Int. J. Anal. Appl. (2025), 23:51

International Journal of Analysis and Applications

L-Mild Normality and L_2-Mild Normality

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Abstract. The purpose of this work is to introduce and study two new topological properties called L-mild normality and L_2-mild normality. A space X is called an *L-mildly normal* space if there exist a mildly normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$. If the space Y is Hausdorff, then the space X is called *L_2-mildly normal*. We investigate these properties and present some examples to illustrate the relationships among L-mild normality and L_2-mild normality with other kinds of topological properties.

1. Introduction

The notions of epi-normality, C-normality and L-normality were introduced by Arhangel'skii during his visiting to Department of Mathematics in King Abdulaziz University, Saudi Arabia on 2012. The notion of C-normality has been studied by Alzahrani and Kalantan in [1]. The notion of L-normality has been studied by Kalantan and Saeed in [2]. In 2022, Al-Awadi and others introduced the notion of C-mild normality in [3]. Alqurashi and Thabit studied the notions of C-almost normality and L-almost normality in [4]. The concepts of CC-Tychonoffness, CCT_3, CC-regularity and CC-almost regularity have been studied in [5]. In this paper, we study two new properties which are L-mild normality and L_2-mild normality. We show that these new properties are different from each other, and they are different from C-normality, L-normality, C-regularity, L-regularity, epi-mild normality and so on. Some properties, counterexample and relationships of these properties are investigated. Two sets A and B of a space X are said to be *separated* if there exist two disjoint open sets U and V in X such that $A \subseteq U$ and $B \subseteq V$ [6–8]. If $\mathcal{T}' \subseteq \mathcal{T}$, then $\mathcal{T}'$ is called a topology that is *coarser* than $\mathcal{T}$ and $\mathcal{T}$ is called *finer* [7]. A subset A of a space X is said to be

Received: Dec. 31, 2024.

2020 *Mathematics Subject Classification.* 54C10, 54D10, 54D20, 54D15, 54D70.

Key words and phrases. epi-mildly normal; epi-normal; C-normal; C-mildly normal; epi-regular; C-regular; L-normal; L-regular.

https://doi.org/10.28924/2291-8639-23-2025-51

ISSN: 2291-8639

a *closed domain* subset if it is the closure of its own interior [9]. A complement of a closed domain set is called *open domain*. The topology on X generated by the family of all open domains denoted by $\mathcal{T}_s$ is coarser than $\mathcal{T}$ and $(X, \mathcal{T}_s)$ is called the *semi regularization* of X. A space $(X, \mathcal{T})$ is called *semi-regular* if $\mathcal{T} = \mathcal{T}_s$ [10]. Any undefined concepts in this work can be found in the introduction section of [4,5,11].

2. Preliminaries

Recall that: a space X is said to be *mildly normal* [12], if any pair of disjoint closed domain subsets A and B of X can be separated. A space X is called *C-normal* [1] (resp. *C-regular* [13], *C-Tychonoff* [14]) if there exist a normal (resp. regular, Tychonoff) space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each compact subspace $A \subseteq X$. A space X is called *L-normal* [2] (resp. *CC-normal* [15]) if there exist a normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf (resp. countably compact) subspace $A \subseteq X$. A space X is called *L-regular* [13] (resp. *L-Tychonoff* [14]) if there exist a regular (resp. Tychonoff) space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$. A space $(X, \mathcal{T})$ is said to be *epi-normal* [16] (resp. *epi-mildly normal* [17], *epi-almost normal* [18], *epi-regular* [19], *epi-quasi normal* [11]), if there exists a topology $\mathcal{T}'$ on X coarser than $\mathcal{T}$ such that $(X, \mathcal{T}')$ is T_4 (resp. Hausdorff mildly-normal, Hausdorff almost-normal, T_3, Hausdorff-quasi-normal).

We give the definitions of L-mild normality and L_2-mild normality.

Definition 2.1. A space X is called *L-mildly normal* space if there exist a mildly normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$. If the space Y is Hausdorff, then the space X is called *L_2-mildly normal*.

From Definition 2.1, it is clear that: every mildly normal space is L-mildly normal and every Hausdorff mildly normal is L_2-mildly normal, where $Y = X$ and the identity function $f : X \to X$ satisfies the requirements. The converse is not true, for example:

Example 2.1. *The modified Dieudonné plank* [15, Example 2.4], is a Tychonoff, L-normal space which is neither mildly normal nor locally compact, see also [2, Example 2.2] and [20, Example 2]. Hence, the modified Dieudonné plank is an L_2-mildly normal space which is neither mildly normal nor locally compact.

Next, we present the following basic results:

Theorem 2.1. *Every L-normal space is L-mildly normal.*

Proof. Let X be an L-normal space. Then, there exist a normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace $A \subseteq X$. Since Y is a normal space, we have Y is mildly normal. Therefore, X is L-mildly normal. □

The converse of Theorem 2.1 is not necessary to be true in general. For example:

Example 2.2. *The finite complement topology* [21, Example 19], $(\mathbb{R}, \mathcal{CF})$ is a T_1-compact space and every subspace of $(\mathbb{R}, \mathcal{CF})$ is compact [21]. Note that: $(\mathbb{R}, \mathcal{CF})$ is not a C-regular space [13]. Since $(\mathbb{R}, \mathcal{CF})$ is a mildly normal space, we get $(\mathbb{R}, \mathcal{CF})$ is L-mildly normal. Therefore, $(\mathbb{R}, \mathcal{CF})$ is an L-mildly normal space, which is neither L-normal, L-regular, epi-mildly normal nor L_2-mildly normal.

Theorem 2.2. *Every Lindelöf L-mildly normal space is mildly normal.*

Proof. Let X be a Lindelöf L-mildly normal space. Then, there exist a mildly normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace A of X. Since X is Lindelöf, put $A = X$. Since f is bijective, we get $f : X \to Y$ is a homeomorphism. Since Y is a mildly normal space, we get X is mildly normal. □

Corollary 2.1. If X is a Lindelöf non mildly normal space, then X cannot be L-mildly normal.

Theorem 2.3. *Every Lindelöf L_2-mildly normal space is Hausdorff mildly normal.*

Proof. It is similar to the proof of Theorem 2.2. □

Corollary 2.2.

(1) Every Lindelöf L_2-mildly normal space is epi-mildly normal.
(2) Every Lindelöf non Hausdorff space cannot be L_2-mildly normal.

The proofs of the next results are similar to that of the corresponding results in [1, 14].

Theorem 2.4. *L-mild normality and L_2-mild normality are topological properties.*

Theorem 2.5. *L-mild normality and L_2-mild normality are additive properties.*

Proposition 2.1. *If X is a T_1 L-mildly normal space, then a witness Y is a T_1-space.*

Proof. It is similar to that of Proposition 1 in [4]. □

Lemma 2.1. *If X is a T_1 L-normal space, then the witness Y is T_4.*

Theorem 2.6. *Every L-mildly normal space is C-mildly normal.*

Proof. Let X be an L-mildly normal space. Then, there exist a mildly normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_A : A \to f(A)$ is a homeomorphism for each Lindelöf subspace A of X. Since every compact subset is Lindelöf, we have each compact subspace C of X is a Lindelöf subspace of X. Thus, the restriction function $f|_C : C \to f(C)$ is a homeomorphism for each compact subspace C of X. Therefore, X is C-mildly normal. □

The converse of Theorem 2.6 may not be true in general. For example:

Example 2.3. Consider the space presented in [4, Example 18]. The space X is a Hausdorff space, which is neither Urysohn, regular, mildly normal, compact, paracompact nor epi-mildly normal [17, Example 16]. Hence, X is neither C_2-mildly normal nor L_2-mildly normal. Note that: X is a Hausdorff Lindelöf second countable C-paracompact space, which is not C_2-paracompact [22, Example 2.25]. Since X is a Lindelöf non mildly normal space, it is not L-mildly normal. Also, the space X is C-almost normal [4]. Hence, it is C-mildly normal. Therefore, the space X is a C-mildly normal space, which is neither mildly normal, L-mildly normal, C_2-mildly normal nor epi-mildly normal.

Example 2.4. *The countable complement topology* [21, Example 20], $(\mathbb{R}, \mathcal{CC})$ is a C-regular space that is not L-regular [13]. Since $(\mathbb{R}, \mathcal{CC})$ is a mildly normal space, we obtain $(\mathbb{R}, \mathcal{CC})$ is L-mildly normal. Also, the countable complement topology is C_2-mildly normal. Since X is Lindelöf non Hausdorff, we get: X cannot be L_2-mildly normal. Therefore, $(\mathbb{R}, \mathcal{CC})$ is an L-mildly normal and C_2-mildly normal space, which is neither L-regular, epi-regular, L_2-mildly normal nor epi-mildly normal.

Note that: if X is L-mildly normal and $f : X \to Y$ is a witness of the L-mild normality of X, then f may not be continuous. For example, the countable complement topology, Example 2.4, is an L-mildly normal space and the witness of the L-mild normality of X is not continuous. But it will be if X is of a countable tightness. A space X is of a *countable tightness* if for each subset B of X and each $x \in \overline{B}$, there exists a countable subset B_0 of B such that $x \in \overline{B_0}$ [7]. Note that: every first countable space is Fréchet, every Fréchet space is sequential and every sequential space is countable tightness.

Theorem 2.7. *If X is an L-mildly normal (resp. L_2-mildly normal) space of a countable tightness and $f : X \to Y$ is a witness of the L-mild normality (resp. L_2-mild normality) of X, then f is continuous.*

Proof. It is similar to that of Theorem 5 in [14] and Theorem 11 in [4]. □

Corollary 2.3. If X is an L-mildly normal (resp. L_2-mildly normal) first countable space and $f : X \to Y$ is a witness of the L-mild normality (resp. L_2-mild normality) of X, then f is continuous.

Theorem 2.8. *If $(X, \mathcal{T})$ is an L-mildly normal countable tightness (resp. Fréchet, first countable) such that the witness $(Y, \mathcal{T}')$ of the L-mild normality of X is Hausdorff, then $(X, \mathcal{T})$ is epi-mildly normal.*

Proof. It is similar to that of Theorem 7 in [4].

□

Theorem 2.9. *If X is a T_3 separable, L-mildly normal (resp. L_2-mildly normal) space and of a countable tightness, then X is mildly normal and epi-mildly normal.*

Proof. It is similar to that of Theorem 12 in [4]. □

Since every second countable space is a Lindelöf separable space [7], and every Lindelöf L-mildly normal (resp. L_2-mildly normal) space is mildly normal, Theorem 2.2) (resp. Hausdorff mildly normal,Theorem 2.3), we get:

Corollary 2.4.

(1) Every Hausdorff second countable L-mildly normal space is epi-mildly normal.
(2) Every second countable L-mildly normal space is mildly normal.
(3) Every second countable L_2-mildly normal space is mildly normal and epi-mildly normal.

It can be observe that: epi-mild normality and L-mild normality are different from each other. For example:

Example 2.5. *The particular point topology* [21, Example 10], $(\mathbb{R}, \mathcal{T}_p)$ is neither a C-regular nor C-normal space [1, 13]. Since the particular point topology $(\mathbb{R}, \mathcal{T}_p)$ is mildly normal, we get $(\mathbb{R}, \mathcal{T}_p)$ is L-mildly normal. Therefore, $(\mathbb{R}, \mathcal{T}_p)$ is an L-mildly normal space, which is neither L-regular, L-normal, epi-mildly normal, epi-normal nor L_2-mildly normal.

Theorem 2.10. *If X is a C-mildly normal space such that every Lindelöf subspace of X is contained in a compact subspace of X, then X is L-mildly normal.*

Proof. Let X be a C-mildly normal space such that if A is a Lindelöf subspace of X, there exists a compact subspace B of X such that $A \subseteq B$. Let Y be any mildly normal space and $f : X \to Y$ be a bijective function such that $f|_C : C \to f(C)$ is a homeomorphism for each compact subspace C of X. Now, let A be any Lindelöf subspace of X. Pick a compact subspace B of X such that $A \subseteq B$. Then, $f|_B : B \to f(B)$ is a homeomorphism. Thus, $f|_A : A \to f(A)$ is a homeomorphism as $(f|_B)|_A = f|_A$. Therefore, X is L-mildly normal. □

Theorem 2.11. *If X is a C_2-mildly normal space such that every Lindelöf subspace of X is contained in a compact subspace of X, then X is L_2-mildly normal.*

Proof. It is similar to the proof of Theorem 2.10. □

3. Some other properties and counterexamples

In this section, we present some other properties, counterexamples and relationships:

Theorem 3.1. *Every L_2-mildly normal space is C_2-mildly normal.*

Proof. It is similar to the proof of Theorem 2.6. □

The converse of Theorem 3.1 is not necessary to be true in general. For example, the countable complement topology, Example 2.4, is a C_2-mildly normal space which is not L_2-mildly normal.

Theorem 3.2. *Every L_2-mildly normal space is L-mildly normal.*

Proof. Since X is an L_2-mildly normal space, there exist a Hausdorff mildly normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_C : C \to f(C)$ is a homeomorphism for each Lindelöf subspace $C \subseteq X$. Since Y is mildly normal, we obtain: X is L-mildly normal. □

The converse of Theorem 3.2 is not necessarily true in general. For example, the finite complement topology, Example 2.2, is an L-mildly normal space which is not L_2-mildly normal. Since every C_2-mildly normal space is C-mildly normal [23], we get:

Corollary 3.1. Every L_2-mildly normal space is C-mildly normal.

Theorem 3.3. *Every T_1 L-normal space is L_2-mildly normal.*

Proof. Let X be a T_1 L-normal space. Then, there exist a normal space Y and a bijective function $f : X \to Y$ such that the restriction function $f|_C : C \to f(C)$ is a homeomorphism for each Lindelöf subspace $C \subseteq X$. By Lemma 2.1, the witness Y is T_4. Hence, Y is Hausdorff mildly normal. Therefore, X is L_2-mildly normal. □

Corollary 3.2. Every T_1 non L_2-mildly normal space cannot be L-normal.

Note that: L-normality and L_2-mild normality are different from each other. For example:

Example 3.1. The left ray topology $(\mathbb{R}, \mathcal{L})$ and the right ray topology $(\mathbb{R}, \mathcal{R})$ are mildly normal spaces because they are normal. Thus, $(\mathbb{R}, \mathcal{L})$ and $(\mathbb{R}, \mathcal{R})$ are L-mildly normal spaces. Since the two spaces are first countable non Hausdorff, they are not L_2-mildly normal. Therefore, $(\mathbb{R}, \mathcal{L})$ and $(\mathbb{R}, \mathcal{R})$ are L-mildly normal spaces, which are neither L-regular, L_2-mildly normal nor epi-mildly normal.

Example 3.2. *The countable complement extension topology* [21, Example 63], is a Hausdorff, Urysohn and Lindelöf space, which is neither regular, completely regular, normal, compact nor first countable [21]. Since a subset A of X is compact if and only if it is finite [21], we get X is C_2-mildly normal. Since X is sub-metrizable, we have X is epi-mildly normal. Observe that: the space X is a mildly normal space, which is not normal. Thus, X is an L_2-mildly normal, epi-mildly normal, epi-regular and epi-completely regular space, but it is neither L-normal nor L-regular.

Theorem 3.4. *Every L_2-mildly normal countable tight (resp.Fréchet, first countable, sequential) space is epi-mildly normal.*

Proof. It is similar to the proof of Theorem 2.8. □

Corollary 3.3.

(1) Every L_2-mildly normal first countable space is Hausdorff.
(2) Every L_2-mildly normal nearly paracompact first countable space is mildly normal.
(3) If X is an L_2-mildly normal first countable space, then there exists a topology $\mathcal{T}^\star$ on X such that $\mathcal{T}^\star \subseteq \mathcal{T}$ and $(X, \mathcal{T}^\star)$ is Hausdorff mildly normal.

Corollary 3.4.

(1) Every first countable non epi-mildly normal space is not L_2-mildly normal.
(2) Every first countable non Hausdorff space is not L_2-mildly normal.

Theorem 3.5. *Every T_1 L-normal first countable space is epi-mildly normal*

Proof. By Lemma 2.1, the witness Y is T_4. Since X is first countable, we get: $f : X \to Y$ is bijective continuous function. By Theorem 3.3, X is L_2-mildly normal. By Corollary 3.3, there exists a topology $\mathcal{T}^\star$ on X such that $\mathcal{T}^\star \subseteq \mathcal{T}$ and $(X, \mathcal{T}^\star)$ is Hausdorff mildly normal. Therefore, X is epi-mildly normal. □

Since any closed extension space $(X^p, \mathcal{T}^*)$ of a given space $(X, \mathcal{T})$ is a π-normal non T_1-space [24], we obtain:

Theorem 3.6. *Every closed extension space $(X^p, \mathcal{T}^*)$ of a given first countable space $(X, \mathcal{T})$ is an L-mildly normal space, but it is not L_2-mildly normal.*

Proof. Since any closed extension space $(X^p, \mathcal{T}^*)$ of a given space $(X, \mathcal{T})$ is π-normal [24, Theorem 9], we get: $(X^p, \mathcal{T}^*)$ is mildly normal. Hence, $(X^p, \mathcal{T}^*)$ is L-mildly normal. Since any closed extension space $(X^p, \mathcal{T}^*)$ of a first countable space $(X, \mathcal{T})$ is first countable [24], we conclude that: $(X^p, \mathcal{T}^*)$ is first countable. Since $(X^p, \mathcal{T}^*)$ is first countable non Hausdorff, by Corollary 3.4, we have: $(X^p, \mathcal{T}^*)$ cannot be L_2-mildly normal. □

Corollary 3.5. L_2-mild normality is not preserved by the closed extension spaces.

Note that: any uncountable indiscrete space is an L-mildly normal space, which is neither C-Tychonoff, L_2-mildly normal nor epi-mildly normal being not Hausdorff. The following example is an L-mildly normal space, which is neither L-regular nor L_2-mildly normal. Note that: L-mild normality does not imply to L_2-mild normality. Here is a counterexample.

Example 3.3. *The excluded point topology* [21, Example 15], $(X, \mathcal{E}_p)$ is a T_0, compact, first countable, paracompact and normal space, which is neither T_1, regular nor semi regular [21]. Since X is compact Lindelöf normal space, which is not regular, the space X is an L-mildly normal space, which is neither L-regular nor L_2-mildly normal. Since X is not T_2, we obtain: X is not epi-mildly normal. Therefore, the space $(X, \mathcal{E}_p)$ is a Lindelöf L-mildly normal space, which is neither L-regular, epi-mildly normal nor L_2-mildly normal.

The following example is an L_2-mildly normal space, which is neither L-normal nor L-regular:

Example 3.4. *The Smirnov's deleted sequence topology* [21, Example 64], is a Urysohn, Lindelöf and second countable space, which is neither regular, normal nor compact [21]. Note that: the space X is sub-metrizable. Thus, it is an epi-mildly normal and C_2-mildly normal space. Since the space X is a mildly normal and C-regular space [11, 13], which is not normal , it is an L_2-mildly normal space, which is neither L-normal nor L-regular.

L_2-mild normality does not imply to L-almost normality. Here is an example:

Example 3.5. *The irregular lattice topology* [21, Example 79], is a Hausdorff, Urysohn, countable, Lindelöf and second countable space, which is neither regular, normal, compact nor paracompact [21]. The irregular lattice topology is a mildly normal space, which is not partially normal [25]. Hence, it is not quasi normal. Thus, the given space is an L_2-mildly normal and C-almost normal space which is neither almost normal, epi-quasi normal nor almost regular [4, 11]. Since X is a Lindelöf space, it is neither L-almost normal, C-regular, C-normal nor C-Tychonoff. Therefore, the irregular lattice topology is an epi-mildly normal, C-almost normal and L_2-mildly normal space which is neither quasi normal, regular, C-regular, epi-regular, epi-quasi normal, C-normal nor L-regular.

Here is an example of an L-normal and L-mildly normal space that is neither C_2-mildly normal nor epi-mildly normal.

Example 3.6. *The integer broom topology* [21, Example 121], is a T_0, normal, semi normal, compact, Lindelöf, separable, countable and paracompact space, which is neither T_1, regular, completely regular nor semi regular [21]. Thus, X is an L-mildly normal space. Since X is a compact Lindelöf normal non Hausdorff space, it is an L-normal space, which is neither L-regular, epi-mildly normal nor C_2-mildly normal.

Note that: every L-mildly normal first countable T_0-space is not necessary to be T_1 and any L-normal space cannot be L_2-mildly normal as shown by the next example:

Example 3.7. *The odd-even topology* [21, Example 6], is a regular, completely regular, normal, Lindelöf and locally compact, but it is neither T_0, compact nor semi regular [21]. The space X is an L-completely regular, L-normal and L-mildly normal space, which is neither epi-mildly normal, C-Tychonoff, see [4, Example 16]. Also, the space X is not L_2-mildly normal. Therefore, the odd-even topology is an L-normal, L-regular and L-mildly normal space, which is neither C_2-mildly normal, L_2-mildly normal nor C-Tychonoff.

Let M be a non-empty subset of a space $(X, \mathcal{T})$. Define a topology $\mathcal{T}_{(M)}$ on X as follows: $\mathcal{T}_{(M)} = \{U \cup K : \; K \subseteq X \setminus M\}$. Then, $(X, \mathcal{T}_{(M)})$ is called a discrete extension of $(X, \mathcal{T})$ denoted by X_M, where $\mathcal{T}_d \subseteq \mathcal{T} \subseteq \mathcal{T}_{(M)}$, see [26].

The following problems are still open: Is there an example of an epi-mildly normal (resp. Hausdorff locally compact) space, which is not L-mildly normal?. Is there an example of an L_2-mildly normal space, which is neither L-normal nor Hausdorff?. Is L-mild normality preserved by the discrete extension?.

4. Conclusion

New topological properties called L-mild normality and L_2-mild normality, have been studied. Some results, properties, relationships and counterexamples have been given and discussed.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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