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Intuitionistic Fuzzy Quasi-Supergraph Integration for Social Network Decision Making

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Abstract. This study explores the complexities of intuitionistic fuzzy (hyper) graphs, considering them as complex (hyper) networks, and presents a unique idea for intuitionistic fuzzy (quasi) superhypergraphs. The extension considers intuitionistic fuzzy superhypergraphs to be complicated superhyper networks to establish particular and general links between labeled items. These intuitionistic fuzzy (quasi) superhypergraphs arrange labeled object groups and analyze them in several relational aspects at the same time, including part-to-part, part-to-whole, and whole-to-whole groupings. The research investigates the characteristics of intuitionistic fuzzy (quasi) superhypergraphs utilizing positive real numbers, such as valued intuitionistic fuzzy (quasi) superhypergraphs and their complements, permutation-based isomorphism notation, and isomorphic (self-complemented) valued intuitionistic fuzzy (quasi) superhypergraphs. It also presents the concept of impact membership value for intuitionistic fuzzy (quasi) superhypergraphs and demonstrates how it may be used to solve real-world problems. Finally, the research demonstrates the use of intuitionistic fuzzy valued quasi superhyper graphs in addressing social network analysis, emphasizing their practical use.

1. Introduction

As a logical development of graph theory, Berge presented the idea of hypergraphs in 1960, which made it possible to express relationships between more than two components [1]. Hypergraphs are very useful for understanding systems that contain group interactions because they may relate any number of vertices, unlike ordinary graphs, which can only link exactly two vertices per edge. Applications for hypergraphs include gene interaction modeling [2], machine learning [3], computer networks [4], chemistry [5], visual classification [6], and social media analysis [7]. Due to its ability to embed, categorize, split, cover, and cluster items in a variety of real-world situations,

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researchers have found that hypergraphs are a valuable tool for examining complicated interactions. Hypergraphs, which function as hypernetworks, have a wide range of applications across several fields, demonstrating their significance and applicability. Some noteworthy examples are the following: using hypergraph convolutional networks for session-based recommendation systems with sequential information embeddings [8], modeling global interactions in social media networks using soft hypergraphs, creating hypergraph-based centrality metrics for global maritime container service networks [9], and applying hypergraph-based analysis for intelligent collaborative manufacturing spaces [10].

Mathematical notions are taught in classical set theory solely, without regard to quality or criterion, which makes them less useful in real-world applications. Real-world uncertainties are addressed by Zadeh's development of fuzzy set theory [11], which is an extension of set theory. Moreover, Smarandache's concept of Plithogenic sets-which includes crisp, fuzzy, intuitionistic fuzzy, and neutrosophic sets-provides a thorough framework for attribute-value-based element classification. Recent work by Edalatpanah et al. has investigated advances in approaches, strategies, and uses of Plithogenic and Neutrosophic sets, addressing real-world issues and proving to be effective extensions of fuzzy sets [12].

Conventional graphs may not be able to provide an accurate analysis of phenomena since many system features are inherently unclear. To address this issue and allow for a more thorough examination of intricate systems with ambiguous characteristics, fuzzy graph theory was introduced. Since its first introduction by Rosenfeld [13], fuzzy graph theory has grown to be a flexible extension of classical graph theory with a wide range of practical uses [14]. But to overcome these drawbacks, fuzzy graphs' intrinsic limits in representing intricate network dynamics have led to the investigation of intricate hypergraph topologies. Kaufmann first introduced the idea of fuzzy hypergraphs, which Lee-Kwanget al. [15] and associates expanded upon, highlighting its use in fuzzy partitioning and system analysis [16]. Akram and others wrote a groundbreaking book on fuzzy hypergraphs through their thorough research, including basic ideas, specifics, and many hypergraph extensions. They study a variety of mathematical models, including q-rung orthopair hypergraphs, bipolar, mpolar fuzzy, complex, intuitionistic, Pythagorean, and single-valued neutrosophic versions. In addition, the book provides useful applications in the fields of computer science, mathematics, and social sciences, making it a priceless tool for researchers and professionals alike [17]. The understanding and implementation of fuzzy (hyper)graphs inside intricate hypernetworks have gone further in recent studies. Research has looked at many topics, including the use of fuzzy soft competition hypergraphs in decision-making processes [19], the deployment of quality functions based on hypergraph and network flow analysis [20], and the modeling of the human nervous system [18], and single-valued neutrosophic soft hypergraphs [22]. Fuzzy hypergraph modeling has also been used for wireless sensor networks [23], stock recommendation systems [24], crime analysis and prediction [21], and interaction assessment in the Bering

Sea [25]. Further research on centrality metrics in fuzzy social networks [26] has improved our understanding of the dynamics and architecture of these networks. The terms "n-superhypergraphs" and "Plithogenic n-superhypergraphs," which are novel extensions of hypergraphs with unique characteristics and practical uses, were recently presented by Smarandache [27]. With the ability to communicate across hyperedges, these n-superhypergraphs offer a more comprehensive idea than conventional hypergraphs, increasing their usefulness. Hamidi et al. [28]developed quasi superhypergraphs as a variant of n-superhypergraphs in a recent development [29]. Shi and Kosari [31] investigate dominating qualities in product ambiguous graphs and their use in medicine. Product ambiguous graphs depict unclear relationships, whereas dominance refers to the capacity of certain aspects to control others. Kosari, Rao et al. [32] examine ambiguous graph topologies and their applications in medical diagnostics. Vague graphs reflect ambiguous or unclear relationships and help people interpret medical data. Kou , Kosari et al. [33] presents a unique description of ambiguous graphs with applications in transport systems. Vague graphs are used in transportation networks to represent ambiguous relationships. Shao, Kosari et al. [34] discuss some confusing graph principles and how they might be used in medical diagnostics. Vague graphs can help portray ambiguous medical facts and later they discuss novel notions in intuitionistic fuzzy graph theory and their application to water supply systems. It most likely describes how fuzzy graphs might model uncertainty in water supply networks [35]. Shi, Kosari et al. [36] discuss about the primary energies of image fuzzy graphs and their applications. It is likely to investigate the application of fuzzy graphs in various computational intelligence systems. Lei, Guan, Jiang, Zou, and Rao [37] describe a machine-proof method for point geometry based on Coq. It most likely describes the creation of a formal proof system for geometric theorems. Rao, Binyamin et al. [38] investigate the planarity of graphs connected with symmetric and pseudo-symmetric numerical semigroups. Kosari, Jiang, et al. [39] investigate connectivity features of ambiguous fuzzy graphs and their implications in university infrastructure design. Khan, Arif et al. [40] discuss interval-valued image fuzzy hypergraphs and their use in decision-making procedures. Alqahtani, Kaviyarasu et al. [41] examine the use of complicated neutrosophic graphs in hospital infrastructure planning. Kaviyarasu, Aslam, Afzal et al. [42] present the notion of connectedness indices in neutrosophic graphs and discuss their use in computer networks, highway systems, and transportation network flows. It is probable to explore how neutrosophic graphs can describe uncertainty in a variety of network systems. Wadei Faris AL-Omeri, Kaviyarasu, and Rajeshwari [43] discuss the internet streaming providers using the max product of complements in neutrosophic graphs. It most likely investigates how neutrosophic graphs may be used to examine online streaming services. Al-Omeri [44] discusses mixed b-fuzzy topological spaces. It likely explores the properties and applications of mixed b-fuzzy topological spaces in fuzzy logic and intelligent systems. Their work tackles the problems and inconsistencies that naturally occur in hypergraph theory since conventional hypergraphs are not able to completely express the interactions between vertices. They want to address these shortcomings in graph and hypergraph structures by presenting superhypergraphs, which will deepen our comprehension of intricate networks. Moreover, Hamidi et al. developed the superhypergraph incidence matrix and deduced its characteristic polynomial, revealing the superhypergraph spectrum. They also came up with ways to count the superedges inside a certain superhypergraph and figure out how many superhypergraphs there are overall on any nonempty set using superedges and partitions. These developments provide important new understandings of the combinatorial and structural characteristics of superhypergraphs.

As a logical extension of intuitionistic fuzzy hypergraphs, we describe the innovative idea of valued intuitionistic fuzzy superhypergraphs in light of these developments. The intuitionistic fuzzy supervertices and intuitionistic fuzzy superedges (also known as intuitionistic fuzzy links), which are precisely described in this paper, constitute the foundation of this system. Valued intuitionistic fuzzy superhypergraphs are motivated by the fact that they can be used to simulate intricate superhypernetworks and address practical issues. Optimal judgments may be reached by converting real-world situations into intuitionistic fuzzy superhypergraphs and utilizing the idea of impact membership values. In investigating notions like strong valued intuitionistic fuzzy superhypergraphs, isomorphic valued intuitionistic fuzzy superhypergraphs, intuitionistic fuzzy link complements, and self-complemented valued intuitionistic fuzzy superhypergraphs, this work explores the intuitionistic fuzzy algebraic structures inherent in valued intuitionistic fuzzy superhypergraphs. In addition, a detailed analysis of the interaction between intuitionistic fuzzy hypergraphs and heavily valued intuitionistic fuzzy superhypergraphs is done to clarify their interrelationships and provide some understanding of their characteristics.

1.1. **Motivation.**

(1) The study attempts to investigate the drawbacks of intuitionistic fuzzy (hyper) graphs, especially in intricate network situations, and suggests a solution to get around them.
(2) The work aims to construct a more complex relationship between labeled items, supporting both specific and broad viewpoints, by adding intuitionistic fuzzy (quasi) superhypergraphs.
(3) The rationale is to offer a framework that is structurally sound and allows labeled object groupings to be simultaneously analyzed from several relational perspectives.

1.2. **Novelty.**

(1) Conceptual Extension: By extending the idea of intuitionistic fuzzy (hyper) graphs beyond its conventional definition, intuitionistic fuzzy (quasi) superhypergraphs are put forth as a unique idea.
(2) The article presents a unique method for expressing and evaluating interactions between labeled items by modeling intuitionistic fuzzy superhypergraphs as complex superhypernetworks.

(3) Part-to-part, part-to-whole, and whole-to-whole links may all be simultaneously understood by the multifaceted analysis of labeled object groups inside intuitionistic fuzzy (quasi) superhypergraphs, which is a unique approach.

2. Preliminary

Definition 2.1. *[1] Let $\mathfrak{Y}$ be a finite set. A hypergraph on $\mathfrak{Y}$ is a pair $I^* = (\mathfrak{Y}, \{T_\mathfrak{l}\}_{\mathfrak{l}=1}^s)$ such that for all $1 \le \mathfrak{l} \le s$, $\emptyset \neq T_\mathfrak{l} \subseteq \mathfrak{Y}$ and*

$$\bigcup_{\mathfrak{l}=1}^{s} T_\mathfrak{l} = \mathfrak{Y}.$$

*The elements $\mathfrak{y}_1, \mathfrak{y}_2, \ldots, \mathfrak{y}_n$ of $\mathfrak{Y}$ are called vertices, and the sets $T_1, T_2, \ldots, T_s$ are called the hyperedges of the hypergraph I^*. In hypergraphs, hyperedges can contain an element (loop), two elements (edge), or more than three elements. A hypergraph $I^* = (\mathfrak{Y}, \{T_\mathfrak{l}\}_{\mathfrak{l}=1}^s)$ is called a **complete hypergraph**, if for any $\mathfrak{y}, \mathfrak{y}' \in \mathfrak{Y}$, there exists $1 \le \mathfrak{l} \le s$ such that*

$$\{\mathfrak{y}, \mathfrak{y}'\} \subseteq T_\mathfrak{l}.$$

*A hypergraph $I^* = (\mathfrak{Y}, \{T_\mathfrak{l}\}_{\mathfrak{l}=1}^s)$ is called a **joint complete hypergraph**, if $|\mathfrak{Y}| = s$, and for all $1 \le \mathfrak{l} \le s$,*

$$|T_\mathfrak{l}| = \mathfrak{l} \quad \text{and} \quad T_\mathfrak{l} \subseteq T_{\mathfrak{l}+1}.$$

If for all $1 \le k \le s$, $|T_k| = 2$, the hypergraph becomes an ordinary (undirected) graph. The incidence matrix of the hypergraph is a binary matrix with n rows representing the vertices $\mathfrak{y}_1, \mathfrak{y}_2, \ldots, \mathfrak{y}_n$, where for all $1 \le \mathfrak{l} \le n$ and for all $1 \le \mathfrak{r} \le s$,

$$m_{\mathfrak{l},\mathfrak{r}} = \begin{cases} 1, & \text{if } \mathfrak{y}_\mathfrak{l} \in T_\mathfrak{r}, \\ 0, & \text{if } \mathfrak{y}_\mathfrak{l} \notin T_\mathfrak{r}. \end{cases}$$

Definition 2.2. *[29] Let $m \in \mathbb{N}$ and $V = \{v_1, v_2, \ldots, v_m\}$ be a set of vertices that contain:*

- *Single vertices (classical ones)*
- *Indeterminate vertices (unclear, vague, unknown)*
- *Null vertices (unknown, empty)*

Consider $P(V)$ as the power set of V, $P^2(V) = P(P(V))$, ..., and $P^{n+1}(V) = P(P^n(V))$. Then the n-superhypergraph (n-SHG) is an ordered pair:

$$n\text{-}SHG = (G_n, T_n)$$

where for any $n \in \mathbb{N}$, $G_n \subseteq P^n(V)$ is the set of vertices, and $T_n \subseteq P^n(V)$ is the set of edges. The set G_n contains some types of vertices: single vertices, indeterminate vertices, null vertices, supervertices i.e., two or more (single, indeterminate, or null) vertices forming a group An n-supervertex is a collection of many vertices such that at least one is an $(n-1)$-supervertex and all other supervertices in the collection have the order $r \le n-1$. The set of edges E_n contains: Single edges, Indeterminate edges (unclear, vague, partially unknown), Null edges (empty, totally unknown), Hyperedges, superedges, n-superedges, Superhyperedges, n-superhyperedges, Multi-edges, Loops.

Definition 2.3. *[30] Let $\mathfrak{Y}$ be a non-empty set. Then:*

(1) *$I^* = (\mathfrak{Y}, T = \{T_l\}_{l=1}^{s}, \{\mathcal{K}_{l,r}\}_{l,r})$ is called a quasi intuitionistic fuzzy superhypergraph, if $\emptyset \neq \mathfrak{Y} = \bigcup_{l=1}^{s} T_l$, where $s \geq 2$, and for all $1 \leq l \leq s$, $T_l \in P^*(\mathfrak{Y})$ is called a supervertex, and for any $l \neq r$, the map $\mathcal{K}_{l,r} : T_l \to T_r$ (denoting T_l links to T_r) is called a superedge.*

(2) *The quasi intuitionistic fuzzy superhypergraph $I^* = (Y, \{T_l\}_{l=1}^{s}, \{\mathcal{K}_{l,r}\}_{l,r})$ is called a intuitionistic fuzzy superhypergraph, if for any $T_l \in P^*(\mathfrak{Y})$, there exists at least one $T_r \in P^*(\mathfrak{Y})$ such that T_l links to T_r (it is not necessary for all supervertices to be linked).*

(3) *The intuitionistic fuzzy superhypergraph $I^* = (\mathfrak{Y}, \{T_l\}_{l=1}^{s}, \{\mathcal{K}_{l,r}\}_{l,r})$ is called a trivial intuitionistic fuzzy superhypergraph, if $s = 1$ (i.e., T_1 cannot link to itself).*

Let $\mathfrak{Y}$ be a non-empty set, then we define:

$$IH(Y) = \{I^* \mid I^* \text{ is an intuitionistic fuzzy superhypergraph on } \mathfrak{Y}\}$$

$$IH(m_1, m_2, \ldots, m_s) = \{(Y, \{T_l\}_{l=1}^{s}, \{\mathcal{K}_{l,r}\}) \in IH \mid |T_l| = m_l\}$$

3. Intuitionistic Fuzzy Quasi Supperhypergraph

Definition 3.1. *Let $r, s \in \mathbb{N}$, $1 \leq r \leq s$, $m \in \mathbb{R}^+$, $I^* = \left(\mathfrak{Y}, \{T_l\}_{\mathfrak{l}=1}^{s}, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ be a quasi fuzzy supper-hypergraph $\mathfrak{p}_\mathfrak{l} = \{(\mathfrak{Y}, \mathfrak{p}_i(\mathfrak{y})) \,|y \in T_l, 0 \leq \mathfrak{p}_\mathfrak{l}(\mathfrak{y}) \leq 1\}$, $\delta_{\mathfrak{l},\mathfrak{r}} : \jmath_{\mathfrak{l},\mathfrak{r}} \to [0,1]$ and $\tau_{\mathfrak{l},\mathfrak{r}} : \jmath_{\mathfrak{l},\mathfrak{r}} \to [0,1]$ be the intuitionistic fuzzy subset then $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is called q- intuitionistic fuzzy quasi super-hypergraph (IFQSHG) on I^* is defined as, $Y = \bigcup_{\mathfrak{l}=1}^{s} supp(\mathfrak{p}_\mathfrak{l})$ $\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \leq \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(E(\mathfrak{p}_\mathfrak{r})))}$, $\tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \leq \frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(E(\mathfrak{p}_\mathfrak{r})))}$, where $\mathfrak{E}(\mathfrak{p}_v) = \Sigma_{y_v \in T_v} \mathfrak{p}(\mathfrak{y}_v)$, are $\mathfrak{p}_\mathfrak{l}'s$ intuitionistic fuzzy supper vertices, $\delta_{\mathfrak{l},\mathfrak{r}}'s$ are membership of intuitionistic fuzzysuppr edges and $\tau_l's$ are non membership of intuitionistic fuzzy superedges. $\mathfrak{p}_\mathfrak{l}$ and $\mathfrak{p}_\mathfrak{r}$, then membership and nonmembership values will be equal to zero. The structure I^* represented as $\left(\mathfrak{Y}, \{T_l\}_{\mathfrak{l}=1}^{s}, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ becomes a intuitionistic fuzzy hypergraph under certain circumstances.*

Theorem 3.1. *Consider $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ as a q-IFSHG on I^*, with q and $\mathfrak{q}'$ being positive real integers. If $\mathfrak{q}'$ is less than q, I remains a $\mathfrak{q}'$-IFQSHG on I^*.*

Proof. Given that q is smaller than q, which are both, positive real values, and $\mathfrak{y} \in \mathfrak{Y}$, where $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a $\mathfrak{q}'$-IFQSHG on I^*, we may deduce

$$\begin{aligned}
\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &\leq \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))} \\
&\leq \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}'(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}'(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}
\end{aligned}$$

$$\begin{aligned}
\tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &\leq \frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))} \\
&\leq \frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}'(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{l}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}'(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}
\end{aligned}$$

Thus,$I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a $\mathfrak{q}'$-IFQSHG on I^*. □

Proposition 3.1. *The following holds,*
(i) *All intuitionistic fuzzy hypergraphs are IFQSHG.*
(ii) *All intuitionistic fuzzy graphs are IFQSHG.*

Proof. The evidence is unambiguous by Definition. □

Theorem 3.2. *Consider a q-IFQSHG,* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *defined on the set* I^{*}.
For every $1 \leq \mathfrak{l} \leq s$, *then*
(i) *for every* $\mathfrak{y} \in \mathfrak{Y}$, $t = \{sup\left(\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right), inf\left(\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)\}$ *is determined.*
(ii) *If* $q \in [v, t]$ *then* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is an intuitionistic fuzzy graph where*
$t = \left\{\bigwedge\left\{\frac{1}{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y})) \bigwedge \delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}\right\}, \bigvee\left\{\frac{1}{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y})) \bigvee \tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}\right\}\right\}$, $\mathfrak{y} \in \mathfrak{Y}$ *and* $l, r \in \mathbb{N}$.

Proof. Let $\mathfrak{y} \in \mathfrak{Y}$. Then (i) $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a q- IFQSHG on I^{*} and for every $1 \leq \mathfrak{l} \leq s$, then

$$\begin{aligned}
\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) &\leq \frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)} \\
&= \frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)} \\
\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) &\leq \frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)} \\
&= \frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}
\end{aligned}$$

(ii) from (i). If $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a intuitionistic fuzzy graph, then
$\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \leq \delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right) \bigwedge \delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)$ and $\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \leq \tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right) \bigvee \tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)$
for all $\mathfrak{y} \in \mathfrak{Y}$ and $l, r \in \mathbb{N}$. □

Corollary 3.1. *If* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is a* $\mathfrak{q}$- *IFQSHG on* I^{*} *and for every* $1 \leq \mathfrak{l} \leq s$, $\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right) > 1$ *then* $\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \leq \delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right) \bigwedge \delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)$ *and* $\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \leq \tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right) \bigvee \tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)$.

Definition 3.2. *Consider two real positive values* $\mathfrak{q}$ *and* $\mathfrak{q}'$ *and two IFQSHGs* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *defined on quasihyper graphs* $I^{*} = \left\{Y, \{T_l\}_{\mathfrak{l}=1}^{s}, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right\}$ *and* $I'^{*} = \left\{Y', \left\{T'_l\right\}_{\mathfrak{l}=1}^{s}, \left\{\jmath'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right\}$, *respectively.*
(i) *An isomorphic* $i : I \to I'$ *is a bijective mapping such that for each* $\mathfrak{y} \in \mathfrak{Y}$, *there exist permutation* α, β *on* $G_s = \{1, 2, 3..., s\}$ *such that* $\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right) = \mathfrak{p}'_{\alpha(l)}\left(i\left(\mathfrak{y}\right)\right)$, $\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \delta'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$ $\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \tau'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$. *We indicate that* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I'^{*} = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *are isomorphic in this situation* $I \cong I'$.
(ii) *If* $I \cong I_c$, *then* I *is called a self complemented IFQSHG.*

Example 3.1. *Consider the set* $Y = \{e, f, g, h\}$ *and* $Y = \{e', f', g', h'\}$. *Defined* $I = \left(\mathfrak{Y}, \{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{2}, \delta_{1,2}, \tau_{1,2}\right)$ *and* $I' = \left(\mathfrak{Y}', \left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{2}, \delta'_{1,2}, \tau'_{1,2}\right)$ *are 2-IFQSHGs, similarly to the Figure 1a,1b in 1. Suppose* $\jmath_{1,2} = \{(e,g), (f,h)\}$ *and* $\jmath'_{1,2} = \{(e',g'), (f',h')\}$. *Consider the mapping* $i : I \to I'$, *given by* $i = \{(e,f'), (f,e'), (g,h'), (h,g')\}$ *with permutation* $\alpha = \beta = (1,2)$. *Then* $I \cong I'$.

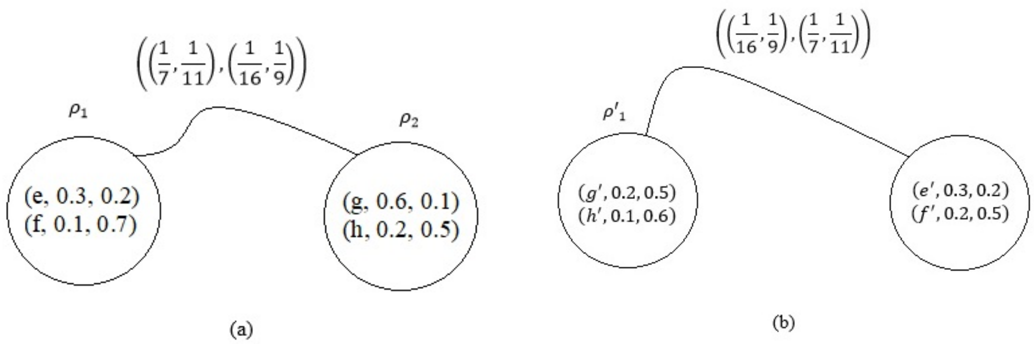

Figure 1. I and I' are isomorphic intuitionistic fuzzy two-quasi superhypergraphs.

Theorem 3.3. *Consider* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *to be* $\mathfrak{q}$*-IFQSHG and* $\mathfrak{q}'$*-IFQSHG on QSHG* $I^* = \left\{Y, \{T_l\}_{\mathfrak{l}=1}^{s}, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right\}$ *and* $I'^* = \left\{Y', \left\{T'_l\right\}_{\mathfrak{l}=1}^{s}, \left\{\jmath'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right\}$, *respectively. If I and I′ are isomorphic, then* $\mathfrak{q}$ *must be the same* $\mathfrak{q}'$.

Proof. An isomorphism is a bijective mapping $i : I \to I'$ that exists when I is isomorphic to I'. For each $\mathfrak{y} \in \mathfrak{Y}$, there exist permutation α, β on $G_s = \{1, 2, 3..., s\}$ such that $\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}) = \mathfrak{p}'_{\alpha(l)}(i(\mathfrak{y}))$, $\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \delta'_{\alpha(l)\beta(r)}\left(i(\mathfrak{y}), \jmath'_{\alpha(l)\beta(r)}(i(\mathfrak{y}))\right)$ and $\tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \tau'_{\alpha(l)\beta(r)}\left(i(\mathfrak{y}), \jmath'_{\alpha(l)\beta(r)}(i(\mathfrak{y}))\right)$. Let us assume an arbitrary intuitionistic fuzzy supervertex $\mathfrak{p}_f$. If $\mathfrak{p}_f = \left\{\left(\mathfrak{y}_l, \mathfrak{p}_f(\mathfrak{y}_l)\right) \mid 1 \le \mathfrak{l} \le e\right\}$ in $\mathfrak{q}$-IFQSHG $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$. Then there exists intuitionistic fuzzy super vertex $\mathfrak{p}'_{f'} = \left\{\left(\mathfrak{y}'_l, \mathfrak{p}_{f'}\left(\mathfrak{y}'_l\right)\right) \mid 1 \le \mathfrak{l} \le e'\right\}$ in $\mathfrak{q}'$-IFQSHG $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ such that $\mathfrak{p}_f(\mathfrak{y}_l) = \mathfrak{p}'_{f'}\left(\mathfrak{y}'_l\right)$ which $f' = \alpha(f)$ and $i(\mathfrak{y}_l) = y'_l$. This means that $e = e'$ and for every given $1 \le \mathfrak{l}, l' \le e$, we have $\frac{y_1}{\mathfrak{q}\left(\sum_{\mathfrak{l}=1}^{e} y_l\right)} = \frac{y'_1}{\mathfrak{q}'\left(\sum_{\mathfrak{l}=1}^{e} y'_l\right)}$. Hence $\sum_{\mathfrak{l}=1}^{f} \frac{y_1}{\mathfrak{q}\left(\sum_{\mathfrak{l}=1}^{e} y_l\right)} = \sum_{\mathfrak{l}=1}^{f} \frac{y'_1}{\mathfrak{q}'\left(\sum_{\mathfrak{l}=1}^{e} y'_l\right)}$ and thus $\mathfrak{q} = \mathfrak{q}'$.
If $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ and $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ to be $\mathfrak{q}$-IFQSHG and $\mathfrak{q}'$-IFQSHG on QSHG $I^* = \left\{Y, \{T_l\}_{\mathfrak{l}=1}^{s}, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right\}$ and $I'^* = \left\{Y', \left\{T'_l\right\}_{\mathfrak{l}=1}^{s}, \left\{\jmath'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right\}$, respectively. The statement $I \cong I'$ implies that $\mathfrak{q} = \mathfrak{q}'$. ◻

Theorem 3.4. *Consider two isomorphic* $\mathfrak{q}$*- IFQSHGs* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$
(i) *There exists a intuitionistic fuzzy supervertex* $\mathfrak{p}_{\mathfrak{l}} \in I$, $\mathfrak{p}'_{\mathfrak{l}} \in I'$ *and* $\Upsilon \in \mathbb{R}$, *such that* $\frac{w(\mathfrak{p}_{\mathfrak{l}})}{w(\mathfrak{p}_{\mathfrak{r}})} = \Upsilon$.
(ii) *For every* $\mathfrak{y} \in \mathfrak{Y}$, *thete exist* $y' \in Y'$ *and* $\Upsilon \in \mathbb{R}$ *such that* $\frac{y}{y'} = \Upsilon$.

Proof. As a intuitionistic fuzzy supervertex in I, $\mathfrak{p}_{\mathfrak{l}} = \left\{\left(\mathfrak{y}_f, \mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}_f\right)\right) \mid 1 \leq f \leq e\right\}$.
Clearly $i\left(\mathfrak{p}_{\mathfrak{l}}\right) = \left\{\left(i\left(\mathfrak{y}_f\right), \mathfrak{p}_{\mathfrak{l}}\left(i\left(\mathfrak{y}_f\right)\right)\right) \mid 1 \leq f \leq e\right\}$ is a intuitionistic fuzzy supervertex in I'. Using theorem 3, we can express $\frac{y_f}{\mathfrak{E}(\mathfrak{p}_l)} = \frac{y'_f}{\mathfrak{E}(i(\mathfrak{p}_l))}$. As a result, $\frac{\mathfrak{E}(\mathfrak{p}_l)}{\mathfrak{E}(i(\mathfrak{p}_l))} = \frac{y_f}{y'_f}$. □

Example 3.2. *Consider 2- IFQSHGs , indicated by* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$*, illustrated in subfigures 1a and1b of Figure 1. It is clear that* $I \cong I'$*. For each* $\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right) \neq \mathfrak{E}\left(\mathfrak{p}'_{\mathfrak{l}}\right)$*. This highlights the fact that even when* $\mathfrak{q}$*-IFQSHGs are isomorphic, there is no guarantee that their intuitionistic fuzzy supervertices will have the same weights.*

Definition 3.3. *Let* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *denote a* $\mathfrak{q}$*-IFQSHG on* I^**.*
If $I^c = \left(\left\{\mathfrak{p}^c_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s}, \left\{\delta^c_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau^c_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is the complement of* $\mathfrak{q}$*-IFQSHG of I, we establish the following conditions*
(i) $\mathfrak{p}^c_{\mathfrak{l}} = \mathfrak{p}_{\mathfrak{l}}$
(ii) $\delta^c_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))} - \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$ *and*
(iii) $\tau^c_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))} - \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$

Lemma 3.1. *For a* $\mathfrak{q}$*-IFQSHG* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *defined over* I^**, the condition*
(i) $\delta^c_{\mathfrak{l},\mathfrak{r}} = \delta_{\mathfrak{l},\mathfrak{r}}$ *holds if and only if* $\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]$
(ii) $\tau^c_{\mathfrak{l},\mathfrak{r}} = \tau_{\mathfrak{l},\mathfrak{r}}$ *holds if and only if* $\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]$
for every $\mathfrak{y} \in \mathfrak{Y}$ *and* $\mathfrak{q} \geq \frac{1}{2}$*.*

Theorem 3.5. *If a* $\mathfrak{q}$*-IFQSHG* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *defined over* I^**, it follows that the* $\left(I^c\right)^c \cong I$*.*

Proof. Assume $\mathfrak{y} \in \mathfrak{Y}$. Then for any $1 \leq \mathfrak{l} \leq s$, $\left(\mathfrak{p}^c_{\mathfrak{l}}\right)^c\left(\mathfrak{y}\right) = \mathfrak{p}^c_{\mathfrak{l}}\left(\mathfrak{y}\right) = \mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)$,

$$\begin{aligned}
\left(\delta^c_{\mathfrak{l},\mathfrak{r}}\right)^c\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) &= \left[\frac{\delta\left(\mathfrak{p}^c_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}^c_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \delta^c_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \\
&= \left[\frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \delta^c_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \\
&= \left[\frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \left[\frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] \\
&+ \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) \\
&= \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)
\end{aligned}$$

and

$$\left(\tau^c_{\mathfrak{l},\mathfrak{r}}\right)^c\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \left[\frac{\tau\left(\mathfrak{p}^c_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}^c_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \tau^c_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$

$$= \left[\frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \tau^{c}_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$
$$= \left[\frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right] - \left[\frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right]$$
$$+ \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$
$$= \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$

Therefore, for every $1 \leq \mathfrak{l} \leq s$, $\left(\mathfrak{p}^{c}_{\mathfrak{l}}\right)^{c} = \mathfrak{p}_{\mathfrak{l}}, \left(\delta^{c}_{\mathfrak{l},\mathfrak{r}}\right)^{c} = \delta_{,rl}$ and $\left(\tau^{c}_{\mathfrak{l},\mathfrak{r}}\right)^{c} = \tau_{,rl}$.
As a result $(I^{c})^{c} \cong I$. □

Theorem 3.6. *If* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *are isomorphic* $\mathfrak{q}$-*IFQSHGs. Their complements* I^{c} *and* I'^{c} *are also isomorphic. Conversely if* I^{c} *and* I'^{c} *are isomorphic* $\mathfrak{q}$-*IFQSHGs, then I and* I' *must also be isomorphic.*

Proof. Given $I \cong I'$, there is a bijective mapping $i : I \to I'$ known as an isomorphism. For each $\mathfrak{y} \in \mathfrak{Y}$, there exist permutation α, β on $G_s = \{1, 2, 3..., s\}$ such that $\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right) = \mathfrak{p}'_{\alpha(l)}\left(i\left(\mathfrak{y}\right)\right)$, $\delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \delta'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$ and $\tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \tau'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$.
Using the complement definition, we can say the following

$$\delta^{c}_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)} - \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$
$$= \frac{\delta\left(\mathfrak{p}'_{\alpha(l)}\left(i\left(y\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}'_{\alpha(l)}\right)\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}'_{\alpha(l)}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(i\left(\mathfrak{y}\right)\right)\right)\right)}{\delta\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}'_{\alpha(l)}\right)\right)\right)} - \delta'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$$
$$= \delta^{c}_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$$

and

$$\tau^{c}_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)} - \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)$$
$$= \frac{\tau\left(\mathfrak{p}'_{\alpha(l)}\left(i\left(y\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}'_{\alpha(l)}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}'_{\alpha(l)}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(i\left(\mathfrak{y}\right)\right)\right)\right)}{\tau\left(\mathfrak{q}\left(\mathfrak{E}\left(\mathfrak{p}'_{\alpha(l)}\right)\right)\right)} - \tau'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath'_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$$
$$= \tau^{c}_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right), \jmath_{\alpha(l)\beta(r)}\left(i\left(\mathfrak{y}\right)\right)\right)$$

Therefore, $I^{c} \cong I'^{c}$. Conversely, if $I^{c} \cong I'^{c}$, Theorem 3.13 states that $I \cong (I^{c})^{c} \cong \left(I'^{c}\right)^{c} \cong I'$. □

Theorem 3.7. *Suppose* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is a self complemented* $\mathfrak{q}$-*IFQSHG on* I^{*}. *Then*
$\sum \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\sum\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \wedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$ *and*
$\sum \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\sum\left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \vee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$

Proof. If $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a self complemented $\mathfrak{q}$-IFQSHG, there exists a mapping $i : Y \to Y$ such that for all $\mathfrak{y} \in \mathfrak{Y}$, $\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}) = \mathfrak{p}'_{l}(i(\mathfrak{y}))$, $\delta'_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))) = \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))$ and $\tau'_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))) = \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))$. Thus,
$\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(i((\mathfrak{y}))))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \wedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \delta_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y})))$ and
$\tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(i((\mathfrak{y}))))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \vee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \tau_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y})))$.
Given that $i : Y \to Y$ is a bijection, we obtain the following conclusion,

$$\begin{aligned}\sum \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \sum \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(i((\mathfrak{y}))))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \sum \delta_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y})))\\ &= \sum \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \sum \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))\\ 2\sum \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \sum \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\end{aligned}$$

As a result $\sum \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \frac{1}{2}\left[\sum\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \vee \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$ and

$$\begin{aligned}\sum \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \sum \left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(i((\mathfrak{y}))))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}))))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \sum \tau_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y})))\\ &= \sum \left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right] - \sum \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))\\ 2\sum \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \sum \left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\end{aligned}$$

As a result $\sum \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = \frac{1}{2}\left[\sum\left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \wedge \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$ □

Example 3.3. *Consider the set $Y = \{y_l\}_{\mathfrak{l}=1}^{6}$. The 4-IFQSHG $I^* = \left(\mathfrak{y}, \{T_l\}_{\mathfrak{l}=1}^{3}, \{\jmath_{1,2}, \jmath_{2,3}\}\right)$ is depicted in Figure 2, where $\jmath_{1,2} = \{(\mathfrak{y}_1, y_3), (\mathfrak{y}_2, y_4)\}$ and $\jmath_{2,3} = \{(\mathfrak{y}_3, y_6), (\mathfrak{y}_4, y_7)\}$.*
Now, LetI $= \left\{Y, \{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{3}, \delta_{1,2}, \delta_{2,3}, \tau_{1,2}, \tau_{2,3}\right\}$ is a IFQSHG as shown in Figure 2. Then

$$\begin{aligned}&\sum \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))\\ &= \frac{1}{2}\left[\sum\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]\\ &= \frac{1}{2}\left[\frac{0.1}{4(0.3)} \bigwedge \frac{0.3}{4(0.7)} + \frac{0.2}{4(0.3)} \bigwedge \frac{0.4}{4(0.7)} + \frac{0.3}{4(0.7)} \bigwedge \frac{0.5}{4(1.1)} + \frac{0.4}{4(0.7)} \bigwedge \frac{0.6}{4(1.1)}\right]\\ &= \frac{1}{2}\left[\frac{0.1}{4(0.3)} + \frac{0.2}{4(0.3)} + \frac{0.3}{4(0.7)} + \frac{0.4}{4(0.7)}\right]\\ &= \frac{1}{4} = 0.25 = \delta_{1,2} + \delta_{2,3}\end{aligned}$$

and

$$\begin{aligned}\sum \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) &= \frac{1}{2}\left[\sum\left[\frac{\tau\left(\mathfrak{p}_{\mathfrak{l}}\left(\mathfrak{y}\right)\right)}{\tau\left(4\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)\right)} \bigvee \frac{\tau\left(\mathfrak{p}_{\mathfrak{r}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right)\right)}{\tau\left(4\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)\right)}\right]\right] \\ &= \frac{1}{2}\left[\frac{0.8}{4\left(1.5\right)} \bigvee \frac{0.5}{4\left(1\right)} + \frac{0.7}{4\left(1.5\right)} \bigvee \frac{0.6}{4\left(1\right)} + \frac{0.5}{4\left(1\right)} \bigvee \frac{0.2}{4\left(0.3\right)} + \frac{0.6}{4\left(1\right)} \bigvee \frac{0.1}{4\left(.3\right)}\right] \\ &= \frac{1}{2}\left[\frac{0.8}{4\left(1.5\right)} + \frac{0.7}{4\left(1.5\right)} + \frac{0.5}{4\left(1\right)} + \frac{0.6}{4\left(1\right)}\right] \\ &= 0.2625 = \tau_{1,2} + \tau_{2,3}\end{aligned}$$

It may be concluded that while evaluating the expression

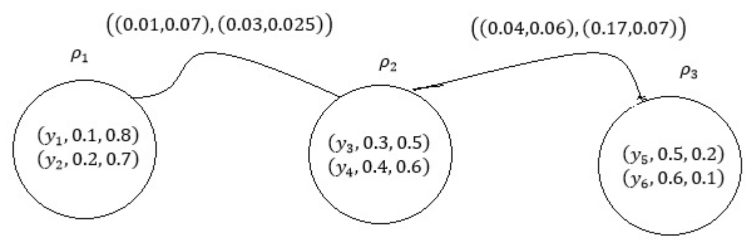

Figure 2. IFQSHG *I*.

$\sum \delta_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\sum\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$ *and*
$\sum \tau_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\sum\left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(4(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]\right]$

where I does not form a self-Complementeed IFQSHG on I^* as indicated in Figure 3. As a result, the converse of Theorem 7 is not necessarily true.

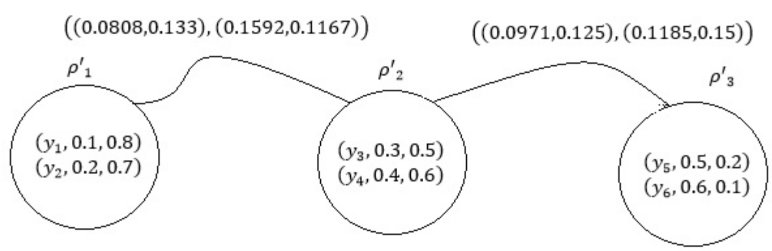

Figure 3. IFQSHG I^c.

Theorem 3.8. *Let* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is q-IFQSHG on I^*. If for all* $\mathfrak{y} \in \mathfrak{Y}$, $\delta^{c}_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]$ *and* $\tau^{c}_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}\left(\mathfrak{y}\right)\right) = \frac{1}{2}\left[\frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]$*, then*
$I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is a self complemented IFQSHG.*

Proof. If $i : \mathfrak{Y} \to \mathfrak{Y}$ by $i(\mathfrak{y}) = y$, where $\mathfrak{y} \in \mathfrak{Y}$. It is clear that $\mathfrak{p}^c(i(y)) = \mathfrak{p}^c(y) = \mathfrak{p}(y)$

$$\begin{aligned}
\delta^c_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \delta^c_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(y))) \\
&= \left[\frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] - \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \\
&= \left[\frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] - \frac{1}{2}\left[\frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] \\
&= \frac{1}{2}\left[\frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] \\
&= \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))
\end{aligned}$$

and

$$\begin{aligned}
\tau^c_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \tau^c_{\mathfrak{l},\mathfrak{r}}(i(\mathfrak{y}), \jmath_{\mathfrak{l},\mathfrak{r}}(i(y))) \\
&= \left[\frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] - \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \\
&= \left[\frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] - \frac{1}{2}\left[\frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] \\
&= \frac{1}{2}\left[\frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}\right] \\
&= \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))
\end{aligned}$$

Therefore, $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^s, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ is a self complemented IFQSHG. □

Definition 3.4. *Consider*$I^* = \left(\mathfrak{Y}, \{T_l\}_{\mathfrak{l}=1}^s, \{\jmath_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *be a quasi fuzzy supperhyper graph* $\mathfrak{p}_\mathfrak{l} = \{(\mathfrak{Y}, \mathfrak{p}_i(\mathfrak{y})) \,|y \in T_l, 0 \le \mathfrak{p}_\mathfrak{l}(\mathfrak{y}) \le 1\}$, $\delta_{\mathfrak{l},\mathfrak{r}} : \jmath_{\mathfrak{l},\mathfrak{r}} \to [0,1]$ *and* $\tau_{\mathfrak{l},\mathfrak{r}} : \jmath_{\mathfrak{l},\mathfrak{r}} \to [0,1]$ *be the intuitionistic fuzzy subset then* $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^s, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is called q- IFQSHG on* I^* *, if* $Y = \bigcup_{\mathfrak{l}=1}^s$ *supp* $(\mathfrak{p}_\mathfrak{l})$ $\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \le \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}$, $\tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \le \frac{\tau(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigvee \frac{\tau(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))}$, *where* $\mathfrak{E}(\mathfrak{p}_v) = \Sigma_{y_v \in T_v} \mathfrak{p}(\mathfrak{y}_v)$

Theorem 3.9. *Let* $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^s, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *be a strong* $\mathfrak{q}$*-IFQSHG on* I^**. Then* $I^c = \left(\left\{\mathfrak{p}^c_\mathfrak{l}\right\}_{\mathfrak{l}=1}^s, \left\{\delta^c_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}} \{\tau_{\mathfrak{l},\mathfrak{r}}\}^c_{\mathfrak{l},\mathfrak{r}}\right)$ *is intuitionistic fuzzy hypergraph.*

Proof. Assume $y \in Y$. $I = \left(\{\mathfrak{p}_\mathfrak{l}\}_{\mathfrak{l}=1}^s, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ be a strong $\mathfrak{q}$-IFQSHG on I^*.
This result in

$$\begin{aligned}
\delta^c_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))} - \delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \\
&= \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))} - \frac{\delta(\mathfrak{p}_\mathfrak{l}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{l})))} \bigwedge \frac{\delta(\mathfrak{p}_\mathfrak{r}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_\mathfrak{r})))} \\
&= 0
\end{aligned}$$

and

$$\begin{aligned}\tau^{c}_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) &= \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))} - \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \\ &= \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))} - \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))} \\ &= 0\end{aligned}$$

Hence for each $\mathfrak{y} \in \mathfrak{Y}$, $\delta^{c}_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = 0$ and $\tau^{c}_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) = 0$. As a result, if $\mathfrak{p}_{\mathfrak{l}}$ and $\mathfrak{p}_{\mathfrak{r}}$ represent different intuitionistic fuzzy supervertices, there are no intuitionistic fuzzy connections linking them. Thus $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ resulting in intuitionistic fuzzy hypergraph as defined in Definition 4. □

Definition 3.5. *Let* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *be a* $\mathfrak{q}$*-IFQSHG on* I^**, the impact membership value of* y *in* I *is denoted as*

$$d(\mathfrak{y}, I) = \begin{cases} \sum_{(\mathfrak{y},\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))\in \jmath_{\mathfrak{l},\mathfrak{r}}} \left[\delta_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})), \tau_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y}))\right] & \text{if } (\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \in \jmath_{\mathfrak{l},\mathfrak{r}}, \\ 0 & \text{if } (\mathfrak{y}, \jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})) \notin \jmath_{\mathfrak{l},\mathfrak{r}}. \end{cases}$$

Theorem 3.10. *Let* $\mathfrak{y} \in \mathfrak{Y}$ *and* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *be a* $\mathfrak{q}$*-IFQSHG on* I^**. Then*
(i) If $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is strong then*
$\mathfrak{q}d(\mathfrak{y}, I) = \sum \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta((\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}}))}, \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau((\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}}))}\right]$,
(ii) $d(\mathfrak{y}, I^c) + d(\mathfrak{y}, I) = \sum \left[\frac{\delta(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigwedge \frac{\delta(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\delta(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}, \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\mathfrak{y}))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}})))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{r}}(\jmath_{\mathfrak{l},\mathfrak{r}}(\mathfrak{y})))}{\tau(\mathfrak{q}(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}})))}\right]$,
(iii) If $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *is a strong then* $d(y, I^c) = 0$.

Proof. The interpretation is simple since it follows Definition 6 and is supported by Theorem 9. □

Corollary 3.2. *Let* $I = \left(\{\mathfrak{p}_{\mathfrak{l}}\}_{\mathfrak{l}=1}^{s}, \{\delta_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}, \{\tau_{\mathfrak{l},\mathfrak{r}}\}_{\mathfrak{l},\mathfrak{r}}\right)$ *and* $I' = \left(\left\{\mathfrak{p}'_{\mathfrak{l}}\right\}_{\mathfrak{l}=1}^{s'}, \left\{\delta'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}, \left\{\tau'_{\mathfrak{l},\mathfrak{r}}\right\}_{\mathfrak{l},\mathfrak{r}}\right)$ *be a q-IFQSHGs. If* $I \cong I'$*, then* $\sum_{y\in I} d(y, I) = \sum_{y\in I'} d(y, I')$.

4. Application of Social Network Decision Making

In this part, we use the notion of intuitionistic fuzzy valued quasi superhypergraphs in real-world circumstances. The following procedure may be used to simplify and explain how fuzzy q-quasi superhypergraphs are applied in actual problems:
Step 1. Begin by visualizing a real-world problem as a complex (super)hypernetwork.
Step 2. Identify and identify the many components and elements in this complex (super)hypernetwork according to their relevance to the given application.
Step 3. Organize these components and constituent elements into separate tables, categorizing them according to their values.
Step 4. Using the information from the tables established in Step 3, design an optimum relationship model between the components with a fuzzy q-quasi superhypergraph.

Step 5. During this phase, choose a suitable value for q and compute the maximum fuzzy superedges using fuzzy supervertices.
Step 6. Determine the effect membership values for the components in the q-fuzzy quasi superhypergraph.
Step 7. Analyze the extreme situation produced from Step 6, taking into account the established requirements required to address and resolve the original real-world problem.

In the framework of Social Network Analysis (SNA), we investigate the complex dynamics of interpersonal interactions inside a corporate setting, similar to a business superhypernetwork. This network represents a complex web of connections between many entities, including persons, goods, and market segments, all working together to achieve strategic and operational objectives within the market environment. In the context of SNA, we define an intuitionistic fuzzy (super)hypernetwork as a q-fuzzy quasi superhypergraph, with entities represented as intuitionistic fuzzy supervertices and connections as intuitionistic fuzzy superedges or links. This illustration depicts the various links and interactions that exist inside the social fabric of a business network. For example, while studying a social network via the lens of an intuitionistic fuzzy quasi superhypergraph, we take into account elements such as persons who represent significant stakeholders in the network, as well as items and their quality, and the buying and selling market and its liquidity. These entities make up the network's intuitionistic fuzzy supervertices, each with its own set of traits and features.

Furthermore, the fuzzy superedges in this social network represent many aspects of contact, such as skill levels, communication competence, and service time between persons, goods, and market sectors. These connections act as conduits for information, resources, and opportunities inside the network, encouraging individuals to collaborate and profit from one another.Within this approach, social network analysis focuses on identifying and optimizing entity interconnection to achieve commercial objectives. It comprises identifying important influencers, forming strategic alliances, and utilizing network dynamics to increase market presence and competitiveness.

Organizations use business intuitionistic fuzzy superhyper networking to build mutually beneficial connections with stakeholders, clients, and consumers, utilizing the network's ability to increase their reach and customer base. The fundamental value is in the capacity to engage with new consumers, create referrals, and eventually expand the client base, fostering long-term growth and success within the network.

Case 1: Analysis of Social Influence and Connectivity

Objective: We aim to investigate the least effect that individuals have on certain social entities, the minimum communication frequency between individuals and the market, and the minimum distance between individuals and significant social hubs.

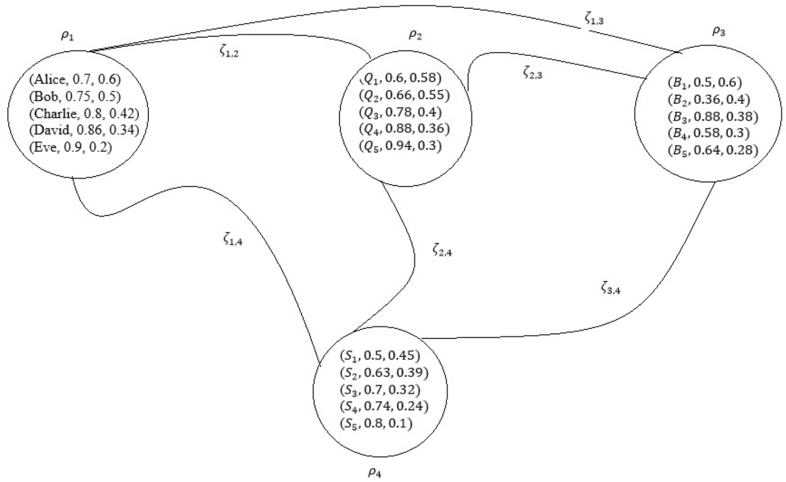

FIGURE 4. IFQSHG I.

$$\delta_{\mathfrak{l},\mathfrak{r}}\left(Alice, Q1\right) \leq \frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(Alice\right)\right)}{\delta\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{l}}\right)\right)} \bigwedge \frac{\delta\left(\mathfrak{p}_{\mathfrak{l}}\left(\jmath_{\mathfrak{l},\mathfrak{r}}\left(Alice\right)\right)\right)}{\delta\left(\mathfrak{E}\left(\mathfrak{p}_{\mathfrak{r}}\right)\right)}$$

$$\begin{aligned}\delta_{\mathfrak{l},\mathfrak{r}}\left(Alice, Q1\right) &\leq \frac{0.7}{4.01} \bigwedge \frac{0.6}{3.86}\\ &= \frac{0.6}{3.86}\end{aligned}$$

$\tau_{\mathfrak{l},\mathfrak{r}}\left(Alice, Q1\right) \leq \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(Alice))}{\tau(\mathfrak{E}(\mathfrak{p}_{\mathfrak{l}}))} \bigvee \frac{\tau(\mathfrak{p}_{\mathfrak{l}}(\jmath_{\mathfrak{l},\mathfrak{r}}(Alice)))}{\tau(\mathfrak{E}(\mathfrak{p}_{\mathfrak{r}}))}$.

$$\begin{aligned}\tau_{\mathfrak{l},\mathfrak{r}}\left(Alice, Q1\right) &\leq \frac{0.6}{2.06} \bigvee \frac{0.58}{2.19}\\ &= \frac{0.6}{2.06}\end{aligned}$$

It follows that,

$\jmath_{1,2} = \{(Alice, Q1), (Bob, Q2), (Charlie, Q3), (David, Q4), (\mathfrak{E}ve, Q5)\}$

$\jmath_{1,3} = \{(Alice, B2), (Bob, B1), (Charlie, B4), (David, B5), (\mathfrak{E}ve, B3)\}$

$\jmath_{1,4} = \{(Alice, S1), (Bob, S3), (Charlie, S2), (David, S5), (\mathfrak{E}ve, S4)\}$

$\jmath_{2,3} = \{(Q1, B2), (Q2, B1), (Q3, B4), (Q4, B5), (Q5, B3)\}$

$\jmath_{2,4} = \{(Q1, S2), (Q2, S1), (Q3, S4), (Q4, S5), (Q5, S3)\}$

$\jmath_{3,4} = \{(B1, S2), (B2, S1), (B3, S4), (B4, S5), (B5, S3)\}$

TABLE 1. Social intuitionistic fuzzy quasi superhypernetworking.

$(Stakeholder, Itemsquality)$	$(Alice, Q1)$	$(Bob, Q2)$	$(Charlie, Q3)$	$(David, Q4)$	$(\mathfrak{E}ve, Q5)$
$(Amountofcapital, quality)$	$\left(\frac{30}{193}, \frac{30}{103}\right)$	$\left(\frac{33}{193}, \frac{55}{219}\right)$	$\left(\frac{39}{193}, \frac{21}{103}\right)$	$\left(\frac{86}{401}, \frac{12}{73}\right)$	$\left(\frac{90}{401}, \frac{10}{73}\right)$

Table 2. Social intuitionistic fuzzy quasi superhypernetworking.

$(Stakeholder, Buying)$	$(Alice, B2)$	$(Bob, B1)$	$(Charlie, B4)$	$(David, B5)$	$(\mathfrak{E}ve, B3)$
$(Amountofcapital, Buyingmarket)$	$\left(\frac{9}{74}, \frac{30}{103}\right)$	$\left(\frac{25}{148}, \frac{30}{103}\right)$	$\left(\frac{29}{148}, \frac{21}{103}\right)$	$\left(\frac{8}{37}, \frac{17}{103}\right)$	$\left(\frac{11}{37}, \frac{19}{98}\right)$

Table 3. Social intuitionistic fuzzy quasi superhypernetworking.

$(Stakeholder, Selling)$	$(Alice, S1)$	$(Bob, S3)$	$(Charlie, S2)$	$(David, S5)$	$(\mathfrak{E}ve, S4)$
$(Amountofcapital, Sellingmarket)$	$\left(\frac{25}{148}, \frac{15}{49}\right)$	$\left(\frac{9}{74}, \frac{3}{10}\right)$	$\left(\frac{74}{337}, \frac{19}{98}\right)$	$\left(\frac{29}{148}, \frac{15}{98}\right)$	$\left(\frac{8}{37}, \frac{16}{75}\right)$

Table 4. Social intuitionistic fuzzy quasi superhypernetworking.

$(Itemsquality, Buying)$	$(Q1, B2)$	$(Q2, B1)$	$(Q3, B4)$	$(Q4, B5)$	$(Q5, B3)$
$(Itemsquality, Buyingmarket)$	$\left(\frac{9}{74}, \frac{58}{219}\right)$	$\left(\frac{25}{193}, \frac{15}{49}\right)$	$\left(\frac{29}{148}, \frac{40}{219}\right)$	$\left(\frac{8}{37}, \frac{12}{73}\right)$	$\left(\frac{11}{37}, \frac{19}{98}\right)$

Table 5. Social intuitionistic fuzzy quasi superhypernetworking.

$(Itemsquality, Selling)$	$(Q1, S2)$	$(Q2, S1)$	$(Q3, S4)$	$(Q4, S5)$	$(Q5, S3)$
$(Itemsquality, Sellingmarket)$	$\left(\frac{30}{193}, \frac{58}{219}\right)$	$\left(\frac{50}{337}, \frac{55}{219}\right)$	$\left(\frac{74}{337}, \frac{40}{219}\right)$	$\left(\frac{80}{337}, \frac{12}{73}\right)$	$\left(\frac{70}{337}, \frac{16}{75}\right)$

Table 6. Social intuitionistic fuzzy quasi superhypernetworking.

$(Buying, Selling)$	$(B1, S2)$	$(B2, S1)$	$(B3, S4)$	$(B4, S5)$	$(B5, S3)$
$(Buyingmarket, Sellingmarket)$	$\left(\frac{25}{148}, \frac{15}{49}\right)$	$\left(\frac{9}{74}, \frac{3}{10}\right)$	$\left(\frac{74}{337}, \frac{19}{98}\right)$	$\left(\frac{29}{148}, \frac{15}{98}\right)$	$\left(\frac{8}{37}, \frac{16}{75}\right)$

Case 2: To select the most influential member of this superhypernetwork, we consider each individual's membership worth. For instance, we compute William's effect membership value using the following method:

$$\begin{aligned} d\,(Alice, I) &= \sum_{(\mathfrak{y}, \jmath_{\mathrm{I},\mathrm{r}}(\mathfrak{y}))\in \jmath_{\mathrm{I},\mathrm{r}}} \left[\delta_{\mathrm{I},\mathrm{r}}\left(\mathfrak{y}, \jmath_{\mathrm{I},\mathrm{r}}\left(\mathfrak{y}\right)\right), \tau_{\mathrm{I},\mathrm{r}}\left(\mathfrak{y}, \jmath_{\mathrm{I},\mathrm{r}}\left(\mathfrak{y}\right)\right)\right] \\ &= \left[\left(\frac{30}{193} + \frac{9}{74} + \frac{50}{337}\right), \left(\frac{30}{103} + \frac{30}{103} + \frac{30}{103}\right)\right] \\ &= (0.4254, 0.8738) \end{aligned}$$

5. Comparison Analysis

There are significant differences between intuitionistic fuzzy quasi superhypergraphs and fuzzy quasi superhypergraphs in terms of both their theoretical foundations and real-world applications.

Table 7. Impact value Social intuitionistic fuzzy quasi superhypernetworking.

y	*Alice*	*Bob*	*Charlie*	*David*	*Eve*
$d(y, I)$	$(0.4254, 0.8737)$	$(0.5476, 0.7851)$	$(0.5849, 0.6116)$	$(0.6680, 0.4944)$	$(0.7413, 0.4908)$

Table 8. Impact value Social intuitionistic fuzzy quasi superhypernetworking.

y	Q_1	Q_2	Q_3	Q_4	Q_5
$d(y, I)$	$(0.2770, 0.5296)$	$(0.2779, 0.5572)$	$(0.4155, 0.3652)$	$(0.4536, 0.3287)$	$(0.5050, 0.4072)$

Table 9. Impact value Social intuitionistic fuzzy quasi superhypernetworking.

y	B_1	B_2	B_3	B_4	B_5
$d(y, I)$	$(0.1483, 0.3061)$	$(0.1216, 0.3)$	$(0.1869, 0.2038)$	$(0.1959, 0.1650)$	$(0.2077, 0.2133)$

The conventional fuzzy hypergraph theory is extended by fuzzy quasi superhypergraphs, which highlight the essential function of fuzzy linkages in establishing impact values in complicated systems. The objective of these structures is to enhance the use of fuzzy hypergraphs in real-world scenarios by resolving their shortcomings in managing imprecision and uncertainty. However, by adding intuitionistic fuzzy logic, which enables a more detailed representation of uncertainty and ambiguity, intuitionistic fuzzy quasi superhypergraphs improve this framework even more. Through highlighting the significance of intuitionistic fuzzy connections, these structures aim to improve the practical application of hypergraph theory, especially in contexts of social network analysis and decision-making. Furthermore, whereas extension dependency and isomorphism are ideas that both frameworks share, intuitionistic fuzzy quasi superhypergraphs go further into influence membership values, demonstrating their usefulness in decision-making process optimization. In summary, whilst fuzzy quasi superhypergraphs provide a foundation for managing fuzziness in hypergraphs, intuitionistic fuzzy quasi superhypergraphs offer a more sophisticated method that can better handle complicated real-world issues with greater levels of uncertainty.

6. Conclusion

The study discusses and explores intuitionistic fuzzy quasi superhypergraphs, emphasizing the importance of intuitionistic fuzzy linkages in determining the impact value of these structures. Its goal is to deploy intuitionistic fuzzy quasi superhypergraphs in actual contexts and increase the application of intuitionistic fuzzy hypergraphs. Notably, the study tackles constraints in intuitionistic fuzzy hypergraph theory by concentrating on optimum situations for sets of components rather than individual elements. Key results include the dependence of a valued intuitionistic fuzzy quasi superhypergraph's extension on its value, the link between intuitionistic fuzzy hypergraphs and intuitionistic fuzzy quasi superhypergraphs, and the introduction of notions like isomorphism, complementation, and self-complementation. Furthermore, the paper investigates the impact membership value of these structures, demonstrating that the impact membership value of a strongly valued intuitionistic fuzzy quasi superhypergraph is zero and that the sum of impact membership values of isomorphic valued intuitionistic fuzzy quasi superhypergraphs is constant. Furthermore, it applies these principles to a real-world situation by providing a social network analysis superhypernetwork and making the best option based on the effect membership value of valued intuitionistic fuzzy quasi superhypergraphs.

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