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Evaluating the Consistency of Neutrosophic Data Using Various Statistical Distributions: Comparative Studies and Applications

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ABSTRACT. In this paper, we primarily have given neutrosophic coefficient of variation, robust neutrosophic coefficient of variation concern to interquartile range, and robust neutrosophic coefficient of variation concern to median absolute deviation. Following the introduction, we have explored the methods of neutrosophic coefficient of variation, which is an effective method for modeling data that is fuzzy, imprecise, and uncertain. For the comparative study, we have given numerical studies based on neutrosophic distributions including discrete and continuous distributions. First, we have compared all three neutrosophic coefficient of variations and then have given the comparative study for all neutrosophic distributions for these neutrosophic coefficient of variations. Also, we have given real data analysis on climate data to highlight the impact of the neutrosophic coefficient of variations. We found that neutrosophic coefficient of variations NCV and based on IQR have near about similar values while the neutrosophic coefficient of variation based on MAD has higher values than other two for all samples and distributions. Further, we observe that with increasing the sample values all three neutrosophic coefficients of variations also increase for all the distributions and provide a general framework over classical methods of coefficient of variations, and the graphical representations also clarify this.

1. Introduction

In comparing variabilities across different data series with varying units, it's more effective to use the coefficient of variation (CV) instead of the standard deviation (SD). The CV is unitless and provides a relative measure of dispersion concerning the mean values. This

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makes it ideal for comparing variability between different data sets. The coefficient of variation (CV), calculated as the ratio of the standard deviation to the mean, is a popular metric for assessing relative dispersion. This measure is useful in a variety of disciplines, including engineering, physics, chemistry, medicine, economics, finance, agriculture, meteorology, sports science, environmental science, education, retail and marketing, quality control, sampling, and so on where it is widely used. In analytical chemistry, the CV is frequently employed to indicate the precision and consistency of an assay, Reed et al [1]. In finance, it is utilized to evaluate relative risk by comparing the CVs of different stocks [2]. In economics, the CV is used as a summary statistic to measure inequality. Other examples of its application include assessing the homogeneity of bone test samples Hammer et al. [3], measuring the strength of ceramics [4], and defining age- and sex-specific cutoff points for body mass index in overweight children [5]. Moment-based measures like the mean and standard deviation are known to be sensitive to outliers. While the mean absolute deviation (MAD) is also affected by outliers, using the interquartile range (IQR) or the median absolute deviation (MAD) robustness can be improved more. To improve robustness, two measures based on IQR and MAD have been developed. Shapiro [6] introduced a robust coefficient of variation (CV) that uses the IQR and median. This measure is calculated as the ratio of the IQR to the median. Similarly, [7] defined a robust CV based on the MAD, which is calculated as the ratio of the MAD to the median. Arachchige [8] examined these robust CV versions, focusing on their properties for interval estimation using data quantiles like the IQR and MAD. However, all these studies for the coefficient of variations are done under classical statistics and do not count indeterminacy/fuzziness in the data, and the coefficient of variations under classical statistics will fail for these types of data.

To deal with uncertainty/fuzziness in the data or uncertain observations, Florentin [9] proposed the concept of descriptive neutrosophic statistics. Neutrosophic statistics have been demonstrated to be more effective than traditional statistics when analyzing imprecise data. When there are no uncertain observations in the dataset, the outcomes of neutrosophic statistical analysis converge with those of classical statistical methods. Neutrosophic statistics is a branch of mathematical science designed to handle imprecise, fuzzy, and uncertain data. It involves the collection, presentation, analysis, and inference of such data. Unlike classical statistics, which deals with precise data, neutrosophic statistics operates in environments with inherent uncertainty. Further, neutrosophic methods explored for analyzing neutrosophic data are discussed in [10] and [11]. Adepoju et al. [12] explores the use of the negative binomial distribution for fuzzy data. [13] and [14] develop discrete and continuous distributions using Neutrosophy. Alvaracín Jarrín et al. [15] applies neutrosophic statistics to social data analysis. Khan et al. [16] extends the Rayleigh distribution with neutrosophic statistics. Neutrosophic statistics are introduced in experimental design by [17] and [18] extends the Kumaraswamy distribution to the neutrosophic Kumaraswamy distribution. Further applications of neutrosophic statistics are provided by Delcea et al. [19]. Algorithms for generating imprecise data in various situations are discussed by [20], [21], Aslam [22-25] and [26]. Also, in sampling theory, Tahir et al. [27], [28], Singh et al. [29], and Singh et al. [30, 31], and have given the estimation of the neutrosophic population parameters utilizing neutrosophic coefficient of variation (the ratio of the standard deviation to the mean) but none has given detailed study on the neutrosophic coefficient of variation (NCV).

After thoroughly reviewing the existing literature, it seems that, to the best of the author's knowledge, there is a significant absence of research addressing the neutrosophic coefficient of variations (NCVs) and robust neutrosophic coefficient of variations based on IQR and MAD in uncertain contexts. To address this gap, we aim to introduce the neutrosophic coefficient of variation, and robust neutrosophic coefficient of variations based on IQR and MAD, and provide a detailed explanation. Like in classical methods of coefficient of variation, the neutrosophic coefficient of variation has also many applications. For example, in manufacturing, measurements of product dimensions can be subject to uncertainties stemming from machine precision, environmental conditions, and human error. By applying NCV, quality control managers can gain a clearer understanding of dimensional variability while factoring in these uncertainties and indeterminate elements. In clinical trials, patient data often contains uncertainties due to inconsistent adherence to treatment protocols or reporting inaccuracies. NCV provides a more thorough assessment of treatment outcome variability, taking these uncertainties and indeterminate data into account. Similarly, in ecological and environmental studies, measurements of pollution levels, temperatures, or species populations can be uncertain due to sampling errors, measurement limitations, and natural variability. Using NCV, researchers can effectively quantify the variation in these measurements, considering inherent uncertainties and indeterminacies. In other fields too, like climate change study as in our manuscript, energy sector, transportation and traffic study, education, and academic performance, financial market analysis, urban planning and development, and so many examples can be given where for the neutrosophic coefficient of variations. Further, to define the neutrosophic coefficient of variations, we have defined the first neutrosophic random variable, neutrosophic methodology, neutrosophic coefficient of variations, NCV, and NCV based on IQR and MAD. For comparative evaluation, we have conducted the simulation study based on generated data on neutrosophic statistical distributions (continuous and discrete distributions) to highlight the impact of the indeterminacy/uncertainty. The real-life climate data is also given to highlight the comparative evaluation of methods of coefficient of variations under uncertainty. To enhance our proposed method, we have also provided graphical representations for the numerical comparisons of the coefficient of variations. Essentially, we expect that our proposed method will offer greater flexibility in analyzing uncertain or indeterminate data, fulfilling the critical need within the domain of statistical analysis.

2. Neutrosophic Random Variable

Suppose that $X_N = X_{1L} + X_{1L}I_N; I_N \epsilon [I_L, I_U]$ be a neutrosophic random variable. The neutrosophic random variable is the combination of two parts known as the determinate part X_{1L} and the indeterminate part $X_{1L}I_N$ and the degree of uncertainty$I_N \epsilon [I_L, I_U]$. Note here that the proposed neutrosophic random variable is the generation of the random variable under classical statistics. The proposed random variable becomes the classical random variable X_{1L} when $I_L = 0$. We assume that the determinate part of the neutrosophic random variable follows the normal distribution mean μ and variance σ^2. According to [13], fuzzy logic is a special case of neutrosophic logic and $I_N^2 = I_N, \ldots, I_N^n = I_N, 0. I_N = 0; n \epsilon \mathrm{N}$. Based on this information, we discuss the expected value of the proposed neutrosophic random variable as follows

$$E(X_N) = E(X_{1L} + X_{1L}I_N) = (1 + I_N)\mu$$

The variance of the proposed neutrosophic random variable is given by

$$Var(X_N) = Var(X_{1L} + X_{1L}I_N) = (1 + I_N)^2\sigma^2$$

The neutrosophic standard deviation is given by

$$S.D(X_N) = (1 + I_N)\sigma$$

2.1 Methodology

Let $X_{1N} = X_{1L} + X_{1L}I_N; I_N \epsilon [I_L, I_U], X_{2N} = X_{2L} + X_{2L}I_N; I_N \epsilon [I_L, I_U], \ldots, X_{nN} = X_{nL} + X_{nL}I_N; I_N \epsilon [I_L, I_U]$ be an independently and identically neutrosophic random variable of size n. Note here that the first variable $X_{1L}, X_{2L}, \ldots, X_{nL}$ presents the random variable under classical statistics and the second part $X_{1L}I_N, X_{2L}I_N, \ldots, X_{nL}I_N$ be the indeterminate part of the neutrosophic random variable and $I_N \epsilon [I_L, I_U]$ be the degree of uncertainty. Then, the neutrosophic sample mean, say $\bar{X}_N$ is derived as

$$\bar{X}_N = \frac{(X_{1L}+X_{1L}I_N)+(X_{2L}+X_{2L}I_N)+\cdots+(X_{nL}+X_{nL}I_N)}{n} = \frac{X_{1L}+X_{2L}+\cdots+X_{nL}}{n} + \frac{(X_{1L}+X_{2L}+\cdots+X_{nL})I_N}{n} \quad (1)$$

After some simplification, we have

$$\bar{X}_N = \bar{X}_{1L} + \bar{X}_{1L}I_N; I_N \epsilon [I_L, I_U] \quad (2)$$

The neutrosophic sample variance, say S_N^2 is derived as follows

$$S_N^2 = \frac{\sum_{i=1}^{n}(X_{iN}-\bar{X}_N)^2}{n-1} = \frac{\sum_{i=1}^{n}\left((1+I_N)X_{iL}-(1+I_N)\bar{X}_L\right)^2}{n-1} = \frac{(1+I_N)^2\sum_{i=1}^{n}(X_{iL}-X_{iL})^2}{n-1} \quad (3)$$

The neutrosophic sample standard deviation is given by

$$S_N = \frac{(1+I_N)\sqrt{\sum_{i=1}^{n}(X_{iL}-X_{iL})^2}}{n-1} \quad (4)$$

Based on the information, the median of odd neutrosophic data $X_{1N}, X_{2N}, \ldots, X_{nN}$ is calculated by

$$m_N = (1 + I_N)\left(\frac{n+1}{2}\right)^{th} value \quad (5)$$

The median of even neutrosophic data $X_{1N}, X_{2N}, \dots, X_{nN}$ is calculated by

$$m_N = (1 + I_N)\frac{\left(\frac{n}{2}\right)^{th} value + \left(\frac{n}{2}+1\right)^{th} value}{2} \quad (6)$$

The neutrosophic quartile for the neutrosophic data $X_{1N}, X_{2N}, \dots, X_{nN}$ is defined by

$$Q_{mN} = m(1 + I_N)\left(\frac{n+1}{4}\right) th\ value,\ m = 1,2,3,4 \quad (7)$$

The first neutrosophic quartile using neutrosophic data $X_{1N}, X_{2N}, \dots, X_{nN}$ is given by

$$Q_{1N} = (1 + I_N)\left(\frac{n+1}{4}\right) th\ value \quad (8)$$

The third neutrosophic quartile using neutrosophic data $X_{1N}, X_{2N}, \dots, X_{nN}$ is given by

$$Q_{3N} = 3(1 + I_N)\left(\frac{n+1}{4}\right) th\ value \quad (9)$$

The neutrosophic interquartile range (IQR) is defined by

$$IQR_N = Q_{3N} - Q_{1N} \quad (10)$$

The neutrosophic median absolute deviation (NMAD) is defined by

$$NMAD = Median\left(\left|\left((1 + I_N)(x_{iL} - \tilde{x}_L)\right)\right|\right) \quad (11)$$

where $\tilde{x}_L$ denote the median of the determinate part of the neutrosophic data.

3. Neutrosophic Coefficient of Variation (NCV)

The coefficient of variation (CV) is a vital metric used to assess the consistency of data. A higher CV value indicates greater dispersion around the mean. Traditional CV, as per classical statistics, is employed to evaluate data consistency when all observations are known. However, this conventional CV has limitations; it is applicable only when the data is recorded under specific conditions and is entirely accurate. In real-world scenarios, achieving precise data is not always feasible. In this section, we aim to enhance the traditional CV by introducing the neutrosophic coefficient of variation (NCV). The NCV is designed to address uncertainties in data. We propose the sample NCV as a potential alternative to the existing CV, incorporating the degree of uncertainty. The proposed NCV is defined as follows:

$$CV_N = \frac{\frac{(1+I_N)\sqrt{\sum_{i=1}^{n}(X_{iL}-X_{iL})^2}}{n-1}}{(\bar{X}_{1L}+\bar{X}_{1L}I_N)} \times 100;\ I_N \epsilon [I_L, I_U] \quad (12)$$

Note that the proposed NCV serves as a generalization of the classical CV. The NCV aligns with the classical CV when I_L=0.

3.1 Robust NCV

Arachchige, Prendergast, & Staudte [8] explored the properties of two robust versions of the CV based on data quantile. In this section, we aim to extend these robust CVs using neutrosophic statistics, anticipating that the proposed NCVs will offer greater efficiency and flexibility in handling imprecise data.

3.2 NCV Based on IQR

The classical CV calculates the ratio of the mean to the standard deviation. Notably, the mean is significantly influenced by extreme values in the data. In contrast, the median remains relatively unaffected by extreme values. Shapiro [6] introduced an alternative robust CV based on the interquartile range (IQR) and median. This robust CV is defined as the ratio of IQR to median. It's worth noting that Shapiro's robust CV applies only when all data observations are accurate. In this section, we aim to adapt Shapiro's robust CV using neutrosophic statistics. We introduce the Neutrosophic Robust Coefficient of Variation (NRCV), based on IQR and median, defined as follows:

$$RCV_{NQ} = 0.75 \times \frac{IQR_N}{m_N}; I_N \epsilon [I_L, I_U] \quad (13)$$

As noted by Arachchige et al. [8], a multiplier of 0.75 aligns RCV_N with CV_N for the neutrosophic normal distribution. It's important to highlight that our proposed RCV_N extends Shapiro's robust CV from 2005. The RCV_N aligns with Shapiro's robust CV when there are no imprecise observations in the data.

3.3 NCV Based on NMAD

The median absolute deviation (MAD) under classical statistics is defined by [7] and given by

$$MAD = Median(|x_{iL} - \tilde{x}_L|)$$

The NMAD, which is the extension of the MAD and is given by

$$NMAD = Median\big(\big|\big((1 + I_N)(x_{iL} - \tilde{x}_L)\big)\big|\big); I_N \epsilon [I_L, I_U]$$

Note the proposed NMAD becomes MAD when I_L=0.

By following [32] and [1], the proposed NCV based on NMAD is defined by

$$RCV_{NM} = \frac{1}{\Phi^{-1}(\frac{3}{4})} \times \frac{NMAD}{m_N}; I_N \epsilon [I_L, I_U] \quad (14)$$

The multiplier $1/\Phi^{-1}(3/4) = 1.4826$, where Φ^{-1} is a quantile function of the standard normal distribution with mean 0 and variance 1. This multiplier is used to make the equivalence between RCV_{NM} and $1.4826 \times NMAD$.

4. Simulation Study

The simulation study is conducted for the numerical comparisons of the neutrosophic coefficient of variations CV_N, RCV_{NQ}, and RCV_{NM} for different neutrosophic distributions along with the classical coefficient of variations CV, RCV_Q, and RCV_M using R Studio. We have used "ntsDists" package for data generation through different neutrosophic distributions. We have calculated the neutrosophic coefficient of variations for the neutrosophic Beta, Normal, Binomial, Discrete Uniform, Exponential, Gamma, and Generalized Exponential distributions, and the results are placed in Table 1, for the neutrosophic Rayleigh, Generalized Rayleigh, Geometric, Kumaraswamy, Laplace, Negative Binomial, Generalized Pareto are placed in Table

2, and for the neutrosophic Uniform, Weibull, Poison distribution are placed in Table 3. The results for each neutrosophic distribution are calculated for samples n=10, 20, 50, 100, and 1000. For the same sample values, we have also calculated the classical coefficient of variations, and the results are placed in Tables 1, 2, and 3 along with the neutrosophic coefficient of variations. Further, the calculated results are visualized by graph and are placed in Figures 1.1 to 17.2.

4.1 Comparison of neutrosophic coefficient of variations CV_N, RCV_{NQ}, and RCV_{NM}

In this sub-section, we compare all three measures of coefficient variations and we see, from Tables 1-3, and Figures 1.1-17.2 that the coefficient of variation RCV_{NM} is higher than both CV_N and RCV_{NQ} for all neutrosophic distributions and all sample sizes n=10, 20, 50, 100, and 1000 except the case of neutrosophic Normal distribution for n=20. Also, we observe an increasing pattern in RCV_{NM} with the increasing sample sizes for all the neutrosophic distributions and a sudden spike observed at n=50 for the neutrosophic distributions Binomial, Normal, Poison, Exponential, Gamma, Generalized Exponential, Generalized Pareto, Kumaraswamy, and for the rest it is observed at n=100. While, the coefficient of variations CV_N and RCV_{NQ} are not much affected by the increasing sample sizes, however, there is an increasing pattern for the neutrosophic Exponential, Gamma, Generalized Exponential, Geometric, and Generalized Pareto distributions (except Negative Binomial and Poison neutrosophic distribution, as there is an increasing pattern for only CV_N) and for the rest of the neutrosophic distributions there is a first increasing and then decreasing (first decreasing and then increasing) pattern (except Negative Binomial and Poison neutrosophic distribution, as it is for only RCV_{NQ}). It is also observed that RCV_{NQ} is greater than CV_N with sample sizes n=50, 100, and 1000, for all the distributions and for the rest of the sample sizes that are n=10, 20, RCV_{NQ} is less/greater than CV_N except for neutrosophic Laplace distribution for which CV_N is greater than RCV_{NQ} at all sample sizes and for neutrosophic Generalized Pareto distribution it is for sample sizes n=50, 100, 1000.

4.2 Comparison of neutrosophic coefficient of variations with the classical coefficient of variations

In this sub-section, we compare the neutrosophic coefficient of variations CV_N, RCV_{NQ}, and RCV_{NM} with the respective classical coefficient of variations CV, RCV_Q, and RCV_M, and we see from Tables 1-3, and Figures 1.1-17.2 that the coefficient of variation RCV_{NM} is higher than respective classical RCV_M for all neutrosophic distributions and all sample sizes n=10, 20, 50, 100, and 1000. In the case of CV_N, we observe that CV_N is higher than respective classical CV for the distributions Laplace, Generalized Pareto, Normal (n=10), Discrete Uniform (n=100, 1000), and for the rest of the distributions it is less than respective classical CV for all sample sizes except the distributions Exponential and Rayleigh for which both the classical and neutrosophic coefficient of variations are same that is there is no effect of indeterminacy. Also, in the case of RCV_{NQ}, we observe that RCV_{NQ} is higher than respective classical RCV_Q for the distributions Laplace, Generalized Pareto, Normal, Discrete Uniform (n=1000), Beta, Kumaraswamy, Poison, and for the rest of the distributions it is less than respective classical RCV_Q for all sample sizes except the distributions Exponential, Uniform, and Rayleigh for which both the classical RCV_Q and neutrosophic RCV_{NQ} are the same that is there is no effect of indeterminacy for these distributions.

5. Real-Data Application

To compile our study numerically, we have taken real-life indeterminate climate data of Alabama state of USA (Multi-Station data summaries for all stations in a state can be computed for any range of dates for all 50 states), recorded in May month. There are several variables, but we are taking into account "Hourly Temperature" vs "Dew Point Temperature" vs "Relative Humidity" variables only here. Along with indeterminate data, classical data is also taken as lower values of the indeterminate data. The data for the three variables is available in Table 5 of Appendix A. Also, one can visit for the data on this link: https://mrcc.purdue.edu/CLIMATE /Hourly/MultiDlyAve2.jsp. Based on the real data mentioned above, for the three variables, we have computed the neutrosophic coefficient of variations CV_N, RCV_{NQ}, and RCV_{NM} along with the classical coefficient of variations CV, RCV_Q, and RCV_M. We see from Table 4 and Figures 18.1 and 18.2, CV_N is least for the Hourly Temperature variable and maximum for Relative Humidity, RCV_{NQ} is least for Dew Point Temperature variable and maximum for Relative Humidity, similarly, RCV_{NM} is least for the Dew Point Temperature variable and maximum for Relative Humidity. Among the three methods of coefficient for all three variables, the values of CV_N are least and RCV_{NM} is maximum except for the case Relative Humidity variable for which RCV_{NQ} is least and RCV_{NM} is maximum. For the comparison of neutrosophic over classical, we see, that the neutrosophic coefficient values are lesser than their respective classical counterpart except coefficient of variation RCV_{NM}, for all three variables.

6. Conclusions

In this paper, we primarily have given neutrosophic coefficient of variation CV_N, robust neutrosophic coefficient of variation RCV_{NQ}, and robust neutrosophic coefficient of variation RCV_{NM}. Following the introduction, we have explored the methods of the neutrosophic coefficient of variation, which is an effective method for modeling data that is fuzzy, imprecise, and uncertain. For the comparative study, we have given the numerical study based on neutrosophic distributions including discrete and continuous distributions. First, we have compared all three neutrosophic coefficient of variations and then have given the comparative study for all neutrosophic distributions for these neutrosophic coefficient of variations. Also, we have given real data analysis on climate data to highlight the impact of the neutrosophic coefficient of variations. From the results and comparative study, we conclude that the neutrosophic coefficient of variations CV_N and RCV_{NQ} have near similar values or properties while the neutrosophic coefficient of variation RCV_{NM} has higher values for all samples and distributions. Further, we observe that with increasing the sample values all three neutrosophic coefficient of variations also increase for all the distributions and provide a general framework over classical methods of coefficient of variations, and the graphical representations also clarify this. The limitation of this study is that it applies only to uncertain data.

In closing, we suggest several avenues of future research as in sampling, for the estimation of neutrosophic population parameters all three neutrosophic coefficient of variation can be used, in the control chart, the study on control chart based on the neutrosophic coefficient of variation can be given, in interval estimation, a neutrosophic coefficient based interval can be studied, and so on utilizing methods neutrosophic coefficient of variation many future studies can be given apart from statistical domain too.

Table 1: Classical vs Neutrosophic CVs based on simulated data

CVs	Beta Distribution: $f_N(x)=\frac{1}{B([1,\ 3],[1,\ 3])}x_N^{[1,\ 3]-1}(1-x_N)^{[1,\ 3]-1}$									
	n=10		n=20		n=50		n=100		n=1000	
	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$
CV_N	0.571	0.473	0.664	0.533	0.638	0.512	0.571	0.467	0.540	0.457
RCV_{NQ}	0.575	0.606	0.857	0.919	0.788	0.848	0.635	0.719	0.596	0.678
RCV_{NM}	0.582	0.809	1.935	2.691	7.465	10.45	16.77	24.88	158.7	237.6
	Normal Distribution: $f_N(x)=\frac{1}{[2,\ 3]\sqrt{2\pi}}exp\left(-\frac{(x_N-[4,\ 5])^2}{2[2,\ 3]^2}\right)$									
CV_N	0.697	0.628	0.591	0.602	0.583	0.601	0.490	0.505	0.516	0.530
RCV_{NQ}	0.780	0.859	0.540	0.565	0.643	0.672	0.430	0.442	0.532	0.550
RCV_{NM}	1.092	3.584	0.064	0.246	1.424	6.485	5.150	25.33	9.340	45.25
	Binomial Distribution: $f_N(x)=\binom{20}{x_N}[0.6, 0.7]^{x_N}(1-[0.6, 0.7])^{20-x_N}$									
CV_N	0.207	0.181	0.196	0.161	0.183	0.147	0.183	0.151	0.184	0.151
RCV_{NQ}	0.172	0.169	0.195	0.188	0.172	0.165	0.188	0.185	0.188	0.186
RCV_{NM}	0.124	1.591	0.119	1.584	3.707	47.64	4.571	58.86	14.58	187.7

	Discrete Uniform Distribution: $f_N(x) = \frac{1}{[10,\ 11]}$									
CV_N	0.451	0.417	0.418	0.410	0.517	0.517	0.527	0.530	0.517	0.520
RCV_{NQ}	0.562	0.482	0.438	0.406	0.667	0.606	0.750	0.714	0.625	0.694
RCV_{NM}	0.297	1.737	1.236	8.560	7.907	42.77	10.97	64.26	107.5	752.4
	Exponential Distribution: $f_N(x) = [0.2,\ 0.3]e^{-[0.2,\ 0.3]x_N}$									
CV_N	0.641	0.641	0.692	0.692	0.848	0.848	0.943	0.943	1.027	1.0270
RCV_{NQ}	0.629	0.629	0.634	0.634	0.967	0.967	1.361	1.361	1.139	1.1390
RCV_{NM}	0.364	1.089	3.484	11.78	33.66	83.00	57.80	170.5	593.5	1912.1
	Gamma Distribution: $f_N(x) = \frac{1}{[7.7,\ 8.8]^{[1.18,\ 2.19]}\Gamma[1.18,\ 2.19]} x_N^{[1.18,\ 2.19]-1} e^{-\frac{x_N}{[7.7,\ 8.8]}}$									
CV_N	0.585	0.526	0.634	0.568	0.775	0.695	0.865	0.773	0.941	0.834
RCV_{NQ}	0.569	0.557	0.577	0.565	0.857	0.823	1.209	1.140	1.022	0.9730
RCV_{NM}	0.141	1.009	2.620	21.44	27.09	156.2	46.11	325.4	474.8	3690.6
	Generalized Exponential Distribution: $f_N(x) = \frac{[1.24, 1.34]}{[7.95,\ 8.36]}\left(1 - e^{-\frac{x_N}{[7.95,\ 8.36]}}\right)^{[1.24,1.34]-1}\left(e^{-\frac{x_N}{[7.95,\ 8.36]}}\right)$									
CV_N	0.576	0.558	0.627	0.608	0.765	0.741	0.857	0.832	0.935	0.9081
RCV_{NQ}	0.559	0.554	0.568	0.563	0.839	0.824	1.187	1.161	1.005	0.9870
RCV_{NM}	0.124	0.921	2.559	21.65	26.32	160.2	45.12	331.8	467.6	3766.6

Table 2: Classical vs Neutrosophic CVs based on simulated data

CVs	Rayleigh Distribution: $f_N(x) = \frac{x_N}{[8.64,9.87]^2}\left(e^{-\left(\frac{x_N}{[8.64,\ 9.87]}\right)^2}\right)$									
	n=10		n=20		n=50		n=100		n=1000	
	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$	I_N=0	$I_N \neq 0$
CV_N	0.483	0.483	0.484	0.484	0.458	0.458	0.448	0.448	0.528	0.528
RCV_{NQ}	0.631	0.631	0.442	0.442	0.447	0.447	0.534	0.534	0.578	0.5780
RCV_{NM}	0.140	1.767	0.731	9.218	0.800	9.610	14.31	155.8	140.8	1523.8
	Generalized Rayleigh Distribution: $f_N(x) = \frac{2[1.24, 1.34]}{[7.67,\ 8.78]} x_N \left(1 - e^{-\frac{x_N^2}{[7.67,\ 8.78]}}\right)^{[1.24,1.34]-1}\left(e^{-\frac{x_N^2}{[7.67,\ 8.78]}}\right)$									

CV_N	0.473	0.389	0.426	0.346	0.463	0.380	0.483	0.398	0.480	0.396
RCV_{NQ}	0.524	0.508	0.469	0.451	0.539	0.514	0.634	0.598	0.541	0.516
RCV_{NM}	0.484	4.234	1.330	10.35	3.960	30.06	16.55	125.6	99.14	754.3
	Geometric Distribution: $f_N(x) = [0.1, 0.2](1-[0.1, 0.2])^{x_N}$									
CV_N	0.810	0.773	0.806	0.782	0.978	0.956	1.055	1.038	1.149	1.128
RCV_{NQ}	0.900	0.853	0.812	0.803	1.125	1.085	1.500	1.457	1.250	1.224
RCV_{NM}	0.593	1.857	4.942	17.97	33.36	88.16	68.79	215.4	574.8	2090
	Kumaraswamy Distribution: $f_N(x) = ([0.24,\ 0.34][1,\ 2])x_N^{[0.24,\ 0.34]-1}\left(1-x_N^{[0.24,\ 0.34]}\right)^{[1,\ 2]-1}$									
CV_N	0.517	0.424	0.481	0.385	0.485	0.392	0.579	0.463	0.572	0.459
RCV_{NQ}	0.611	0.644	0.472	0.494	0.488	0.501	0.848	0.805	0.741	0.729
RCV_{NM}	1.758	2.784	3.233	5.120	6.610	10.12	10.17	13.57	49.45	70.32
	Laplace Distribution: $f_N(x) = \frac{1}{2[1,\ 2]}\left(e^{-\frac{\lvert x_N-[0.23,\ 0.34]\rvert}{[1,\ 2]}}\right)$									
CV_N	0.745	0.760	0.956	0.980	0.933	0.953	0.822	0.840	0.850	0.870
RCV_{NQ}	0.442	0.443	0.597	0.602	0.464	0.467	0.541	0.545	0.471	0.475
RCV_{NM}	1.558	2.587	10.84	15.76	19.80	29.56	28.58	42.57	435.2	605.1
	Negative Binomial Distribution: $f_N(x) = \binom{3+x_N-1}{x_N}[0.1, 0.2]^3(1-[0.1, 0.2])^{x_N}$									
CV_N	0.404	0.396	0.439	0.433	0.545	0.535	0.609	0.598	0.660	0.6480
RCV_{NQ}	0.411	0.406	0.375	0.375	0.583	0.574	0.825	0.807	0.682	0.6800
RCV_{NM}	0.424	4.721	0.645	7.829	10.38	100.1	21.20	225.5	170.2	1981.6
	Generalized Pareto Distribution: $f_N(x) = \frac{1}{[2.67,\ 2.88]}\left(1+\frac{[1.19,\ 1.29]}{[2.67,\ 2.88]}x_N\right)^{\frac{1}{[1.19,\ 1.29]}-1}$									
CV_N	0.845	0.864	1.209	1.255	2.004	2.115	2.257	2.380	18.314	18.686
RCV_{NQ}	0.776	0.783	1.113	1.131	2.091	2.161	1.415	1.446	2.3910	2.4900
RCV_{NM}	5.822	14.98	15.93	46.06	155.1	468.3	294.2	917.5	111440	421002

Table 3: Classical vs Neutrosophic CVs based on simulated data

CVs	Uniform Distribution: $f_N(x) = \frac{1}{[0,\ 2]-[4,\ 8]}$									
	n=10		n=20		n=50		n=100		n=1000	
	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0
CV_N	0.543	0.457	0.502	0.423	0.506	0.424	0.514	0.430	0.577	0.481
RCV_{NQ}	0.682	0.661	0.532	0.520	0.549	0.534	0.683	0.654	0.759	0.727
RCV_{NM}	1.220	3.711	2.261	6.873	3.997	11.53	9.129	23.50	50.41	128.9
	Weibull Distribution: $f_N(x) = \frac{[1.05,\ 2.05]\, x_N{}^{[1.05,\ 2.05]-1}}{[8.34,\ 9.45]^{[1.05,2.05]}}\left(e^{-\left(\frac{x_N}{[8.34,\ 9.45]}\right)^{[1.05,\ 2.05]}}\right)$									
CV_N	0.132	0.126	0.122	0.116	0.128	0.123	0.136	0.130	0.142	0.136
RCV_{NQ}	0.152	0.152	0.111	0.111	0.108	0.108	0.154	0.153	0.138	0.138
RCV_{NM}	0.335	0.574	0.483	0.828	1.519	2.582	2.096	3.503	26.39	44.48
	Poison Distribution: $f_N(x) = \frac{e^{-[2,\ 3]}[2,\ 3]^{x_N}}{x_N!}$									
CV_N	0.598	0.471	0.576	0.481	0.651	0.574	0.635	0.567	0.679	0.609
RCV_{NQ}	0.525	0.559	0.375	0.402	0.750	0.700	0.750	0.703	0.750	0.703
RCV_{NM}	1.186	3.888	1.186	3.982	4.448	11.86	22.24	61.16	43.74	120.3

Table 4: Classical vs Neutrosophic CVs based on real data

CVs	Hourly Temp.		Dew Point Temp.		Relative Humidity	
	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0	I_N=0	$I_N \neq$0
CV_N	0.084	0.034	0.110	0.036	0.214	0.183
RCV_{NQ}	0.108	0.105	0.072	0.070	0.154	0.150
RCV_{NM}	0.321	18.02	0.315	15.06	1.167	36.27

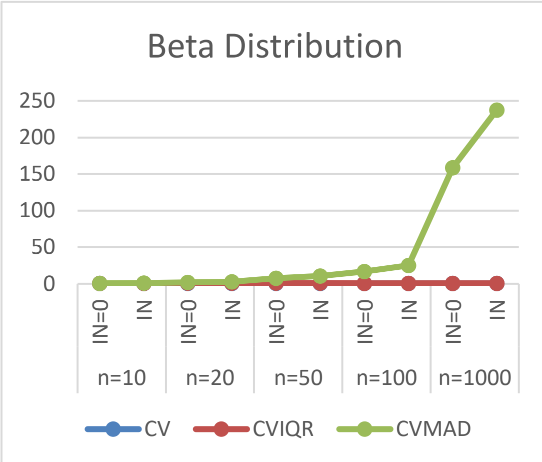


Figure 1.1

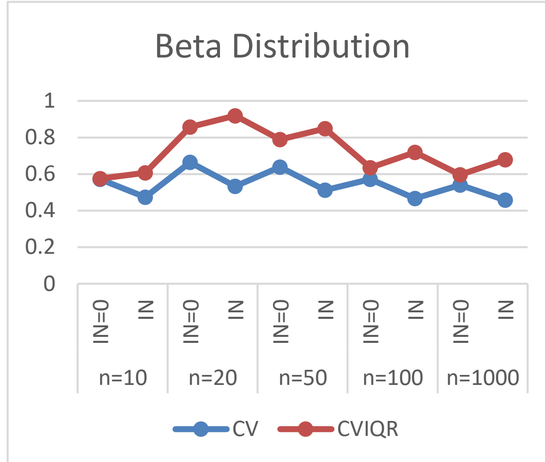


Figure 1.2

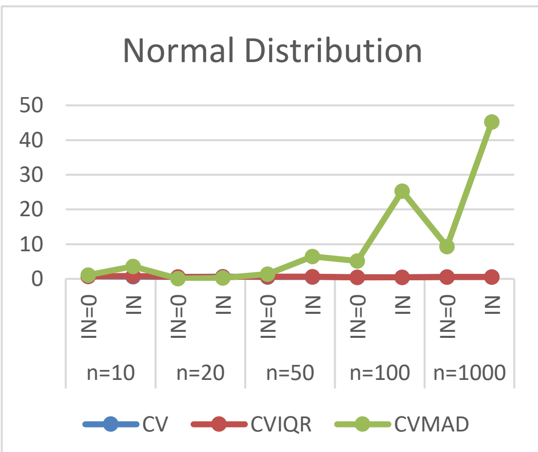


Figure 2.1

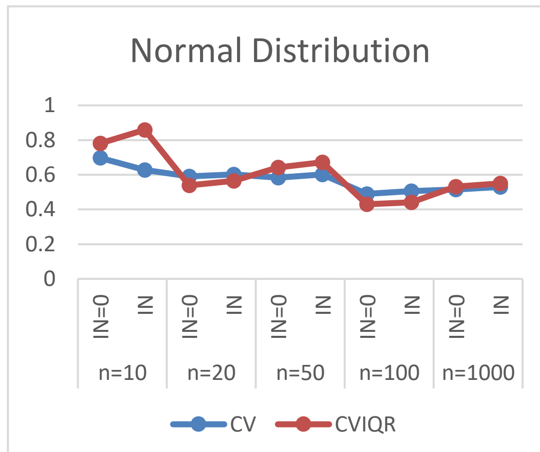


Figure 2.2

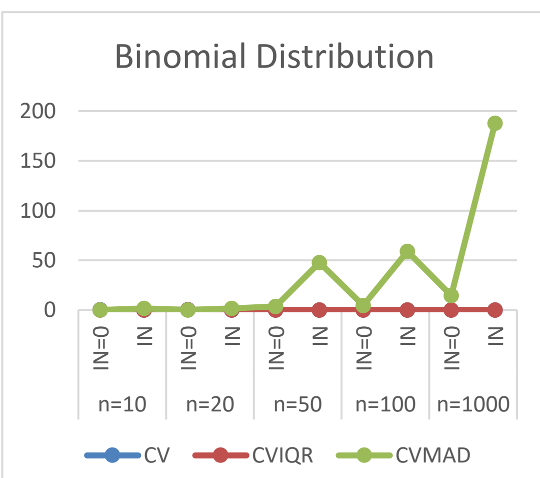


Figure 3.1

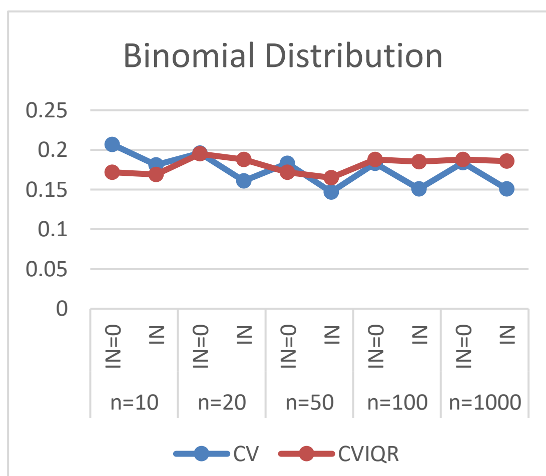
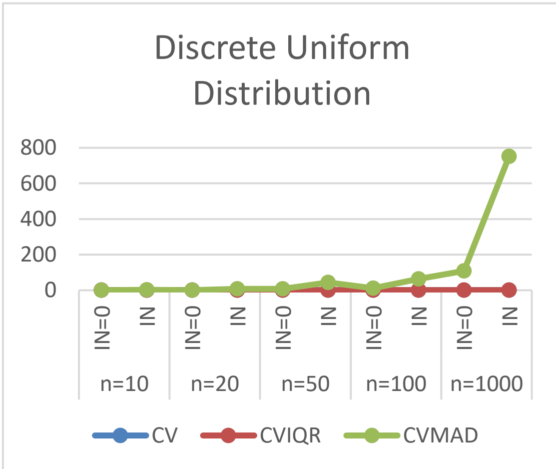


Figure 3.2



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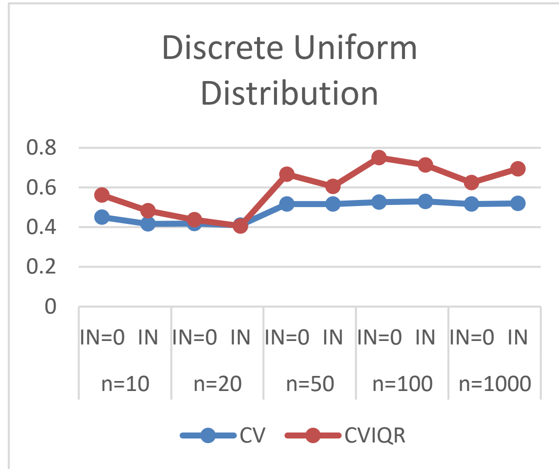


Figure 4.2

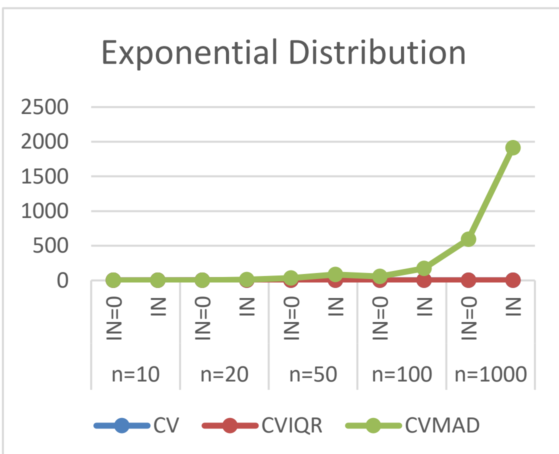


Figure 5.1

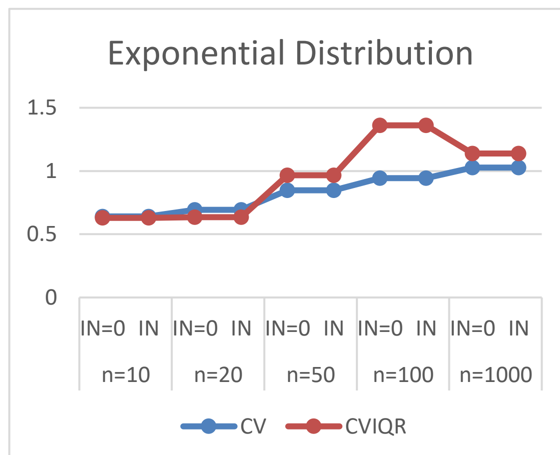


Figure 5.2

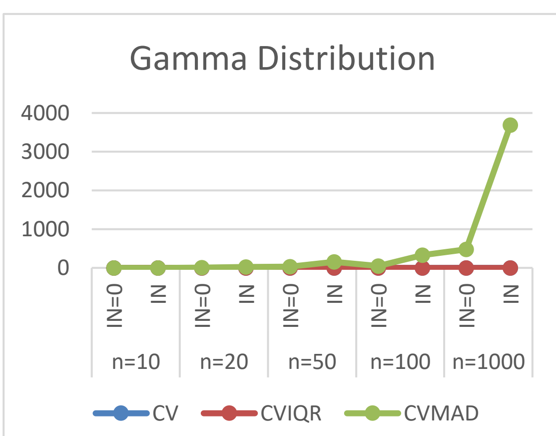
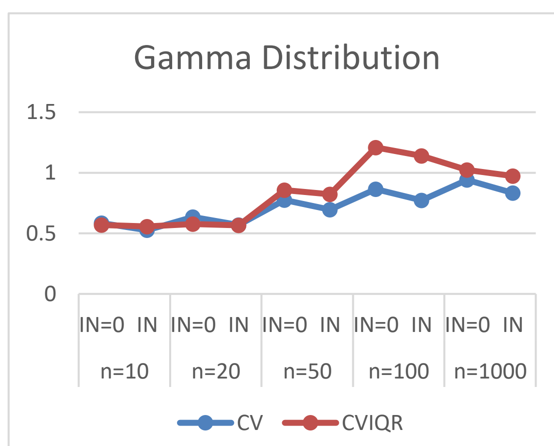


Figure 6.1



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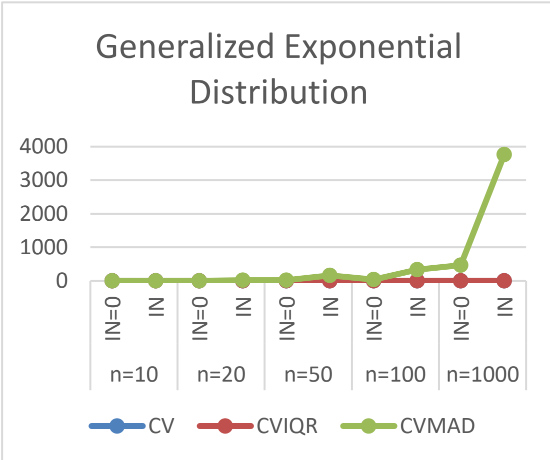

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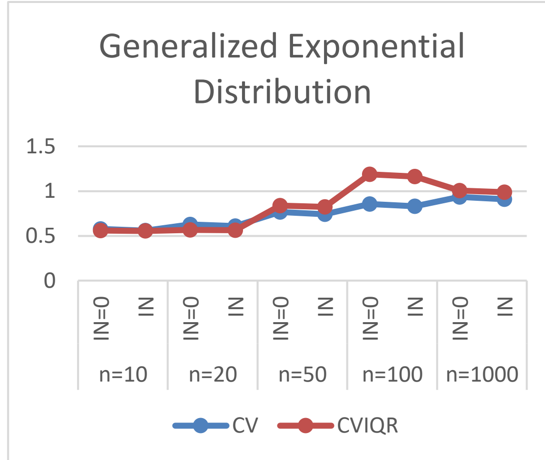


Figure 7.2

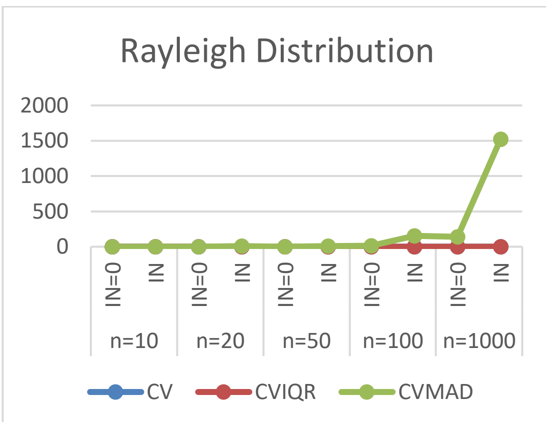


Figure 8.1

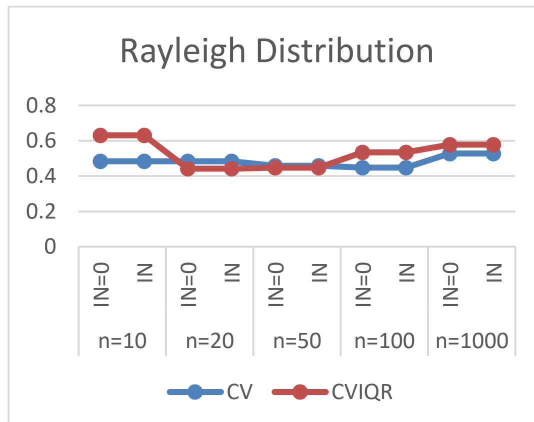
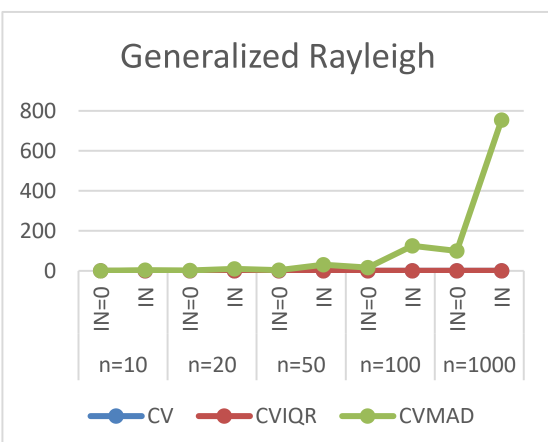


Figure 8.2



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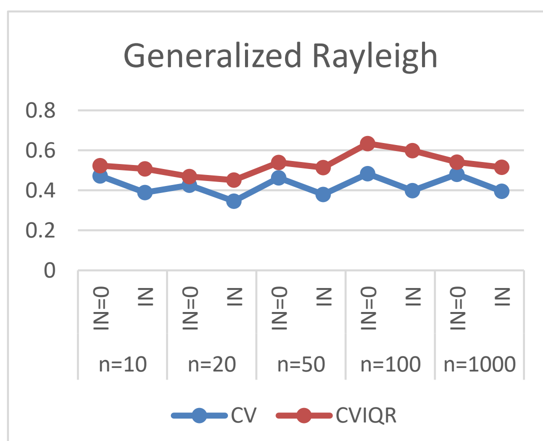


Figure 9.2

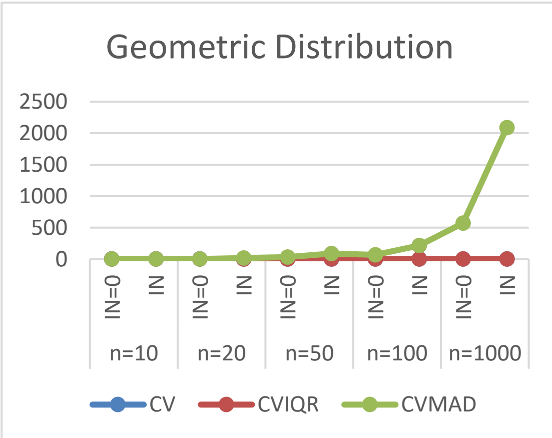


Figure 10.1

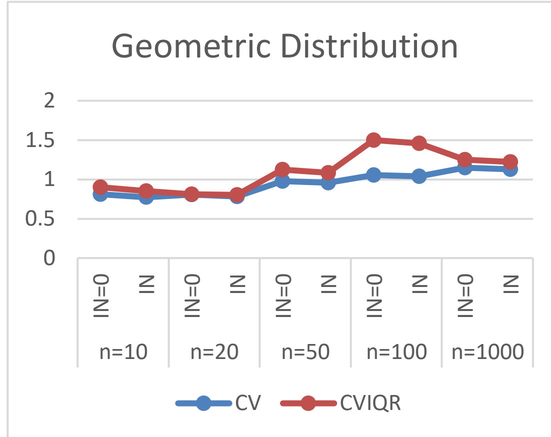


Figure 10.2

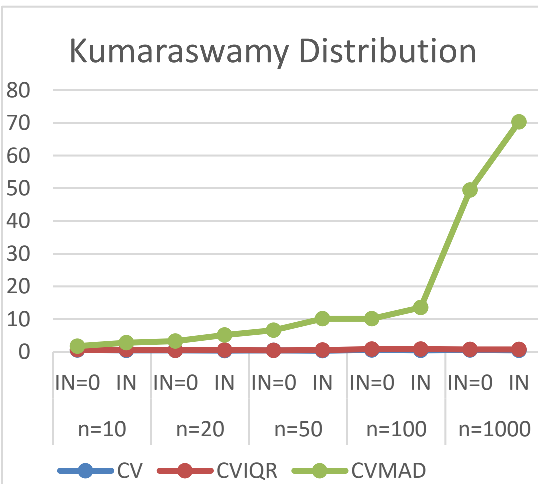


Figure 11.1

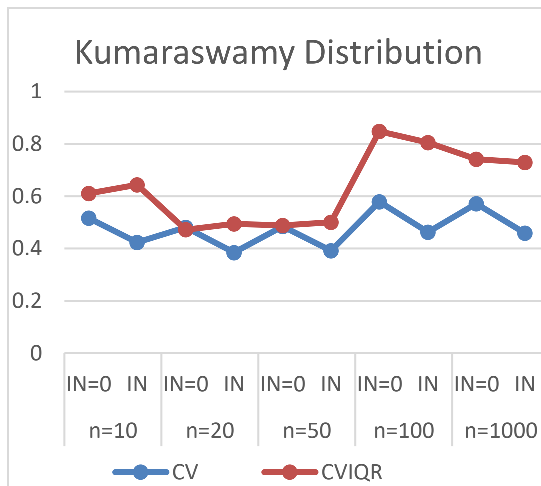


Figure 11.2

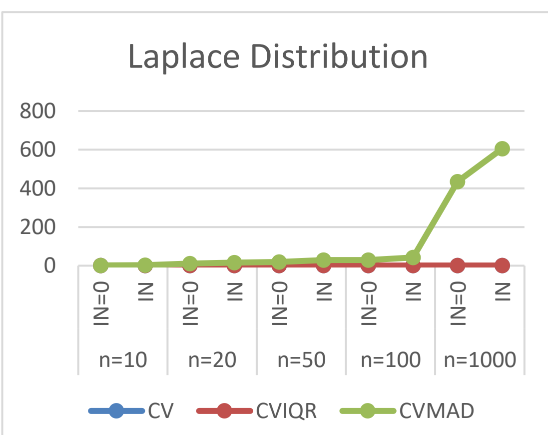


Figure 12.1

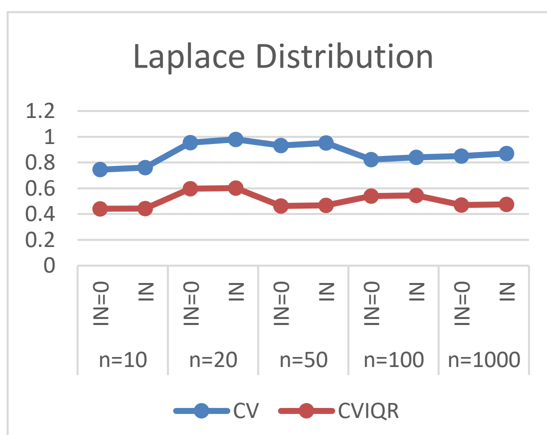


Figure 12.2

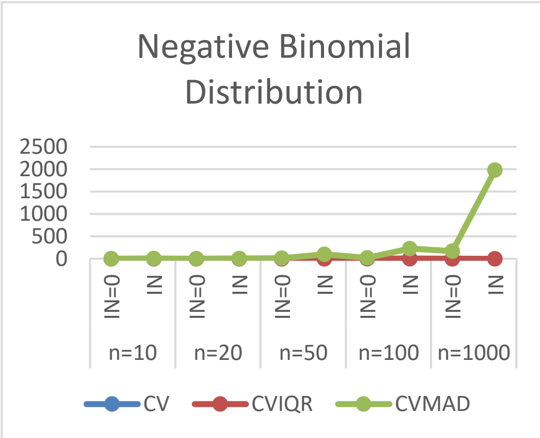


Figure 13.1

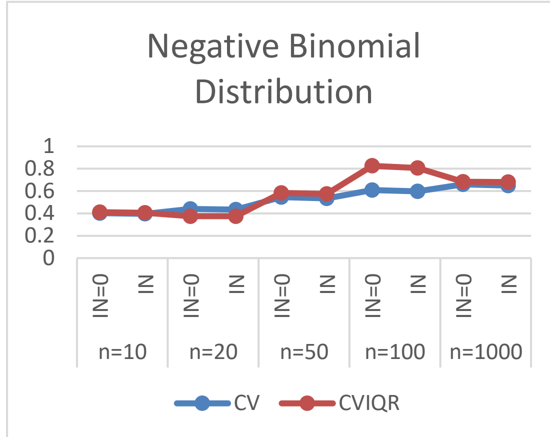


Figure 13.2

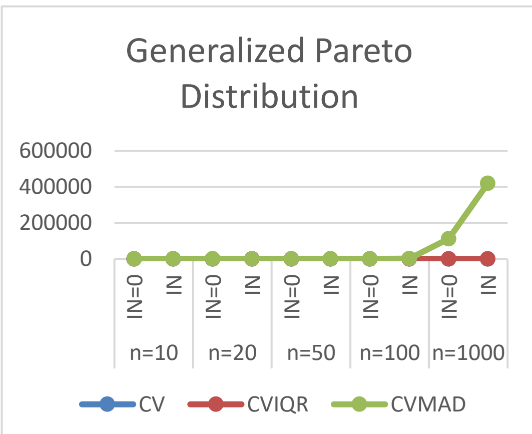
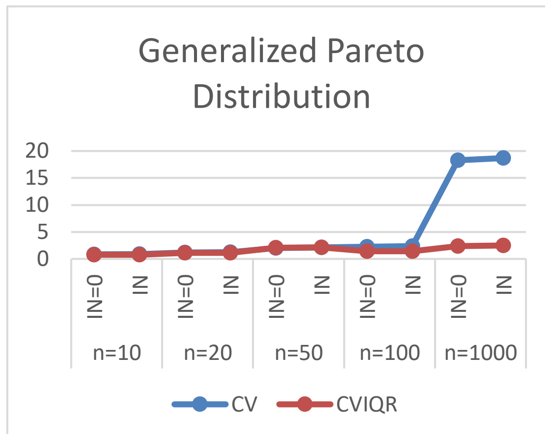


Figure 14.1



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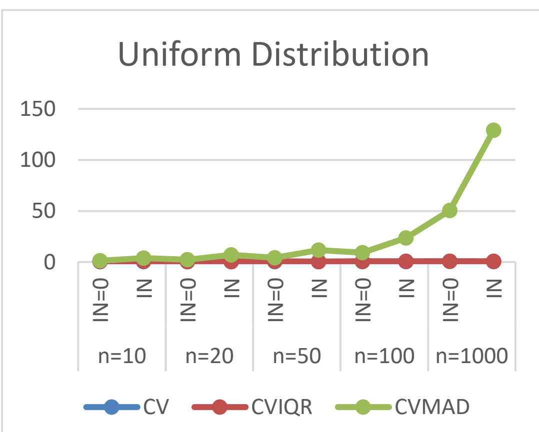

Figure 15.1

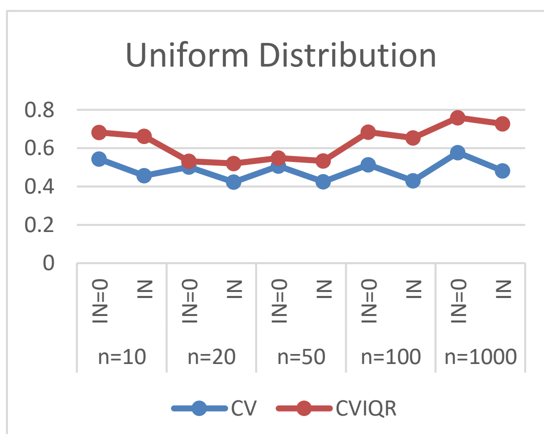

Figure 15.2

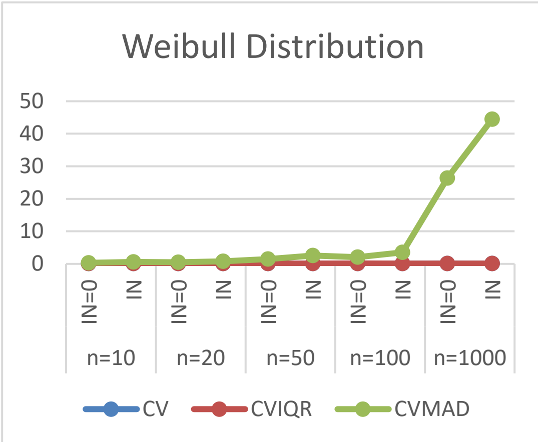

Figure 16.1

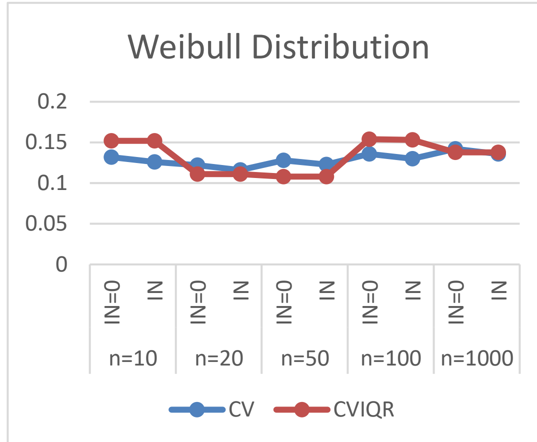

Figure 16.2

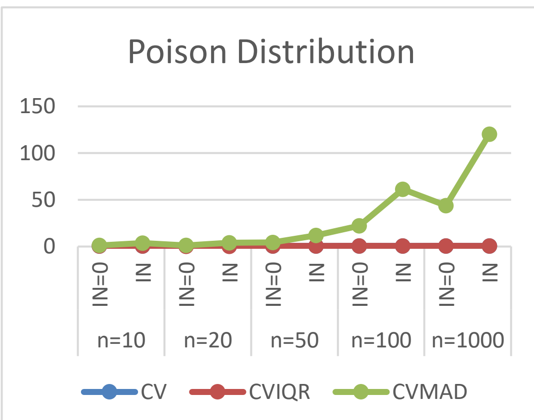

Figure 17.1

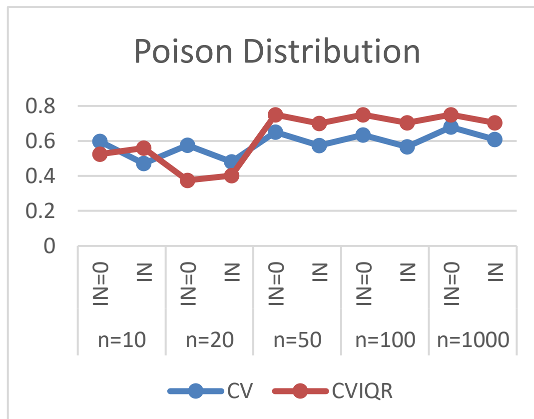

Figure 17.2

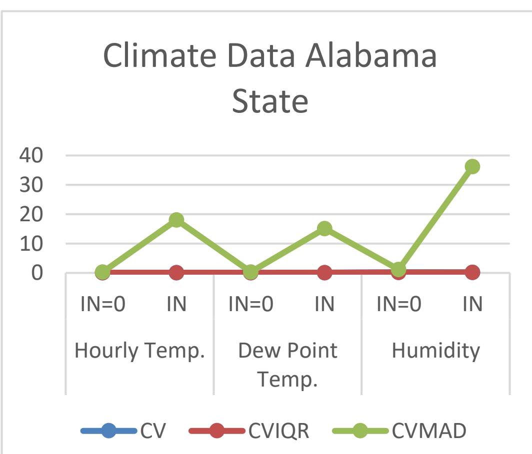

Figure 18.1

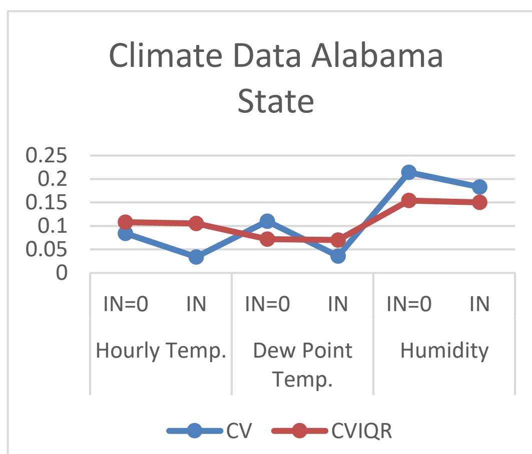

Figure 18.2

Appendix A

Table 5: Climate data of Alabama state, May 2024

Hourly Temp.		Dew Point Temp.		Relative Humidity	
Min	Max	Min	Max	Min	Max
52	88	46	73	33	100
65	93	47	76	28	94
52	90	45	72	30	94
50	89	44	74	28	100
61	93	52	76	26	100
56	90	34	75	19	100
58	92	49	76	35	100
57	91	47	77	29	100
64	94	52	77	35	100
53	89	47	74	30	100
61	81	61	63	54	100
57	93	46	79	31	100
54	92	47	75	29	100
54	90	48	76	31	100
62	90	53	76	38	97
49	83	43	71	36	99
55	91	50	77	33	97
51	90	45	74	28	100

Data availability: The author confirms that the data backing the findings of this study are included in this manuscript and are available in Appendix A.

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Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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