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Optimal Utilization of Port Free-of-Charge Storage for Non-Instantaneously Deteriorating Items with Time-Varying Order Quantity Dependent Demand with Green Technology Investment during Transit

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Abstract. This paper presents an innovative inventory model for non-instantaneously deteriorating items with time-varying order quantity-dependent demand. The model strategically utilizes port-provided free storage periods to optimize inventory management across a distribution system comprising one port storage and an owned storage. While most industries rely on owned or rented warehouses to store goods before distributing them to retailers, this model proposes a more cost-efficient approach by leveraging the port's free storage period as a temporary warehouse until the free duration expires. By investing in energy-efficient green equipment, this approach decreases carbon emissions during product transit between the port and warehouse, as well as to industries. This allows companies to delay the incurrence of holding costs and optimize resource allocation, thereby minimizing total inventory costs while maintaining service levels. The model's theoretical foundation is verified using numerical examples, which emphasize significant findings on cost optimization and efficient inventory dynamics. Additionally, a comprehensive sensitivity analysis conducted using MATLAB reveals the impact of various parameter modifications, offering valuable insights for decision-makers across diverse industrial settings.

1. Introduction

Inventory management plays a crucial role in optimizing the operations of supply chains, especially in industries dealing with non-instantaneous deteriorating(NID) items. Such items, unlike instantaneously perishable goods, maintain their usability for a certain period before starting to degrade, presenting unique challenges in storage and distribution. Additionally, demand often

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varies with time and can be influenced by factors such as order quantity, creating a complex environment for inventory optimization.

In the context of global trade, ports act as pivotal hubs in the supply chain, offering temporary storage solutions that, if utilized strategically, can significantly reduce overall inventory costs. Many ports provide a free-of-charge storage period, typically lasting several weeks, which can be leveraged to defer expenses before transferring goods to owned storages. Effective utilization of this cost-free period is particularly beneficial for businesses handling time-sensitive or deteriorating products, allowing them to reduce holding costs while still meeting customer demand efficiently.

Incorporating carbon emission considerations into inventory management reflects the growing importance of sustainability. Investments in energy-efficient green technologies can significantly lower the carbon footprint associated with transportation activities. This study extends previous research on sustainable inventory systems by proposing a model that integrates green technology investment for reducing carbon emission during the transportation from port to industries or from port to warehouse and from there to industry.

Accurately forecasting demand is essential for effective inventory management and meeting customer needs in any business. Demand is often influenced by a combination of factors, including a constant base level of need, changes over time, and the size of orders placed. Understanding how these elements interact is crucial for anticipating future demand. For instance, a pharmaceutical company may experience a steady baseline demand for essential medications, but during a health crisis, demand could increase due to heightened public awareness and urgency. Additionally, when healthcare providers place larger orders to stock up, this can drive further demand by reducing unit costs through bulk purchasing. With these factors in mind, we have assumed demand to be time-varying and dependent on order quantity.

This paper presents a novel inventory model that addresses these issues by incorporating time-varying order quantity-dependent demand into a system that integrates both port and owned storage facilities. The primary objective of the model is to minimize total inventory costs by strategically managing the movement of goods between the port and owned storage locations during the free storage period. The proposed approach considers the dynamics of NID and aims to optimize resource allocation across the supply chain.

The Figure 1 illustrates the logistics process for goods arriving at a port and their subsequent distribution. When goods reach the port, they are stored in port storage, which offers a free storage period. During this time, the goods can be stored without any additional cost. A portion of the inventory, denoted as "M," is sent to an owned storage facility (warehouse). When demand arises, the goods stored in the port are utilized first, within the free storage period, to optimize costs. Once the free storage is fully used or the demand exceeds the available stock at the port, the inventory from the owned storage is then used to fulfill the demand. Finally, the goods are delivered to the industry or buyer as needed. This strategy ensures efficient inventory management by prioritizing cost-effective storage while maintaining a steady supply chain. Figure 1 exposes graphically the entire system.

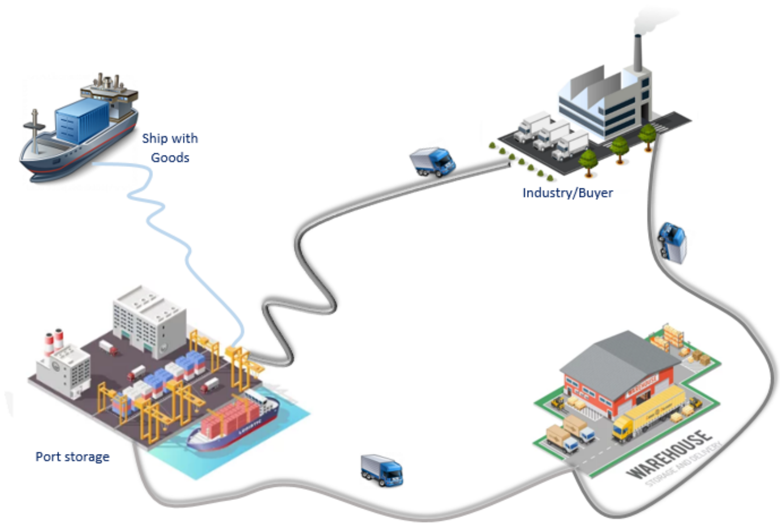

Figure 1. Graphical representation of the proposed strategy (Source: Author's Own).

1.1. **Research Question.** Based on the above discussion, our objective is to explore the following research questions:

(a) How can port-provided free storage be optimized to minimize inventory costs for NID items with time-varying order quantity dependent demand?

(b) What is the role of demand forecasting in managing inventory levels and minimizing costs in systems utilizing port-provided free storage?

(c) How do green technology investments and transportation optimization impact cost efficiency and sustainability in inventory systems using port-provided free storage?

1.2. **Research Objective.** The objective of this research is to develop and validate an inventory model that optimizes the use of free-of-charge port storage for NID items. The model aims to determine the optimal allocation of inventory between port and owned storage facilities while considering time-varying order quantity-dependent demand and optimizes carbon emission reduction. By minimizing total inventory costs and maximizing resource efficiency, the model seeks to improve overall supply chain performance and decision-making.

1.3. **Novelty.** The novelty of this research lies in its integration of multiple real-world complexities—such as NID, time-varying order quantity-dependent demand, carbon emission reduction and free-of-charge port storage—into a unified inventory model. While many existing studies focus on inventory management for deteriorating items or time-varying demand separately, this research uniquely addresses how port storage, with its temporary cost-free advantage, can be strategically utilized to minimize overall inventory costs. Furthermore, this study introduces an innovative methodology that balances inventory dynamics between port and owned storage, offering a cost-effective solution for industries that rely on global supply chains. The comprehensive sensitivity analysis conducted provides deeper insights into how different parameters influence the system's performance, enhancing the model's practical relevance for diverse industrial applications.

1.4. **Orientation.** The document is structured as follows: In Section 2, the motivation for the study is discussed, a brief assessment of the literature is presented, research gaps are noted, and the main contributions of the paper are highlighted. The proposed framework's assumptions, notations, mathematical formulation and computational algorithm are introduced in Section 3. Section 4 provides the mathematical model's solution numerically. To evaluate the effectiveness of the model, Section 5 uses MATLAB R2024a to perform sensitivity analysis. The managerial findings from this study are discussed in Section 6. Section 7 offers a concluding summary of the work.

2. Background

Inventory management for deteriorating items has been a critical focus in supply chain management since the early work of Ghare and Schrader [1], who developed models for exponentially decaying inventories. Building on this foundation, subsequent studies like those by Buzacott [2] and Misra [3] examined the effects of inflation on inventory systems, highlighting the economic challenges associated with managing perishable goods under fluctuating financial conditions. Hartely [4] emphasized the managerial aspects of operations research, which has since evolved to include complex inventory models for a variety of deteriorating items.

When cargo arrives at the port, it is typically allowed to stay without charge for a certain period, as studied by Kim and Kim [5] and Martín et al. [6]. For instance, major container ports in Europe offer free storage for 3 to 9 days, while in Asia, free storage usually ranges from 3 to 15 days, and in Egypt, it is about 10 days (http://www.cma-cgm.com/ebusiness/tariffs/demurrage-detention). At Jawaharlal Nehru Port Trust (JNPT), Mumbai, import containers have a free storage period of around 7 days, while export containers are allowed 5 days. Mumbai Port Trust (MbPT) generally grants 5 days of free storage before demurrage charges apply. Chennai Port Trust provides a free storage period of approximately 5-15 days for containers. To capitalize on these free storage periods, distributors need to consider these time frames to better align their distribution center (DC) and retailer inventory replenishment activities, thereby reducing overall inventory costs. Floating stock policies and their impact on supply chains have been extensively studied by researchers such as Ochtman et al. [7], Dekker et al. [8], and Pourakbar et al. [9]. These studies highlight the benefits of using intermodal transport and advance deployment of floating stocks in fast-moving consumer goods (FMCG) supply chains. Van Asperen and Dekker [10] further expanded on this by evaluating port-of-entry choices, emphasizing the importance of centrality and flexibility in inventory management.

Haralambides [11] examined competition and pricing of port infrastructure, providing a broader context for understanding port-related storage policies. Haralambides and Gujar [12] discussed the pricing strategies and opportunities for public-private partnerships in the Indian dry ports sector, which align with effective inventory management practices. Later, Haralambides [13] highlighted the role of ports in global supply chains, emphasizing their strategic importance in enhancing efficiency and reducing costs.

The strategic utilization of port-provided free storage periods has been explored by Li [14, 15], who demonstrated how these periods can be optimized within a distribution system comprising one distribution center and multiple ports. These studies underscore the critical role of ports in inventory management, especially in scenarios involving non-instantaneously deteriorating items with time-varying demand.

Several researchers have developed advanced inventory models to address these complexities. Chang [16] introduced models with stock-dependent demand and nonlinear holding costs, while Singh and Malik [17] focused on two-storage systems for NID items. Jaggi and Tiwari [18] extended these concepts to include price-dependent demand and time-varying holding costs, providing further insights into managing deteriorating inventory under flexible pricing strategies.

Recent studies have integrated environmental and sustainability considerations into inventory models. Datta [19] examined the impact of green technology investments on production-inventory systems, reflecting a growing trend towards environmentally sustainable practices. Rangarajan and Karthikeyan have made significant contributions to this field [20–22], developing EOQ models for deteriorating items with various demand rates and inflationary conditions.

Further extending the scope of inventory models, researchers like Bishi et al. [23] and Yadav and Swami [24] developed two-warehouse models for NID items, incorporating exponential demand rates and variable holding costs. Chakraborty et al. [25] introduced multi-warehouse systems with partial backlogging, accounting for inflation and uncertain demand conditions.

The integration of carbon emissions considerations into inventory models has also gained attention, as demonstrated by Wee and Daryanto [26], who explored imperfect quality item inventory models. Taleizadeh et al. [27] further developed joint pricing and inventory decisions with carbon emission considerations, highlighting the balance between operational efficiency and environmental responsibility. Mashud et al. [28] proposed sustainable inventory models with controllable carbon emissions, reinforcing the importance of green technology in modern supply chains.

Recent studies by Das et al. [29] and Lok et al. [30] have focused on preservation technology investments and the impact of carbon emissions in inventory systems, respectively. Limi et al. [31–33] have provided comprehensive reviews and new models analyzing factors like quadratic demand, time-varying holding costs, and inflation. These studies offer robust solutions for optimizing inventory levels and costs in multi-warehouse systems affected by external factors such as advertising and selling prices.

Overall, the literature demonstrates the importance of strategically utilizing free storage periods at ports, managing non-instantaneously deteriorating items, and optimizing order quantities in response to time-varying demand. The integration of environmental considerations and advanced inventory models continues to evolve, offering new approaches to cost-effective and sustainable inventory management in diverse industrial settings.

2.1. **Motivation.** The increasing complexities of global supply chains and the push towards sustainable practices have motivated the need for innovative inventory management strategies. The strategic use of port-provided free-of-charge storage offers an underutilized opportunity for cost optimization in inventory management. Most industries store their goods in owned or rented warehouses, incurring significant holding costs from the moment the inventory is stored. However, leveraging the temporary free storage provided by ports can delay these costs, thereby optimizing overall resource allocation. Moreover, the integration of green technology investments, such as energy-efficient equipment during transit, aligns with the growing emphasis on reducing carbon emissions, further enhancing sustainability in supply chain operations. These dual objectives—cost efficiency and environmental sustainability—drive the need for a comprehensive inventory model that considers both aspects.

2.2. **Research gap.** A selection of research publications are analyzed and contrasted with the suggested model in Table 1. Despite extensive research on inventory management for NID items having time-varying demand, there remains a significant gap in the literature concerning the optimal utilization of port-provided free-of-charge storage. Most existing studies focus on managing warehouse storage costs or controlling deterioration rates without leveraging the cost advantages of temporary, free port storage. Additionally, while some research has explored the integration of green technologies and carbon emission reductions in supply chain systems, there is a lack of models that combine these sustainability practices with the strategic use of port storage. No current model simultaneously addresses the complexities of NID, time-varying order quantity-dependent demand, and the utilization of port free storage, alongside green technology investments during transit. This gap underscores the need for a comprehensive approach that minimizes total inventory costs and promotes environmental sustainability through innovative transport and storage strategies. The proposed study aims to fill this gap by developing a unified inventory model that integrates these diverse elements, providing new insights for sustainable supply chain management.

Table 1. Comparison of key aspects in the current study and other works.

Author(s)	Time varying order quantity dependent demand	NID	Port free-of-charge	Carbon Emission	Green Technology
Haralambides [13]	no	no	yes	no	no
Li [14]	no	no	yes	no	no
Li [15]	no	no	yes	no	no
Datta [19]	no	no	no	yes	yes
Rangarajan & Karthikeyan [21]	no	yes	no	no	no
Yadav & Swami [24]	no	yes	no	no	no
Taleizadeh et al. [27]	no	no	no	yes	yes
Mashud et al. [28]	no	yes	no	yes	yes
Lok et al. [30]	no	yes	no	yes	no
Limi et al. [33]	no	yes	no	no	no
Pervin [34]	no	no	no	yes	yes
Proposed study	yes	yes	yes	yes	yes

2.3. **Contribution.** To address this gap, the proposed study aims to introduce a comprehensive two-warehouse inventory model that integrates all critical aspects identified in the comparison table. This study introduces a novel inventory model that integrates the strategic use of port-provided free-of-charge storage with non-instantaneously deteriorating items, considering time-varying order quantity-dependent demand and green technology investments during transit. By combining these critical factors, the model offers a comprehensive solution for minimizing total inventory costs while enhancing sustainability in supply chain management. The study also provides a detailed sensitivity analysis to demonstrate the impact of key parameters, offering valuable insights for decision-makers seeking to optimize inventory management in alignment with both economic and environmental objectives.

3. Mathematical formulation

Here, we present the mathematical model along with the associated symbols

3.1. **Notations.** This work uses the following notations throughout.

Notations	**Description**
A	The order amount
μ	The deterioration rate in PS
γ	The deterioration rate in OS
M	Quantity arranged per cycle
N	The OS's capacity
$\mathcal{I}_{p_1}(t)$	Inventory levels in PS during the interval $0 \le t \le t_N$
$\mathcal{I}_{p_2}(t)$	Inventory levels in PS during the interval $t_N \le t \le t_p$

Notations	**Description**
$\mathcal{I}_{o_1}(t)$	Inventory levels in OS during the interval $0 \leq t \leq t_N$
$\mathcal{I}_{o_2}(t)$	Inventory levels in OS during the interval $t_N \leq t \leq t_p$
$\mathcal{I}_{o_3}(t)$	Inventory levels in OS during the interval $t_p \leq t \leq t_o$
l	The base demand rate
m	Coefficient of rate of change of demand over time
n	Coefficient of the sensitivity of demand to the order quantity Q
t_p	Time span during which the inventory in PS can be stored for free of charge
c_p	Purchasing cost
ht	Holding cost at any time t in OW
v	Number of trips
q_1	Minimum transportation costs required to transport items
q_2	Fuel consumption with empty truck
q_3	Supplementary fuel consumption of the truck per ton of payload
d_t	Distance traveled from PS to customer or from PS to OW then OW to customer
c_e	Carbon emission produced by the vehicle
w	Product weight
e_x	Extra carbon emission cost for transporting one unit of an item
a	Constant market size
V_t	Variable transportation cost, which is equal to fuel price
b	Shape parameter
G_T	Green technology investment
Decision Variables	
t_N	Time span during which there is no deterioration
t_o	Duration of the inventory cycle
Q	The order quantity

3.2. **Assumptions.** The following presumptions form the foundation of the inventory model.

(i) One item is the focus of the inventory model's development.
(ii) Lead time is zero.
(iii) Demand is considered as time varying order quantity Q dependent demand. i.e., $D = l + mt + nQ$ where l is the base demand rate, m is the coefficient of rate of demand over time and n is the coefficient of sensitivity of demand to the order quantity Q.
(iv) Inventory replenishment does not allow for shortages.
(v) We only consider free storage durations supplied by merchants' ports, not those affiliated with the DC. That is, we solely consider free storage periods for inbound freight.
(vi) The inventory in port storage is fully utilized during the free-of-charge storage period.
(vii) The retailer's budget sets the maximum amount for a green technology effort. The fraction of emission reduction caused by G_T is F= $\delta(1-e^{-\chi G_T})$, where δ reflects the amount of carbon emissions if green technology is invested in, and χ impacts green technology's potential to cut emissions same as used by Mashud et al. [28].
(vii) Both PS and OW have a limited capacity.

(ix) Once an item reaches its designated lifespan, degradation takes place. This indicates that the product will deteriorate at a constant rate once its lifespan has passed, but it will not deteriorate during that time.

(x) The first inventory to be used up is from the PS and then from OW.

(xi) Throughout the cycle, the model's decaying components are neither replaced nor fixed.

(xi) The holding cost is a variable that follows a function of time.

3.3. **Mathematical model.** For non-instantaneously deteriorating items, orders are placed from other states or countries. These items are shipped by sea and arrive at the port. Once they reach the port, we receive port storage free of charge for a half month period.

In this model, items are initially dispatched from port storage to meet demand, while the remaining inventory is moved to owned storage. As long as free port storage is available, products are fulfilled from there. Once port storage is depleted, the owned storage is used to continue fulfilling demand. This approach optimizes the use of free port storage, reducing storage costs and ensuring a smooth transition between the two storage types.

Figure 2 explicitly shows the model graphically, illustrating the flow of items from the initial shipment, through port storage, and finally to the owned storage.

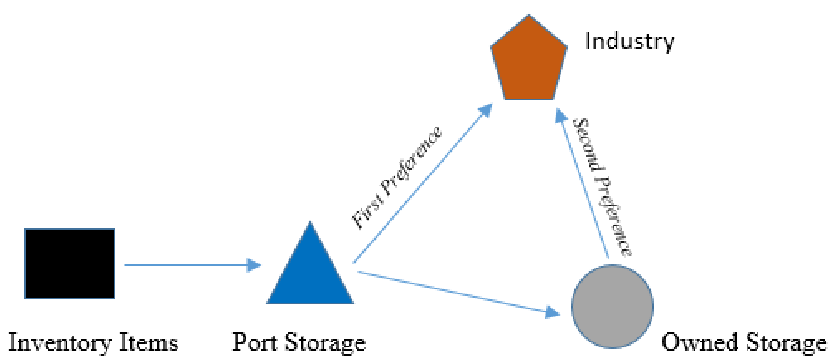

Figure 2. A distribution system with a port, owned storage and industry

Figure 3 presents the inventory time diagram, which depicts the behavior of the model throughout the period 't' using the notations and assumptions described above. During the inventory cycle $[0, t_o]$, the level of stock at any time t is given by the function $\mathcal{I}(t)$. The rate of the inventory level decreases in accordance with the demand rate and the deterioration rate of the available inventory level.

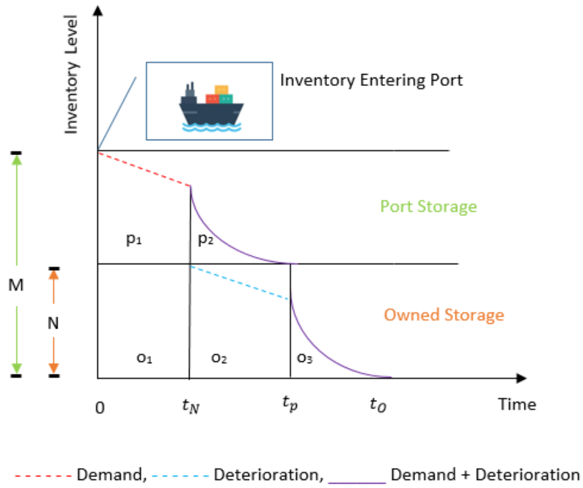

FIGURE 3. Two warehouse inventory system with Advance payment (Source: Author's Own).

In the port storage, until the non-instantaneous period t_N, no deterioration occurs, and only demand is considered. The differential equation for the inventory level during the interval $0 < t < t_N$ in the port storage is

$$\frac{d\mathcal{I}_{p_1}(t)}{dt} = -(l + mt + nQ) \tag{3.1}$$

Solving equation 3.1 and At the beginning of the cycle ($t = 0$), the inventory level is equal to the order quantity $(M - N)$ i.e., at $t = 0$ the inventory level is given by $\mathcal{I}_{p_1}(0) = M - N$. Substituting this value we get,

$$\mathcal{I}_{p_1}(t) = \Big[M - (l + mt + nQ)t - N\Big] \tag{3.2}$$

Inventory level at t_N at the port is given as follows,

$$\mathcal{I}_{p_1}(t_N) = \Big[M - (l + mt_N + nQ)t_N - N\Big]$$

During the interval $t_N < t < t_p$, the items starts to deteriorate and demand is met in the port. Hence the governing differential equation is

$$\frac{d\mathcal{I}_{p_2}(t)}{dt} + \mu\mathcal{I}_{p_2}(t) = -(l + mt + nQ) \tag{3.3}$$

solving equation 3.3 and using $\mathcal{I}_{p_2}(t_p) = 0$, we get

$$\mathcal{I}_{p_2}(t) = \left[\left(\frac{m}{\mu^2} - \frac{l + mt + nQ}{\mu}\right) - e^{\mu(t_p - t)}\left(\frac{m}{\mu^2} - \frac{l + mt_p + nQ}{\mu}\right)\right] \tag{3.4}$$

During the interval $0 < t < t_N$, the items in the owned storage is not utilized nor any deterioration occurs. Hence the governing differential equation is

$$\frac{d\mathcal{I}_{o_1}(t)}{dt} = 0 \tag{3.5}$$

solving equation 3.5 and using $\mathcal{I}_{o_1}(0) = N$, we get

$$\mathcal{I}_{o_1}(t) = N \tag{3.6}$$

During the interval $t_N < t < t_p$, in the owned storage the items starts to deteriorate but no demand occurs. Hence the governing differential equation is

$$\frac{d\mathcal{I}_{o_2}(t)}{dt} + \gamma\mathcal{I}_{o_2}(t) = 0 \tag{3.7}$$

solving equation 3.7 and using $\mathcal{I}_{o_2}(t_p) = N$, we get

$$\mathcal{I}_{o_2}(t) = \left[Ne^{\gamma(t_p-t)}\right] \tag{3.8}$$

During the interval $t_p < t < t_o$, the items undergoes an deterioration and demand is met in the owned storage. Hence the governing differential equation is

$$\frac{d\mathcal{I}_{o_3}(t)}{dt} + \gamma\mathcal{I}_{o_3}(t) = -(l + mt + nQ) \tag{3.9}$$

solving equation 3.9 and using $\mathcal{I}_{o_3}(t_p) = 0$, we get

$$\mathcal{I}_{o_3}(t) = \left[\left(\frac{m}{\gamma^2} - \frac{l + mt + nQ}{\gamma}\right) - e^{\gamma(t_o-t)}\left(\frac{m}{\gamma^2} - \frac{l + mt_o + nQ}{\gamma}\right)\right] \tag{3.10}$$

Using the continuity at $t = t_N$, $\mathcal{I}_{p_1}(t_N) = \mathcal{I}_{p_2}(t_N)$ and from 3.2 and 3.4 we get the maximum inventory level as follows.

$$M = N + (l + mt_N + nQ)t_N + \left(\frac{m}{\mu^2} - \frac{l + mt_N + nQ}{\mu}\right) - e^{\mu(t_p-t_N)}\left(\frac{m}{\mu^2} - \frac{l + mt_p + nQ}{\mu}\right) \tag{3.11}$$

Using the continuity at $t = t_p$, $\mathcal{I}_{o_2}(t_p) = \mathcal{I}_{o_3}(t_p)$ and from 3.8 and 3.10 we get the maximum inventory level in OS as follows.

$$N = \left[\left(\frac{m}{\gamma^2} - \frac{l + mt_p + nQ}{\gamma}\right) - e^{\gamma(t_o-t_p)}\left(\frac{m}{\gamma^2} - \frac{l + mt_o + nQ}{\gamma}\right)\right] \tag{3.12}$$

therefore, the inventory level for next replenishment using equation 3.11 and 3.12 is

$$\begin{aligned} M =& \left(\frac{m}{\gamma^2} - \frac{l + mt_p + nQ}{\gamma}\right) - e^{\gamma(t_o-t_p)}\left(\frac{m}{\gamma^2} - \frac{l + mt_o + nQ}{\gamma}\right) + (l + mt_N + nQ)t_N \\ &+ \left(\frac{m}{\mu^2} - \frac{l + mt_N + nQ}{\mu}\right) - e^{\mu(t_p-t_N)}\left(\frac{m}{\mu^2} - \frac{l + mt_p + nQ}{\mu}\right) \end{aligned} \tag{3.13}$$

In this scenario, the total cost during the replenishment cycle is:

(i) Ordering Cost= A

(ii) Purchasing Cost = c_pM

$$= c_p\Bigg[\left(\frac{m}{\gamma^2} - \frac{l+mt_p+nQ}{\gamma}\right) - e^{\gamma(t_o-t_p)}\left(\frac{m}{\gamma^2} - \frac{l+mt_o+nQ}{\gamma}\right) + (l+mt_N+nQ)t_N + \left(\frac{m}{\mu^2} - \frac{l+mt_N+nQ}{\mu}\right) - e^{\mu(t_p-t_N)}\left(\frac{m}{\mu^2} - \frac{l+mt_p+nQ}{\mu}\right)\Bigg]$$

(iii) Holding Cost in OW

$$= \int_0^{t_N}(ht)\mathcal{I}_{o_1}(t)dt + \int_{t_N}^{t_p}(ht)\mathcal{I}_{o_2}(t)dt + \int_{t_p}^{t_o}(ht)\mathcal{I}_{o_3}(t)dt$$

$$=h\Bigg[\frac{Nt_N^2}{2} + \frac{N}{\gamma^2}\left(e^{\gamma(t_p-t_N)}(\gamma t_N+1) - (\gamma t_p+1)\right) + \frac{m-\gamma(l+nQ)}{2\gamma^2}(t_o^2-t_p^2) - \frac{m}{3\gamma}(t_o^3-t_p^3) + \left(\frac{m}{\gamma^2} - \frac{l+mt_o+nQ}{\gamma}\right)\left(\frac{1}{\gamma}\left(t_o - t_pe^{\gamma(t_o-t_p)}\right)\right) - \left(\frac{1}{\gamma^2}\left(1-e^{\gamma(t_o-t_p)}\right)\right)\Bigg]$$

(iv) Deteriorating Cost PS

$$=c_p\int_{t_N}^{t_p}\mu\mathcal{I}_{p_2}(t)dt$$

$$=c_p\left[\frac{m-(l+nQ)\mu}{\mu}(t_p-t_N) - \frac{m}{2}\left(t_p^2-t_N^2\right) + \left(\frac{m-(l+mt_p+nQ)\mu}{\mu^2}\right)\left(1-e^{\mu(t_p-t_N)}\right)\right]$$

(v) Deteriorating Cost OW

$$=c_p\left[\int_{t_N}^{t_d}\gamma\mathcal{I}_{o_2}(t)dt + \int_{t_p}^{t_o}\gamma\mathcal{I}_{o_3}(t)dt\right]$$

$$=c_p\Bigg[N\left(e^{\gamma(t_p-t_N)}-1\right) + \frac{m+(l+nQ)\gamma}{\gamma}(t_o-t_p) - \frac{m}{2}\left(t_o^2-t_p^2\right) + \left(\frac{m-(l+mt_o+nQ)\gamma}{\gamma^2}\right)\left(1-e^{\gamma(t_o-t_p)}\right)\Bigg]$$

(vi) The Transportation Cost (TC)

The transportation cost comprises a fixed and variable transport cost together with a carbon emission cost. Here, the emission cost hinges on the delivery quantity (Q). The emission cost variable is calculated based on the weight of the shipment and relates to the size of the shipment. The suggested approach involves transporting the cargo size (Q) from the PS to client or from the PS to the OW and then from the OW to the client. The retailer covers the cost of transportation/shipping. The $3d_t$ factor includes the return travel distance plus the PS to retailer.The emission cost of a product is determined by its delivery amount (Q) or vehicle payload (also known as truck capacity).The variable transportation cost, V_t, is multiplied by the total vehicle fuel use (q_2). When we load the with products, the distance d will be increased by the addition of fuel consumption q_3 required for the journey. The increased carbon emission penalty applies only while the vehicle is loaded with merchandise and not when returning empty. Therefore, the transportation cost is [28]

$$TC = v\Big[q_1 + (3d_tV_tq_2 + d_tq_3wQ) + (2d_tc_e + d_te_xQ)\Big]$$

(vii) The Green Technology Investment Cost (GTIC)
To calculate this multiply the entire cycle length (t_o) by the total green technology investment (G_T).

$$GTIC = G_Tt_o$$

(viii) Reduced Transportation Cost (RTC)
Implementing green technology (Assumption (viii)) can result in decreased transportation costs.

$$RTC = v\Big[q_1 + (3d_tV_tq_2 + d_tq_3wQ) + (3d_tc_e + d_te_xQ) * (1 - \delta(1 - e^{-\chi G_T}))\Big]$$

Total inventory cost:

The overall total cost includes ordering, purchasing, holding, deteriorating, reduced transportation and green technology investment cost. As a formula:

$$T^{IC}\Big(t_N, t_o, Q\Big) = \frac{1}{t_o}\Big[OC + +PC + HC_{OW} + DC_{PS} + DC_{OW} + RTC + GTIC\Big]$$

$$\begin{aligned}
= \frac{1}{t_o}\Bigg[&A + c_p\Bigg[\Bigg(\frac{m}{\gamma^2} - \frac{l + mt_p + nQ}{\gamma}\Bigg) - e^{\gamma(t_o - t_p)}\Bigg(\frac{m}{\gamma^2} - \frac{l + mt_o + nQ}{\gamma}\Bigg) + (l + mt_N + nQ)t_N \\
&+ \Bigg(\frac{m}{\mu^2} - \frac{l + mt_N + nQ}{\mu}\Bigg) - e^{\mu(t_p - t_N)}\Bigg(\frac{m}{\mu^2} - \frac{l + mt_p + nQ}{\mu}\Bigg)\Bigg]\\
&+ h\Bigg[\frac{Nt_N^2}{2} + \frac{N}{\gamma^2}\Bigg(e^{\gamma(t_p - t_N)}(\gamma t_N + 1) - (\gamma t_p + 1)\Bigg) + \frac{m - \gamma(l + nQ)}{2\gamma^2}(t_o^2 - t_p^2)\\
&- \frac{m}{3\gamma}(t_o^3 - t_p^3) + \Bigg(\frac{m}{\gamma^2} - \frac{l + mt_o + nQ}{\gamma}\Bigg)\Bigg(\frac{1}{\gamma}\Big(t_o - t_pe^{\gamma(t_o - t_p)}\Big)\Bigg) - \Bigg(\frac{1}{\gamma^2}\Big(1 - e^{\gamma(t_o - t_p)}\Big)\Bigg)\Bigg]\\
&+ c_p\Bigg[\frac{m - (l + nQ)\mu}{\mu}(t_p - t_N) - \frac{m}{2}\Big(t_p^2 - t_N^2\Big) + \Bigg(\frac{m - (l + mt_p + nQ)\mu}{\mu^2}\Bigg)\Big(1 - e^{\mu(t_p - t_N)}\Big)\Bigg]\\
&+ c_p\Bigg[N\Big(e^{\gamma(t_p - t_N)} - 1\Big) + \frac{m + (l + nQ)\gamma}{\gamma}(t_o - t_p) - \frac{m}{2}\Big(t_o^2 - t_p^2\Big)\\
&+ \Bigg(\frac{m - (l + mt_o + nQ)\gamma}{\gamma^2}\Bigg)\Big(1 - e^{\gamma(t_o - t_p)}\Big)\Bigg]
\end{aligned}$$

$$+ G_T t_o + \nu\Big[q_1 + (2d_tV_tq_2 + d_tq_3wQ) + (2d_tc_e + d_te_xQ) * (1 - \delta(1 - e^{-\chi G_T}))\Big]\Big] \tag{3.14}$$

3.4. **Computational Algorithm:** **Step 1:** Consider the parameter values for $A, c_p, m, \gamma, l, \mu, h, N, G_T, n, q_1, q_2, q_3, d_t, V_t, w, c_e, t_p, e_x, \delta, \chi$

Step 2: Using equation 3.14, find

$$\frac{\partial T^{IC}}{\partial t_N}, \ \frac{\partial T^{IC}}{\partial t_o} \ \text{ and } \ \frac{\partial T^{IC}}{\partial Q}.$$

Step 3: From steps 1 & 2, determine the values of t_N, t_o, and Q.
Step 4: The Hessian matrix H, which consists of second-order partial derivatives, establishes the necessary conditions for minimizing T^{IC}.

$$H = \begin{bmatrix} \frac{\partial^2 T^{IC}}{\partial t_N^2} & \frac{\partial^2 T^{IC}}{\partial t_N \partial t_o} & \frac{\partial^2 T^{IC}}{\partial t_N \partial Q} \\ \frac{\partial^2 T^{IC}}{\partial t_o \partial t_N} & \frac{\partial^2 T^{IC}}{\partial t_o^2} & \frac{\partial^2 T^{IC}}{\partial t_o \partial Q} \\ \frac{\partial^2 T^{IC}}{\partial Q \partial t_N} & \frac{\partial^2 T^{IC}}{\partial t_o \partial Q} & \frac{\partial^2 T^{IC}}{\partial Q^2} \end{bmatrix},$$

$$H_1 = \frac{\partial^2 T^{IC}}{\partial t_N^2} > 0, \quad H_2 = det \begin{bmatrix} \frac{\partial^2 T^{IC}}{\partial t_N^2} & \frac{\partial^2 T^{IC}}{\partial t_N \partial t_o} \\ \frac{\partial^2 T^{IC}}{\partial t_o \partial t_N} & \frac{\partial^2 T^{IC}}{\partial t_o^2} \end{bmatrix} > 0,$$

and

$$H_3 = detH = det \begin{bmatrix} \frac{\partial^2 T^{IC}}{\partial t_N^2} & \frac{\partial^2 T^{IC}}{\partial t_N \partial t_o} & \frac{\partial^2 T^{IC}}{\partial t_N \partial Q} \\ \frac{\partial^2 T^{IC}}{\partial t_o \partial t_N} & \frac{\partial^2 T^{IC}}{\partial t_o^2} & \frac{\partial^2 T^{IC}}{\partial t_o \partial Q} \\ \frac{\partial^2 T^{IC}}{\partial Q \partial t_N} & \frac{\partial^2 T^{IC}}{\partial t_o \partial Q} & \frac{\partial^2 T^{IC}}{\partial Q^2} \end{bmatrix} > 0.$$

H_1, H_2 and H_3 represent the minors of the Hessian matrix.
Step 5: The optimal values for t_N, t_o and Q are determined. These values are used to calculate the overall optimal inventory cost, T^{IC}.

3.5. **Special Cases.** The model discussed above is a NID inventory model with free-of-charge port storage and time-varying order quantity-dependent demand. This model can be extended or simplified under the following specific cases:

Case (a): When $n = 0$ is considered, the demand becomes linearly time-dependent. Consequently, the model simplifies to a NID model with a demand that is linear time-dependent .

Case (b): When $m = 0$ is taken, the demand becomes solely dependent on the order quantity. Thus, the model transforms into a NID model with order quantity-dependent demand.

Case (c): When $m = 0$ and $n = 0$ are considered, the demand becomes constant. This leads to a further simplification where the model becomes a NID model with constant demand.

Case (d): When $G_T = 0$, the proposed model excludes the effect of green technology investment, thereby focusing on the traditional aspects of the inventory model without green technology considerations.

4. Numerical analysis

In a real-life application of the suggested inventory model, consider Diagnostic Care and Trading Private Limited Adayar, Chennai, India (DCT). Here we attempt to simplify inventory management while decreasing costs by using the model's ideal replenishment timings as determined by MATLAB. DCT saves money and improves supply chain efficiency by carefully examining elements including degradation rates, transportation costs, holding costs, and storage capacity.

Model 1: Inventory model with time-varying order quantity dependent demand for NID goods. In this section, we apply the suggested inventory model to a real-world situation in DCT involving the item Maglumi Molcision SARS-CoV-2. Using numerical numbers based on price fluctuations detected in real-time data from the firm. Consider an inventory system with the following data in appropriate units to demonstrate the aforesaid solution procedure:
$A = \$500,\ c_p = \$150,\ m = 10\ units,\ \gamma = 0.4,\ l = 7\ units,\ \mu = 8,\ h = \$0.7,\ N = \$165,$
$G_T = 7,\ n = 0.07\ units,\ q_1 = \$0.3/Shipment,\ d_t = 4Km,\ V_t = 8,\ q_2 = \$0.85L/100km,$
$q_3 = 3.8\ L/100km/ton\ of\ payload,\ w = 8kg/unit,\ c_e = \$3.75/km,\ t_p = 1.5\ months,$
$e_x = \$2.4/unit/km,\ \nu = 12, \delta = 7,\ \chi = 6.$
The value of $t_N = 2.5174$, $t_o = 3.6117$, and $Q = \$213.9509$ are obtained using MATLAB. As a result, the optimal total cost for inventory management considering NID, is determined to be $T^{IC} = \$7792.5$. The aforementioned model may be further classified as follows.

Model 2: Inventory Model for Case (a)
The proposed model can be adapted to an inventory model with linear time-dependent demand for NID items. When we assume $n = 0$ in the given model with the aforementioned data values, the optimum total cost is determined to be $T^{IC} = \$5437.3$.

Model 3: Inventory Model for Case (b)

The proposed model can be adapted to an inventory model with order quantity-dependent demand for NID items. When we assume $m = 0$ in the given model with the aforementioned data values, the optimum total cost is determined to be $T^{IC} = \$4519.7$.

Model 4: Inventory Model for Case (c)

The proposed model can be adapted for an inventory model having constant demand for NID items. When we assume $n = 0$ and $m = 0$ in the given model with the aforementioned data values, the optimum total cost is determined to be $T^{IC} = \$2164.5$.

Model 5: Inventory Model for Case (d)

The proposed model can be adapted to an inventory model without green technology investment for NID items. When we assume $G_T = 0$ in the given model with the aforementioned data values, the optimum total cost is determined to be $T^{IC} = \$8590.3$.

4.1. **Observation.** The suggested inventory model is used to Diagnostic Care and Trading Private Limited (DCT) for the item Maglumi Molcision SARS-CoV-2, resulting in considerable cost reductions and increased supply chain efficiency. By considering various scenarios such as linear time-dependent demand, order quantity-dependent demand, constant demand, and the absence of green technology investment, we observe varying impacts on the total inventory cost. This highlights the model's versatility and the importance of tailoring inventory strategies to specific operational conditions to achieve optimal performance. Model 1 and Model 5 without green technology are compared and shown in Figure 4

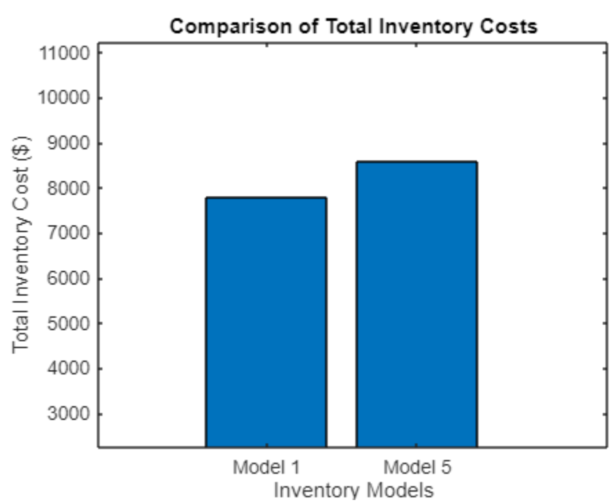

Figure 4. Total inventory costs comparison across different inventory models.

5. Sensitivity analysis

Examine the sensitivity of the inventory model's parameters. Varying the parameters in Table 3 by −50%, −25%,
+ 25%, +50% leads to substantial changes in the optimal values for the total cost (T^{IC}) of the proposed model.

Table 3: Analysis of parameter sensitivity.

Parameter	**Initial value**	**Percentage variation**	t_N	t_o	Q	T^{IC}	% change in T^{IC}
c_p	150	-50%	2.5163	3.6169	215.7628	4482.2	-42.48
		-25%	2.5169	3.6142	214.2475	6136.5	-21.25
		+25%	2.5177	3.6106	213.5969	9465.7	21.47
		+50%	2.5182	3.6099	213.3623	11103.2	42.48
m	10	-50%	2.5387	3.6099	157.0716	5551.9	-28.75
		-25%	2.5264	3.6104	192.1067	6744.6	-13.45
		+25%	2.5135	3.6118	242.3651	8915.2	14.41
		+50%	2.5109	3.6119	270.7741	10039.3	28.83
l	7	-50%	2.5174	3.6117	163.9509	6691.6	-14.13
		-25%	2.5174	3.6117	194.5026	7303.2	-6.28
		+25%	2.5174	3.6117	238.9509	8342.9	7.06
		+50%	2.5174	3.6117	269.7614	8957.2	14.95
n	0.07	-50%	2.5174	3.6117	327.9019	7242.1	-7.06
		-25%	2.5174	3.6117	285.2679	7562.5	-2.95
		+25%	2.5174	3.6117	171.1608	7992.5	2.28
		+50%	2.5174	3.6117	142.6339	8342.4	7.05
γ	0.4	-50%	2.5106	4.0432	251.7682	10655.4	36.74
		-25%	2.5136	3.9982	229.6531	8630.9	10.76
		+25%	2.5204	3.4269	206.5219	6972.1	-10.53
		+50%	2.5258	3.2323	200.2006	5194.5	-33.63
μ	8	-50%	2.5344	3.6112	213.8939	7100.9	-8.87
		-25%	2.5232	3.6115	213.9316	7476.2	-4.06
		+25%	2.5140	3.6118	213.9628	8083.1	3.73
		+50%	2.5062	3.6121	213.9942	8350.6	7.16
h	0.7	-50%	2.5177	3.6116	213.9489	7790.9	-0.02
		-25%	2.5176	3.6116	213.9499	7791.6	-0.01
		+25%	2.5173	3.6117	213.9516	7793.2	0.01
		+50%	2.5171	3.6117	213.9529	7793.8	0.02
N	165	-50%	2.5277	3.6109	213.8632	7766.9	-0.32
		-25%	2.5204	3.6112	213.9162	7775.6	-0.22
		+25%	2.5126	3.6119	213.9796	7804.9	0.16
		+50%	25078	3.6120	213.9989	7816.9	0.31
G_T	7	-50%	2.5174	3.6117	213.0835	7810.9	0.23
		-25%	2.5174	3.6117	213.4521	7798.7	0.07
		+25%	2.5174	3.6117	213.9363	7984.5	-0.07

Parameter	**Initial value**	**Percentage variation**	t_N	t_o	Q	T^{IC}	% change in T^{IC}
		+50%	2.5174	3.6117	213.9962	7976.2	-0.23
t_p	1.5	-50%	1.7421	2.7624	162.4326	6246.5	-19.84
		-25%	1.9005	2.9837	195.2421	7483.4	-3.96
		+25%	3.1381	4.2392	232.2192	8030.1	3.05
		+50%	3.7604	4.8667	250.0624	9314.7	19.52

Figure 5 show the behaviour changes in different parameters vs change in t_N, t_o and Q for the above table.

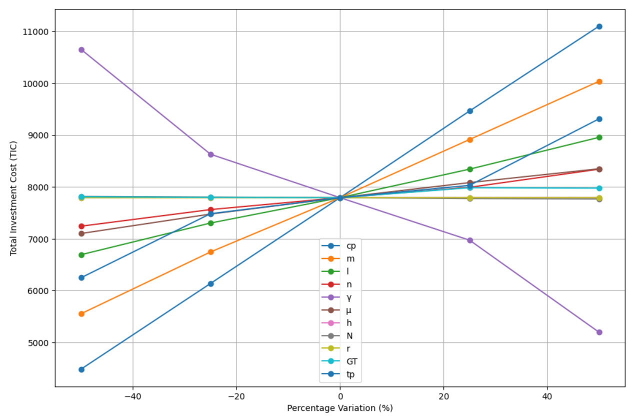

FIGURE 5. Behaviour of total cost.

In Table 4, the sensitivity analysis of the parameters is provided for the values $t_N = 2.5174,\ t_o = 3.6117$ and $Q = 213.9507$ as computed using MATLAB.

TABLE 4. Sensitivity analysis concerning the parameters having $t_N = 2.5174,\ t_o = 3.6117$ and $Q = 213.9507$

Parameter	Initial value	T^{IC}				↑/↓
		-50%	-25%	25%	50%	
A	500	7723.2	7757.9	7827.1	7861.7	↑
d_t	4	7293.6	7543.1	8041.9	8291.3	↑
q_1	0.3	7792.1	7792.3	7792.6	7792.9	↑
q_2	0.85	7742.8	7762.3	7822.6	7852.7	↑
q_3	3.8	6580.4	7186.4	8398.5	9004.5	↑
ν	12	5789.6	6826.8	8569.5	9271.5	↑
V_t	8	7732.2	7762.3	7822.6	7852.7	↑
w	8	6580.4	7186.4	8398.5	9004.5	↑
c_e	3.75	7991.8	7892.1	7692.8	7593.1	↓
e_x	2.4	8366.7	8079.5	7505.4	7218.3	↓
δ	7	8694.9	8243.7	7341.3	6890.1	↓
χ	6	7790.5	7791.5	7793.5	7794.5	↑

Increasing(↑) and Decreasing(↓)

Figure 6 shows the variation in total cost of parameters in table 4.

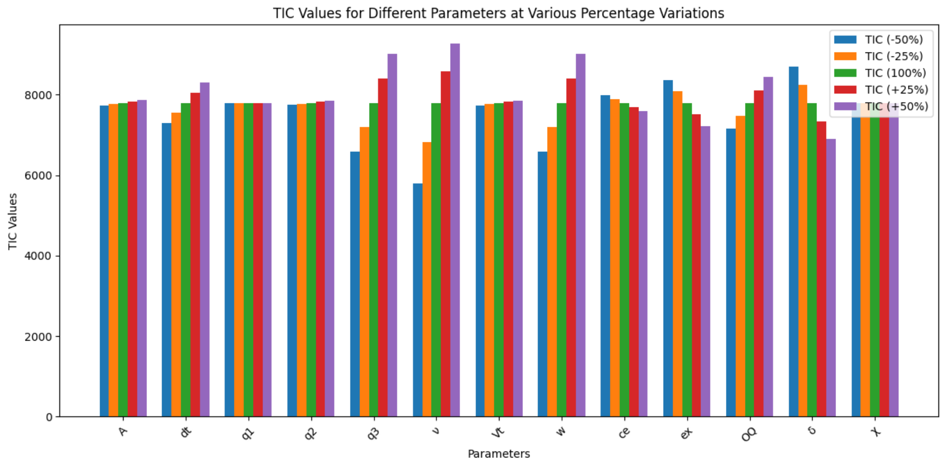

Figure 6. Variation of total cost.

5.1. **Insights from the sensitivity analysis.** According to Table 3- Table 4, the following characteristics are observed from the sensitivity analysis of the parameter values.

(i) Increase in the parameter t_p leads to an increase in the total inventory cost T^{IC}. Extending the port storage period allows inventory to be held longer, increasing holding costs, and thereby raising total inventory expenses.

(ii) Increase in the parameters l, m, and n results in higher total inventory costs T^{IC}. As the base demand rate and sensitivity to order quantities rise, the inventory levels required also increase, leading to a significant rise in associated costs.

(iii) Higher deterioration rate γ in OS decreases total inventory costs T^{IC}. Rapid deterioration improves turnover and reduces holding costs in OS, while a higher deterioration rate μ in PS can increase replenishment frequency and holding costs, leading to higher overall expenses.

(iv) Increase in the holding cost parameter h moderately increases T^{IC}. As holding costs rise, it becomes more expensive to store inventory over time, directly impacting the total costs.

(v) Increase in the parameter N shows a rise in total inventory cost T^{IC}. As owned storage capacity increases, inventory-related costs, including storage and management, grow accordingly, leading to overall total costs.

(vi) Increase in the parameter G_T results in a decrease T^{IC}. Greater investment in green technology reduces carbon emissions and associated costs, leading to more cost-effective inventory management over time.

(vii) Increase in the parameter A leads to higher total inventory costs T^{IC}. Larger order quantities result in higher ordering costs, which then inflate the overall inventory expenses.

(viii) Increase in transportation-related parameters such as ν (number of trips), d_t (distance), q_1 (minimum transportation costs), and V_t (variable transportation costs) increases total inventory costs T^{IC}. However, optimizing parameters like fuel consumption q_2, carbon emissions c_e, and extra emission costs e_x can reduce these costs, making transportation more efficient.

6. Managerial insight

Following are some significant findings from this study:

- **Optimal Utilization of Free Port Storage:** Managers should carefully balance the storage duration in port facilities. While extending the port storage period t_p might reduce immediate logistics costs, it can lead to increased holding costs if the inventory stays too long. Efficient planning is crucial to avoid unnecessary expenses.
- **Demand Management and Forecasting:** The sensitivity of inventory costs to demand-related parameters l, m, and n highlights the need for accurate demand forecasting. Investing in demand prediction tools can help in optimizing inventory levels and reduce the risk of excess costs due to overstocking or understocking.
- **Focus on Inventory Turnover and Deterioration Rates:** Higher deterioration rates γ in OS improve inventory turnover, lowering overall holding costs. However, in PS, higher deterioration rates μ may increase replenishment frequency and holding expenses. Managers should focus on optimizing stock levels and minimizing deterioration through better inventory control strategies.
- **Green Technology and Sustainable Practices:** Investing in green technologies G_T can lead to significant long-term savings by reducing carbon emissions and associated costs. Companies that prioritize sustainability not only benefit from reduced costs but also enhance their brand reputation and comply with environmental regulations.
- **Transportation Cost Optimization:** Transportation parameters such as ν (number of trips), q_1 (transportation cost), and d_t (distance) significantly impact total inventory costs. Managers should focus on optimizing routes, reducing fuel consumption, and leveraging efficient logistics strategies to minimize transportation-related expenses and improve overall cost efficiency.

7. Conclusion, Limitations and Future Scope

7.1. **Conclusion.** This study introduces a novel model that optimizes the use of port-provided free storage periods for non-instantaneously deteriorating items with time-varying order quantity-dependent demand. By leveraging the temporary free storage at ports as a cost-efficient alternative to traditional warehousing and incorporating green technology investments to reduce carbon emissions, the model provides a comprehensive approach to minimizing total inventory costs and enhancing resource allocation. The model's effectiveness is demonstrated through numerical examples, and a detailed sensitivity analysis further highlights the impact of various parameters on inventory costs, offering valuable insights for decision-makers in diverse industrial contexts. This approach not only reduces costs but also aligns with sustainability goals, providing a strategic framework for companies aiming to optimize their supply chain performance in a competitive market environment.

7.2. **Real-life applications.** The proposed inventory model has significant real-world relevance, particularly in industries that handle non-instantaneously deteriorating products, such as packed food, pharmaceuticals, and chemicals. These industries often deal with products that degrade over time but not immediately, requiring precise inventory management to avoid losses while meeting customer demand. The model can be effectively applied in global supply chains where companies import goods and need to strategically utilize port storage to reduce holding costs. Retailers and manufacturers importing products can optimize the allocation of inventory between port and owned storage to minimize costs during fluctuating demand periods. Additionally, the model is particularly useful for businesses dealing with products that have time-sensitive deterioration, helping them ensure cost efficiency while maintaining service levels.

7.3. **Future scope.** There are several promising directions for future research and development based on the current model. One potential area is incorporating stochastic elements, such as random demand and variable deterioration rates, to make the model more adaptable to real-world uncertainties. Another extension could involve expanding the model to multi-echelon inventory systems, where multiple ports, warehouses, and distribution centers are considered, reflecting the complexity of large-scale supply chains. Additionally, refining the model to account for nonlinear cost structures could improve its applicability in scenarios where costs like holding, transportation, and deterioration are not strictly linear. Integration with real-time data and IoT technologies is another avenue for future exploration, enabling dynamic adjustments to inventory strategies based on live market data or supply chain disruptions. Lastly, more extensive incorporation of green technology investments could explore the balance between sustainability initiatives and cost optimization, a growing concern in modern industries.

7.4. **Limitations.** Despite its contributions, the model has certain limitations that should be acknowledged. The deterministic assumptions used for demand and deterioration rates may limit the model's adaptability to environments with significant uncertainty or volatility. Additionally, the assumption of fixed storage capacities does not fully reflect real-world situations where storage availability can fluctuate due to factors like seasonal demand or temporary expansions. The focus on a single distribution network with one port and one warehouse also restricts the model's applicability to more complex, multi-echelon supply chains. Another limitation is the use of simplified, linear cost relationships, which may not capture the nuanced, nonlinear cost dynamics encountered in practice. Lastly, the model does not explicitly account for external factors such as geopolitical risks, market volatility, or sudden supply chain disruptions, which can have significant impacts on inventory management decisions.

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References

[1] P.N. Ghare, G.F. Schrader, A Model for Exponentially Decaying Inventories, J. Ind. Eng. 15 (1963), 238–243.

[2] J.A. Buzacott, Economic Order Quantities with Inflation, J. Oper. Res. Soc. 26 (1975), 553–558. https://doi.org/10.1057/jors.1975.113.

[3] R.B. Misra, A Note on Optimal Inventory Management under Inflation, Naval Res. Logist. Quart. 26 (1979), 161–165. https://doi.org/10.1002/nav.3800260116.

[4] R.V. Hartley, Operations Research: A Managerial Emphasis, Good Year Publishing Company, (1976).

[5] K.H. Kim, K.Y. Kim, Optimal Price Schedules for Storage of Inbound Containers, Transp. Res. Part B Methodol. 41 (2007), 892–905. https://doi.org/10.1016/j.trb.2007.02.001.

[6] E. Martín, J. Salvador, S. Saurí, Storage Pricing Strategies for Import Container Terminals under Stochastic Conditions, Transp. Res. Part E Logist. Transp. Rev. 68 (2014), 118–137. https://doi.org/10.1016/j.tre.2014.05.009.

[7] G. Ochtman, R. Dekker, E. van Asperen, W. Kusters, Floating Stocks in FMCG Supply Chains, Report No. EI 2004-17, Econometric Institute, Erasmus University Rotterdam, (2004). http://hdl.handle.net/1765/1276.

[8] R. Dekker, E. Van Asperen, G. Ochtman, W. Kusters, Floating Stocks in FMCG Supply Chains: Using Intermodal Transport to Facilitate Advance Deployment, Int. J. Phys. Distrib. Logist. Manag. 39 (2009), 632–648. https://doi.org/10.1108/09600030910996297.

[9] M. Pourakbar, A. Sleptchenko, R. Dekker, The Floating Stock Policy in Fast Moving Consumer Goods Supply Chains, Transp. Res. Part E Logist. Transp. Rev. 45 (2009), 39–49. https://doi.org/10.1016/j.tre.2008.06.001.

[10] E. Van Asperen, R. Dekker, Centrality, Flexibility and Floating Stocks: A Quantitative Evaluation of Port-of-Entry Choices, Marit. Econ. Logist. 15 (2013), 72–100. https://doi.org/10.1057/mel.2012.22.

[11] H.E. Haralambides, Competition, Excess Capacity, and the Pricing of Port Infrastructure, Int. J. Marit. Econ. 4 (2002), 323–347. https://doi.org/10.1057/palgrave.ijme.9100053.

[12] H. Haralambides, G. Gujar, The Indian Dry Ports Sector, Pricing Policies and Opportunities for Public-Private Partnerships, Res. Transp. Econ. 33 (2011), 51–58. https://doi.org/10.1016/j.retrec.2011.08.006.

[13] H. Haralambides, Globalization, Public Sector Reform, and the Role of Ports in International Supply Chains, Marit. Econ. Logist. 19 (2017), 1–51. https://doi.org/10.1057/s41278-017-0068-6.

[14] Z. Li, Inventory Management for One Warehouse Multi-Port Systems With Free Storage Periods, in: Proceedings of the 2nd International Conference on Mechanical Control and Automation, 2017, pp. 96-100.

[15] Z. Li, Optimal Utilization of Ports' Free-of-Charge Times in One Distribution Center and Multiple Ports Inventory Systems, Complexity 2019 (2019), 4510586. https://doi.org/10.1155/2019/4510586.

[16] C.-T. Chang, Inventory Models With Stock-Dependent Demand and Nonlinear Holding Costs for Deteriorating Items, Asia-Pac. J. Oper. Res. 21 (2004), 435–446. https://doi.org/10.1142/S0217595904000321.

[17] S.R. Singh, A.K. Malik, An Inventory Model With Stock-Dependent Demand With Two Storages Capacity for Non-Instantaneous Deteriorating Items, Int. J. Math. Sci. Appl. 1 (2011), 1255-1260.

[18] C.K. Jaggi, S. Tiwari, Two-Warehouse Inventory Model for Non-Instantaneous Deteriorating Items With Price Dependent Demand and Time-Varying Holding Cost, in: Mathematical Modelling and Applications, LAMBERT Academic Publishers, 2014, pp. 225-238.

[19] T.K. Datta, Effect of Green Technology Investment on a Production-Inventory System with Carbon Tax, Adv. Oper. Res. 2017 (2017), 4834839. https://doi.org/10.1155/2017/4834839.

[20] K. Rangarajan, K. Karthikeyan, An Optimal EOQ Inventory Model for Non-Instantaneous Deteriorating Items with Various Time Dependent Demand Rates and Time Dependent Holding Cost, IOP Conf. Ser.: Mater. Sci. Eng. 263 (2017), 042152. https://doi.org/10.1088/1757-899X/263/4/042152.

[21] K. Rangarajan, K. Karthikeyan, EOQ Models for Non-Instantaneous/Instantaneous Deteriorating Items with Cubic Demand Rate Under Inflation and Permissible Delay in Payments, Ital. J. Pure Appl. Math. 37 (2017), 197-218.

[22] K. Rangarajan, K. Karthikeyan, Two Warehouse Optimal Inventory Model for Non-Instantaneous Deteriorating Items, Int. J. Eng. Technol. 7 (2018), 946–948. https://doi.org/10.14419/ijet.v7i4.10.26633.

[23] B. Bishi, J. Behera, S.K. Sahu, Two-Warehouse Inventory Model for Non-instantaneous deteriorating Items with Exponential Demand Rate, Int. J. Appl. Eng. Res. 14 (2019), 495-515.

[24] A.S. Yadav, A. Swami, An Inventory Model for Non-Instantaneous Deteriorating Items with Variable Holding Cost under Two-Storage, Int. J. Procure. Manag. 12 (2019), 690. https://doi.org/10.1504/IJPM.2019.102928.

[25] D. Chakraborty, D.K. Jana, T.K. Roy, Multi-Warehouse Partial Backlogging Inventory System with Inflation for Non-Instantaneous Deteriorating Multi-Item under Imprecise Environment, Soft Comput. 24 (2020), 14471–14490. https://doi.org/10.1007/s00500-020-04800-3.

[26] H.M. Wee, Y. Daryanto, Imperfect Quality Item Inventory Models Considering Carbon Emissions, in: N.H. Shah, M. Mittal (Eds.), Optimization and Inventory Management, Springer, Singapore, 2020: pp. 137–159. https://doi.org/10.1007/978-981-13-9698-4_8.

[27] A.A. Taleizadeh, B. Hazarkhani, I. Moon, Joint Pricing and Inventory Decisions with Carbon Emission Considerations, Partial Backordering and Planned Discounts, Ann. Oper. Res. 290 (2020), 95–113. https://doi.org/10.1007/s10479-018-2968-y.

[28] A.H. Md Mashud, M. Pervin, U. Mishra, Y. Daryanto, M.-L. Tseng, M.K. Lim, A Sustainable Inventory Model with Controllable Carbon Emissions in Green-Warehouse Farms, J. Clean. Prod. 298 (2021), 126777. https://doi.org/10.1016/j.jclepro.2021.126777.

[29] S.K. Das, M. Pervin, S.K. Roy, G.W. Weber, Multi-Objective Solid Transportation-Location Problem with Variable Carbon Emission in Inventory Management: A Hybrid Approach, Ann. Oper. Res. 324 (2023), 283–309. https://doi.org/10.1007/s10479-020-03809-z.

[30] Y.W. Lok, S.S. Supadi, K.B. Wong, Optimal Investment in Preservation Technology for Non-Instantaneous Deteriorating Items under Carbon Emissions Consideration, Comput. Ind. Eng. 183 (2023), 109446. https://doi.org/10.1016/j.cie.2023.109446.

[31] P. Rajadurai, K. Rangarajan, A. Limi, Recent Review on Non-Instantaneous Deteriorating Items in Inventory Models (2006-2022), Int. J. Serv. Oper. Manag. 1 (2023), 10061336. https://doi.org/10.1504/IJSOM.2023.10061336.

[32] A. Limi, K. Rangarajan, P. Rajadurai, A. Akilbasha, K. Parameswari, Three Warehouse Inventory Model for Non-Instantaneous Deteriorating Items with Quadratic Demand, Time-Varying Holding Costs and Backlogging over Finite Time Horizon, Ain Shams Eng. J. 15 (2024), 102826. https://doi.org/10.1016/j.asej.2024.102826.

[33] A. Limi, K. Rangarajan, C. Jana, E. Ghith, T. Lamoudan, G.-W. Weber, A.A. Abdelhamid, Analyzing How Inflation Affects Non-Instantly Decaying Goods with Demand Linked to Ads and Selling Price in a Dual-Warehouse Setup, Alexandria Eng. J. 106 (2024), 517–529. https://doi.org/10.1016/j.aej.2024.08.014.

[34] M. Pervin, A Sustainable Deteriorating Inventory Model with Backorder and Controllable Carbon Emission by Using Green Technology, Environ. Dev. Sustain. (2024). https://doi.org/10.1007/s10668-024-04717-z.