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Certain Coefficient Inequalities of Functions Using a Linear Multiplier Fractional q-Differintegral Operator with Conic Domain

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Abstract. In this article, we study the concept of a linear multiplier fractional q-differintegral operator associated with the symmetric conic domain. This work aims to define new subclasses of $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$ with Janowski functions, their coefficient bounds, and their consequences result are derived.

1. Introduction

Let $\mathcal{A}$ be the set of functions of the structure.

$$\mathfrak{f}(\tau) = \tau + \sum_{s=2}^{\infty} a_s \tau^s, \tag{1.1}$$

which are analytically contained inside the unit disc $\mathcal{E} = \{\tau : |\tau| < 1\}$. Let $\mathcal{S}$ be the subclass of $\mathcal{A}$ comprising functions that are univalent in the unit disc.

The q- difference operator of a complex-valued functional $\mathfrak{f}(\tau)$, defined on the domain of $\mathcal{E}$, be shown as

$$\partial_q \mathfrak{f}(\tau) = \frac{\mathfrak{f}(q\tau) - \mathfrak{f}(\tau)}{(q-1)\,\tau}, \qquad \tau \neq 0, \tau \in \mathcal{E}, \text{and } q \in (0,1).$$

Clearly,

$$\lim_{q\to 1^-} \left(\partial_q \mathfrak{f}\right)(\tau) = \lim_{q\to 1^-} \frac{\mathfrak{f}(q\tau) - \mathfrak{f}(\tau)}{(q-1)\,\tau} = \mathfrak{f}'(\tau).$$

The q-derivative corresponding to equation (1.1) is expressed as

$$\left(\partial_q \mathfrak{f}\right)(\tau) = 1 + \sum_{s=2}^{\infty} [s]_q \, a_s \tau^{s-1},$$

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where

$$[s]_q = \frac{1-q^s}{1-q}, \qquad (s \in \mathbb{C}).$$

For a detailed examination of q-derivatives, readers are advised to refer to ([9], [3], [4], [13]).

Definition 1.1. *[14] A linear multiplier fractional q-differintegral operator is formally defined as*

$$\mathcal{L}_{q,\beta}^{\sigma,0}\mathfrak{f}(\tau) = \mathfrak{f}(\tau),$$

$$\mathcal{L}_{q,\beta}^{\sigma,1}\mathfrak{f}(\tau) = (1-\beta)\Omega_q^{\sigma}\mathfrak{f}(\tau) + \beta\tau\mathcal{L}_q\left(\Omega_q^{\sigma}\mathfrak{f}(\tau)\right),$$

$$\mathcal{L}_{q,\beta}^{\sigma,2}\mathfrak{f}(\tau) = \mathcal{L}_{q,\,\beta}^{\sigma,1}\left(\mathcal{L}_{q,\beta}^{\sigma,1}\mathfrak{f}(\tau)\right),$$

$$\vdots$$

in general

$$\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau) = \mathcal{L}_{q,\beta}^{\sigma,1}\left(\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}-1}\mathfrak{f}(\tau)\right). \tag{1.2}$$

If $\mathfrak{f}(\tau)$ belong to $\mathcal{A}$ is represented as specified in (1.1)*, then according to* (1.2)*, we derive*

$$\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau) = \tau + \sum_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)a_s\tau^s,$$

where

$$\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right) = \left(\frac{\Gamma_q(s+1)\Gamma_q(2-\sigma)}{\Gamma_q(s+1-\sigma)}\left[1+\left([s]_q-1\right)\beta\right]\right)^{\mathfrak{m}} \tag{1.3}$$

and $0 \le \sigma < 2,\ \mathfrak{m} \in \mathbb{N}_0 = \mathbb{N} \cup \{0\},\ \beta > 0,\ 0 < q < 1.$

Definition 1.2. *[10] A function $\mathfrak{f}(\tau) \in \mathcal{A}$ is classified as belonging to the $k-ST$ class if and only if*

$$\Re e\left(\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)}\right) > k\left|\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)} - 1\right|.$$

Alternatively,

$$\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)} \prec \tilde{\tilde{p}}_k(\tau), \qquad (k \ge 0, \tau \in \mathcal{E})$$

where

$$\tilde{\tilde{p}}_k(\tau) = \begin{cases} 1 + \frac{1}{k^2-1}sin\left(\frac{\pi}{2Q(\kappa)}\int_0^{\frac{u(\tau)}{\sqrt{\kappa}}}\frac{dx}{\sqrt{1-x^2}\sqrt{1-\kappa^2x^2}}\right) + \frac{1}{k^2-1}, & if \quad k > 1, \\ 1 + \frac{2}{1-k^2}sinh^2\left[\left(\frac{2}{\pi}cos^{-1}k\right)tan^{-1}h\sqrt{\tau}\right], & if \quad 0 < k < 1, \\ 1 + \frac{2}{\pi^2}\left(log\frac{1+\sqrt{\tau}}{1-\sqrt{\tau}}\right)^2, & if \quad k = 1, \\ \frac{1+\tau}{1-\tau}, & if \quad k = 0. \end{cases} \tag{1.4}$$

where $u(\tau) = \frac{\tau-\sqrt{\kappa}}{1-\sqrt{\kappa\tau}}, \kappa \in (0,1), \tau \in \mathcal{E}$ and τ is selected. Thus, $k = cosh(\frac{\pi Q'(\kappa)}{4Q(\kappa)})$, where $Q(\kappa)$ denotes Legendre's full elliptic integral of the first kind with $Q'(\kappa)$ represents its complement integral of $Q(\kappa)$. For *further information, check out ([7], [6], [11]). It has been established in [8] that if* $\tilde{\tilde{p}}_k(\tau) = 1 + \delta_k \tau + \cdots$*, then the value of* δ_k *is determined by equation* (1.4)*, is given by*

$$\delta_k = \begin{cases} \frac{\pi^2}{4(k^2-1)\sqrt{\kappa}(1+\kappa)Q^2(\kappa)}, & \text{if} \quad k > 1, \\ \frac{8}{\pi^2}, & \text{if} \quad k = 1, \\ \frac{8(\cos^{-1}k)^2}{\pi^2(1-k^2)}, & \text{if} \quad 0 \le k < 1. \end{cases}$$

Noor and Malik [12] expanded the $k-ST$ class by introducing a generalized version, $k-ST[\mathfrak{X},\mathfrak{Y}]$, utilizing the concept of Janowski functions. For more information on Janowski functions, please refer to [5].

Definition 1.3. *[10] A function* $\mathfrak{f}(\tau)$ *belong to* $\mathcal{A}$ *is classified as belonging to the class* $k-ST\,[\mathfrak{X},\mathfrak{Y}]$*, where* $k \ge 0$ *along with* $-1 \le \mathfrak{Y} < \mathfrak{X} < 1$*, if and only if*

$$\Re e\left(\frac{(\mathfrak{Y}-1)\left(\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)}\right)-(\mathfrak{X}-1)}{(\mathfrak{Y}+1)\left(\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)}\right)-(\mathfrak{X}+1)}\right) > k\left|\frac{(\mathfrak{Y}-1)\left(\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)}\right)-(\mathfrak{X}-1)}{(\mathfrak{Y}+1)\left(\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)}\right)-(\mathfrak{X}+1)}-1\right|,$$

in other words

$$\frac{\tau\mathfrak{f}'(\tau)}{\mathfrak{f}(\tau)} \prec \frac{(1+\mathfrak{X})\,\tilde{\tilde{p}}_k(\tau)-(\mathfrak{X}-1)}{(1+\mathfrak{Y})\,\tilde{\tilde{p}}_k(\tau)-(\mathfrak{Y}-1)}.$$

Definition 1.4. *[10] A function* $\tilde{p}(\tau)$ *is classified within the set of* $k-P_q\,[\mathfrak{X},\mathfrak{Y}]$ *, where* $k \ge 0,\ 0 < q < 1$ *and* $-1 \le \mathfrak{Y} < \mathfrak{X} < 1$*, if and only if*

$$\tilde{p}\,(\tau) \prec \frac{((3-q)+\mathfrak{X}\,(q+1))\,\tilde{\tilde{p}}_k(\tau)-(\mathfrak{X}\,(q+1)-(3-q))}{((3-q)+\mathfrak{Y}\,(q+1))\,\tilde{\tilde{p}}_k(\tau)-(\mathfrak{Y}\,(q+1)-(3-q))},$$

and $\tilde{\tilde{p}}_k\,(\tau)$ *given by* (1.4)*.*

From a geometric perspective, the function $\tilde{p}\,(\tau) \in k-P_q[\mathfrak{X},\mathfrak{Y}]$ *takes values in the domain* $\Omega_{q,k}[\mathfrak{X},\mathfrak{Y}]$*, where ,*$-1 \le \mathfrak{Y} < \mathfrak{X} \le 1, k \ge 0$*, which is characterized by*

$$\Omega_{q,k}[\mathfrak{X},\mathfrak{Y}] = \left\{\hbar_1 : \Re e(\phi) > k\left|\phi - 1\right|\right\}$$

where

$$\phi = \frac{((q-3)+\mathfrak{Y}\,(q+1))\,\hbar_1\,(\tau)+((3-q)-\mathfrak{X}\,(q+1))}{((3-q)+\mathfrak{Y}\,(q+1))\,\hbar_1\,(\tau)-((3-q)+\mathfrak{X}\,(q+1))}.$$

Definition 1.5. *A function* $\mathfrak{f}(\tau)$ *belong to* $\mathcal{A}$ *is classified as belonging to the class* $k-ST_q[\mathfrak{X},\mathfrak{Y}]$*, where* $k \ge 0$*,* $-1 \le \mathfrak{Y} < \mathfrak{X} \le 1$*, if and only if*

$$\Re e\left[\frac{((q-3)+\mathfrak{Y}\,(q+1))\,\frac{\tau\partial_q\mathfrak{f}(\tau)}{\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}\,(q+1))}{((3-q)+\mathfrak{Y}\,(q+1))\,\frac{\tau\partial_q\mathfrak{f}(\tau)}{\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}\,(q+1))}\right]$$
$$> k\left|\frac{((q-3)+\mathfrak{Y}\,(q+1))\,\frac{\tau\partial_q\mathfrak{f}(\tau)}{\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}\,(q+1))}{((3-q)+\mathfrak{Y}\,(q+1))\,\frac{\tau\partial_q\mathfrak{f}(\tau)}{\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}\,(q+1))}-1\right|.$$

Alternatively stated

$$\frac{\tau\partial_q\mathfrak{f}(\tau)}{\mathfrak{f}(\tau)} \in k - P_q[\mathfrak{X},\mathfrak{Y}].$$

Definition 1.6. *A function $\mathfrak{f}(\tau)$ belong to $\mathcal{A}$ is classified as belonging to the class $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$, where $k \geq 0$, $-1 \leq \mathfrak{Y} < \mathfrak{X} \leq 1$, if and only if*

$$\Re\mathrm{e}\left[\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}\right]$$
$$> k\left|\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}-1\right|.$$

Alternatively stated

$$\frac{\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)} \in k - P_q[\mathfrak{X},\mathfrak{Y}].$$

Clearly,

(i) *If $\mathfrak{m} = 0, \beta = 1$ and $\sigma = 1$, we obtain $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}] = k - ST_q[\mathfrak{X},\mathfrak{Y}]$ which was presented by Mahmood et al. [10].*

(ii) *If $\mathfrak{m} = 0, \beta = 1, \sigma = 0$ and $k = 0$, the class $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$ reduces to $\mathcal{S}_q^*[\mathfrak{X},\mathfrak{Y}]$, which investigated by Srivastava et al. [16].*

(iii) *If $\mathfrak{m} = 0, \beta = 1, \sigma = 1$ as well as $q \to 1^-$, the class $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$ reduces to the class $k - ST[\mathfrak{X},\mathfrak{Y}]$, which pertains to the established category of Janowski k-starlike functions, as Noor and Malik stated [12].*

We require the subsequent lemma to show our principal result.

Lemma 1.1. *[15] Let $h(\tau) = 1 + \sum_{s=2}^{\infty} v_s\tau^s$ be subordinate to $\mathcal{H}(\tau) = 1 + \sum_{s=2}^{\infty} V_s\tau^s$. Let $\mathcal{H}(\tau)$ remain univalent in the unit disk as well as $\mathcal{H}(\mathcal{E})$ be convex. Then*

$$|v_s| \leq |V_1|, \quad s \geq 1. \tag{1.5}$$

2. Main Results

Theorem 2.1. *Let $\mathfrak{f}$ belong to $\mathcal{A}$ and be of the form* (1.1)*. It is in the class $k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$, $q \in (0,1)$, $0 \leq \sigma < 2$, $\beta > 0$, $k \geq 0$, $\mathfrak{m} \in \mathbb{N}_0$ along with $-1 \leq \mathfrak{Y} < \mathfrak{X} \leq 1$, if it satisfies*

$$\sum_{s=2}^{\infty} \frac{R_s}{(q+1)|\mathfrak{Y}-\mathfrak{X}|}|a_s| < 1, \tag{2.1}$$

where

$$R_s = \mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\left\{\Big|((3-q)+\mathfrak{Y}(q+1))[s]_q - ((3-q)+\mathfrak{X}(q+1))\Big| + 2(1+k)(3-q)[s-1]_q q\right\}$$

and $\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)$ *given by* (1.3).

Proof. Considering that equation (2.1) holds, it is sufficient to demonstrate that

$$k\left|\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}-1\right| -\Re\mathrm{e}\left[\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}-1\right]<1,$$

we suppose

$$\begin{aligned}
&k\left|\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}-1\right| -\Re\mathrm{e}\left[\frac{((q-3)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}+((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\frac{\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}-((3-q)+\mathfrak{X}(q+1))}-1\right]\\
&\leq(1+k)\left|\frac{((q-3)+\mathfrak{Y}(q+1))\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)+\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)((3-q)-\mathfrak{X}(q+1))}{((3-q)+\mathfrak{Y}(q+1))\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)-\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)((3-q)+\mathfrak{X}(q+1))}-1\right|\\
&=2(1+k)(3-q)\left|\frac{\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)-\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)}{((3-q)+\mathfrak{Y}(q+1))\tau\partial_q\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)-\mathcal{L}^{\sigma,\mathfrak{m}}_{q,\beta}\mathfrak{f}(\tau)((3-q)-\mathfrak{X}(q+1))}\right|\\
&=2(1+k)(3-q)\left|\frac{\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\left(1-[s]_q\right)a_s\tau^s}{(q+1)(\mathfrak{Y}-\mathfrak{X})\tau+\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\begin{bmatrix}((3-q)+\mathfrak{Y}(q+1))[s]_q\\-((3-q)+\mathfrak{X}(q+1))\end{bmatrix}a_s\tau^s}\right|\\
&=2(1+k)(3-q)\left|\frac{\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\left(-q[s-1]_q\right)a_s\tau^s}{(q+1)(\mathfrak{Y}-\mathfrak{X})\tau+\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\begin{bmatrix}((3-q)+\mathfrak{Y}(q+1))[s]_q\\-((3-q)+\mathfrak{X}(q+1))\end{bmatrix}a_s\tau^s}\right|\\
&\leq\frac{2(1+k)(3-q)q\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)[s-1]_q|a_s|}{(q+1)|\mathfrak{Y}-\mathfrak{X}|-\sum_{s=2}^{\infty}\mathscr{C}(s,\sigma,\mathfrak{m},\beta,q)\left|((3-q)+\mathfrak{Y}(q+1))[s]_q-((3-q)+\mathfrak{X}(q+1))\right||a_s|},
\end{aligned}$$

since

$$\frac{2\left(1+k\right)q\left(3-q\right)\sum\limits_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left[s-1\right]_{q}\left|a_{s}\right|}{\left(q+1\right)\left|\mathfrak{Y}-\mathfrak{X}\right|-\sum\limits_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left|\left(\left(3-q\right)+\mathfrak{Y}\left(q+1\right)\right)\left[s\right]_{q}-\left(\left(3-q\right)+\mathfrak{X}\left(q+1\right)\right)\right|\left|a_{s}\right|},$$

bounded above by one, so

$$2\left(1+k\right)q\left(3-q\right)\sum_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left[s-1\right]_{q}\left|a_{s}\right|<\left|\mathfrak{Y}-\mathfrak{X}\right|\left(q+1\right)$$
$$-\sum_{s=2}^{\infty}\left|\left(\left(3-q\right)+\mathfrak{Y}\left(q+1\right)\right)\left[s\right]_{q}-\left(\left(3-q\right)+\mathfrak{X}\left(q+1\right)\right)\right|\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left|a_{s}\right|,$$

which yields,

$$\sum_{s=2}^{\infty}\left\{2\left(1+k\right)\left(3-q\right)\left[s-1\right]_{q}\left(q\right)+\left|\left(\left(3-q\right)+\mathfrak{Y}\left(q+1\right)\right)\left[s\right]_{q}-\left(\left(3-q\right)+\mathfrak{X}\left(q+1\right)\right)\right|\right\}$$
$$.\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left|a_{s}\right|<\left(q+1\right)\left|\mathfrak{Y}-\mathfrak{X}\right|.$$

We attain the required outcome. □

For $\mathfrak{m}=0$, we arrive at the following corollary, which has already been proved by Mahmood et al. [10].

Corollary 2.1. *Suppose $\mathfrak{f}$ belongs to $\mathcal{A}$ and is expressed in the same way as* (1.1), *then it belongs to $k-ST_{q}[\mathfrak{X},\mathfrak{Y}]$, where $k\geq 0$, $-1\leq\mathfrak{Y}<\mathfrak{X}\leq 1$, if the following condition is satisfied:*

$$\sum_{s=2}^{\infty}\left\{\left|\left(\left(3-q\right)+\mathfrak{Y}(q+1)\right)\left[s\right]_{q}-\left(\left(3-q\right)+\mathfrak{X}(q+1)\right)\right|+2(1+k)(3-q)\left[s-1\right]_{q}q\right\}\left|a_{s}\right|$$
$$<\left(q+1\right)\left|\mathfrak{Y}-\mathfrak{X}\right|.$$

For $q\to 1^{-},\mathfrak{m}=0$, we reach the well-known corollary which has been studied by Noor and Malik [12].

Corollary 2.2. *Let $\mathfrak{f}$ be an element of $\mathcal{A}$ and of the type* (1.1), *then it belongs to $k-ST[\mathfrak{X},\mathfrak{Y}]$, if it fulfills*

$$\sum_{s=2}^{\infty}\frac{\left\{\left|s\left(1+\mathfrak{Y}\right)-\left(1+\mathfrak{X}\right)\right|+2(1+k)\left(s-1\right)\right\}}{\left|\mathfrak{Y}-\mathfrak{X}\right|}\left|a_{s}\right|<1,$$

where $k\geq 0$, $-1\leq\mathfrak{Y}<\mathfrak{X}\leq 1$.

For $\mathfrak{m}=0, q\to 1^{-}, \mathfrak{Y}=-1$ along with $\mathfrak{X}=1$, we obtain the following corollary, which has been investigated by Kanas and Wisniowska [7].

Corollary 2.3. *Let $\mathfrak{f}$ be an element of $\mathcal{A}$ and of the type* (1.1), *then it belongs to $k-ST$, $k\geq 0$, if it satisfies*

$$\sum_{s=2}^{\infty}\left\{s\left(1+k\right)-k\right\}\left|a_{s}\right|<1.$$

Selverman [17] established the following well-known result for $\mathfrak{X} = 1-2\alpha$, $\mathfrak{Y} = -1$, along with $k = 0$.

Corollary 2.4. *Let $\mathfrak{f}$ be an element of $\mathcal{A}$ and of the type* (1.1)*, then it belongs to $S^*(\alpha)$, if it meets the subsequent criteria*

$$\sum_{s=2}^{\infty} \{s-\alpha\} |a_s| < 1-\alpha,$$

where, $0 \le \alpha < 1$.

Theorem 2.2. *Let $\mathfrak{f}(\tau) \in k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$, $k \ge 0$, $-1 \le \mathfrak{Y} < \mathfrak{X} \le 1$, and be of the form given by* (1.1)*, then*

$$|a_s| \le \prod_{\iota=0}^{s-2} \frac{\left|(q+1)(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}(1+\iota,\sigma,\mathfrak{m},\beta,q)\,\delta_k - 4\mathfrak{Y}\,[\iota]_q\, q\mathscr{C}(1+\iota,\sigma,\mathfrak{m},\beta,q)\right|}{4\,[1+\iota]_q\, q\mathscr{C}(2+\iota,\sigma,\mathfrak{m},\beta,q)}, \qquad (s \ge 2), \qquad (2.2)$$

where

$$\mathscr{C}(1+\iota,\sigma,\mathfrak{m},\beta,q) = \left(\frac{\Gamma_q(2-\sigma)\,\Gamma_q(2+\iota)}{\Gamma_q(2+\iota-\sigma)}\left[1+\left([1+\iota]_q-1\right)\beta\right]\right)^{\mathfrak{m}},$$

$$\mathscr{C}(2+\iota,\sigma,\mathfrak{m},\beta,q) = \left(\frac{\Gamma_q(3+\iota)\,\Gamma_q(2-\sigma)}{\Gamma_q(3+\iota-\sigma)}\left[1+\left([2+\iota]_q-1\right)\beta\right]\right)^{\mathfrak{m}}.$$

Proof. By definition, if $\mathfrak{f} \in k - ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$, it follows

$$\frac{\tau\partial_q \mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)}{\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)} = \tilde{p}_k(\tau), \qquad (2.3)$$

such that

$$\tilde{p}_k(\tau) \prec \frac{((3-q)+\mathfrak{X}(q+1))\,\tilde{\tilde{p}}_k(\tau) - (\mathfrak{X}(q+1)-(3-q))}{((3-q)+\mathfrak{Y}(q+1))\,\tilde{\tilde{p}}_k(\tau) + ((3-q)-\mathfrak{Y}(q+1))}.$$

If $\tilde{\tilde{p}}_k(\tau) = 1 + \delta_k\tau + \ldots$, it implies

$$\frac{((3-q)+\mathfrak{X}(q+1))\,\tilde{\tilde{p}}_k(\tau) - (\mathfrak{X}(q+1)-(3-q))}{((3-q)+\mathfrak{Y}(q+1))\,\tilde{\tilde{p}}_k(\tau) + ((3-q)-\mathfrak{Y}(q+1))}$$
$$= 1 + \frac{1}{4}(\mathfrak{X}-\mathfrak{Y})(q+1)\,\delta_k + \frac{1}{4}\left[\left(-\frac{q}{4}\mathfrak{X} - \frac{1}{4}\mathfrak{X} + \frac{q}{4}\mathfrak{Y} + \frac{1}{4}\mathfrak{Y}\right)((q+1)(1+\mathfrak{Y}) - 2q + 2)\right]\delta_k^2 + \cdots. \qquad (2.4)$$

Consider that

$$\tilde{p}(\tau) = 1 + \sum_{s=1}^{\infty} v_s\tau^s.$$

Then, using (1.5) and (2.4), we achieve

$$|v_s| \le \frac{1}{4}(\mathfrak{X}-\mathfrak{Y})(q+1)\,|\delta_k|, \qquad (s \ge 1). \qquad (2.5)$$

Based on equation (2.3), it follows

$$\tau\partial_q\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)=\mathcal{L}_{q,\beta}^{\sigma,\mathfrak{m}}\mathfrak{f}(\tau)\tilde{p}(\tau).$$

Let

$$\tilde{p}(\tau)=1+\sum_{s=1}^{\infty}v_s\tau^s,$$

which leads to

$$\tau+\sum_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s]_q\,a_s\tau^s=\left(\tau+\sum_{s=2}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)a_s\tau^s\right)\left(1+\sum_{s=1}^{\infty}v_s\tau^s\right)$$

$$\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s]_q\,a_s\tau^s=\left(\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)a_s\tau^s\right)\left(1+\sum_{s=1}^{\infty}v_s\tau^s\right),(v_0=1),$$

which yields,

$$\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s]_q\,a_s\tau^s=\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)a_s\tau^s+\left(\sum_{s=1}^{\infty}v_s\tau^s\right)\left(\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)a_s\tau^s\right).$$

Therefore,

$$\sum_{s=1}^{\infty}\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left([s]_q-1\right)a_s\tau^s=\sum_{s=1}^{\infty}\left(\sum_{\iota=1}^{s-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)a_\iota v_{s-\iota}\right)\tau^s.$$

Now, equating the coefficients of τ^s, yields the following

$$\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)\left([s]_q-1\right)a_s=\sum_{\iota=1}^{s-\iota}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)a_\iota v_{s-\iota}.$$

This indicates

$$a_s=\frac{1}{\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s-1]_q\,q}\sum_{\iota=1}^{s-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)a_\iota v_{s-\iota}.$$

Utilizing (2.5), we achieve

$$|a_s|\leq\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)|\delta_k|}{4\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s-1]_q\,q}\sum_{\iota=1}^{s-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)|a_\iota|. \tag{2.6}$$

We now proceed to show

$$\begin{aligned}&\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)|\delta_k|}{4\mathscr{C}\left(s,\sigma,\mathfrak{m},\beta,q\right)[s-1]_q\,q}\sum_{\iota=1}^{s-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)|a_\iota|\\&\leq\prod_{\iota=0}^{s-2}\frac{\left|(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)\delta_k-4\mathfrak{Y}\,[\iota]_q\,q\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)\right|}{4\,[1+\iota]_q\,q\mathscr{C}\left(2+\iota,\sigma,\mathfrak{m},\beta,q\right)}.\end{aligned} \tag{2.7}$$

Using induction, for $s=2$ and from (2.6), we fulfill

$$|a_2|\leq\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)|\delta_k|}{4\mathscr{C}\left(2,\sigma,\mathfrak{m},\beta,q\right)[1]_q\,q}\sum_{\iota=1}^{2-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)|a_\iota|, \tag{2.8}$$

which leads to

$$|a_2| \leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|\delta_k|}{4\mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)\,[1]_q\,q}, \qquad (a_1 = 1, \mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q) = 1)\,.$$

From (2.2), we conclude

$$|a_2| \leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|\delta_k|}{4\mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)\,[1]_q\,q}.$$

For $s=3$, from (2.6), we attain

$$\begin{aligned}|a_3| &\leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\mathscr{C}\,(3,\sigma,\mathfrak{m},\beta,q)\,[2]_q\,q}\sum_{\iota=1}^{2}\mathscr{C}\,(\iota,\sigma,\mathfrak{m},\beta,q)\,|a_\iota| \\ &= \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\mathscr{C}\,(3,\sigma,\mathfrak{m},\beta,q)\,[2]_q\,q}\left(\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|a_1| + \mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)\,|a_2|\right). \\ &\leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|\delta_k|}{4\mathscr{C}\,(3,\sigma,\mathfrak{m},\beta,q)\,[2]_q\,q}\left(1+\frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\,[1]_q\,q}\right).\end{aligned}$$

From equation (2.2), we attain

$$\begin{aligned}|a_3| &\leq \prod_{\iota=0}^{1}\frac{\left|(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\,\delta_k - 4\mathfrak{Y}\,[\iota]_q\,q\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\right|}{4\,[1+\iota]_q\,q\mathscr{C}\,(2+\iota,\sigma,\mathfrak{m},\beta,q)} \\ &= \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|\delta_k|}{4\,[1]_q\,q\mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)}\left(\frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)\,|\delta_k| + 4\,[1]_q\,q\mathscr{C}\,(2,\sigma,\mathfrak{m},\beta,q)}{4\mathscr{C}\,(3,\sigma,\mathfrak{m},\beta,q)\,[2]_q\,q}\right). \\ &\leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1,\sigma,\mathfrak{m},\beta,q)\,|\delta_k|}{4\mathscr{C}\,(3,\sigma,\mathfrak{m},\beta,q)\,[2]_q\,q}\left(1+\frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\,[1]_q\,q}\right).\end{aligned}$$

Now, let us consider the inequality (2.7) is true for $s=\mathfrak{n}$.
From (2.6), we obtain

$$|a_\mathfrak{n}| \leq \frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\mathscr{C}\,(\mathfrak{n},\sigma,\mathfrak{m},\beta,q)\,[\mathfrak{n}-1]_q\,q}\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\,(\iota,\sigma,\mathfrak{m},\beta,q)\,|a_\iota|,$$

Moreover, from (2.2), we observe

$$|a_\mathfrak{n}| \leq \prod_{\iota=0}^{\mathfrak{n}-2}\frac{\left|(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\,\delta_k - 4\mathfrak{Y}\,[\iota]_q\,q\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\right|}{4\,[1+\iota]_q\,q\mathscr{C}\,(2+\iota,\sigma,\mathfrak{m},\beta,q)}.$$

According to the induction hypothesis,

$$\begin{aligned}&\frac{(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,|\delta_k|}{4\mathscr{C}\,(\mathfrak{n},\sigma,\mathfrak{m},\beta,q)\,[\mathfrak{n}-1]_q\,q}\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\,(\iota,\sigma,\mathfrak{m},\beta,q)\,|a_\iota| \\ &\qquad \leq \prod_{\iota=0}^{\mathfrak{n}-2}\frac{\left|(q+1)\,(\mathfrak{X}-\mathfrak{Y})\,\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\,\delta_k - 4\mathfrak{Y}\,[\iota]_q\,q\mathscr{C}\,(1+\iota,\sigma,\mathfrak{m},\beta,q)\right|}{4\,[1+\iota]_q\,q\mathscr{C}\,(2+\iota,\sigma,\mathfrak{m},\beta,q)}.\end{aligned} \tag{2.9}$$

Next, both sides of the equation (2.9) are multiplied by

$$\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left|\delta_{k}\right|+4\left[\mathfrak{n}-1\right]_{q}q\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)}{4\left[\mathfrak{n}\right]_{q}q\mathscr{C}\left(\mathfrak{n}+1,\sigma,\mathfrak{m},\beta,q\right)},$$

yielding

$$\begin{aligned}
&\prod_{\iota=0}^{\mathfrak{n}-1}\frac{\left|(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)\delta_{k}-4\mathfrak{Y}\left[\iota\right]_{q}q\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)\right|}{4\left[1+\iota\right]_{q}q\mathscr{C}\left(2+\iota,\sigma,\mathfrak{m},\beta,q\right)}\\
&\quad\geq\left(\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left|\delta_{k}\right|+4\left[\mathfrak{n}-1\right]_{q}q\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)}{4\left[\mathfrak{n}\right]_{q}q\mathscr{C}\left(\mathfrak{n}+1,\sigma,\mathfrak{m},\beta,q\right)}\right)\\
&\qquad\cdot\left(\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|}{4\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}-1\right]_{q}q}\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|\right)\\
&\quad=\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|}{4\mathscr{C}\left(\mathfrak{n}+1,\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}\right]_{q}q}\left(\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)}{4\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}-1\right]_{q}q}\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|+\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|\right)\\
&\quad\geq\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|}{4\mathscr{C}\left(\mathfrak{n}+1,\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}\right]_{q}q}\left(\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left|a_{\mathfrak{n}}\right|+\sum_{\iota=1}^{\mathfrak{n}-1}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|\right)\\
&\quad=\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|}{4\mathscr{C}\left(\mathfrak{n}+1,\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}\right]_{q}q}\sum_{\iota=1}^{\mathfrak{n}}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|.
\end{aligned}$$

We conclude

$$\begin{aligned}
&\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\left|\delta_{k}\right|}{4\mathscr{C}\left(\mathfrak{n},\sigma,\mathfrak{m},\beta,q\right)\left[\mathfrak{n}\right]_{q}q}\sum_{\iota=1}^{\mathfrak{n}}\mathscr{C}\left(\iota,\sigma,\mathfrak{m},\beta,q\right)\left|a_{\iota}\right|\\
&\qquad\leq\prod_{\iota=0}^{\mathfrak{n}-1}\frac{(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)\left|\delta_{k}\right|+4\left[\iota\right]_{q}q\mathscr{C}\left(1+\iota,\sigma,\mathfrak{m},\beta,q\right)}{4\left[1+\iota\right]_{q}q\mathscr{C}\left(2+\iota,\sigma,\mathfrak{m},\beta,q\right)}.
\end{aligned}$$

This illustrates that inequality (2.7) is valid for $s=\mathfrak{n}+1$. Consequently, the desired result has been achieved. □

For $\mathfrak{m}=0$, this leads to the result previously investigated by Mahmood et al. [10].

Corollary 2.5. *Let $\mathfrak{f}$ belong to $k-ST_{q}\left[\mathfrak{X},\mathfrak{Y}\right]$ and be of the kind* (1.1), *then*

$$\left|a_{s}\right|\leq\prod_{\iota=0}^{s-2}\frac{\left|(q+1)\left(\mathfrak{X}-\mathfrak{Y}\right)\delta_{k}-4\mathfrak{Y}\left[\iota\right]_{q}q\right|}{4\left[1+\iota\right]_{q}q},\qquad(s\geq2).$$

For $k=0$ and $\mathfrak{m}=0$. Theorem 2.2 simplifies to the subsequent corollary, as articulated by Srivastava et al. [16].

Corollary 2.6. *Let* $\mathfrak{f}$ *belong to* $\mathcal{S}_q^*[\mathfrak{X},\mathfrak{Y}]$ *and be of the kind* (1.1), *then*

$$|a_s| \leq \prod_{\iota=0}^{s-2} \frac{\left|(\mathfrak{X}-\mathfrak{Y})(q+1)-2\mathfrak{Y}q[\iota]_q\right|}{2[1+\iota]_q q}, \qquad (s \geq 2).$$

For $q \to 1^-$ along with $\mathfrak{m}=0$. Theorem 2.2 reduced to the established result, as presented by Noor and Malik. [12].

Corollary 2.7. *Let* $\mathfrak{f}$ *belong to* $k-ST[\mathfrak{X},\mathfrak{Y}]$ *and be of the form presented by* (1.1) *. Then*

$$|a_s| \leq \prod_{\iota=0}^{s-2} \frac{\left|(\mathfrak{X}-\mathfrak{Y})\delta_k-2\mathfrak{Y}\iota\right|}{2(1+\iota)}, \qquad (s \geq 2).$$

For $k=0, \mathfrak{m}=0$, and $q \to 1^-$ Theorem 2.2 reduces to the following corollary [16].

Corollary 2.8. *Let* $\mathfrak{f}$ *belong to* $\mathcal{S}^*[\mathfrak{X},\mathfrak{Y}]$ *and be of the form presented by* (1.1)*. Then*

$$|a_s| \leq \prod_{\iota=0}^{s-2} \frac{\left|(\mathfrak{X}-\mathfrak{Y})-\mathfrak{Y}\iota\right|}{(1+\iota)}, \qquad (s \geq 2, -1 \leq \mathfrak{Y} < \mathfrak{X} \leq 1).$$

For $k=0, \mathfrak{m}=0, q \to 1^-, \mathfrak{X}=1-2\alpha$, and $\mathfrak{Y}=-1$, the following established conclusion is obtained; refer to [2].

Corollary 2.9. *Let* $\mathfrak{f}(\tau) \in \mathcal{S}^*(\alpha)$ *and be of the form* (1.1)*. Then*

$$|a_s| \leq \frac{\prod_{\iota=2}^{s}(\iota-2\alpha)}{(s-1)!}, \qquad (0 \leq \alpha < 1, s \geq 2).$$

Corollary 2.10. *[1] For* $\mathfrak{X}=1$, $\mathfrak{Y}=-1$, $\mathfrak{m}=0$, *and* $q \to 1^-$, *then* (2.2) *reduces to*

$$|a_s| \leq \prod_{\iota=0}^{s-2} \frac{|\delta_k+\iota|}{(1+\iota)}, \qquad (s \geq 2).$$

Corollary 2.11. *[1] For* $\mathfrak{X}=1-2\alpha$, $\mathfrak{Y}=-1$, $\mathfrak{m}=0$, *and* $q \to 1^-$, *then* (2.2) *reduces to*

$$|a_s| \leq \prod_{\iota=0}^{s-2} \frac{\left|(1-\alpha)\delta_k+\iota\right|}{(1+\iota)}, \qquad (s \geq 2, 0 \leq \alpha < 1).$$

3. Conclusion

We have studied the concept of a linear multiplier fractional q-differintegral operator and using this we defined new subclasses $k-ST_{q,\beta}^{\sigma,\mathfrak{m}}[\mathfrak{X},\mathfrak{Y}]$ associated with conic domain. In addition, we obtained coefficient inequalities, bounds, and some consequences.

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