Med. Res. Methodol.* 4 (2004), 13. <https://doi.org/10.1186/1471-2288-4-13>.
- [13] Z. Ali, Sb. Bhaskar, Basic Statistical Tools in Research and Data Analysis, *Indian J. Anaesth.* 60 (2016), 662. <https://doi.org/10.4103/0019-5049.190623>.
- [14] S. Greenland, S.J. Senn, K.J. Rothman, et al. Statistical Tests, P Values, Confidence Intervals, and Power: A Guide to Misinterpretations, *Eur. J. Epidemiol.* 31 (2016), 337–350. <https://doi.org/10.1007/s10654-016-0149-3>.
- [15] J. In, S. Lee, Statistical Data Presentation, *Korean J. Anesthesiol.* 70 (2017), 267–276. <https://doi.org/10.4097/kjae.2017.70.3.267>.
- [16] X. Yang, J. Sun, R.C. Patel, et al. Associations between HIV Infection and Clinical Spectrum of COVID-19: A Population Level Analysis Based on US National COVID Cohort Collaborative (N3C) Data, *The Lancet HIV* 8 (2021), e690–e700. [https://doi.org/10.1016/S2352-3018\(21\)00239-3](https://doi.org/10.1016/S2352-3018(21)00239-3).
- [17] R. Viertl, Univariate Statistical Analysis with Fuzzy Data, *Comput. Stat. Data Anal.* 51 (2006), 133–147. <https://doi.org/10.1016/j.csda.2006.04.002>.
- [18] P. Filzmoser, R. Viertl, Testing Hypotheses with Fuzzy Data: The Fuzzy p-Value, *Metrika* 59 (2004), 21–29. <https://doi.org/10.1007/s001840300269>.
- [19] C.-C. Tsai, C.-C. Chen, Tests of Quality Characteristics of Two Populations Using Paired Fuzzy Sample Differences, *Int. J. Adv. Manuf. Technol.* 27 (2006), 574–579. <https://doi.org/10.1007/s00170-004-2212-6>.
- [20] S.M. Taheri, M. Arefi, Testing Fuzzy Hypotheses Based on Fuzzy Test Statistic, *Soft Comput.* 13 (2009), 617–625. <https://doi.org/10.1007/s00500-008-0339-3>.

- [21] E.B. Jamkhaneh, A.N. Ghara, Testing Statistical Hypotheses with Fuzzy Data, in: 2010 International Conference on Intelligent Computing and Cognitive Informatics, IEEE, Kuala Lumpur, Malaysia, 2010: pp. 86–89. <https://doi.org/10.1109/ICICCI.2010.56>.
- [22] J. Chachi, S.M. Taheri, R. Viertl, Testing Statistical Hypotheses Based on Fuzzy Confidence Intervals, *Austrian J. Stat.* 41 (2016), 267–286. <https://doi.org/10.17713/ajs.v41i4.168>.
- [23] D. Kalpanapriya, P. Pandian, Statistical Hypotheses Testing with Imprecise Data, *Appl. Math. Sci.* 6 (2012), 5285–5292.
- [24] S. Parthiban, P. Gajivaradhan, A Comparative Study of Two-Sample t-Test Under Fuzzy Environments Using Trapezoidal Fuzzy Numbers, *Int. J. Math. Stat. Invent.* 4 (2016), 39–54.
- [25] M. Montenegro, Two-Sample Hypothesis Tests of Means of a Fuzzy Random Variable, *Inf. Sci.* 133 (2001), 89–100. [https://doi.org/10.1016/S0020-0255\(01\)00078-0](https://doi.org/10.1016/S0020-0255(01)00078-0).
- [26] S. Park, S.-J. Lee, S. Jun, Patent Big Data Analysis Using Fuzzy Learning, *Int. J. Fuzzy Syst.* 19 (2017), 1158–1167. <https://doi.org/10.1007/s40815-016-0192-y>.
- [27] H. Garg, R. Arora, Generalized Maclaurin Symmetric Mean Aggregation Operators Based on Archimedean T-Norm of the Intuitionistic Fuzzy Soft Set Information, *Artif. Intell. Rev.* 54 (2021), 3173–3213. <https://doi.org/10.1007/s10462-020-09925-3>.
- [28] F. Smarandache, *Neutrosophy. Neutrosophic Probability, Set, and Logic*, ProQuest Information & Learning, Ann Arbor, Michigan, 1998.
- [29] S.K. Das, S. Edalatpanah, A New Ranking Function of Triangular Neutrosophic Number and Its Application in Integer Programming, *Int. J. Neutrosoph. Sci.* 4 (2020), 82–92. <https://doi.org/10.54216/IJNS.040202>.
- [30] O. G. El Barbary, R. Abu Gdairi, Neutrosophic Logic-Based Document Summarization, *J. Math.* 2021 (2021), 9938693. <https://doi.org/10.1155/2021/9938693>.
- [31] F. Smarandache, *Introduction to Neutrosophic Statistics*, Sitech & Education Publishing, 2014.
- [32] J. Chen, J. Ye, S. Du, Scale Effect and Anisotropy Analyzed for Neutrosophic Numbers of Rock Joint Roughness Coefficient Based on Neutrosophic Statistics, *Symmetry* 9 (2017), 208. <https://doi.org/10.3390/sym9100208>.
- [33] J. Chen, J. Ye, S. Du, R. Yong, Expressions of Rock Joint Roughness Coefficient Using Neutrosophic Interval Statistical Numbers, *Symmetry* 9 (2017), 123. <https://doi.org/10.3390/sym9070123>.
- [34] R.A.K. Sherwani, H. Shakeel, M. Saleem, et al. A New Neutrosophic Sign Test: An Application to COVID-19 Data, *PLOS ONE* 16 (2021), e0255671. <https://doi.org/10.1371/journal.pone.0255671>.
- [35] M. Aslam, Neutrosophic Statistical Test for Counts in Climatology, *Sci. Rep.* 11 (2021), 17806. <https://doi.org/10.1038/s41598-021-97344-x>.
- [36] M. Albassam, N. Khan, M. Aslam, Neutrosophic D’Agostino Test of Normality: An Application to Water Data, *J. Math.* 2021 (2021), 5582102. <https://doi.org/10.1155/2021/5582102>.

- [37] M. Aslam, The Neutrosophic Negative Binomial Distribution: Algorithms and Practical Application, REVSTAT-Stat. J. in Press.
- [38] M. Aslam, Bayes Estimator Under Neutrosophic Statistics: A Robust Approach for Handling Uncertainty and Imprecise Data, Methodol. Comput. Appl. Probab. 26 (2024), 56.
<https://doi.org/10.1007/s11009-024-10126-6>.
- [39] W.B.V. Kandasamy, F. Smarandache, Fuzzy and Neutrosophic Analysis of Women with HIV/AIDS, arXiv:math/0507037 [math.GM] (2005). <https://doi.org/10.48550/arXiv.math/0507037>.