International Journal of Analysis and Applications

Neutrosophic Method for Identifying Extreme Values in Imprecise Data Using Median Absolute Deviation

Muhammad Saleem[1], Muhammad Aslam[2,*]

[1]Department of Industrial Engineering, Faculty of Engineering, King Abdulaziz University, Rabigh, 21911, Saudi Arabia; msaleim1@kau.edu.sa

[2]Department of Statistics, Faculty of Science, King Abdulaziz University, Jeddah 21589, Saudi Arabia

[*]Corresponding author: aslam_ravian@hotmail.com, magmuhammad@kau.edu.sa

ABSTRACT. The traditional calculation of Z-scores for outlier detection is highly sensitive to extreme data points, making it unsuitable under conditions of uncertainty. In this study, we propose a novel approach to modify the Z-score method using neutrosophic statistics. Key statistical measures, including the median, neutrosophic standard deviation, and median absolute deviation, will be computed based on neutrosophic random variables. An extensive simulation study will evaluate the impact of varying uncertainty levels on the adaptation of Z-scores for outlier detection and their effectiveness in identifying outliers. Comparative analysis of Z-scores derived from different methods will also be performed. The proposed methodology will be applied to neutrosophic GG25 gray cast iron data, demonstrating its practical utility. We hypothesize that uncertainty levels will significantly affect Z-score computations and, consequently, outlier detection in the dataset.

1. Introduction

In statistical analysis, an outlier refers to a data point that significantly deviates from the rest of the dataset. These anomalies often arise from measurement variations or experimental errors. Outliers can skew descriptive statistics, such as the mean, potentially misleading analysts and decision-makers about the dataset's true characteristics. Identifying and addressing outliers is crucial in statistical analysis, often necessitating their elimination or adjustment to align with the dataset's typical values. The Z-score method is a traditional tool for spotting outliers, measuring how many standard deviations a data point is from the mean. It calculates this by dividing the difference between the data point and the mean by the standard deviation. Conventionally, data points with a Z-score beyond the range of -3 to 3 are considered outliers.

Received Feb. 18, 2025

2020 Mathematics Subject Classification. 62H10.

Key words and phrases. outliers; fuzzy data; simulation; uncertainty; classical statistics.

https://doi.org/10.28924/2291-8639-23-2025-144

ISSN: 2291-8639

However, this method is sensitive to outliers itself, as the mean can be distorted by extreme values. To mitigate this, an adjusted Z-score formula has been introduced, relying on the median—a more robust central tendency measure—rather than the mean. This revised formula, which considers the distance between data points and the median relative to the standard deviation, deems data points as outliers if their Z-score falls outside the -3.5 to 3.5 ranges. This method, known as a more robust approach for outlier detection, addresses the vulnerability of the traditional Z-score to extreme values [1]. [2] conducted a survey on outlier detection methods. [3] utilized the Z-score method to identify outliers in high-dimensional data. [4] employed an enhanced Z-score method for outlier detection. [5] introduced a method for detecting univariate outliers tailored for nurse researchers. [6] presented expedited approaches for outlier detection. [7] applied the Z-score method to detect outliers in time series signal strength data. [8] developed an approach for detecting outliers in surface water temperature data. [9] proposed a method for outlier detection in groundwater data. [10] conducted a review on outlier detection in geotechnical data. More detail can be seen in [11].

Neutrosophic statistics is a mathematical framework utilized for analyzing data gathered amidst uncertainty. This discipline focuses on handling imprecise, fuzzy, and interval data, encompassing their collection, analysis, and interpretation. Introduced by **[12]**, neutrosophic statistics extends beyond classical statistics by offering additional insights, notably the degree of indeterminacy, which classical statistics often fails to provide. Recent studies, such as the work by [13], have demonstrated the efficacy of neutrosophic statistics compared to interval statistics. [14] and [15] introduced the method to analysis the neutrosophic data. [16] presented the neutrosophic Dixon's test to detect outliers in the data. [17] presented the outliers method in neutrosophic. [18] and [19] presented the exponential distribution and Rayleigh distribution using the neutrosophic statistics. [20] presented the application of neutrosophic correlation coefficient. [21] presented the analysis of covariance using the neutrosophic statistics. [22] applied the neutrosophic logistic model for fuzzy data.

The existing Z-score methods within classical statistics are commonly employed for outlier detection in uncertain data. Despite an extensive literature review, there appears to be a dearth of research regarding the application of the Z-score method within neutrosophic statistics. To address this gap, this paper aims to introduce the design of the Z-score method within the framework of neutrosophic statistics. We will conduct a comprehensive simulation study to investigate how varying degrees of uncertainty influence outlier detection using the proposed method. Additionally, we will provide comparative analyses and practical demonstrations of the proposed approach using neutrosophic datasets. It is anticipated that the proposed method for outlier detection will exhibit improved performance in situations characterized by uncertainty.

2. The Proposed Method

Suppose that $X_N = X_L + X_L I_N; I_N \epsilon [I_L, I_U]$ be a neutrosophic random variable and is based on two parts. The first random variable X_L denotes the random variable for the determinate part of neutrosophic data and $X_L I_N$ be the indeterminate part of the neutrosophic number and $I_N \epsilon [I_L, I_U]$ be the indeterminacy. Suppose that X_L follows the normal distribution with mean μ and variance σ^2. Note that the proposed neutrosophic random variable $X_N \epsilon [X_L, X_U]$ reduces to X_L if I_L=0. Note here that $I_N^2 = I_N$. The neutrosophic expectation of the proposed neutrosophic random variable is given by

$$E(X_N) = E(X_L + X_L I_N) = \mu + \mu I_N \tag{1}$$

The neutrosophic expectation of the proposed neutrosophic random variable is given by

$$Var(X_N) = Var(X_L + X_L I_N) = (1 + I_N)^2 \, \sigma^2 \tag{2}$$

Note that when I_L=0, the expectation and variance of the proposed neutrosophic random variable converge to those of classical statistics. Let $x_{1N}, x_{2N}, \dots, x_{nN}$ represent a neutrosophic random sample of size n. The neutrosophic mean $\bar{x}_N$ is computed as follows

$$\bar{x}_N = \frac{(1+I_N)x_{1L} + (1+I_N)x_{2L} + \dots (1+I_N)x_{nL}}{n} = \frac{(1+I_N)\sum_{i=1}^{n} x_{iL}}{n} = (1 + I_N)\bar{x}_{iL} \tag{3}$$

The neutrosophic sample variance s_N^2 is calculated by

$$s_N^2 = \frac{\sum_{i=1}^{n}(x_{iN} - \bar{x}_{iN})^2}{n-1} = \frac{\sum_{i=1}^{n}\left((1+I_N)x_{iL} - (1+I_N)\bar{x}_{iL}\right)^2}{n-1} = \frac{\sum_{i=1}^{n}(x_{iL} - \bar{x}_{iL})^2 (1+I_N)^2}{n-1} \tag{4}$$

The sample standard deviation is given by

$$s_N = (1 + I_N)\sqrt{\frac{\sum_{i=1}^{n}(x_{iL} - \bar{x}_{iL})^2}{n-1}} \tag{5}$$

The mean absolute deviation (meanAD) quantifies variability by representing the average distance between values and their mean. Extending this concept, we define the neutrosophic mean absolute deviation (NmeanAD) as

$$\text{NmeanAD} = \frac{\sum\left|\left((1+I_N)(x_{iL} - \bar{x}_{iL})\right)\right|}{n} \tag{6}$$

The median absolute deviation (MAD) is used to measure the dispersion in the data. It is known as the robust statistic as it is not affected by the extreme value as compare to standard deviation. Extending this concept, we define the neutrosophic median absolute deviation (NMAD) as

$$NMAD = Median\left(\left|\left((1 + I_N)(x_{iL} - \tilde{x}_L)\right)\right|\right) \tag{7}$$

where $\tilde{x}_L$ is the median of lower part of the neutrosophic data.

We will present the modification of the Z-score using the neutrosophic idea. In this modified Z-score, we will incorporate the idea of neutrosophy and the proposed test will be based on the median instead of mean. It is expected that the proposed test will be robust than the existing Z-score under classical statistics. We also expect the proposed test will be less affected by the outlier as compared to the existing test. In addition, the existing Z-score cannot be applied under uncertain environment. Depending on the value of MAD, the proposed Z-score will be given as

$$z_N = \frac{0.6745*(X_N - \tilde{x}_N)}{NMAD} \quad (8)$$

The proposed method is implemented in the following steps

Step-1: specify I_N and compute $\tilde{x}_L$

Step-2: Compute $NMAD$ using Eq. (7).

Step-3: Compute z_N using Eq. (8).

Note the proposed Z-score reduces to the Z-score under classical statistics when there is no imprecise value in the data.

3. Simulation

In this section, we will perform the simulation study and we will generate the values of Z-score using the proposed method for various values of $\tilde{x}_L$ and I_N. We will study the effect of the degree of indeterminacy on the generation of Z-score for the detecting of the outlier in the data. We will present the values of Z-score using the proposed method for various I_N in Tables 1-4. The values of Z-score when $\tilde{x}_L$=16 are shown in Table 1. The values of Z-score when $\tilde{x}_L$=17.5 are shown in Table 2. The values of Z-score when $\tilde{x}_L$=23 are shown in Table 3. The values of Z-score when $\tilde{x}_L$=25 are shown in Table 4. From these tables, it can be seen that there is decreasing trends in the values of Z-score as the values of I_N increases. For example, for Table 1 (the first row), it can be seen that the value of Z-score is -0.766 when I_N=0.1 and the value of Z-score is -0.422 when I_N=1. The behavior in the values of Z-score for various values of I_N when $\tilde{x}_L$=16 is shown in Figure 1. The behavior in the values of Z-score for various values of I_N when $\tilde{x}_L$=25 is shown in Figure 2. From Figures 1-2, it can be seen that there is significant effect of the degree of indeterminacy on the generation of Z-score.

Table 1: Modified Z-score when $\tilde{x}_L$=16 for various values of I_N

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
-0.843	-0.766	-0.703	-0.649	-0.602	-0.562	-0.527	-0.496	-0.468	-0.444	-0.422
-0.759	-0.690	-0.632	-0.584	-0.542	-0.506	-0.474	-0.446	-0.422	-0.399	-0.379
-0.759	-0.690	-0.632	-0.584	-0.542	-0.506	-0.474	-0.446	-0.422	-0.399	-0.379
-0.675	-0.613	-0.562	-0.519	-0.482	-0.450	-0.422	-0.397	-0.375	-0.355	-0.337
-0.337	-0.307	-0.281	-0.259	-0.241	-0.225	-0.211	-0.198	-0.187	-0.178	-0.169
-0.169	-0.153	-0.141	-0.130	-0.120	-0.112	-0.105	-0.099	-0.094	-0.089	-0.084
-0.084	-0.077	-0.070	-0.065	-0.060	-0.056	-0.053	-0.050	-0.047	-0.044	-0.042
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
0.253	0.230	0.211	0.195	0.181	0.169	0.158	0.149	0.141	0.133	0.126
0.506	0.460	0.422	0.389	0.361	0.337	0.316	0.298	0.281	0.266	0.253
0.675	0.613	0.562	0.519	0.482	0.450	0.422	0.397	0.375	0.355	0.337
0.843	0.766	0.703	0.649	0.602	0.562	0.527	0.496	0.468	0.444	0.422
0.843	0.766	0.703	0.649	0.602	0.562	0.527	0.496	0.468	0.444	0.422
1.096	0.996	0.913	0.843	0.783	0.731	0.685	0.645	0.609	0.577	0.548
2.529	2.299	2.108	1.946	1.807	1.686	1.581	1.488	1.405	1.331	1.265

Table 2: Modified Z-score when $\tilde{x}_L$=17.5 for various values of I_N

$I_N=0$	$I_N=0.1$	$I_N=0.2$	$I_N=0.3$	$I_N=0.4$	$I_N=0.5$	$I_N=0.6$	$I_N=0.7$	$I_N=0.8$	$I_N=0.9$	$I_N=1$
-0.862	-0.784	-0.718	-0.663	-0.616	-0.575	-0.539	-0.507	-0.479	-0.454	-0.431
-0.787	-0.715	-0.656	-0.605	-0.562	-0.525	-0.492	-0.463	-0.437	-0.414	-0.393
-0.787	-0.715	-0.656	-0.605	-0.562	-0.525	-0.492	-0.463	-0.437	-0.414	-0.393
-0.712	-0.647	-0.593	-0.548	-0.509	-0.475	-0.445	-0.419	-0.396	-0.375	-0.356
-0.412	-0.375	-0.343	-0.317	-0.294	-0.275	-0.258	-0.242	-0.229	-0.217	-0.206
-0.262	-0.238	-0.219	-0.202	-0.187	-0.175	-0.164	-0.154	-0.146	-0.138	-0.131
-0.187	-0.170	-0.156	-0.144	-0.134	-0.125	-0.117	-0.110	-0.104	-0.099	-0.094
6.183	5.621	5.152	4.756	4.416	4.122	3.864	3.637	3.435	3.254	3.091
-0.112	-0.102	-0.094	-0.086	-0.080	-0.075	-0.070	-0.066	-0.062	-0.059	-0.056
0.112	0.102	0.094	0.086	0.080	0.075	0.070	0.066	0.062	0.059	0.056
0.337	0.307	0.281	0.259	0.241	0.225	0.211	0.198	0.187	0.178	0.169
0.937	0.852	0.781	0.721	0.669	0.625	0.586	0.551	0.520	0.493	0.468
0.637	0.579	0.531	0.490	0.455	0.425	0.398	0.375	0.354	0.335	0.319
0.637	0.579	0.531	0.490	0.455	0.425	0.398	0.375	0.354	0.335	0.319
0.862	0.784	0.718	0.663	0.616	0.575	0.539	0.507	0.479	0.454	0.431
2.136	1.942	1.780	1.643	1.526	1.424	1.335	1.256	1.187	1.124	1.068

Table 3: Modified Z-score when $\tilde{x}_L$=23 for various values of I_N

$I_N=0$	$I_N=0.1$	$I_N=0.2$	$I_N=0.3$	$I_N=0.4$	$I_N=0.5$	$I_N=0.6$	$I_N=0.7$	$I_N=0.8$	$I_N=0.9$	$I_N=1$
-1.529	-1.390	-1.274	-1.176	-1.092	-1.019	-0.956	-0.899	-0.849	-0.805	-0.764
-1.439	-1.308	-1.199	-1.107	-1.028	-0.959	-0.899	-0.846	-0.799	-0.757	-0.719
-1.439	-1.308	-1.199	-1.107	-1.028	-0.959	-0.899	-0.846	-0.799	-0.757	-0.719
-0.809	-0.736	-0.675	-0.623	-0.578	-0.540	-0.506	-0.476	-0.450	-0.426	-0.405
-0.989	-0.899	-0.824	-0.761	-0.707	-0.660	-0.618	-0.582	-0.550	-0.521	-0.495
0.090	0.082	0.075	0.069	0.064	0.060	0.056	0.053	0.050	0.047	0.045
-0.719	-0.654	-0.600	-0.553	-0.514	-0.480	-0.450	-0.423	-0.400	-0.379	-0.360
6.925	6.295	5.771	5.327	4.946	4.617	4.328	4.073	3.847	3.645	3.462
-0.630	-0.572	-0.525	-0.484	-0.450	-0.420	-0.393	-0.370	-0.350	-0.331	-0.315
0.630	0.572	0.525	0.484	0.450	0.420	0.393	0.370	0.350	0.331	0.315
-0.090	-0.082	-0.075	-0.069	-0.064	-0.060	-0.056	-0.053	-0.050	-0.047	-0.045
0.630	0.572	0.525	0.484	0.450	0.420	0.393	0.370	0.350	0.331	0.315
0.270	0.245	0.225	0.208	0.193	0.180	0.169	0.159	0.150	0.142	0.135
0.270	0.245	0.225	0.208	0.193	0.180	0.169	0.159	0.150	0.142	0.135
0.540	0.491	0.450	0.415	0.385	0.360	0.337	0.317	0.300	0.284	0.270
2.068	1.880	1.724	1.591	1.477	1.379	1.293	1.217	1.149	1.089	1.034

Table 4: Modified Z-score when $\tilde{x}_L$=25 for various values of I_N

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
-1.602	-1.456	-1.335	-1.232	-1.144	-1.068	-1.001	-0.942	-0.890	-0.843	-0.801
-1.518	-1.380	-1.265	-1.167	-1.084	-1.012	-0.949	-0.893	-0.843	-0.799	-0.759
-1.518	-1.380	-1.265	-1.167	-1.084	-1.012	-0.949	-0.893	-0.843	-0.799	-0.759
-0.927	-0.843	-0.773	-0.713	-0.662	-0.618	-0.580	-0.546	-0.515	-0.488	-0.464
-1.096	-0.996	-0.913	-0.843	-0.783	-0.731	-0.685	-0.645	-0.609	-0.577	-0.548
-0.084	-0.077	-0.070	-0.065	-0.060	-0.056	-0.053	-0.050	-0.047	-0.044	-0.042
-0.422	-0.383	-0.351	-0.324	-0.301	-0.281	-0.263	-0.248	-0.234	-0.222	-0.211
6.323	5.749	5.270	4.864	4.517	4.216	3.952	3.720	3.513	3.328	3.162
1.686	1.533	1.405	1.297	1.204	1.124	1.054	0.992	0.937	0.888	0.843
0.422	0.383	0.351	0.324	0.301	0.281	0.263	0.248	0.234	0.222	0.211
-0.253	-0.230	-0.211	-0.195	-0.181	-0.169	-0.158	-0.149	-0.141	-0.133	-0.126
0.422	0.383	0.351	0.324	0.301	0.281	0.263	0.248	0.234	0.222	0.211
0.084	0.077	0.070	0.065	0.060	0.056	0.053	0.050	0.047	0.044	0.042
0.084	0.077	0.070	0.065	0.060	0.056	0.053	0.050	0.047	0.044	0.042
0.337	0.307	0.281	0.259	0.241	0.225	0.211	0.198	0.187	0.178	0.169
1.771	1.610	1.475	1.362	1.265	1.180	1.107	1.042	0.984	0.932	0.885

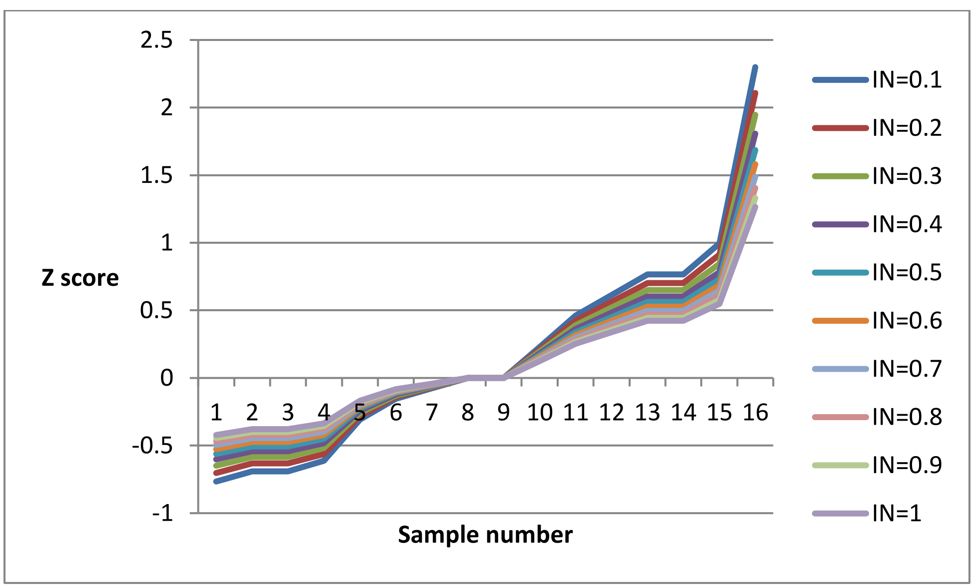

Figure 1: Z score when $\tilde{x}_L$=16

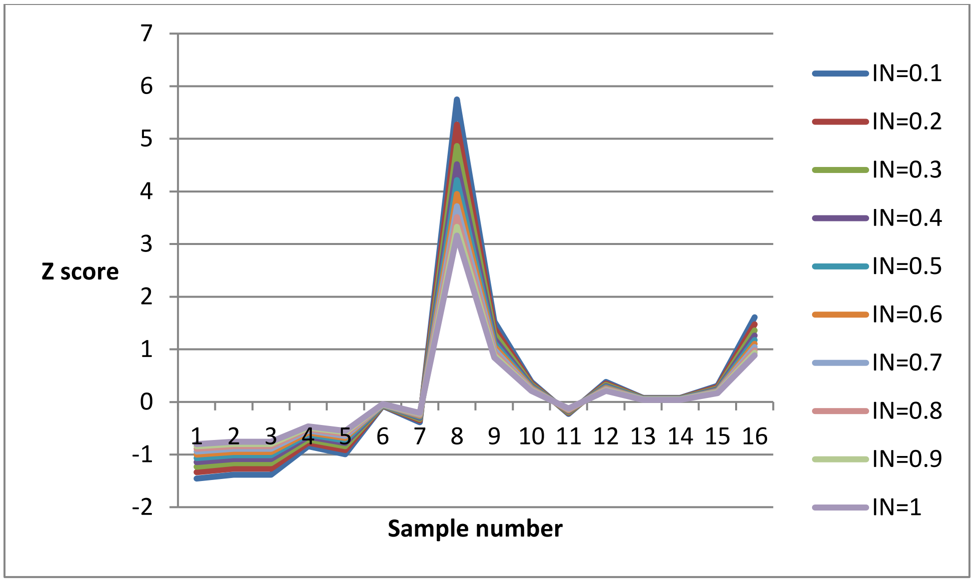

Figure 2: Z score when $\tilde{x}_L$=25

4. Effect on Detecting Outliers

Now, we will discuss the effect of degree of indeterminacy on the detecting the outlier in the data. We will generate Z-score using the method under the classical statistics, modified Z-score using the classical statistics and the proposed Z-score using the neutrosophic statistics. For the easy reference, we express, the Z-score under the classical statistics as follows

$$Z = \frac{(X-\mu)}{\sigma} \tag{9}$$

where μ and σ are the mean and standard deviation of the data. The Z-score using the modified Z-score is expressed by

$$z_N = \frac{0.6745*(X-\tilde{x})}{MAD} \tag{10}$$

where $\tilde{x}$ and MAD are the median and median absolute deviation of the data. Note here that according to the existing method under classical statistics, if the value of Z-score beyond -3 and 3 is declared as the outlier. According to [1], for the modified case, if the value of Z-score beyond -3.5 and 3.5 is declared as the outlier.

Figure 3 is presented here to illustrate the outlier detection methods employed in our analysis. Z-scores were computed using three distinct methods, and the results are depicted in Figure 3. Notably, the Z-score values derived from our proposed method were calculated under the condition where I_N equals 0.5. Examination of Figure 3 reveals that while both the proposed method within the framework of neutrosophic statistics and a modified approach within classical statistics identify observation 8 as an outlier, the proposed method suggests that a point closer to the upper limit is also noteworthy, in contrast to the modified Z-score computed using classical statistics. Moreover, this same point falls within the specified limits when employing Z-score calculations based on classical statistics alone. The visual representation in Figure 3 highlights the influence of indeterminacy on outlier identification within the dataset. Consequently, relying solely on existing outlier detection methods in the presence of uncertainty may lead decision-makers astray.

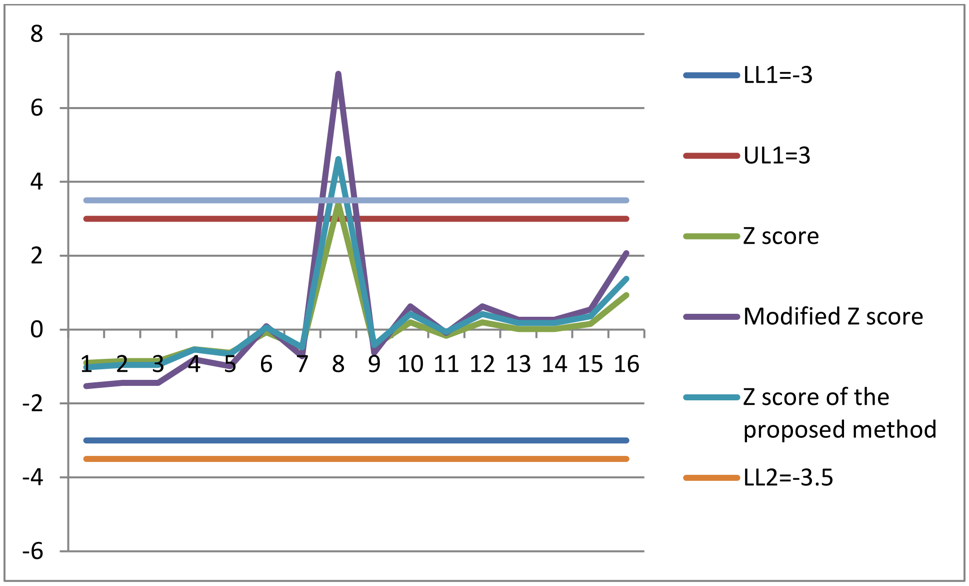

Figure 3: Z score when $\tilde{x}_L$=23 and I_N=0.50

5. Comparative Study

In this section, we aim to compare the efficacy of three distinct methods for generating Z-scores and explore the impact of varying degrees of indeterminacy on their computation. As previously outlined, we will juxtapose the proposed method for generating Z-scores, tailored for outlier detection, with both a modified Z-score approach and the conventional method rooted in classical statistics. For our proposed method, we will maintain a fixed value of I_N at 0.1. The Z-score values are shown in Table 5. Subsequently, Z-scores were computed using these methods and plotted in Figure 4. Upon inspection of Figure 4, it becomes apparent that the Z-scores derived from our proposed method under conditions of uncertainty diverge from those generated by the modified Z-score method and the classical statistics-based approach. Specifically, the Z-score curve obtained from the classical statistics-based method appears lower compared to the other two curves. Additionally, the Z-score curve stemming from our proposed method under uncertainty exhibits a lower trajectory compared to the modified Z-score curve under classical statistics. This analysis unequivocally demonstrates that Z-scores computed under neutrosophic statistics manifest distinct characteristics from those computed using the other two methodologies. Consequently, the influence of indeterminacy on Z-score generation significantly impacts outlier identification within the dataset. In essence, this study concludes that decisions regarding outliers diverge when employing neutrosophic statistics as opposed to traditional methods rooted in classical statistics.

Table 5: Comparisons between Z-score when $I_N = 0.1$

Z score using classical statistics	Modified Z-score	Z score using neutrosophic statistics
-1.529	-0.89941	-1.390
-1.439	-0.85358	-1.308
-1.439	-0.85358	-1.308
-0.809	-0.53277	-0.736
-0.989	-0.62443	-0.899
0.090	-0.07447	0.082
-0.719	-0.48694	-0.654
6.925	3.408581	6.295
-0.630	-0.44111	-0.572
0.630	0.200505	0.572
-0.090	-0.16613	-0.082
0.630	0.200505	0.572
0.270	0.017186	0.245
0.270	0.017186	0.245
0.540	0.154675	0.491
2.068	0.933779	1.880

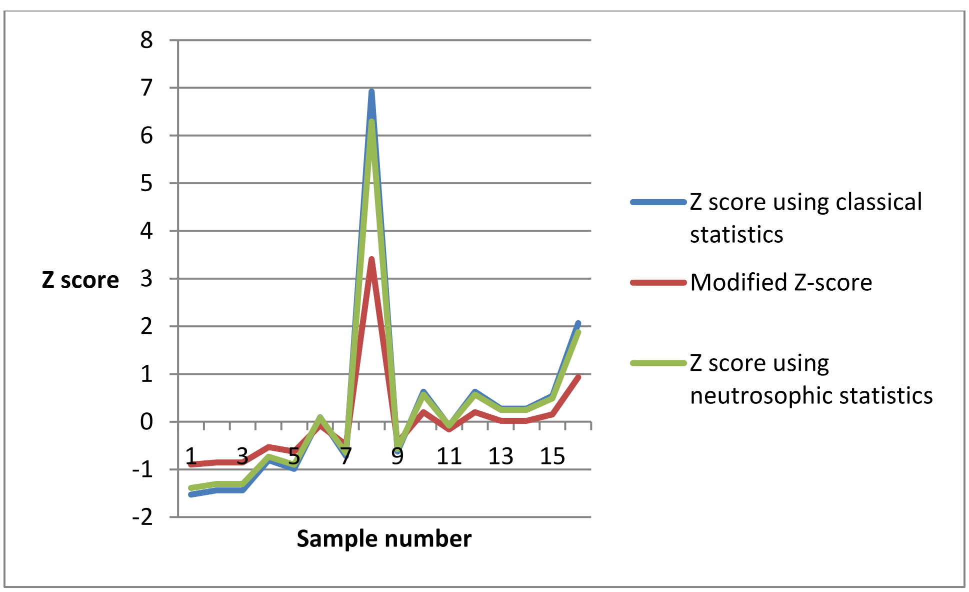

Figure 4: Comparisons between Z-scores for various methods when I_N=0.1

6 Application using GG25 Gray Cast Iron Data

This section delves into the application of our proposed method using the GG25 gray cast iron dataset obtained from **[23]**. It's worth noting that **[23]** initially presented the fuzzy GG25 gray cast iron data, which we have compiled and displayed in Table 6. Analysis of this neutrosophic data reveals the inadequacy of existing methods rooted in classical statistics for outlier detection within the GG25 gray cast iron dataset. Conversely, our proposed method demonstrates suitability for this task. The median values derived from the GG25 gray cast iron data indicate a lower bound of 4 and an upper bound of 4.01, with a degree of indeterminacy calculated at 0.0025. Utilizing this information, we computed Z-score values employing our proposed method and illustrated them in Figure 5. Notably, examination of Figure 5 indicates that the Z-score values fall comfortably within the specified limits. Consequently, based on our analysis, we conclude that no outliers are detected within the GG25 gray cast iron dataset utilizing the proposed method. In summary, our proposed approach suggests the absence of outliers within the GG25 gray cast iron data.

Table 6: The real data

[4.149,4.159]	[4.219,4.229]	[4.243,4.253]	[4.359,4.369]
[3.957,3.967]	[3.999,4.009]	[3.948,3.958]	[3.990,4.000]
[3.886,3.896]	[4.001,4.011]	[3.713,3.723]	[3.759,3.769]
[3.817,3.827]	[4.081,4.091]	[4.348,4.358]	[4.225,4.235]

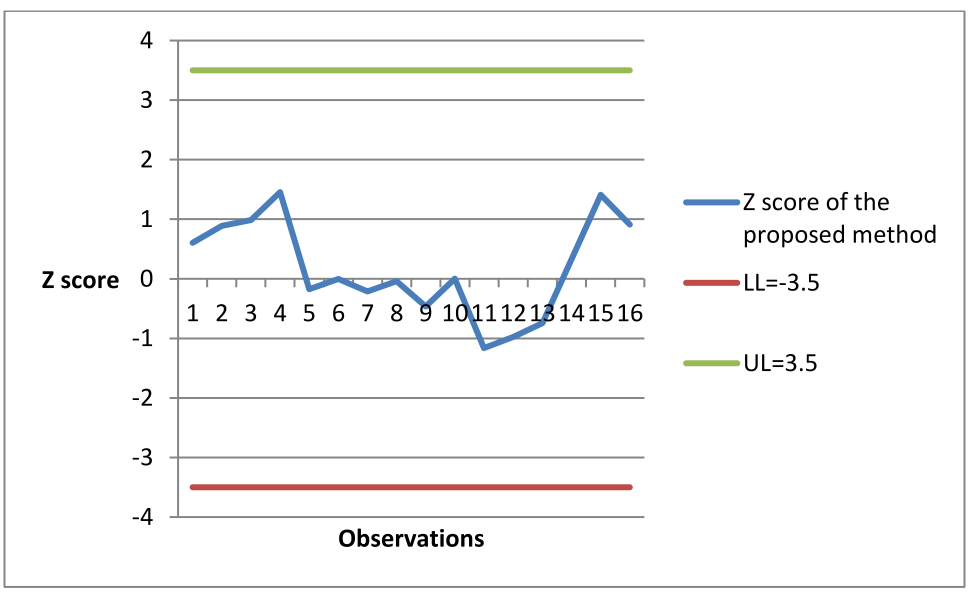

Figure 5: Z score using the real data

7. Concluding Remakes

This paper introduces several statistical techniques within the framework of neutrosophic statistics. Specifically, we explore the application of the median absolute deviation and the Z-score method for generating imprecise values in uncertain environments. Furthermore, we conduct an extensive simulation study examining the impact of various median values, as well as a simulation method to identify outliers within datasets. Additionally, we provide a real-world example from the field of chemistry to illustrate practical applications. Through these investigations, it becomes evident that the computation of Z-scores is notably influenced by the level of uncertainty present. Consequently, our study suggests that the proposed method holds promise for outlier detection in uncertain datasets. However, it's important to note that the proposed method is limited to scenarios where data uncertainty exists, such as interval, imprecise, or fuzzy data. Moving forward, future research could explore the application of the proposed method with different sampling schemes. Additionally, there are various aspects of the proposed method that warrant further investigation and could serve as avenues for future research.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] B. Iglewicz, D.C. Hoaglin, How to Detect and Handle Outliers, ASQC Quality Press, Milwaukee, 1993.

[2] V. Hodge, J. Austin, A Survey of Outlier Detection Methodologies, Artif. Intell. Rev. 22 (2004), 85-126. https://doi.org/10.1023/b:aire.0000045502.10941.a9.

[3] P. Venkataanusha, C. Anuradha, P.S.R.C. Murty, S.K. Chebrolu, Detecting Outliers in High Dimensional Data Sets Using Z-Score Methodology, Int. J. Innov. Technol. Explor. Eng. 9 (2019), 48–53. https://doi.org/10.35940/ijitee.A3910.119119.

[4] V. Aggarwal, V. Gupta, P. Singh, K. Sharma, N. Sharma, Detection of Spatial Outlier by Using Improved Z-Score Test, in: 2019 3rd International Conference on Trends in Electronics and Informatics (ICOEI), IEEE, Tirunelveli, India, 2019: pp. 788–790. https://doi.org/10.1109/ICOEI.2019.8862582.

[5] F.I. Mowbray, S.M. Fox-Wasylyshyn, M.M. El-Masri, Univariate Outliers: a Conceptual Overview for the Nurse Researcher, Canad. J. Nurs. Res. 51 (2018), 31-37. https://doi.org/10.1177/0844562118786647.

[6] E.O. Merza, N.J. Mohammed, Fast Ways to Detect Outliers, J. Techn. 3 (2021), 66-73. https://doi.org/10.51173/jt.v3i1.287.

[7] A.S. Yaro, F. Maly, P. Prazak, Outlier Detection in Time-series Receive Signal Strength Observation Using Z-score Method with Sn Scale Estimator for Indoor Localization, Appl. Sci. 13 (2023), 3900. https://doi.org/10.3390/app13063900.

[8] E.J. Jamshidi, Y. Yusup, J.S. Kayode, M.A. Kamaruddin, Detecting Outliers in a Univariate Time Series Dataset Using Unsupervised Combined Statistical Methods: a Case Study on Surface Water Temperature, Ecol. Inform. 69 (2022), 101672. https://doi.org/10.1016/j.ecoinf.2022.101672.

[9] W. Berendrecht, M. van Vliet, J. Griffioen, Combining Statistical Methods for Detecting Potential Outliers in Groundwater Quality Time Series, Environ. Monitor. Assess. 195 (2022), 85. https://doi.org/10.1007/s10661-022-10661-0.

[10] B. Dastjerdy, A. Saeidi, S. Heidarzadeh, Review of Applicable Outlier Detection Methods to Treat Geomechanical Data, Geotechnics 3 (2023), 375-396. https://doi.org/10.3390/geotechnics3020022.

[11] O.Y. Rodionova, A.L. Pomerantsev, Detection of Outliers in Projection-based Modeling, Anal. Chem. 92 (2019), 2656-2664. https://doi.org/10.1021/acs.analchem.9b04611.

[12] F. Smarandache, Introduction to Neutrosophic Statistics, Sitech & Education Publishing, 2014.

[13] F. Smarandache, Neutrosophic Statistics is an Extension of Interval Statistics, While Plithogenic Statistics is the Most General Form of Statistics, Infinite Study, 2022.

[14] J. Chen, J. Ye, S. Du, Scale Effect and Anisotropy Analyzed for Neutrosophic Numbers of Rock Joint Roughness Coefficient Based on Neutrosophic Statistics, Symmetry 9 (2017), 208. https://doi.org/10.3390/sym9100208.

[15] J. Chen, J. Ye, S. Du, R. Yong, Expressions of Rock Joint Roughness Coefficient Using Neutrosophic Interval Statistical Numbers, Symmetry 9 (2017), 123. https://doi.org/10.3390/sym9070123.

[16] M. Aslam, On Detecting Outliers in Complex Data Using Dixon's Test Under Neutrosophic Statistics, J. King Saud Univ. - Sci. 32 (2020), 2005-2008. https://doi.org/10.1016/j.jksus.2020.02.003.

[17] T. Sangeetha, G.M. Amalanathan, Outlier Detection in Neutrosophic Sets by Using Rough Entropy Based Weighted Density Method, CAAI Trans. Intell. Technol. 5 (2020), 121-127. https://doi.org/10.1049/trit.2019.0093.

[18] W. Duan, Z. Khan, M. Gulistan, A. Khurshid, Neutrosophic Exponential Distribution: Modeling and Applications for Complex Data Analysis, Complexity 2021 (2021), 5970613. https://doi.org/10.1155/2021/5970613.

[19] Z. Khan, M. Gulistan, N. Kausar, C. Park, Neutrosophic Rayleigh Model with Some Basic Characteristics and Engineering Applications, IEEE Access 9 (2021), 71277-71283. https://doi.org/10.1109/access.2021.3078150.

[20] J.E. Ricardo, A.J.R. Fernández, T.T.C. Martínez, W.A.C. Calle, Analysis of Sustainable Development Indicators through Neutrosophic Correlation Coefficients, Neutrosophic Sets Syst. 52 (2022), 355-362.

[21] A. AlAita, M. Aslam, Analysis of Covariance Under Neutrosophic Statistics, J. Stat. Comput. Simul. 93 (2022), 397-415. https://doi.org/10.1080/00949655.2022.2108423.

[22] L.A. Al-Essa, Z. Khan, F.S. Alduais, Neutrosophic Logistic Model with Applications in Fuzzy Data Modeling, J. Intell. Fuzzy Syst. 46 (2024), 3867-3880. https://doi.org/10.3233/jifs-233357.

[23] M. Çolak, M. Uçurum, M. Çinar, Ü. Atici, Application of Fuzzy Statistical Process Control for a Manufacturing of GG25 Gray Cast Iron Material, Omer Halisdemir Univ. J. Eng. Sci. 7 (2018), 427-437. https://doi.org/10.28948/ngumuh.387316.23.