Int. J. Anal. Appl. (2025), 23:222

International Journal of Analysis and Applications

Mastering Complexity: Fuzzy Logic-Driven Optimization for Multi-Objective Transport Solutions Using LINGO Software

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Abstract. This work introduces a new method for transportation optimisation decision-making that utilizes LINGO software and fuzzy logic-powered optimisation. The main aim is to minimize variance in accounting for expenses. A sophisticated three-stage stratified random sampling procedure supported by randomised response mechanisms is utilized to achieve this. It primarily contributes a framework through which policymakers can make significant enhancement in the method of collecting data, especially for such cases in which privacy among respondents is very critical. It addresses challenges that face data collection involving sensitive issues and remains within data economy as well as integrity by bringing in fuzzy logic seamlessly in cooperation with randomized response technique.

1. Introduction

A staple of modern research, statistical inference provides flexible methods tailored to the specific needs of many investigations and contexts. Among the most popular is the combination of probabilistic frameworks with analytical and enumeration-based inference. This has been the basis of the modern approaches to statistics and is essentially required for reliable conclusions from empirical data. Although enumeration inference and analytical inference share the same goals, their assumptions and probability structures are quite different, and hence there is a need for creative

Received: Mar. 6, 2025.

2020 *Mathematics Subject Classification.* 94D05, 62D05.

Key words and phrases. multi index transportation problem; linear programming; optimization algorithms; stratified random sample; sensitive attribute; decision-making; fuzzy logic.

https://doi.org/10.28924/2291-8639-23-2025-222

ISSN: 2291-8639

and flexible statistical modeling approaches. The development of statistical techniques, especially in the area of sampling, underwent a sea change in the years following World War II. Research activity exploded during this time, revolutionizing data collection methods. Sampling techniques became revolutionary tools with which it was possible to collect information from subsets of larger populations in an efficient manner. That development greatly advanced the profession by enhancing statistical resolution and redefining the ways in which analyses and interpretations were carried out. In particular, surveys have been of crucial importance in this evolution. They are now indispensable to collect data in a scientific, systematic way with supplementary sources like census data, previous surveys, and pilot projects. These add-on resources provide sound insight into the issues being studied and thus make the survey more accurate and informative. This randomised response technique introduced by Warner is a breakthrough in the development of survey methodology. By protecting the respondent's anonymity and still obtaining the right data, the innovative method resolves the problem of collecting data regarding sensitive characteristics. This interest has been raised because of its capability to protect data integrity as researchers can estimate the percentage of a population with sensitive features. Major works like those in [2–9] and [10-14] expand on Warner's groundbreaking work by delving deeper into the implications of RR approaches for sampling and statistical inference. The above contributions establish RR as an indispensable field of study by demonstrating the transformative impact for managing sensitive data and considering its broad applicability to modern statistical research in pursuit of more ethical and trustworthy data collection methods.

2. Problem Formulation

Electrical systems have become much larger and more efficient over time with the help of optimisation techniques. These techniques enhance the performance of both linear and nonlinear systems by providing flexibility and real-time optimisation. Optimisation processes, which modify system parameters and identify ideal values for maximising or minimising results, provide a comprehensive method for system improvement. A single design for an electrical system to be declared the best choice is still too premature for the ever-changing technical landscape of today. Opportunities are constantly being reshaped by continued technical developments, making the traditional approaches outdated and providing new avenues for exploration.

In this case, LINGO software was used to overcome the challenge of rounding continuous data to integers. The problem is reformulated into a better and more useful electrical system design technique by rephrasing it as a fuzzy integer nonlinear programming model. Besides the solution to the immediate problem, this innovative approach showed how fuzzy nonlinear programming can tackle difficult optimisation problems.

This strategy's effectiveness highlights the importance of interdisciplinary collaboration in solving difficult problems at the nexus of technology and optimization. By combining ideas from electrical engineering, fuzzy logic, and optimisation theory, researchers developed new solutions that extended beyond conventional academic boundaries. This type of cooperation enhances problem-solving techniques and further expands our understanding of the underlying systems and processes.

Multidisciplinary collaboration should be fostered in order to make up with innovative ideas that drive the progress of innovation, and particularly in a society where new problems are constantly being brought about by technology. Science can push the boundaries that might have been thought to restrict them by embracing a variety of viewpoints and areas of knowledge related to contemporary electrical system designs.

A finite population of size N, say U, is split into L strata of size N_h ($h = 1, 2, 3,$
$dots, L$). For each stratum, use SRSWR to randomly sample n_h responders. Assume that each stratum's size,
pi_{sh2}, is known. The following is the three-stage randomisation device provided to the n_h responders from the hth stratum:

First-Stage Randomization Device.

Statements	Selection Probability
Statement 1: Do you belong to the uncommonly sensitive A_1 group?	α_h
Statement 2: Go to randomization device R_{2h}	$(1-\alpha_h)$

Second-Stage Randomization Device R_{2h}.

Statements	Selection Probability
Statement 1: Do you belong to the uncommonly sensitive A_1 group?	p_h
Statement 2: Go to randomization device R_{3h}	$(1-p_h)$

Three statements were used by the randomisation device R_3h. The number of cards the respondent selected from the first and second decks to get the cards that stated his or her personal status is denoted by X_h and Y_h. If ϕ_{hi} is the i^{th} respondent in the h^{th} stratum, then ϕ_{hi} could be written as:

$$\phi_h = \alpha_h \pi_{sh1} + (1-\alpha_h)\left[P_h \pi_{sh1} + (1-P_h)(Q_{1h}\pi_{sh1} + Q_{2h}\pi_{sh2})\right], \tag{1}$$

$$\hat{\pi}_{sh1} = \frac{\hat{\phi}_h - Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)}{\alpha_h + (1-\alpha_h)P_h + (1-\alpha_h)(1-P_h)Q_{1h}}, \tag{2}$$

$$V(\pi_{sh1}) = \frac{\pi_{sh1}(1-\pi_{sh1})}{n_h} + \frac{\left[Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)\left(1 - Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)\right)\right]}{n_h\left[\alpha_h + (1-\alpha_h)P_h + (1-\alpha_h)(1-P_h)Q_{1h}\right]^2}. \tag{3}$$

and

$$\hat{\pi}_s = \sum_{h=1}^{L} W_h \hat{\pi}_{sh1} \tag{4}$$

Using (2), we obtain:

$$\hat{\pi}_s = \sum_{h=1}^{L} W_h \left[\frac{\hat{\phi}_{hi} - Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)}{\alpha_h + (1-\alpha_h)P_h + (1-\alpha_h)(1-P_h)Q_{1h}}\right] \tag{5}$$

Sampling variance is given by:

$$V(\hat{\pi}_s) = \sum_{h=1}^{L} W_h^2 V(\hat{\pi}_{sh1}) \tag{6}$$

Or equivalently:

$$V(\hat{\pi}_s) = \sum_{h=1}^{L} W_h^2 \Bigg[\frac{\pi_{sh1}(1-\pi_{sh1})}{n_h} + \frac{Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)\left(1-Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)\right)}{n_h\left[\alpha_h+(1-\alpha_h)P_h+(1-\alpha_h)(1-P_h)Q_{1h}\right]^2} \Bigg]. \tag{7}$$

where,

$\beta_h = \left[\frac{Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)(1-Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h))}{\left[\alpha_h+(1-\alpha_h)P_h+(1-\alpha_h)(1-P_h)Q_{1h}\right]^2} \right]$

with variance

$$V(\hat{\pi}_s) = \sum_{h=1}^{L} \frac{W_h^2}{n_h}\beta_h$$

having cost function

$$= c_0 + \sum_{h=1}^{k} c_h n_h \tag{8}$$

Whereas c_0 represents the survey's available fixed budget and thus overhead cost. This is how the problem formulation for fixed-cost nonlinear programming is:

$$\text{Minimize} V(\hat{\pi}_s) = \sum_{h=1}^{k} \frac{w_h^2}{n_h}\beta_h$$

subject to

$$\sum_{h=1}^{k} n_h c_h \leq c_0,$$

and

$$1 \leq n_h \leq N_h, \quad and n_h integers h = \{1,2,\ldots,k\},$$

The limitations $1 \leq n_h$ and $n_h \leq N_h$ are put in place, respectively.

3. Fuzzy Formulation

As an adverse aspect, this difficulty motivated one crucial field called Privacy-Preserving Data Mining. A significant strategy used towards this is randomisation whereby such information is supposed to disguise all the sensitive information hidden inside it before its access from analysts to data. A kind of fuzzy number utilized herein under the context of its own existence is Triangular Fuzzy Number, or TFN, shortly.

$$\text{Minimize} \sum_{h=1}^{k} \frac{w_h^2}{n_h}\beta_h$$

subject to

$$\sum_{h=1}^{k}(c_h^1, c_h^2, c_h^3)n_h \leq (c_0^1, c_0^2, c_0^3)$$

and

$$1 \leq n_h \leq N_h,$$

and $n_h integers h = \{1, 2, \ldots, k\}$, where β_h=$\left[\frac{Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)(1-Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h))}{[\alpha_h+(1-\alpha_h)P_h+(1-\alpha_h)(1-P_h)Q_{1h}]^2}\right]$

and $\tilde{c_h} = (c_h^1, c_h^2, c_h^3)$ is triangular fuzzy numbers with membership function:

$$\mu_{\tilde{c}_{h_i}}(x) = \begin{cases} \frac{x-c_h^1}{c_h^2-c_h^1}, & \text{if } c_h^1 \leq x \leq c_h^2, \\ \frac{c_i^3-x}{c_h^3-c_h^2}, & \text{if } c_h^2 \leq x \leq c_h^3, \\ 0, & \text{else.} \end{cases}$$

It explains, in detail, the membership function corresponding to the budget available:

$$\mu_{\tilde{c}_0}(x) = \begin{cases} \frac{x-c_0^1}{c_h^2-c_0^1}, & \text{if } c_0^1 \leq x \leq c_0^2, \\ \frac{c_0^3-x}{c_0^3-c_0^2}, & \text{if } c_0^2 \leq x \leq c_0^3, \\ 0, & \text{else.} \end{cases}$$

Moreover, we discuss trapezoidal fuzzy numbers (TrFN).

$$\text{Minimize}\sum_{h=1}^{k}\frac{w_h^2}{n_h}\beta_h$$

subject to

$$\sum_{h=1}^{k}(c_h^1, c_h^2, c_h^3, c_h^4)n_h \leq (c_0^1, c_0^2, c_0^3, c_0^4)$$

and

$$1 \leq n_h \leq N_h$$

, and $n_h integers h = \{1, 2, \ldots, k\}$

where

$$\beta_h = \left[\frac{Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h)\,(1-Q_{2h}\pi_{sh2}(1-P_h)(1-\alpha_h))}{[\alpha_h+(1-\alpha_h)P_h+(1-\alpha_h)(1-P_h)Q_{1h}]^2}\right]$$

and $\tilde{c}_i = (c_h^1, c_h^2, c_h^3, c_h^4)$ is trapezoidal fuzzy numbers with membership function:

$$\mu_{\tilde{c}_i}(x) = \begin{cases} 0, & \text{if } x \leq c_h^1, \\ \frac{x-c_h^1}{c_h^2-c_h^1}, & \text{if } c_h^1 \leq x \leq c_h^2, \\ 1, & \text{if } c_h^2 \leq x \leq c_h^3, \\ \frac{c_h^4-x}{c_h^4-c_h^3}, & \text{if } c_h^3 \leq x \leq c_h^4, \\ 0, & \text{if } c_h^4 \leq x. \end{cases}$$

With this, we explore the fuzzy formulation further to develop applicable methodologies for privacy-preserving data mining.

$$\mu_{\tilde{c_0}}(x) = \begin{cases} 0, & \text{if } x \leq c_0^1, \\ \frac{x-c_0^1}{c_0^2-c_0^1}, & \text{if } c_0^1 \leq x \leq c_0^2, \\ 1, & \text{if } c_0^2 \leq x \leq c_0^3, \\ \frac{c_0^4-x}{c_0^4-c_0^3}, & \text{if } c_0^3 \leq x \leq c_0^4, \\ 0, & \text{if } c_0^4 \leq x. \end{cases}$$

4. Lagrange Multipliers Formulation

The Lagrangian function is

$$\phi(n_h, \lambda) = \sum_{i=1}^{k} \frac{w_i^2}{n_i}\beta_{hi} + \lambda\left(\sum_{i=1}^{k}(c_i^{(1)}, c_i^{(2)}, c_i^{(3)})n_i - (c_0^{(1)}, c_0^{(2)}, c_0^{(3)})\right)$$

with

$$\frac{\partial\phi}{\partial n_i} = 0 \implies n_i = -\frac{w_i^2}{n_h^2}\beta_i + \lambda(c_i^{(1)}, c_i^{(2)}, c_i^{(3)})$$

$$n_i = \frac{1}{\sqrt{\lambda}}\frac{\sqrt{\beta_i}}{\sqrt{\left(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}\right)}}$$

also

$$\frac{\partial\phi}{\partial\lambda} = \left(\sum_{i=1}^{k}(c_i^{(1)}, c_i^{(2)}, c_i^{(3)})n_i - (c_0^{(1)}, c_0^{(2)}, c_0^{(3)})\right) = 0$$

This gives

$$\sum_{i=1}^{k} w_i\left(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}\right)\sqrt{\frac{\beta_i}{\lambda\left(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}\right)}} - \left(c_0^{(1)}, c_0^{(2)}, c_0^{(3)}\right) = 0$$

or

$$\frac{1}{\sqrt{\lambda}} = \frac{(c_0^{(1)}, c_0^{(2)}, c_0^{(3)})}{\sum_{i=1}^{k} w_i\sqrt{\beta_i(c_i^{(1)}, c_i^{(2)}, c_i^{(3)})}}$$

and

$$n_i^* = \frac{(c_0^{(1)}, c_0^{(2)}, c_0^{(3)})w_i\sqrt{\frac{\beta_i}{(c_i^{(1)}, c_i^{(2)}, c_i^{(3)})}}}{\sum_{i=1}^{k} w_i\sqrt{\beta_i(c_i^(1), c_i^(2), c_(3))}}$$

For the trapezoidal fuzzy number case, the optimal allocation is

$$n_i^* = \frac{(c_0^{(1)}, c_0^{(2)}, c_0^{(3)}, c_0^{(4)})w_i\sqrt{\frac{\beta_i}{(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}, c_i^{(4)})}}}{\sum_{i=1}^{k} w_i\sqrt{\beta_i(c_i^{(1)}, c_i^{(2)}, c_i^{(3)}, c_i^{(4)})}}$$

5. Conversion of Fuzzy Numbers

The equivalent crisp allocation is

$$n_i^* = \frac{(c_0^{(3)} - (c_0^{(3)} - c_0^{(2)}))w_i \sqrt{\frac{\beta_i}{(c_h^(1) + (c_h^(2) - c_h^(1))}}}{\sum_{i=1}^k w_i \sqrt{\beta_i(c_h^{(1)} + (c_h^{(2)} - c_h^{(1)}))}}$$

For equal disbursement, divide the total sample size equally among strata, resulting in

$$n_i^* = \frac{(c_0^{(4)} - (c_0^{(4)} - c_0^{(3)}))w_i \sqrt{\frac{\beta_i}{(c_h^(1) + (c_h^(2) - c_h^(1)))}}}{\sum_{i=1}^k w_i \sqrt{\beta_i(c_h^{(1)} + (c_h^{(2)} - c_h^{(1)}))}}$$

with

$$n_i = \frac{n}{k}$$

where n is computed as

$$\sum_{i=1}^k w_i \sqrt{\beta_i\left(c_h^{(1)} + (c_h^{(2)} - c_h^{(1)})\right) n_i} = \left(c_0^{(4)} - (c_0^{(4)} - c_0^{(3)})\right)$$

$$n_i \propto w_i$$

$$n_i = n w_i$$

and for n_i

$$n_i = \frac{N\left(c_0^{(4)} - (c_0^{(4)} - c_0^{(3)})\right)}{\sum_{i=1}^k w_i \sqrt{(c_h^{(1)} + (c_h^{(2)} - c_h^{(1)}))N_h}}$$

having

$$n_i = n\frac{N_i}{N}$$

6. Proposed Algorithm

Step 1: Formulate a FTP
Step 2: The multi-objective transportation problem (MOTP) is solved by solving h-times for each objective. For each solution, the relevant values are obtained using a matrix form:

$$\begin{matrix} X_1 \\ X_2 \\ \vdots \\ X_i \end{matrix} \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1k} \\ Z_{21} & Z_{22} & \cdots & Z_{2k} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{h1} & Z_{h2} & \cdots & Z_{hk} \end{bmatrix},$$

For each X_i $(i = 1, 2, \ldots, h)$, it is the isolated optimal solution to the h-separate transportation problems with h-separate objective functions $Z_{ij} = Z_j(X_i)$. Since it is the i-th objective function at the j-th solution, the expression for Z_{ij} is obtained by the i-th row and j-th column elements.

Step 3: The membership function defines the range of values for Step 2, which is to establish how acceptable or unacceptable the solution is. Upper and lower boundaries are set up for each objective as follows:

$$U_h^{\mu} = \max(Z_t(X_t)), \quad L_h^{\mu} = \min(Z_t(X_t)), \quad 0 \leq t \leq h.$$

where U_h^{μ} and L_h^{μ} are respectively the upper and lower bound for the (h^{th} objective function Z_h) $h = 1, 2, ..., k$

Step 4: The membership function is defined as:

$$\mu_{h_i}(Z(X)_h) = \begin{cases} 1, & Z_h(X) \leq L_h^{\mu}, \\ 1 - \frac{Z_h(X) - L_h^{\mu}}{d_h}, & L_h^{\mu} \leq Z_h(X) \leq U_h^{\mu}, \\ 0, & Z_h(X) \geq U_h^{\mu}, \end{cases}$$

where $d_h = U_h^{\mu} - L_h^{\mu}$.

Step 5: An intuitionistic fuzzy optimization for MOLP is defined as:

$$\tilde{\mu}_{h_i}(Z(X)_h) = \begin{cases} 1, & Z_h(X) \leq L_h^{\mu}, \\ e^{-\frac{1}{2}(1 - \frac{Z_h(X) - L_h^{\mu}}{d_h})}, & L_h^{\mu} \leq Z_h(X) \leq U_h^{\mu}, \\ 0, & Z_h(X) \geq U_h^{\mu}, \end{cases}$$

where $d_h = U_h^{\mu} - L_h^{\mu}$.

7. Numerical Illustration

The necessary FNLLP is calculated using population size of 1000 with total survey budgets of 3500, 4000, and 4800 units for TFNs, and 3500, 4000, 4400, and 4600 units for TrFNs:

$$\text{Minimize} V(\hat{\pi}_S) = \frac{0.0224937869}{n_1} + \frac{0.1236160385}{n_2}$$

subject to

$$(1, 2, 4)n_1 + (18, 20, 24)n_2 \leq (3500, 4000, 4800)$$

$$1 \leq n_1 \leq 300$$

$$1 \leq n_2 \leq 700$$

The optimal allocations are obtained using the values from Tables 1 and 2 at $\alpha = 0.5$:

$$n_1 = \frac{(4800 - 800\alpha) \cdot 0.3 \sqrt{\frac{0.4(1-0.4)+0.005}{\alpha+1}}}{0.3\sqrt{(0.24 + 0.005)(\alpha + 1)} + 0.7\sqrt{(0.24 + 0.017)(2\alpha + 180)}}$$

$$n_2 = \frac{(4800 - 800\alpha) \cdot 0.7 \sqrt{\frac{(0.24+0.017)}{2\alpha+18}}}{0.3\sqrt{(0.24 + 0.005)(\alpha + 1)} + 0.7\sqrt{(0.24 + 0.17)(2\alpha + 18)}}$$

$n_1 = 308.5986; n_2 = 211.858$

Similarly, the optimal allocation problem will be resolved by inserting values from Tables 1–3 at a threshold of 0.50.

$$n_1 = \frac{(4400-200\alpha)\cdot 0.3\sqrt{\frac{0.4(1-0.4)+0.005}{\alpha+1}}}{0.3\sqrt{(0.24+0.005)(\alpha+1)}+0.7\sqrt{(0.24+0.017)(2\alpha+180)}}$$

$$n_2 = \frac{(4400-200\alpha)\cdot 0.7\sqrt{\frac{(0.24+0.17)}{(2\alpha+18)}}}{0.3\sqrt{(0.24+0.005)(\alpha+1)}+0.7\sqrt{(0.24+0.017)(2\alpha+18)}}$$

$n_1 = 301.585; n_2 = 207.043$

Now, Z_1 and Z_2 for $X(2), X(1)$ can be written in the form of a matrix as given below:

$$\begin{bmatrix} Z_1(X_1) & Z_2(X_1) \\ Z_1(X_2) & Z_2(X_2) \end{bmatrix} = \begin{bmatrix} 308 & 207 \\ 301 & 211 \end{bmatrix}.$$

From the above, we have:

$$U_1^{\mu} = 308, \quad U_2^{\mu} = 301, \quad L_1^{\mu} = 207, \quad L_2^{\mu} = 211.$$

Table 1. Two different strata in a stratified population.

Stratum	(c_h^1, c_h^2, c_h^3)	(c_0^1, c_0^2, c_0^3)
1	$(1,2,4)$	$(1,2,4,7)$
2	$(18,20,24)$	$(18,20,24,26)$

Table 2. Calculated values of β_i

Stratum	β_i
1	0.0224937869
2	0.1236160385

Table 3. Optimum allocation and variance values.

Case	Variance
TFN	0.000000002734
TrFN	0.000000004199

8. Conclusion

This study presents a significant achievement in creating and testing a three-stage randomised response model carefully designed to address an important problem of both theoretical and practical interest. The characteristics and recommendations of the model received much attention, which proved that this model could solve complex problems in statistical sampling.

The paper aimed to investigate and assess critically the methods taken inside the model in solving an optimal allocation problem in a three-stage stratified random sampling framework. The study used a robust solution strategy based on fuzzy nonlinear programming techniques by integrating fuzzy cost considerations. Although the problem is rather complex, the application of Lagrange multipliers provided an error-free way to find the optimal distribution. The repeated outcome always proves the validity and reliability of the model and shows how it has more strength than the recently established estimates. The results support the proposed approach and illustrate how it may enhance three-stage randomized response models' decision-making. The paper highlights the importance of new methods in advancing statistical research, making them a competitive substitute for traditional approaches.

Acknowledgements: The authors acknowledge the Deanship of Graduate Studies and Scientific Research, Taif University, for funding this work.

Data Availability: All data generated or analysed during this study are included in this published article.

Author Contributions: Formal analysis: Z.A.G. Project administration: Z.G.K. Supervision, validation, and writing-review: T.A.T. Writing-original draft preparation: G.S. Data curation, Funding Acquization: E.S.A. Funding Acquization, Visualization: N.E.A.
All authors have read and agreed to the published version of the manuscript.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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