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A Novel Concept of Bipolar Fuzzy Sets in UP (BCC)-Algebras: Bipolar Fuzzy Implicative UP (BCC)-Filters

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Abstract. This paper introduces the innovative concept of bipolar fuzzy implicative UP (BCC)-filters (BFIBCCFs) within the framework of UP (BCC)-algebras, aiming to enhance the theoretical understanding of fuzzy algebraic structures. Building upon established principles of bipolar fuzzy sets and UP (BCC)-algebras, we rigorously define BFIBCCFs and investigate their essential properties. The study explores the intricate relationships between BFIBCCFs and their associated cut sets, providing a comprehensive mathematical framework for their analysis. By extending existing theories, this research bridges a critical gap in the field and sets a foundation for further exploration in logical algebra and computational applications. The findings contribute not only to advancing theoretical mathematics but also to enabling practical applications in areas requiring precise modeling of uncertainty and dual-valued logic.

1. Introduction

Many researchers have explored algebraic structural concepts, including BCK-algebras [9], BCI-algebras [8], BCH-algebras [4], KU-algebras [17], UP-algebras [6], and others. They have a close relationship with logic. For example, BCI-algebras, which were introduced by Iséki [8] in 1966, have relations with BCI-logic, which is the BCI-system in combinatory logic and has applications in

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the language of functional programming. BCK and BCI-algebras are two types of logical algebras. They were introduced in 1966 by Imai and Iséki [8, 9] and have been thoroughly researched by numerous researchers. The class of BCK-algebras is well-known to be a proper subclass of the class of BCI-algebras. Prabpayak and Leerawat [17] established the notion of KU-algebras in 2009. In 2017, Iampan [6] proposed the notion of UP-algebras as an extension of KU-algebras. The notion of UP-algebras (see [6]) and the notion of BCC-algebras (see [15]) are the same, as shown by Jun et al. [10] in 2022. In this article, we will refer to it as BCC instead of UP as a sign of respect for Komori, who first described it in 1984.

Fuzzy sets are tools that use a mathematical framework to overcome uncertainties to supplant a group of things whose boundaries are ambiguous, as studied by Zadeh in 1965 [19]. In fuzzy set theory, there are numerous types of fuzzy set extensions, such as intuitionistic fuzzy sets, interval-valued fuzzy sets, vague sets, and so on. The fuzzy set is expanded by a bipolar-valued fuzzy set with a membership degree range of $[-1, 0] \cup [0, 1]$, which was studied by Zhang in 1994 [20]. Jun and Song [13] utilized the notion of bipolar fuzzy sets in BCH-algebras in 2008. Many researchers studied bipolar fuzzy sets in algebraic structures such that in 2011, Lee and Jun [16] studied bipolar fuzzy a-ideals of BCI-algebras. Jun et al. [12] investigated bipolar fuzzy CI-algebras in 2012. In 2018, Kawila et al. [14] introduced the concept of bipolar fuzzy BCC-algebras. The concept of doubt bipolar fuzzy H-ideals of BCK/BCI algebras was presented by Al-Masarwah and Ahmad [1]. They distinguished between the BCK/BCI-algebras' strong doubt positive t-level cut set and strong doubt negative s-level cut set. At present, Gaketem et al. [2] have introduced a new concept of bipolar fuzzy comparative BCC-filters (BFCBCCFs) and investigated their essential properties.

The structure of this article is meticulously organized to guide the reader through the foundational concepts and innovative contributions. Section 2 establishes the groundwork by presenting key definitions, illustrative examples, and fundamental properties of BCC-algebras, which serve as a basis for subsequent discussions. Section 3 introduces the groundbreaking concept of bipolar fuzzy implicative BCC-filters (BFIBCCFs), delving into their theoretical properties and intricate relationships. Finally, Section 4 provides a comprehensive synthesis of the findings, highlighting their significance and proposing directions for future research to extend the impact of this study.

2. Preliminaries

The concept of BCC-algebras (see [15]) can be redefined without the condition (2.6) as follows:

Definition 2.1. *[5] A BCC-algebra is defined as $\mathcal{A} = (\mathcal{A}, *, 0)$ of type $(2, 0)$, where $\mathcal{A}$ is a nonempty set, $*$ is a binary operation on $\mathcal{A}$, and 0 is a fixed element of $\mathcal{A}$ if it satisfies the axioms:*

$$(\text{for all } x, y, z \in \mathcal{A})((y * z) * ((x * y) * (x * z)) = 0), \tag{2.1}$$

$$(\text{for all } x \in \mathcal{A})(0 * x = x), \tag{2.2}$$

$$(\text{for all } x \in \mathcal{A})(x * 0 = 0), \tag{2.3}$$

$$(\text{for all } x, y \in \mathcal{A})((x * y = 0, y * x = 0) \Rightarrow x = y). \tag{2.4}$$

According to [6], BCC-algebras are a generalization of KU-algebras, as we well know (see [17]). Unless otherwise indicated, we will assume that $\mathcal{A}$ is a BCC-algebra $(\mathcal{A}, *, 0)$.

The binary relation $\leq$ on $\mathcal{A}$ is defined as follows:

$$(\text{for all } x, y \in \mathcal{A})(x \leq y \Leftrightarrow x * y = 0) \tag{2.5}$$

and the statements that follow are true (see [6,7]).

$$(\text{for all } x \in \mathcal{A})(x \leq x), \tag{2.6}$$

$$(\text{for all } x, y, z \in \mathcal{A})((x \leq y, y \leq z) \Rightarrow x \leq z), \tag{2.7}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow z * x \leq z * y), \tag{2.8}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow y * z \leq x * z), \tag{2.9}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y * x, \text{ in particular, } y * z \leq x * (y * z)), \tag{2.10}$$

$$(\text{for all } x, y \in \mathcal{A})(y * x \leq x \Leftrightarrow x = y * x), \tag{2.11}$$

$$(\text{for all } x, y \in \mathcal{A})(x \leq y * y), \tag{2.12}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})(x * (y * z) \leq x * ((a * y) * (a * z))), \tag{2.13}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})(((a * x) * (a * y)) * z \leq (x * y) * z), \tag{2.14}$$

$$(\text{for all } x, y, z \in \mathcal{A})((x * y) * z \leq y * z), \tag{2.15}$$

$$(\text{for all } x, y, z \in \mathcal{A})(x \leq y \Rightarrow x \leq z * y), \tag{2.16}$$

$$(\text{for all } x, y, z \in \mathcal{A})((x * y) * z \leq x * (y * z)), \tag{2.17}$$

$$(\text{for all } a, x, y, z \in \mathcal{A})((x * y) * z \leq y * (a * z)). \tag{2.18}$$

Iampan [6], Guntasow et al. [3], and Jun and Iampan [11] introduced the concepts of BCC-subalgebras, BCC-ideals, BCC-filters, and implicative BCC-filters of BCC-algebras as the following definition.

Definition 2.2. *A nonempty subset $\mathcal{S}$ of $\mathcal{A}$ is called*

(1) a BCC-subalgebra of $\mathcal{A}$ if

$$(\textit{for all } x, y \in \mathcal{S})(x * y \in \mathcal{S}), \tag{2.19}$$

(2) a BCC-ideal of $\mathcal{A}$ if

$$0 \in \mathcal{S}, \tag{2.20}$$

$$(\textit{for all } x, y, z \in \mathcal{A})((x * (y * z) \in \mathcal{S}, y \in \mathcal{S}) \Rightarrow x * z \in \mathcal{S}), \tag{2.21}$$

(3) a BCC-filter of $\mathcal{A}$ if (2.20) *and*

$$(\textit{for all } x, y \in \mathcal{A})((x \in \mathcal{S}, x * y \in \mathcal{S}) \Rightarrow y \in \mathcal{S}), \tag{2.22}$$

(4) an implicative BCC-filter (IBCCF) of $\mathcal{A}$ if (2.20) *and*

$$(\textit{for all } x, y, z \in \mathcal{A})((x * (y * z) \in \mathcal{S}, x * y \in \mathcal{S}) \Rightarrow x * z \in \mathcal{S}). \tag{2.23}$$

We know that every IBCCF is a BCC-filter, but the converse is not generally valid, as shown in the following example.

Example 2.1. *[11] Consider a BCC-algebra $\mathcal{A} = (\mathcal{A}, *, 0)$, where $\mathcal{A} = \{0, 1, 2, 3\}$ is defined in the following Cayley table.*

$*$	0	1	2	3
0	0	1	2	3
1	0	0	1	2
2	0	0	0	2
3	0	0	0	0

Then $\{0\}$ is a BCC-filter of $\mathcal{A}$, but it is not an IBCCF since $2 * (2 * 3) = 0 \in \{0\}$ and $2 * 2 = 0 \in \{0\}$, but $2 * 3 = 2 \notin \{0\}$.

The concept of bipolar fuzzy sets in a nonempty set is now reviewed.

Definition 2.3. *[20] A bipolar fuzzy set (BFS) ω in a nonempty set $\mathcal{S}$ is an object having the form*

$$\omega := \{(x, \omega^-(x), \omega^+(x)) \mid x \in \mathcal{S}\},$$

where $\omega^- : \mathcal{S} \to [-1, 0]$ and $\omega^+ : \mathcal{S} \to [0, 1]$. We'll use the symbol $\omega = (\omega^-, \omega^+)$ for the BFS $\omega = \{(x, \omega^-(x), \omega^+(x)) \mid x \in \mathcal{S}\}$ for the purpose of simplicity.

In 2018, Kawila et al. [14] introduced the concepts of bipolar fuzzy BCC-subalgebras, bipolar fuzzy BCC-filters, and bipolar fuzzy BCC-ideals of BCC-algebras as the following definition.

Definition 2.4. *A BFS $\omega = (\omega^-, \omega^+)$ in $\mathcal{A}$ is called*

(1) a bipolar fuzzy BCC-subalgebra of $\mathcal{A}$ if

$$(\text{for all } x, y \in \mathcal{A})(\omega^-(x * y) \leq \max\{\omega^-(x), \omega^-(y)\}), \tag{2.24}$$

$$(\text{for all } x, y \in \mathcal{A})(\omega^+(x * y) \geq \min\{\omega^+(x), \omega^+(y)\}), \tag{2.25}$$

(2) a bipolar fuzzy BCC-ideal of $\mathcal{A}$ if

$$(\text{for all } x \in \mathcal{A})(\omega^-(0) \leq \omega^-(x)), \tag{2.26}$$

$$(\text{for all } x \in \mathcal{A})(\omega^+(0) \geq \omega^+(x)), \tag{2.27}$$

$$(\text{for all } x, y, z \in \mathcal{A})(\omega^-(x * z) \leq \max\{\omega^-(x * (y * z)), \omega^-(y)\}), \tag{2.28}$$

$$(\text{for all } x, y, z \in \mathcal{A})(\omega^+(x * z) \geq \min\{\omega^+(x * (y * z)), \omega^+(y)\}), \tag{2.29}$$

(3) a bipolar fuzzy BCC-filter (BFBCCF) of $\mathcal{A}$ if if (2.26), (2.27), *and*

$$(\text{for all } x, y \in \mathcal{A})(\omega^-(y) \leq \max\{\omega^-(x * y), \omega^-(x)\}), \tag{2.30}$$

$$(\text{for all } x, y \in \mathcal{A})(\omega^+(y) \geq \min\{\omega^+(x * y), \omega^+(x)\}). \tag{2.31}$$

3. Bipolar fuzzy implicative BCC-filters

In this section, we introduce the novel concept of bipolar fuzzy implicative BCC-filters (BFIBCCFs), an innovative extension within the realm of BCC-algebras. This exploration delves into their formal definitions and uncovers key properties, providing fresh insights into their structural behavior and logical underpinnings. By building upon foundational theories, this section aims to establish BFIBCCFs as a pivotal element in advancing the study of fuzzy algebraic systems.

Definition 3.1. *A BFS $\omega = (\omega^-, \omega^+)$ in $\mathcal{A}$ is called a bipolar fuzzy implicative BCC-filter (BFIBCCF) of $\mathcal{A}$ if* (2.26), (2.27), *and*

$$(\text{for all } x, y, z \in \mathcal{A})(\omega^-(x * z) \leq \max\{\omega^-(x * (y * z)), \omega^-(x * y)\}), \tag{3.1}$$

$$(\text{for all } x, y, z \in \mathcal{A})(\omega^+(x * z) \geq \min\{\omega^+(x * (y * z)), \omega^+(x * y)\}). \tag{3.2}$$

The following theorem is easy to prove.

Theorem 3.1. *If $\omega = (\omega^-, \omega^+)$ is a BFS in $\mathcal{A}$ with ω^- and ω^+ are constant, then it is a BFIBCCF of $\mathcal{A}$.*

The following example shows that the converse of the above theorem is generally not true.

Example 3.1. *Consider a BCC-algebra $\mathcal{A} = (\mathcal{A}, *, 0)$, where $\mathcal{A} = \{0, 1, 2, 3, 4\}$ is defined in the Cayley table below.*

$*$	0	1	2	3	4
0	0	1	2	3	4
1	0	0	2	3	4
2	0	1	0	0	1
3	0	1	3	0	1
4	0	0	3	3	0

Define a BFS $\omega = (\omega^-, \omega^+)$ in $\mathcal{A}$ as follows:

$\mathcal{A}$	0	1	2	3	4
ω^-	-0.9	-0.5	0.3	-0.2	-0.1
ω^+	0.7	0.5	0.6	0.4	0.2

Then ω is a BFIBCCF of $\mathcal{A}$.

Theorem 3.2. *Every BFIBCCF of $\mathcal{A}$ is a BFBCCF.*

Proof. Let $\omega = (\omega^-, \omega^+)$ be a BFIBCCF of $\mathcal{A}$. We are left only to show that (2.30) and (2.31) are true. Let $x, y \in \mathcal{A}$. Then

$$\begin{aligned}
\omega^-(y) &= \omega^-(0 * y) && \text{(by (2.2))}\\
&\leq \max\{\omega^-(0 * (x * y)), \omega^-(0 * x)\} && \text{(by (3.1))}\\
&= \max\{\omega^-(x * y), \omega^-(x)\} && \text{(by (2.2))}
\end{aligned}$$

and

$$\begin{aligned}\omega^{+}(y) &= \omega^{+}(0*y) && \text{(by (2.2))}\\ &\geq \min\{\omega^{+}(0*(x*y)), \omega^{+}(0*x)\} && \text{(by (3.2))}\\ &= \min\{\omega^{+}(x*y), \omega^{+}(x)\}. && \text{(by (2.2))}\end{aligned}$$

Hence, ω is a BFBCCF of $\mathcal{A}$. □

Example 3.2. *Consider a BCC-algebra $\mathcal{A} = (\mathcal{A}, *, 0)$, where $\mathcal{A} = \{0, 1, 2, 3\}$ is defined in the Cayley table below.*

*	0	1	2	3
0	0	1	2	3
1	0	0	2	2
2	0	1	0	3
3	0	0	0	0

Define a BFS $\omega = (\omega^{-}, \omega^{+})$ in $\mathcal{A}$ as follows:

$\mathcal{A}$	0	1	2	3
ω^{-}	−0.6	−0.4	−0.2	−0.2
ω^{+}	0.9	0.6	0.3	0.3

Then ω is a BFBCCF of $\mathcal{A}$, but it is not a BFIBCCF of $\mathcal{A}$. Indeed,

$$\omega^{-}(2*3) = \omega^{-}(3) = -0.2 > -0.4 = \max\{\omega^{-}(2*(1*3)), \omega^{-}(2*1)\}.$$

Definition 3.2. *Let $\omega = (\omega^{-}, \omega^{+})$ be a BFS in $\mathcal{A}$. We define a subset $\omega^{-1}(0, 0)$ of $\mathcal{A}$ by*

$$\omega^{-1}(0, 0) = \{x \in \mathcal{A} \mid \omega^{-}(x) = \omega^{-}(0) \text{ and } \omega^{+}(x) = \omega^{+}(0)\}.$$

Theorem 3.3. *Let $\omega = (\omega^{-}, \omega^{+})$ be a BFIBCCF of $\mathcal{A}$. Then $\omega^{-1}(0, 0)$ is an IBCCF of $\mathcal{A}$.*

Proof. Clearly, $0 \in \omega^{-1}(0, 0)$. Let $x, y, z \in \mathcal{A}$ be such that $x*(y*z) \in \omega^{-1}(0, 0)$ and $x*y \in \omega^{-1}(0, 0)$. Then $\omega^{-}(x*(y*z)) = \omega^{-}(0), \omega^{-}(x*y) = \omega^{-}(0), \omega^{+}(x*(y*z)) = \omega^{+}(0), \omega^{+}(x*y) = \omega^{+}(0)$. Thus

$$\begin{aligned}\omega^{-}(0) &\leq \omega^{-}(x*z) && \text{(by (2.26))}\\ &\leq \max\{\omega^{-}(x*(y*z)), \omega^{-}(x*y)\} && \text{(by (3.1))}\\ &= \max\{\omega^{-}(0), \omega^{-}(0)\}\\ &= \omega^{-}(0)\end{aligned}$$

and

$$\begin{aligned}\omega^{+}(0) &\geq \omega^{+}(x*z) && \text{(by (2.27))}\\ &\geq \min\{\omega^{+}(x*(y*z)), \omega^{+}(x*y)\} && \text{(by (3.2))}\\ &= \min\{\omega^{+}(0), \omega^{+}(0)\}\\ &= \omega^{+}(0).\end{aligned}$$

That is, $\omega^-(x*z) = \omega^-(0)$ and $\omega^+(x*z) = \omega^+(0)$, so $x*z \in \omega^{-1}(0,0)$. Hence, $\omega^{-1}(0,0)$ is an IBCCF of $\mathcal{A}$. □

Definition 3.3. *Let $\omega = (\omega^-, \omega^+)$ be a BFS in $\mathcal{A}$. For $r^- \in [-1,0]$ and $r^+ \in [0,1]$, the sets*

$$L_N(\omega; r^-) = \{x \in \mathcal{A} \mid \omega^-(x) \leq r^-\},$$

$$U_N(\omega; r^-) = \{x \in \mathcal{A} \mid \omega^-(x) \geq r^-\},$$

$$L_P(\omega; r^+) = \{x \in \mathcal{A} \mid \omega^+(x) \leq r^+\},$$

$$U_P(\omega; r^+) = \{x \in \mathcal{A} \mid \omega^+(x) \geq r^+\}$$

are called the negative lower r^--cut, the negative upper r^--cut, the positive lower r^+-cut and the positive upper r^+-cut of ω, respectively. The set

$$C(\omega; (r^-, r^+)) = L_N(\omega; r^-) \cap U_P(\omega; r^+)$$

is called the (r^-, r^+)-cut of ω. For any $k \in [0,1]$, we denote the set

$$C(\omega; k) = C(\omega; (-k, k))$$

is called the k-cut of ω.

Theorem 3.4. *Let $\omega = (\omega^-, \omega^+)$ be a BFS in $\mathcal{A}$. Then ω is a BFIBCCF of $\mathcal{A}$ if and only if the followings are true:*

(i) for all $r^- \in [-1,0]$, $L_N(\omega; r^-)$ is an IBCCF of $\mathcal{A}$ if $L_N(\omega; r^-)$ is nonempty,
(ii) for all $r^+ \in [0,1]$, $U_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$ if $U_P(\omega; r^+)$ is nonempty.

Proof. Assume ω is a BFIBCCF of $\mathcal{A}$. Let $r^- \in [-1,0]$ be such that $L_N(\omega; r^-) \neq \emptyset$ and let $a \in L_N(\omega; r^-)$. Then $\omega^-(a) \leq r^-$. By (2.26), we have $\omega^-(0) \leq \omega^-(a) \leq r^-$. Thus $0 \in L_N(\omega; r^-)$.

Let $x, y, z \in \mathcal{A}$ be such that $x*(y*z) \in L_N(\omega; r^-)$ and $x*y \in L_N(\omega; r^-)$. Then $\omega^-(x*(y*z)) \leq r^-$ and $\omega(x*y) \leq r^-$. By (3.1), we have $\omega^-(x*z) \leq \max\{\omega^-(x*(y*z)), \omega^-(x*y)\} \leq r^-$. Thus $x*z \in L_N(\omega; r^-)$. Hence, $L_N(\omega; r^-)$ is an IBCCF of $\mathcal{A}$.

Let $r^+ \in [0,1]$ be such that $U_P(\omega; r^+) \neq \emptyset$ and let $a \in U_P(\omega; r^+)$. Then $\omega^+(a) \geq r^+$. By (2.27), we have $\omega^+(0) \geq \omega^+(a) \geq r^+$. Thus $0 \in U_P(\omega; r^+)$.

Let $x, y, z \in \mathcal{A}$ be such that $x*(y*z) \in U_P(\omega; r^+)$ and $x*y \in U_P(\omega; r^+)$. Then $\omega^+(x*(y*z)) \geq r^+$ and $\omega^+(x*y) \geq r^+$. By (3.2), we have $\omega^+(x*z) \geq \min\{\omega^+(x*(y*z)), \omega^+(x*y)\} \geq r^+$. Thus $x*z \in U_P(\omega; r^+)$. Hence, $U_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$.

Conversely, assume for all $r^- \in [0,1], L_N(\omega; r^-)$ is an IBCCF of $\mathcal{A}$ if $L_N(\omega; r^-)$ is nonempty and for all $r^+ \in [0,1], U_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$ if $U_P(\omega; r^+)$ is nonempty.

Let $x \in \mathcal{A}$. Then $\omega^-(x) \in [-1,0]$. Put $r^- = \omega^-(x)$. Then $\omega^-(x) \leq r^-$. Thus $x \in L_N(\omega; r^-) \neq \emptyset$, so $L_N(\omega; r^-)$ is an IBCCF of $\mathcal{A}$. By (2.20), we have $0 \in L_N(\omega; r^-)$. Thus $\omega^-(0) \leq r^- = \omega^-(x)$.

Let $x, y, z \in \mathcal{A}$. Then $\omega^-(x*(y*z)), \omega^-(x*y) \in [-1,0]$. Put $r^- = \max\{\omega^-(x*(y*z)), \omega^-(x*y)\}$. Then $\omega^-(x*(y*z)) \leq r^-$ and $\omega^-(x*y) \leq r^-$. Thus $x*(y*z), x*y \in L_N(\omega; r^-) \neq \emptyset$, so $L_N(\omega; r^-)$ is an

IBCCF of $\mathcal{A}$. By (2.23), we have $x * z \in L_N(\omega; r^-)$. Thus $\omega^-(x * z) \leq r^- = \max\{\omega^-(x * (y * z)), \omega^-(x * y)\}$.

Let $x \in \mathcal{A}$. Then $\omega^+(x) \in [0, 1]$. Put $r^+ = \omega^+(x)$. Then $\omega^+(x) \geq r^+$. Thus $x \in U_P(\omega; r^+) \neq \emptyset$, so $U_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$. By (2.20), we have $0 \in U_P(\omega; r^+)$. Thus $\omega^+(0) \geq r^+ = \omega^+(x)$.

Let $x, y, z \in \mathcal{A}$. Then $\omega^+(x * (y * z)), \omega^+(x * y) \in [0, 1]$. Put $r^+ = \min\{\omega^+(x * (y * z)), \omega^+(x * y)\}$. Then $\omega^+(x * (y * z)) \geq r^+$ and $\omega^+(x * y) \geq r^+$. Thus $x * (y * z), x * y \in U_P(\omega; r^+) \neq \emptyset$, so $U_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$. By (2.23), we have $x * z \in U_P(\omega; r^+)$. Thus $\omega^+(x * z) \geq r^+ = \min\{\omega^+(x * (y * z)), \omega^+(x * y)\}$.

Hence, ω is a BFIBCCF of $\mathcal{A}$. □

Corollary 3.1. *If $\omega = (\omega^-, \omega^+)$ is a BFIBCCF of $\mathcal{A}$, then for all $k \in [0, 1]$, $C(\omega; k)$ is an IBCCF of $\mathcal{A}$ if $C(\omega; k)$ is a nonempty.*

Definition 3.4. *[18] Let $\omega = (\omega^-, \omega^+)$ be a BFS in $\mathcal{A}$. The BFS $\overline{\omega} = (\overline{\omega^-}, \overline{\omega^+})$ is defined by for all $x \in \mathcal{A}$, $\overline{\omega^-}(x) = -1 - \omega^-(x)$ and $\overline{\omega^+}(x) = 1 - \omega^+(x)$ which is called the complement of ω in $\mathcal{A}$.*

The following lemma is easy to prove.

Lemma 3.1. *Let $\omega = (\omega^-, \omega^+)$ be a BFS in $\mathcal{A}$. For all $r^- \in [-1, 0]$ and $r^+ \in [0, 1]$, the followings are true:*

(i) $L_N(\overline{\omega}; r^-) = U_N(\omega; -1 - r^-)$,
(ii) $U_N(\overline{\omega}; r^-) = L_N(\omega; -1 - r^-)$,
(iii) $L_P(\overline{\omega}; r^+) = U_P(\omega; 1 - r^+)$,
(iv) $U_P(\overline{\omega}; r^+) = L_P(\omega; 1 - r^+)$.

Theorem 3.5. *Let $\omega = (\omega^-, \omega^+)$ be a BFS in $\mathcal{A}$. Then $\overline{\omega} = (\overline{\omega^-}, \overline{\omega^+})$ is a BFIBCCF of $\mathcal{A}$ if and only if the followings are true:*

(i) for all $r^- \in [-1, 0]$, $U_N(\omega; r^-)$ is an IBCCF of $\mathcal{A}$ if $U_N(\omega; r^-)$ is nonempty,
(ii) for all $r^+ \in [0, 1]$, $L_P(\omega; r^+)$ is an IBCCF of $\mathcal{A}$ if $L_P(\omega; r^+)$ is nonempty.

Proof. It follows from Theorem 3.4 and Lemma 3.1. □

4. Conclusion

This paper introduces and explores the novel concept of bipolar fuzzy implicative UP (BCC)-filters (BFIBCCFs) within the framework of UP (BCC)-algebras, offering significant advancements in understanding their algebraic properties and relations. Through detailed theoretical analysis, we establish connections between BFIBCCFs and their associated cuts, extending the applicability of bipolar fuzzy set theory in algebraic structures. The results presented here not only enrich the theoretical foundation of fuzzy algebra but also pave the way for future studies exploring practical applications and further generalizations of bipolar fuzzy filters in complex logical systems and computational models.

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