Int. J. Anal. Appl. (2025), 23:132

International Journal of Analysis and Applications

Interval-Valued Complex Neutrosophic Sets and Complex Neutrosophic Soft Topological Spaces

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ABSTRACT. Network performance is the evaluation and assessment of collective network statistics, to define the quality of services offered by the computer network. It is a qualitative and quantitative technique that measures and defines the performance level for a network. Networking provides a link between different factors (bandwidth, number of devices, network traffic and latency) for performing multiple tasks. These factors affect the network speed and quality. Some errors occur due to network traffic and latency can produce uncertain results. These results provide low quality and speed in the network that caused time wasting with no required results. In this regard, the notion of interval valued complex neutrosophic relation (IVCNR) is developed to handle this situation. Modeling problems by using the idea of interval valued complex neutrosophic sets (IVCNSs) and interval valued complex neutrosophic relations (IVCNRs) will not only formulate the effects of one factor to other but also defines the grades of membership, abstains and non-membership. The cartesian product among two IVCNSs and the types of IVCNRs is discussed. By applying the methods of IVCNRs on the factors of network performance that can produce better network speed and improved quality in the network. In continuation this study introduced and investigates the structure of complex neutrosophic soft topological spaces. The foundational definitions of complex neutrosophic soft topology, open and closed sets, interior, closure, and boundary are formally established. The study also explores the concept of complex neutrosophic soft bases and subspace topologies, along with criteria for basis generation and

Received Feb. 20, 2025

2020 Mathematics Subject Classification. 68M10.

Key words and phrases. complex neutrosophic set; complex neutrosophic relation; network quality; fuzzy modelling; complex neutrosophic soft topology.

https://doi.org/10.28924/2291-8639-23-2025-132

ISSN: 2291-8639

topological refinement. Several theorems elucidate the relationships among topological constructs and operations such as union, intersection, and complementation under complex neutrosophic soft conditions. We apply pervious methods on these problems and collect some results. But through this method, the required results achieved more reliable than the previous methods. So, the proposed method is the best method for modeling uncertain complexities in the required results. Some applications are also given that can be applied in our day to day life.

1. Introduction

Uncertainty refers to the cognitive conditions involving unknown information. The term uncertainty defines lack of exact information that helps to find correct and clear results. Before 1965, it was dreadful to deal with uncertainty for different real-life problems. In 1965, Zadeh [1] introduced the conception of fuzzy set theory for solving these ambiguities. Fuzzy set (FS) used to assign membership grades in the partition from of the unit interval. Atanassov [2] gave the idea of intuitionistic fuzzy set (IFS). IFS deal with both membership and non-membership grades and their sum must be in the interval [0,1]. F Samarandache [3] introduced the generalization of IFS, known as Neutrosophic set (NS). NS provides three possibilities of membership, abstinence, and non-membership grades and the sum of these three grades must be in the interval ranging in [0,3]. NS is the generalization of IFS. NS provides the membership, non-membership values as well as neutral values rather than IFS. Rough Neutrosophic sets (RNS) introduced by Broumi et al. [4]. The notion of using complex numbers in FSs is known as complex fuzzy sets (CFSs). Ramot et al. [5] developed the idea of CFS. CFSs are used for modeling multi-dimensional problems. CFSs describes the membership degrees in the complex valued mappings. Alkouri et al. [6] introduce the concept of complex intuitionistic fuzzy set (CIFS). CIFS uses complex valued mappings and their sum also in the unit interval. Complex neutrosophic set (CNS) proposed by Ali and Smarandache [7]. A CNS is characterized by three complex valued mapping i.e., membership grade, abstinence and non-membership grades and their sum is in the interval [0,3]. Complex valued mappings have two terms these are phase term and amplitude term. These terms represent two different entities. Broumi et al [8] described the Bipolar complex neutrosophic set (BCNS).

FS in the form of intervals known as interval valued fuzzy set (IVFS). IVFS is developed by zedah [9] in 1975. IVFS represents the membership degrees in the form of interval values that reflect uncertainty by assigning membership degrees. In 1999, Atanassov [10] introduce Interval valued intuitionistic fuzzy sets (IVIFSs). IVIFS is an improved from rather than IVFS. IVIFS describes the intervals of membership as well as non-membership degrees. Modeling real life problems with the help of using IVIFS is very useful and interesting. C Cornelis et al. [11] develop the implication in IFS and IVFS. H Zhang et al. [12] Introduced the interval valued neutrosophic set (IVNS) in decision-making problems. S Broumi and F Smarandache [13] develop the idea of similarity measure of IVNS. The conception of using complex numbers in the intervals is called interval valued complex fuzzy set (IVCFS). IVCFS introduced by S Greenfield et al. [14]. IVCFS defines the intervals of complex valued mapping that discuss only the membership. H Grag and D Rani [15] develop the idea of interval valued complex intuitionistic fuzzy set (IVCIFS). IVCIFS discuss the membership as well as non-membership grades and the sum of both complex values is ranging from zero to one. Ali et al. [16] introduce the interval valued complex neutrosophic set (IVCNS)

Mandel [17] gave the idea of fuzzy relations (FRs). FRs are the extension of crisp relations (CR). FRs indicates the strength of relationships by the degree of membership. If the value of membership degree near to 1 present the strong relationship, and if the membership degree nearer to 0 then it indicates the weaker relationship. A Nasir et al. [18] discussed the medical diagnosis and life span of sufferer by using IVCFRs. Burillo and Bustince [19] develop the idea of intuitionistic fuzzy relation (IFR). IFR discuss the membership and non-membership degrees, respectively. A Nasir et al. [20] gave the idea of IVIFRs in cybersecurity against loopholes in industrial control. Neutrosophic relation (NR) developed by Yang et al. [21]. NRs used to deal with intermediate and inconsistent information. Ramot et al. [22] gave the idea of complex fuzzy relation (CFR). CFR provides the complex valued mappings in the membership degrees. Complex intuitionistic fuzzy relation (CIFR) developed by N Jan et al. [23]. Complex neutrosophic relation (CNR) introduced by Nasir et al. [24]. AL-Quran and Alkhazaleh [25] introduced the relations between complex neutrosophic sets with decision making application. Broumi et al [26] defined the Bipolar complex neutrosophic sets on the decision-making problem. Interval valued fuzzy relation (IVFR) defines the fuzzy relations in the intervals. IVFR introduced by Bustince and Burillo [27]. Interval valued complex fuzzy relations (IVCFRs) developed by Nasir et al. [28]. IVCFRs provides the complex valued mappings in the intervals. The improved form f IVFR is interval valued intuitionistic fuzzy relation (IVIFR) introduced by Wu et al. [29]. IVIFR discuss both membership and non-membership for any query. Interval valued complex intuitionistic fuzzy relation (IVCIFR) provides the complex values in the form of intervals Zhang et al. [30] gave the idea of interval valued neutrosophic relation (IVNR). IVNRs discussed the three possibilities like: membership, abstinence, and non-membership degrees of different problems.

The purpose of this paper is to define the conception of interval valued complex neutrosophic relations (IVCNRs) and its different types such as IVCN-inverse relation, IVCN-reflexive relation, IVCN-irreflexive relation, IVCN-symmetric relation, IVCN-asymmetric relation, IVCN-antisymmetric relation, IVCN-transitive relation, IVCN-order relation, IVCN-composite relation and IVCN-equivalence relation. Different properties and important results also proved for solving different queries. IVCNRs carry three degrees, these degrees show the qualities of different relations. The membership degree shows better performance, the abstinence shows no effect on the performance and the non-membership degree provides low performance on the proposed application. The limitations of the Networks are dynamic, intricate systems made up of many interconnected parts. The intricacy and dynamic nature of networks make it difficult to study and comprehend every facet of their behavior. Also, certain network components may occasionally be hidden from researchers' view, particularly in distributed or cloud-based environments. This lack of visibility may make it more difficult to comprehend network performance in its entirety. More advanced techniques may make data gathering and analysis more efficient and give researchers a deeper understanding of network behavior. This might entail data analytics, machine learning algorithms, or sophisticated monitoring tools.

This paper arranged as the section 2 discusses some basic notions. Section 3 defines the main results and theorems proved. Section 4 introduces the concept of complex neutrosophic soft topological spaces and their properties. It also discusses some important results. Section 5 discusses some more results on complex neutrosophic soft topological spaces.

Section 6 propose the application of IVCNRs and IVCNSs that investigate the performance of networking in a network using different factors that affect the network for bad or a good quality and speed. Section 7 compares the proposed method with existing methods. The paper ends with a conclusion. Section 8 discusses the conclusion and future direction.

2. Preliminaries

Definition 2.1. [1] A fuzzy set $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, \alpha(\nu): \nu \epsilon \mathcal{U}\}$

Where $\alpha(\nu)$ is a membership degree of fuzzy set defined as $\alpha: \mathcal{U} \to [0,1]$.

Definition 2.2. [5] A complex fuzzy set $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, \alpha_c(\nu): \nu \epsilon \mathcal{U}\}$, where $\alpha_c(\nu)$ is a membership degree of CFS defined as $\alpha_c: \mathcal{U} \to \{\mathcal{Z} | \mathcal{Z} \epsilon \mathbb{C}, |\mathcal{Z}| \leq 1\}$. Moreover, $\mathcal{Z}(\nu) = \mu_c(\nu) e^{\rho_c{}^{(\nu)2\pi i}}$ and $0 \leq \mu_c, \rho_c \leq 1$ are known as amplitude and phase terms, respectively.

Definition 2.3. [5] Cartesian Product (CP) is the set of all possible ordered combinations consisting of one member from each of those sets. The CP of two CFSs $\mathfrak{A} = \left\{\nu_i, \mu_c(\nu_i) e^{\rho_c{}^{(\nu_i)2\pi i}}: \nu_i \epsilon \mathcal{U}\right\}$ and $\mathfrak{B} = \left\{\nu_j, \mu_c(\nu_j) e^{\rho_c{}^{(\nu_j)2\pi i}}: \nu_j \epsilon \mathcal{U}\right\}$, $i, j \epsilon \mathbb{N}$ is given by

$$\mathfrak{A} \times \mathfrak{B} = \left\{(\nu_i, \nu_j), \mu_c(\nu_i, \nu_j) e^{\rho_c(\nu_i, \nu_j)2\pi i}: \nu_i \epsilon \mathcal{U}, \nu_j \epsilon \mathcal{U}\right\}$$

The degree of membership of CP $\mathfrak{A} \times \mathfrak{B}$ is defined as

$$\mathcal{Z}_{\mathfrak{A} \times \mathfrak{B}}(\nu_i, \nu_j) = \mu_{(\mathfrak{A} \times \mathfrak{B})}(\nu_i, \nu_j) e^{\rho_{(\mathfrak{A} \times \mathfrak{B})}{}^{(\nu_i, \nu_j)2\pi i}} = \alpha i \gamma\{\mu_{\mathfrak{A}}(\nu_i), \mu_{\mathfrak{B}}(\nu_j)\} e^{\{\rho_{\mathfrak{A}}(\nu_i), \rho_{\mathfrak{B}}(\nu_j)\}2\pi i}$$

and $\mu_{(\mathfrak{A} \times \mathfrak{B})}(\nu_i, \nu_j), \rho_{(\mathfrak{A} \times \mathfrak{B})}(\nu_i, \nu_j) \epsilon [0,1]$.

Definition 2.4. [2] An intuitionistic fuzzy set $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form

$$\mathfrak{A} = \{\nu, \alpha(\nu), \beta(\nu): \nu \epsilon \mathcal{U}\}$$

where $\alpha(\nu)$ and $\gamma(\nu)$ are the membership and non-membership degree of IFS defined as $\alpha, \gamma: \mathcal{U} \to [0,1]$ and $0 \leq \alpha(\nu) + \gamma(\nu) \leq 1$.

Definition 2.5. [6] A CIFS $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, \alpha(\nu), \gamma(\nu): \nu\epsilon\mathcal{U}\}$ Where $\alpha(\nu)$ and $\gamma(\nu)$ are the membership and non-membership degree of CIFS defined as $\alpha: \mathcal{U} \to \{\mathcal{Z}_\alpha | \mathcal{Z}_\alpha \epsilon \mathbb{C}, |\mathcal{Z}_\alpha| \leq 1\}$ and $\gamma: \mathcal{U} \to \{\mathcal{Z}_\gamma | \mathcal{Z}_\gamma \epsilon \mathbb{C}, |\mathcal{Z}_\gamma| \leq 1\}$ where $\mathcal{Z}_\alpha(\nu) = \mu_\alpha(\nu) e^{\rho_\alpha(\nu)2\pi i}$and $\mathcal{Z}_\gamma(\nu) = \mu_\gamma(\nu) e^{\rho_\gamma(\nu)2\pi i}$ CIFS has the conditions

$0 \leq \mu_\alpha(\nu) + \mu_\gamma(\nu) \leq 1$ and $0 \leq \rho_\alpha(\nu) + \rho_\gamma(\nu) \leq 1$ are known as amplitude terms and phase terms, respectively.

Definition 2.6. [3] An NS $\mathfrak{A}$ on a universe $\mathcal{U}$ is in the form of real valued function $\alpha(\nu), \beta(\nu), \gamma(\nu): \mathcal{U} \to [0,1]$

Where $\alpha(\nu), \beta(\nu), \gamma(\nu)$ known as the degree of membership, abstinence, and non-membership, respectively. NS has the condition $0 \leq \alpha(\nu) + \beta(\nu) + \gamma(\nu) \leq 3$

Definition 2.7. [7] A CNS $\mathfrak{A}$ on a universe $\mathcal{U}$ is in the form of real valued function $: \mathcal{U} \to \{\mathcal{Z}_\alpha | \mathcal{Z}_\alpha \epsilon \mathbb{C}, |\mathcal{Z}_\alpha| \leq 1\}$, $\beta: \mathcal{U} \to \{\mathcal{Z}_\beta | \mathcal{Z}_\beta \epsilon \mathbb{C}, |\mathcal{Z}_\beta| \leq 1\}$, and $\gamma: \mathcal{U} \to \{\mathcal{Z}_\gamma | \mathcal{Z}_\gamma \epsilon \mathbb{C}, |\mathcal{Z}_\gamma| \leq 1\}$ are known as membership, abstinence, and non-membership, respectively. Where $\mathcal{Z}_\alpha(\nu) = \mu_\alpha(\nu) e^{\rho_\alpha(\nu)2\pi i}$, $\mathcal{Z}_\beta(\nu) = \mu_\beta(\nu) e^{\rho_\beta(\nu)2\pi i}$ and $\mathcal{Z}_\gamma(\nu) = \mu_\gamma(\nu) e^{\rho_\gamma(\nu)2\pi i}$.The conditions of CNFS are $0 \leq \mu_\alpha(\nu) + \mu_\beta(\nu) + \mu_\gamma(\nu) \leq 3$ and $0 \leq \rho_\alpha(\nu) + \rho_\beta(\nu) + \rho_\gamma(\nu) \leq 3$. Where n is natural number. $\mu_\alpha(\nu)$, $\mu_\beta(\nu)$, and $\mu_\gamma(\nu)$ are called amplitude terms and $\rho_\alpha(\nu)$, $\rho_\beta(\nu)$, and $\rho_\gamma(\nu)$ are known as phase terms.

Definition 2.8. [9] An IVFS $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, \alpha(\nu)^-, \alpha(\nu)^+: \nu\epsilon\mathcal{U}\}$

Where $\alpha(\nu)^-: \mathfrak{A} \to [0,1]$ and $\alpha(\nu)^+: \mathfrak{A} \to [0,1]$ are called the lower and upper degrees of membership respectively.

Definition 2.9. [14] An IVCFS $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, [\mu_c(\nu)^-, \mu_c(\nu)^+] e^{[\rho_c(\nu)^-, \rho_c(\nu)^+]2\pi i}: \nu\epsilon\mathcal{U}\}$

Where ${\mu_c}^-: \mathfrak{A} \to [0,1]$, ${\mu_c}^+: \mathfrak{A} \to [0,1]$, ${\rho_c}^-: \mathfrak{A} \to [0,1]$ and ${\rho_c}^+: \mathfrak{A} \to [0,1]$ are the mappings called the lower amplitude term, upper amplitude term, lower phase term and upper phase term f degree of membership and $i = \sqrt{-1}$.

Definition 2.10. [10] An IVIFS $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \{\nu, [\alpha(\nu)^-, \alpha(\nu)^+], [\gamma(\nu)^-, \gamma(\nu)^+]: \nu\epsilon\mathcal{U}\}$

Where $\alpha(\nu)^-: \mathfrak{A} \to [0,1]$ and $\alpha(\nu)^+: \mathfrak{A} \to [0,1]$ are called the lower and upper degrees of membership and $\gamma(\nu)^-: \mathfrak{A} \to [0,1]$ and $\gamma(\nu)^+: \mathfrak{A} \to [0,1]$ are called the lower and upper degrees of non-membership degrees, respectively.

Definition 2.11. [15] An IVCIFS $\mathfrak{A}$ on a universe $\mathcal{U}$ is of the form $\mathfrak{A} = \left\{\nu, [\mu_\alpha(\nu)^-, \mu_\alpha(\nu)^+] e^{[\rho_\alpha(\nu)^-, \rho_\alpha(\nu)^+]2\pi i}, [\mu_\gamma(\nu)^-, \mu_\gamma(\nu)^+] e^{[\rho_\gamma(\nu)^-, \rho_\gamma(\nu)^+]2\pi i}: \nu\epsilon\mathcal{U}\right\}$

$\mu_\alpha(\nu)^-: \mathfrak{A} \to [0,1]$, $\mu_\gamma(\nu)^-: \mathfrak{A} \to [0,1]$, $\rho_\alpha(\nu)^-: \mathfrak{A} \to [0,1]$ and $\rho_\gamma(\nu)^-: \mathfrak{A} \to [0,1]$ are mappings called the lower and upper amplitude term and lower and upper phase term of membership and non-membership degrees, respectively. And $i = \sqrt{-1}$. An IVCIFS have the conditions $0 \leq {\mu_\alpha}^+ {\mu_\gamma}^+ \leq 1$ and $0 \leq {\rho_\alpha}^+ {\rho_\gamma}^+ \leq 1$.

Definition 2.12. [12] An IVNS $\mathfrak{A}$ on a universe $\mathcal{U}$ is in the form of real valued function $\alpha(\nu)^-, \beta(\nu)^-, \gamma(\nu)^-: \mathcal{U} \to [0,1]$ and $\alpha(\nu)^+, \beta(\nu)^+, \gamma(\nu)^+: \mathcal{U} \to [0,1.$ Where $\alpha(\nu), \beta(\nu), \gamma(\nu)$ known as the degree of membership, abstinence, and non-membership, respectively. NS have the conditions $0 \leq \alpha(\nu)^- + \beta(\nu)^- + \gamma(\nu)^- \leq 3$, $0 \leq \alpha(\nu)^+ + \beta(\nu)^+ + \gamma(\nu)^+ \leq 3$

Definition 2.13. [17] An IVCNS $\mathfrak{A}$ on a universe $\mathcal{U}$ is in the form of real valued function $\alpha(\nu)^-, \alpha(\nu)^+: \mathcal{U} \to \{\mathcal{Z}_\alpha | \mathcal{Z}_\alpha \epsilon \mathbb{C}, |\mathcal{Z}_\alpha| \leq 1\}$, $\beta(\nu)^-, \beta(\nu)^+: \mathcal{U} \to \{\mathcal{Z}_\beta | \mathcal{Z}_\beta \epsilon \mathbb{C}, |\mathcal{Z}_\beta| \leq 1\}$, and $\gamma(\nu)^-, \gamma(\nu)^+: \mathcal{U} \to \{\mathcal{Z}_\gamma | \mathcal{Z}_\gamma \epsilon \mathbb{C}, |\mathcal{Z}_\gamma| \leq 1\}$ are known as membership, abstinence, and non-membership, respectively. Where $\mathcal{Z}_\alpha(\nu) = \mu_\alpha(\nu) e^{\rho_\alpha(\nu) 2\pi i}$, $\mathcal{Z}_\beta(\nu) = \mu_\beta(\nu) e^{\rho_\beta(\nu) 2\pi i}$ and $\mathcal{Z}_\gamma(\nu) = \mu_\gamma(\nu) e^{\rho_\gamma(\nu) 2\pi i}$. The conditions of CNFS are $0 \leq \mu_\alpha(\nu)^- + \mu_\beta(\nu)^- + \mu_\gamma(\nu)^- \leq 3$ and $0 \leq \rho_\alpha(\nu)^- + \rho_\beta(\nu)^- + \rho_\gamma(\nu)^- \leq 3$. Where n is natural number. $\mu_\alpha(\nu)$, $\mu_\beta(\nu)$, and $\mu_\gamma(\nu)$ are called amplitude terms and $\rho_\alpha(\nu)$, $\rho_\beta(\nu)$, and $\rho_\gamma(\nu)$ are known as phase terms.

Definition 2.14. [22] A CFR Ω is a non-empty subset of $\mathfrak{A} \times \mathfrak{B}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are CFSs.

Example 1. For a CFS, $\mathfrak{A} = \{(\nu_1, 0.6e^{(0.3)2\pi i}), (\nu_2, 0.3e^{(0.5)2\pi i}), (\nu_3, 0.8e^{(0.7)2\pi i})\}$

$$\mathfrak{A} \times \mathfrak{A} = \begin{Bmatrix} ((\nu_1,\nu_1), 0.6e^{(0.3)2\pi i}), ((\nu_1,\nu_2), 0.3e^{(0.3)2\pi i}), ((\nu_1,\nu_3), 0.6e^{(0.3)2\pi i}), \\ ((\nu_2,\nu_1), 0.3e^{(0.3)2\pi i}), ((\nu_2,\nu_2), 0.3e^{(0.5)2\pi i}), ((\nu_2,\nu_3), 0.3e^{(0.5)2\pi i}), \\ ((\nu_3,\nu_1), 0.6e^{(0.3)2\pi i}), ((\nu_3,\nu_2), 0.3e^{(0.5)2\pi i}), ((\nu_3,\nu_3), 0.8e^{(0.7)2\pi i}) \end{Bmatrix}$$

The CFR is $\Omega = \{((\nu_1,\nu_2), 0.3e^{(0.3)2\pi i}), ((\nu_1,\nu_3), 0.6e^{(0.3)2\pi i}), ((\nu_2,\nu_3), 0.3e^{(0.5)2\pi i}), ((\nu_3,\nu_1), 0.6e^{(0.3)2\pi i}),\}$

Definition 2.15. [28] An IVCFR Ω is a non-empty subset of $\mathfrak{A} \times \mathfrak{B}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are CFSs, i.e., $\Omega \subseteq \mathfrak{A} \times \mathfrak{B}$.

Definition 2.16. [24] A CNR Ω is a non-empty subset of $\mathfrak{A} \times \mathfrak{B}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are CFSs. where $\Omega \subseteq \mathfrak{A} \times \mathfrak{B}$.

Example 2.17. [24] For a given CNS, $\mathfrak{A} = \begin{Bmatrix} (\nu_1, 0.6e^{(0.3)2\pi i}, 1e^{(0,7)2\pi i}, 2e^{(0.6)2\pi i}), \\ (\nu_2, 0.3e^{(0.5)2\pi i}, 0.9e^{(0.7)2\pi i}, 0e^{(0.2)2\pi i}), \\ (\nu_3, 0.8e^{(0.7)2\pi i}, 1e^{(0.5)2\pi i}, 1e^{(0.7)2\pi i}) \end{Bmatrix}$

$$\mathfrak{A} \times \mathfrak{A} = \begin{Bmatrix} ((\nu_1,\nu_1), 0.6e^{(0.3)2\pi i}, 1e^{(0,7)2\pi i}, 2e^{(0.6)2\pi i}), ((\nu_1,\nu_2), 0.3e^{(0.5)2\pi i}, 0.9e^{(0.7)2\pi i}, 2e^{(0.6)2\pi i}), \\ ((\nu_1,\nu_3), 0.6e^{(0.3)2\pi i}, 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}), \\ ((\nu_2,\nu_1), 0.3e^{(0.5)2\pi i}, 0.9e^{(0,7)2\pi i}, 2e^{(0.6)2\pi i}), ((\nu_2,\nu_2), 0.3e^{(0.5)2\pi i}, 0.9e^{(0.7)2\pi i}, 0e^{(0.2)2\pi i}), \\ ((\nu_2,\nu_3), 0.6e^{(0.3)2\pi i}, 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}), \\ ((\nu_3,\nu_1), 0.6e^{(0.3)2\pi i}, 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}), ((\nu_3,\nu_2), 0.6e^{(0.3)2\pi i} 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}), \\ ((\nu_3,\nu_3), 0.8e^{(0.7)2\pi i}, 1e^{(0.5)2\pi i}, 1e^{(0.7)2\pi i}) \end{Bmatrix}$$

The CNR Ω is $\Omega = \begin{Bmatrix} ((\nu_1,\nu_3), 0.6e^{(0.3)2\pi i}, 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}), \\ ((\nu_2,\nu_2), 0.3e^{(0.5)2\pi i}, 0.9e^{(0.7)2\pi i}, 0e^{(0.2)2\pi i}), \\ ((\nu_3,\nu_2), 0.6e^{(0.3)2\pi i} 1e^{(0.5)2\pi i}, 2e^{(0.7)2\pi i}) \end{Bmatrix}$

3. Some results on Interval-Valued Complex Neutrosophic Relation (IVCNR)

In this section some results related to interval valued complex neutrosophic relations are given. Examples are given for better understanding.

Definition 3.1. An IVCNR Ω is a non-empty subset of $\mathfrak{A} \times \mathfrak{B}$, where $\mathfrak{A}$ and $\mathfrak{B}$ are IVCNSs. where $\Omega \subseteq \mathfrak{A} \times \mathfrak{B}$.

Example 3.2. For a given IVCNS

$$\mathfrak{A} = \left\{ \begin{matrix} \left(\nu_1, [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0,5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}\right), \\ \left(\nu_2, [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}\right), \\ \left(\nu_3, [0.8, 1]e^{[0.8,2]2\pi i}, [0.7,1]e^{[0,1]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}\right) \end{matrix} \right\}$$

The CP of $\mathfrak{A}$ to itself

$$\mathfrak{A} \times \mathfrak{A} = \left\{ \begin{matrix} \left((\nu_1, \nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0,5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_1, \nu_2), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_1, \nu_3), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_2, \nu_1), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_2, \nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}\right), \\ \left((\nu_2, \nu_3), [0.3, 1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}\right), \\ \left((\nu_3, \nu_1), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_3, \nu_2), [0.3, 1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}\right), \\ \left((\nu_3, \nu_3), [0.8, 1]e^{[0.8,2]2\pi i}, [0.7,1]e^{[0,1]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}\right) \end{matrix} \right\}$$

The IVCNR Ω is $\Omega = \left\{ \begin{matrix} \left((\nu_1, \nu_3), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}\right), \\ \left((\nu_2, \nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}\right), \\ \left((\nu_3, \nu_2), [0.3, 1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}\right) \end{matrix} \right\}$

Definition 3.3. Let $\mathfrak{A}$ be an IVCNS on a universe $\mathcal{U}$ and Ω be an IVCNR on $\mathfrak{A}$. Then,

i. An IVCN-inverse relation Ω^{-1} of an IVCNR on $\mathfrak{A}$

$$\Omega = \left\{ (\nu_i, \nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i, \nu_j), \mu^+{}_\alpha(\nu_i, \nu_j)] e^{[\rho^-{}_\alpha(\nu_i,\nu_j), \rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i, \nu_j), \mu^+{}_\beta(\nu_i, \nu_j)] e^{[\rho^-{}_\beta(\nu_i,\nu_j), \rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i, \nu_j), \mu^+{}_\gamma(\nu_i, \nu_j)] e^{[\rho^-{}_\gamma(\nu_i,\nu_j), \rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{pmatrix} : (\nu_i, \nu_j) \epsilon\, \Omega \right\},$$

is defined as

$$\Omega^{-1} = \left\{ (\nu_j, \nu_i), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j, \nu_i), \mu^+{}_\alpha(\nu_j, \nu_i)] e^{[\rho^-{}_\alpha(\nu_j,\nu_i), \rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j, \nu_i), \mu^+{}_\beta(\nu_j, \nu_i)] e^{[\rho^-{}_\beta(\nu_j,\nu_i), \rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j, \nu_i), \mu^+{}_\gamma(\nu_j, \nu_i)] e^{[\rho^-{}_\gamma(\nu_j,\nu_i), \rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{pmatrix} : (\nu_j, \nu_i) \epsilon\, \Omega \right\}$$

ii. An IVCNR Ω is an IVCN-reflexive relation on $\mathfrak{A}$ if

$$\forall\left\{\nu,\begin{pmatrix}[\mu^-{}_{\alpha}(\nu),\mu^+{}_{\alpha}(\nu)]e^{[\rho^-{}_{\alpha}(\nu),\rho^+{}_{\alpha}(\nu)]2\pi i},\\ [\mu^-{}_{\beta}(\nu),\mu^+{}_{\beta}(\nu)]\,e^{[\rho^-{}_{\beta}(\nu),\rho^+{}_{\beta}(\nu)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu),\mu^+{}_{\gamma}(\nu)]\,e^{[\rho^-{}_{\gamma}(\nu),\rho^+{}_{\gamma}(\nu)]2\pi i}\end{pmatrix}\right\}\epsilon\ \mathfrak{A}$$

$$\Rightarrow\left\{(\nu,\nu),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu,\nu),\mu^+{}_{\alpha}(\nu,\nu)]e^{[\rho^-{}_{\alpha}(\nu,\nu),\rho^+{}_{\alpha}(\nu,\nu)]2\pi i},\\ [\mu^-{}_{\beta}(\nu,\nu),\mu^+{}_{\beta}(\nu,\nu)]\,e^{[\rho^-{}_{\beta}(\nu,\nu),\rho^+{}_{\beta}(\nu,\nu)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu,\nu),\mu^+{}_{\gamma}(\nu,\nu)]\,e^{[\rho^-{}_{\gamma}(\nu,\nu),\rho^+{}_{\gamma}(\nu,\nu)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega$$

iii.An IVCNR Ω is an IVCN-irreflexive relation on $\mathfrak{A}$ if

$$\forall\left\{\nu,\begin{pmatrix}[\mu^-{}_{\alpha}(\nu),\mu^+{}_{\alpha}(\nu)]e^{[\rho^-{}_{\alpha}(\nu),\rho^+{}_{\alpha}(\nu)]2\pi i},\\ [\mu^-{}_{\beta}(\nu),\mu^+{}_{\beta}(\nu)]\,e^{[\rho^-{}_{\beta}(\nu),\rho^+{}_{\beta}(\nu)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu),\mu^+{}_{\gamma}(\nu)]\,e^{[\rho^-{}_{\gamma}(\nu),\rho^+{}_{\gamma}(\nu)]2\pi i}\end{pmatrix}\right\}\epsilon\ \mathfrak{A}$$

$$\Rightarrow\left\{(\nu,\nu),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu,\nu),\mu^+{}_{\alpha}(\nu,\nu)]e^{[\rho^-{}_{\alpha}(\nu,\nu),\rho^+{}_{\alpha}(\nu,\nu)]2\pi i},\\ [\mu^-{}_{\beta}(\nu,\nu),\mu^+{}_{\beta}(\nu,\nu)]\,e^{[\rho^-{}_{\beta}(\nu,\nu),\rho^+{}_{\beta}(\nu,\nu)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu,\nu),\mu^+{}_{\gamma}(\nu,\nu)]\,e^{[\rho^-{}_{\gamma}(\nu,\nu),\rho^+{}_{\gamma}(\nu,\nu)]2\pi i}\end{pmatrix}\right\}\notin\Omega$$

iv.An IVCNR Ω is an IVCN-symmetric relation on $\mathfrak{A}$ if

$$\forall\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu_i,\nu_j),\mu^+{}_{\alpha}(\nu_i,\nu_j)]e^{[\rho^-{}_{\alpha}(\nu_i,\nu_j),\rho^+{}_{\alpha}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\beta}(\nu_i,\nu_j),\mu^+{}_{\beta}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\beta}(\nu_i,\nu_j),\rho^+{}_{\beta}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu_i,\nu_j),\mu^+{}_{\gamma}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\gamma}(\nu_i,\nu_j),\rho^+{}_{\gamma}(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\ \mathfrak{A}$$

$$\Rightarrow\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu_i,\nu_j),\mu^+{}_{\alpha}(\nu_i,\nu_j)]e^{[\rho^-{}_{\alpha}(\nu_i,\nu_j),\rho^+{}_{\alpha}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\beta}(\nu_i,\nu_j),\mu^+{}_{\beta}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\beta}(\nu_i,\nu_j),\rho^+{}_{\beta}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu_i,\nu_j),\mu^+{}_{\gamma}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\gamma}(\nu_i,\nu_j),\rho^+{}_{\gamma}(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega$$

v.An IVCNR Ω is an IVCN-antisymmetric relation on $\mathfrak{A}$ if

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu_i,\nu_j),\mu^+{}_{\alpha}(\nu_i,\nu_j)]e^{[\rho^-{}_{\alpha}(\nu_i,\nu_j),\rho^+{}_{\alpha}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\beta}(\nu_i,\nu_j),\mu^+{}_{\beta}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\beta}(\nu_i,\nu_j),\rho^+{}_{\beta}(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu_i,\nu_j),\mu^+{}_{\gamma}(\nu_i,\nu_j)]\,e^{[\rho^-{}_{\gamma}(\nu_i,\nu_j),\rho^+{}_{\gamma}(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}:(\nu_i,\nu_j)\epsilon\ \Omega$$

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_{\alpha}(\nu_j,\nu_i),\mu^+{}_{\alpha}(\nu_j,\nu_i)]e^{[\rho^-{}_{\alpha}(\nu_j,\nu_i),\rho^+{}_{\alpha}(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_{\beta}(\nu_j,\nu_i),\mu^+{}_{\beta}(\nu_j,\nu_i)]\,e^{[\rho^-{}_{\beta}(\nu_j,\nu_i),\rho^+{}_{\beta}(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_{\gamma}(\nu_j,\nu_i),\mu^+{}_{\gamma}(\nu_j,\nu_i)]\,e^{[\rho^-{}_{\gamma}(\nu_j,\nu_i),\rho^+{}_{\gamma}(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}(\nu_j,\nu_i)\epsilon\ \Omega$$

$$\Rightarrow\left\{(\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i),\mu^+{}_\alpha(\nu_i)]e^{[\rho^-{}_\alpha(\nu_i),\rho^+{}_\alpha(\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_i),\mu^+{}_\beta(\nu_i)]\,e^{[\rho^-{}_\beta(\nu_i),\rho^+{}_\beta(\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i),\mu^+{}_\gamma(\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_i),\rho^+{}_\gamma(\nu_i)]2\pi i}\end{pmatrix}\right\}:(\nu_i)\epsilon\,\Omega$$

$$\Rightarrow\left\{(\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j),\mu^+{}_\alpha(\nu_j)]e^{[\rho^-{}_\alpha(\nu_j),\rho^+{}_\alpha(\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_j),\mu^+{}_\beta(\nu_j)]\,e^{[\rho^-{}_\beta(\nu_j),\rho^+{}_\beta(\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j),\mu^+{}_\gamma(\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_j),\rho^+{}_\gamma(\nu_j)]2\pi i}\end{pmatrix}\right\}:(\nu_j)\epsilon\,\Omega$$

vi.An IVCNR Ω is an IVCN-asymmetric relation on $\mathfrak{A}$ if

$$\forall\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i),\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}(\nu_j,\nu_i)\epsilon\,\Omega$$

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}:(\nu_i,\nu_j)\notin\;\Omega$$

vii.An IVCNR Ω is an IVCN-transitive relation on $\mathfrak{A}$ if

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}:(\nu_i,\nu_j)\epsilon\,\Omega$$

$$\left\{(\nu_j,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_k),\mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_k),\mu^+{}_\beta(\nu_j,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_k),\mu^+{}_\gamma(\nu_j,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i}\end{pmatrix}\right\}:(\nu_j,\nu_k)\epsilon\,\Omega$$

$$\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}:(\nu_k,\nu_i)\epsilon\,\Omega$$

viii.An IVCNR Ω is an IVCN- composite relation on $\mathfrak{A}$ if

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}:(\nu_i,\nu_j)\epsilon\,\Omega_1$$

and

$$\left\{ (\nu_j, \nu_k), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j,\nu_k), \mu^+{}_\alpha(\nu_j,\nu_k)] e^{[\rho^-{}_\alpha(\nu_j,\nu_k), \rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j,\nu_k), \mu^+{}_\beta(\nu_j,\nu_k)] e^{[\rho^-{}_\beta(\nu_j,\nu_k), \rho^+{}_\beta(\nu_j,\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j,\nu_k), \mu^+{}_\gamma(\nu_j,\nu_k)] e^{[\rho^-{}_\gamma(\nu_j,\nu_k) \rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i} \end{pmatrix} \right\} : (\nu_j, \nu_k) \epsilon\, \Omega_2$$

Then

$$\left\{ (\nu_i, \nu_k), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i,\nu_k), \mu^+{}_\alpha(\nu_i,\nu_k)] e^{[\rho^-{}_\alpha(\nu_i,\nu_k), \rho^+{}_\alpha(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i,\nu_k), \mu^+{}_\beta(\nu_i,\nu_k)] e^{[\rho^-{}_\beta(\nu_i,\nu_k), \rho^+{}_\beta(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i,\nu_k), \mu^+{}_\gamma(\nu_i,\nu_k)] e^{[\rho^-{}_\gamma(\nu_i,\nu_k), \rho^+{}_\gamma(\nu_i,\nu_k)]2\pi i} \end{pmatrix} \right\} : (\nu_i, \nu_k) \epsilon \Omega_1 \circ \Omega_2$$

ix. An IVCNR Ω is an IVCN-order relation on $\mathfrak{A}$ if Ω is IVCN-transitive relation, IVCN-antisymmetric relation, and IVCN-reflexive relation.

x. An IVCNR Ω is an IVCN-equivalence relation on $\mathfrak{A}$ if Ω is IVCN-transitive relation, IVCN-symmetric relation, and IVCN-reflexive relation.

Example 3.4. Let an IVCNS

$$\mathfrak{A} = \begin{Bmatrix} (\nu_1, [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0,5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ (\nu_2, [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}), \\ (\nu_3, [0.8,1]e^{[0.8,2]2\pi i}, [0.7,1]e^{[0,1]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}) \end{Bmatrix}$$

The CP of $\mathfrak{A}$ to itself

$$\mathfrak{A} \times \mathfrak{A} = \begin{Bmatrix} ((\nu_1,\nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0,5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_1,\nu_2), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_1,\nu_3), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \\ ((\nu_2,\nu_1), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_2,\nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}), \\ ((\nu_2,\nu_3), [0.3,1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}), \\ ((\nu_3,\nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \\ ((\nu_3,\nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}), \\ ((\nu_3,\nu_3), [0.8,1]e^{[0.8,2]2\pi i}, [0.7,1]e^{[0,1]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}) \end{Bmatrix}$$

- The IVCNR Ω_1 is IVCN-equivalence relation on $\mathfrak{A}$

$$\Omega_1 = \begin{Bmatrix} ((\nu_1,\nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0,5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_1,\nu_3), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \\ ((\nu_3,\nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \\ ((\nu_3,\nu_3), [0.8,1]e^{[0.8,2]2\pi i}, [0.7,1]e^{[0,1]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}), \\ ((\nu_2,\nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}) \end{Bmatrix}$$

- The IVCNR Ω_1 is IVCN-order relation on $\mathfrak{A}$

$$\Omega_2 = \left\{ \begin{array}{c} ((\nu_2, \nu_2), [0.3,1]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.5]e^{[0,0.3]2\pi i}), \\ ((\nu_1, \nu_2), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_2, \nu_3), [0.3, 1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}), \\ ((\nu_3, \nu_1), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \end{array} \right\},$$

- The IVCNR Ω_3 is IVCN-composite relation on $\mathfrak{A}$

$$\Omega_3 = \left\{ \begin{array}{c} ((\nu_1, \nu_2), [0.3,0.9]e^{[0,0.5]2\pi i}, [0.9,2]e^{[0.2,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_2, \nu_3), [0.3, 1]e^{[0,0.5]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.1,0.6]2\pi i}), \\ ((\nu_1, \nu_3), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}) \end{array} \right\},$$

- The IVCNR Ω_4 is IVCN-transitive relation on $\mathfrak{A}$

$$\Omega_4 = \left\{ \begin{array}{c} ((\nu_1, \nu_1), [0.6,0.9]e^{[0.3,0.6]2\pi i}, [1,2]e^{[0.5,0.9]2\pi i}, [0,0.8]e^{[0.9,1]2\pi i}), \\ ((\nu_1, \nu_3), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}), \\ ((\nu_3, \nu_1), [0.6, 0.9]e^{[0.3,0.6]2\pi i}, [0.7,1]e^{[0,0.9]2\pi i}, [1,2]e^{[0.9,1]2\pi i}) \end{array} \right\}$$

Theorem 3.5. An IVCNR Ω is an IVCN-symmetric relation if and only if $\Omega = \Omega^{-1}$.

Proof. Consider $\Omega = \Omega^{-1}$, then

$$\left\{ (\nu_i, \nu_j), \left(\begin{array}{c} [\mu^-{}_\alpha(\nu_i, \nu_j), \mu^+{}_\alpha(\nu_i, \nu_j)] e^{[\rho^-{}_\alpha(\nu_i,\nu_j), \rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i, \nu_j), \mu^+{}_\beta(\nu_i, \nu_j)] e^{[\rho^-{}_\beta(\nu_i,\nu_j), \rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i, \nu_j), \mu^+{}_\gamma(\nu_i, \nu_j)] e^{[\rho^-{}_\gamma(\nu_i,\nu_j), \rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{array} \right) \right\} : (\nu_i, \nu_j) \epsilon\, \Omega$$

Implies

$$\left\{ (\nu_j, \nu_i), \left(\begin{array}{c} [\mu^-{}_\alpha(\nu_j, \nu_i), \mu^+{}_\alpha(\nu_j, \nu_i)] e^{[\rho^-{}_\alpha(\nu_j,\nu_i), \rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j, \nu_i), \mu^+{}_\beta(\nu_j, \nu_i)] e^{[\rho^-{}_\beta(\nu_j,\nu_i), \rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j, \nu_i), \mu^+{}_\gamma(\nu_j, \nu_i)] e^{[\rho^-{}_\gamma(\nu_j,\nu_i), \rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{array} \right) \right\} : (\nu_j, \nu_i) \epsilon\, \Omega^{-1}$$

But $\Omega = \Omega^{-1}$, Therefore

$$\left\{ (\nu_j, \nu_i), \left(\begin{array}{c} [\mu^-{}_\alpha(\nu_j, \nu_i), \mu^+{}_\alpha(\nu_j, \nu_i)] e^{[\rho^-{}_\alpha(\nu_j,\nu_i), \rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j, \nu_i), \mu^+{}_\beta(\nu_j, \nu_i)] e^{[\rho^-{}_\beta(\nu_j,\nu_i), \rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j, \nu_i), \mu^+{}_\gamma(\nu_j, \nu_i)] e^{[\rho^-{}_\gamma(\nu_j,\nu_i), \rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{array} \right) \right\} : (\nu_j, \nu_i) \epsilon \Omega$$

So, Ω is an IVCN-symmetric relation.

Again, suppose that Ω is an IVCN-symmetric relation, then

$$\left\{ (\nu_i, \nu_j), \left(\begin{array}{c} [\mu^-{}_\alpha(\nu_i, \nu_j), \mu^+{}_\alpha(\nu_i, \nu_j)] e^{[\rho^-{}_\alpha(\nu_i,\nu_j), \rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i, \nu_j), \mu^+{}_\beta(\nu_i, \nu_j)] e^{[\rho^-{}_\beta(\nu_i,\nu_j), \rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i, \nu_j), \mu^+{}_\gamma(\nu_i, \nu_j)] e^{[\rho^-{}_\gamma(\nu_i,\nu_j), \rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{array} \right) \right\} : (\nu_i, \nu_j) \epsilon\, \Omega$$

Implies

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)\right]2\pi i}\end{pmatrix}\right\}:(\nu_j,\nu_i)\epsilon\ \Omega$$

But

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)\right]2\pi i}\end{pmatrix}\right\}:(\nu_i,\nu_j)\epsilon\ \Omega$$

Implies

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)\right]2\pi i}\end{pmatrix}\right\}:(\nu_j,\nu_i)\epsilon\ \Omega^{-1}$$

This Implies that $\Omega=\Omega^{-1}$. □

Theorem 3.6. For IVCN symmetric relation $\Omega_1$and Ω_2, the intersection $\Omega_1 \cap \Omega_2$ is also an IVCN symmetric relation.

Proof. Consider $\Omega_1$and Ω_2 are two IVCN symmetric relations on an IVCNS $\mathfrak{A}$. Using the definition of IVCNR, $\Omega_1 \subseteq \mathfrak{A}\times\mathfrak{A}$ and $\Omega_2 \subseteq \mathfrak{A}\times\mathfrak{A}$ implies $\Omega_1 \cap \Omega_2 \subseteq \mathfrak{A}\times\mathfrak{A}$.

This implies that $\Omega_1 \cap \Omega_2$ is IVCNR on $\mathfrak{A}$.

Now, again if

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)\right]2\pi i}\end{pmatrix}\right\}\epsilon\Omega_1 \cap \Omega_2$$

implies

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)\right]2\pi i}\end{pmatrix}\right\}\epsilon\Omega_1,$$

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ \left[\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)\right]2\pi i}\end{pmatrix}\right\}\epsilon\Omega_2.$$

As Ω_1 and Ω_2 are IVCN symmetric relations,

$$\left\{(\nu_j,\nu_i), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j,\nu_i), \mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j,\nu_i), \mu^+{}_\beta(\nu_j,\nu_i)]\, e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j,\nu_i), \mu^+{}_\gamma(\nu_j,\nu_i)]\, e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{pmatrix}\right\} : \epsilon \Omega_1,$$

$$\left\{(\nu_j,\nu_i), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j,\nu_i), \mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j,\nu_i), \mu^+{}_\beta(\nu_j,\nu_i)]\, e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j,\nu_i), \mu^+{}_\gamma(\nu_j,\nu_i)]\, e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{pmatrix}\right\} : \epsilon \Omega_2,$$

Implies

$$\left\{(\nu_j,\nu_i), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j,\nu_i), \mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j,\nu_i), \mu^+{}_\beta(\nu_j,\nu_i)]\, e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j,\nu_i), \mu^+{}_\gamma(\nu_j,\nu_i)]\, e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{pmatrix}\right\} : \epsilon \Omega_1 \cap \Omega_2.$$

Hence proved that the intersection of two IVCN symmetric relations is also IVCN symmetric relation. □

Theorem 3.7. An IVCNR Ω is an IVCN transitive relation if and only if $\Omega \circ \Omega \subseteq \Omega$.

Proof. Consider that Ω is an IVCN transitive relation, then

$$\left\{(\nu_i,\nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i,\nu_j), \mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i,\nu_j), \mu^+{}_\beta(\nu_i,\nu_j)]\, e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i,\nu_j), \mu^+{}_\gamma(\nu_i,\nu_j)]\, e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{pmatrix}\right\} \in \Omega$$

and

$$\left\{(\nu_j,\nu_k), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j,\nu_k), \mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j,\nu_k), \mu^+{}_\beta(\nu_j,\nu_k)]\, e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j,\nu_k), \mu^+{}_\gamma(\nu_j,\nu_k)]\, e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i} \end{pmatrix}\right\} \in \Omega,$$

implies that

$$\left\{(\nu_i,\nu_k), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i,\nu_k), \mu^+{}_\alpha(\nu_i,\nu_k)]e^{[\rho^-{}_\alpha(\nu_i,\nu_k),\rho^+{}_\alpha(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i,\nu_k), \mu^+{}_\beta(\nu_i,\nu_k)]\, e^{[\rho^-{}_\beta(\nu_i,\nu_k),\rho^+{}_\beta(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i,\nu_k), \mu^+{}_\gamma(\nu_i,\nu_k)]\, e^{[\rho^-{}_\gamma(\nu_i,\nu_k),\rho^+{}_\gamma(\nu_i,\nu_k)]2\pi i} \end{pmatrix}\right\} \in \Omega.$$

But

$$\left\{(\nu_i,\nu_k), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i,\nu_k), \mu^+{}_\alpha(\nu_i,\nu_k)]e^{[\rho^-{}_\alpha(\nu_i,\nu_k),\rho^+{}_\alpha(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i,\nu_k), \mu^+{}_\beta(\nu_i,\nu_k)]\, e^{[\rho^-{}_\beta(\nu_i,\nu_k),\rho^+{}_\beta(\nu_i,\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i,\nu_k), \mu^+{}_\gamma(\nu_i,\nu_k)]\, e^{[\rho^-{}_\gamma(\nu_i,\nu_k),\rho^+{}_\gamma(\nu_i,\nu_k)]2\pi i} \end{pmatrix}\right\} \in \Omega^\circ\Omega.$$

Hence $\Omega \circ \Omega \subseteq \Omega$.

On the other hand, let $\Omega \circ \Omega \subseteq \Omega$, then

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega$$

and

$$\left\{(\nu_j,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_k),\mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_k),\mu^+{}_\beta(\nu_j,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_k),\mu^+{}_\gamma(\nu_j,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega,$$

implies

$$\left\{(\nu_j,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_k),\mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_k),\mu^+{}_\beta(\nu_j,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_k),\mu^+{}_\gamma(\nu_j,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega^\circ\Omega,$$

But $\Omega^\circ\Omega \subseteq \Omega$ implies that

$$\left\{(\nu_j,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_k),\mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_k),\mu^+{}_\beta(\nu_j,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_k),\mu^+{}_\gamma(\nu_j,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega$$

So Ω is an IVCN transitive relation. □

Theorem 3.8. An IVCN equivalence relation implies that $\Omega = \Omega \circ \Omega$.

Proof. Since an IVCN equivalence relation Ω is also an IVCN transitive relation, then by the pervious theorem, $\Omega \circ \Omega \subseteq \Omega$.

(1) Now assume that

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega \quad (2)$$

As Ω is an IVCN equivalence relation, Ω satisfies the properties of IVCN symmetric relation and IVCN transitive relation, implies that

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}\in\Omega \quad (3)$$

Equation (2), (3) implies that

$$\left\{(\nu,\nu),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu,\nu),\mu^+{}_\alpha(\nu,\nu)\right]e^{\left[\rho^-{}_\alpha(\nu,\nu),\rho^+{}_\alpha(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu,\nu),\mu^+{}_\beta(\nu,\nu)\right]e^{\left[\rho^-{}_\beta(\nu,\nu),\rho^+{}_\beta(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu,\nu),\mu^+{}_\gamma(\nu,\nu)\right]e^{\left[\rho^-{}_\gamma(\nu,\nu),\rho^+{}_\gamma(\nu,\nu)\right]2\pi i}\end{pmatrix}\right\}\in\Omega \tag{4}$$

But with IVCN composite relation,

$$\left\{(\nu,\nu),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu,\nu),\mu^+{}_\alpha(\nu,\nu)\right]e^{\left[\rho^-{}_\alpha(\nu,\nu),\rho^+{}_\alpha(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu,\nu),\mu^+{}_\beta(\nu,\nu)\right]e^{\left[\rho^-{}_\beta(\nu,\nu),\rho^+{}_\beta(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu,\nu),\mu^+{}_\gamma(\nu,\nu)\right]e^{\left[\rho^-{}_\gamma(\nu,\nu),\rho^+{}_\gamma(\nu,\nu)\right]2\pi i}\end{pmatrix}\right\}\in\Omega^\circ\Omega \tag{5}$$

So, this implies that $\Omega=\Omega\circ\Omega$ (6)

From (1) and (6), we have $\Omega=\Omega\circ\Omega$. □

Theorem 3.9. The inverse IVCNR Ω^{-1} of an IVCN order relation Ω is also an IVCN order relation.

Proof. The inverse IVCNR Ω^{-1} of an IVCN order relation Ω is also an IVCN order relation if it satisfies the three properties:

$$\forall\ \nu\epsilon\mathfrak{A},\left\{(\nu,\nu),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu,\nu),\mu^+{}_\alpha(\nu,\nu)\right]e^{\left[\rho^-{}_\alpha(\nu,\nu),\rho^+{}_\alpha(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu,\nu),\mu^+{}_\beta(\nu,\nu)\right]e^{\left[\rho^-{}_\beta(\nu,\nu),\rho^+{}_\beta(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu,\nu),\mu^+{}_\gamma(\nu,\nu)\right]e^{\left[\rho^-{}_\gamma(\nu,\nu),\rho^+{}_\gamma(\nu,\nu)\right]2\pi i}\end{pmatrix}\right\}\in\Omega$$

since Ω is also an IVCN reflexive relation:

Implies

$$\left\{(\nu,\nu),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu,\nu),\mu^+{}_\alpha(\nu,\nu)\right]e^{\left[\rho^-{}_\alpha(\nu,\nu),\rho^+{}_\alpha(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu,\nu),\mu^+{}_\beta(\nu,\nu)\right]e^{\left[\rho^-{}_\beta(\nu,\nu),\rho^+{}_\beta(\nu,\nu)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu,\nu),\mu^+{}_\gamma(\nu,\nu)\right]e^{\left[\rho^-{}_\gamma(\nu,\nu),\rho^+{}_\gamma(\nu,\nu)\right]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1} \tag{7}$$

Thus, Ω^{-1} is an IVCN reflexive relation.

- Consider

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)\right]e^{\left[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)\right]2\pi i}\end{pmatrix}\right\}\in\Omega,$$

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}\left[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)\right]2\pi i},\\ \left[\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)\right]2\pi i},\\ \left[\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)\right]e^{\left[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)\right]2\pi i}\end{pmatrix}\right\}\in\Omega,$$

Implies

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1},$$

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1} \quad (8)$$

But Ω is also an IVCN antisymmetric relation. So

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}=$$

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\} \quad (9)$$

Therefore, Ω^{-1} is IVCN antisymmetric relation.

- Assume $\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega,$

$$\left\{(\nu_j,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_k),\mu^+{}_\alpha(\nu_j,\nu_k)]e^{[\rho^-{}_\alpha(\nu_j,\nu_k),\rho^+{}_\alpha(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_k),\mu^+{}_\beta(\nu_j,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_k),\rho^+{}_\beta(\nu_j,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_k),\mu^+{}_\gamma(\nu_j,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_k)\rho^+{}_\gamma(\nu_j,\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega,$$

Implies that $\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1},$

$$\left\{(\nu_k,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_j),\mu^+{}_\alpha(\nu_k,\nu_j)]e^{[\rho^-{}_\alpha(\nu_k,\nu_j),\rho^+{}_\alpha(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_j),\mu^+{}_\beta(\nu_k,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_j),\rho^+{}_\beta(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_j),\mu^+{}_\gamma(\nu_k,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_j)\rho^+{}_\gamma(\nu_k,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1} \quad (10)$$

But Ω is also an IVCN transitive relation. Consequently,

$$\left\{(\nu_i,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_k),\mu^+{}_\alpha(\nu_i,\nu_k)]e^{[\rho^-{}_\alpha(\nu_i,\nu_k),\rho^+{}_\alpha(\nu_i,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_k),\mu^+{}_\beta(\nu_i,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_k),\rho^+{}_\beta(\nu_i,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_k),\mu^+{}_\gamma(\nu_i,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_k),\rho^+{}_\gamma(\nu_i,\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega, \text{ this implies that}$$

$$\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}\in\Omega^{-1} \tag{11}$$

Hence, Ω^{-1} is an IVCN transitive relation. Thus with (7), (9), and (11), Ω^{-1} is also an IVCN order relation. □

Theorem 3.10. For an IVCN equivalence relation Ω.

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\in\Omega \quad \text{if and only if } \Omega^{\nu_i}=\Omega^{\nu_j}.$$

Proof. Consider $\Omega^{\nu_i}=\Omega^{\nu_j}$, then $\nu_k\epsilon\mathfrak{A}$,

$$\left\{\nu_k,\begin{pmatrix}[\mu^-{}_\alpha(\nu_k),\mu^+{}_\alpha(\nu_k)]e^{[\rho^-{}_\alpha(\nu_k),\rho^+{}_\alpha(\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_k),\mu^+{}_\beta(\nu_k)]\,e^{[\rho^-{}_\beta(\nu_k),\rho^+{}_\beta(\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k),\mu^+{}_\gamma(\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_k),\rho^+{}_\gamma(\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega^{\nu_i}$$

$$\text{Implies}\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega$$

$$\text{Implies}\left\{(\nu_i,\nu_k),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_k),\mu^+{}_\alpha(\nu_i,\nu_k)]e^{[\rho^-{}_\alpha(\nu_i,\nu_k),\rho^+{}_\alpha(\nu_i,\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_k),\mu^+{}_\beta(\nu_i,\nu_k)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_k),\rho^+{}_\beta(\nu_i,\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_k),\mu^+{}_\gamma(\nu_i,\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_k),\rho^+{}_\gamma(\nu_i,\nu_k)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega \tag{12}$$

Since Ω is an IVCN symmetric relation.

In the same manner,

$$\left\{\nu_k,\begin{pmatrix}[\mu^-{}_\alpha(\nu_k),\mu^+{}_\alpha(\nu_k)]e^{[\rho^-{}_\alpha(\nu_k),\rho^+{}_\alpha(\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_k),\mu^+{}_\beta(\nu_k)]\,e^{[\rho^-{}_\beta(\nu_k),\rho^+{}_\beta(\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k),\mu^+{}_\gamma(\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_k),\rho^+{}_\gamma(\nu_k)]2\pi i}\end{pmatrix}\right\}\in\Omega^{\nu_j}$$

Implies $\left\{(\nu_k,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_j),\mu^+{}_\alpha(\nu_k,\nu_j)]e^{[\rho^-{}_\alpha(\nu_k,\nu_j),\rho^+{}_\alpha(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_j),\mu^+{}_\beta(\nu_k,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_j),\rho^+{}_\beta(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_j),\mu^+{}_\gamma(\nu_k,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_j),\rho^+{}_\gamma(\nu_k,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega$

Implies $\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega$

Because Ω is an IVCN transitive relation.

Conversely, assume

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega,$$

$$\left\{\nu_k,\begin{pmatrix}[\mu^-{}_\alpha(\nu_k),\mu^+{}_\alpha(\nu_k)]e^{[\rho^-{}_\alpha(\nu_k),\rho^+{}_\alpha(\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_k),\mu^+{}_\beta(\nu_k)]\,e^{[\rho^-{}_\beta(\nu_k),\rho^+{}_\beta(\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k),\mu^+{}_\gamma(\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_k),\rho^+{}_\gamma(\nu_k)]2\pi i}\end{pmatrix}\right\}\epsilon\ \Omega^{\nu_i},\text{ implies}$$

$$\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega. \tag{13}$$

Then

$$\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega,$$

$$\left\{(\nu_i,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_i,\nu_j),\mu^+{}_\alpha(\nu_i,\nu_j)]e^{[\rho^-{}_\alpha(\nu_i,\nu_j),\rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_i,\nu_j),\mu^+{}_\beta(\nu_i,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_i,\nu_j),\rho^+{}_\beta(\nu_i,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_i,\nu_j),\mu^+{}_\gamma(\nu_i,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_i,\nu_j),\rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega,\text{ implies}$$

$$\left\{(\nu_k,\nu_j),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_j),\mu^+{}_\alpha(\nu_k,\nu_j)]e^{[\rho^-{}_\alpha(\nu_k,\nu_j),\rho^+{}_\alpha(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_j),\mu^+{}_\beta(\nu_k,\nu_j)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_j),\rho^+{}_\beta(\nu_k,\nu_j)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_j),\mu^+{}_\gamma(\nu_k,\nu_j)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_j),\rho^+{}_\gamma(\nu_k,\nu_j)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega,$$

Because Ω is an IVCN transitive relation

$$\left\{\nu_k, \begin{pmatrix} [\mu^-{}_\alpha(\nu_k), \mu^+{}_\alpha(\nu_k)] e^{[\rho^-{}_\alpha(\nu_k), \rho^+{}_\alpha(\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_k), \mu^+{}_\beta(\nu_k)] e^{[\rho^-{}_\beta(\nu_k), \rho^+{}_\beta(\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_k), \mu^+{}_\gamma(\nu_k)] e^{[\rho^-{}_\gamma(\nu_k), \rho^+{}_\gamma(\nu_k)]2\pi i} \end{pmatrix}\right\} \epsilon\ \Omega^{\nu_j} \tag{14}$$

$$\Omega^{\nu_i} \subseteq \Omega^{\nu_j} \tag{15}$$

Similarly, suppose

$$\left\{(\nu_i, \nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i, \nu_j), \mu^+{}_\alpha(\nu_i, \nu_j)] e^{[\rho^-{}_\alpha(\nu_i,\nu_j), \rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i, \nu_j), \mu^+{}_\beta(\nu_i, \nu_j)] e^{[\rho^-{}_\beta(\nu_i,\nu_j), \rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i, \nu_j), \mu^+{}_\gamma(\nu_i, \nu_j)] e^{[\rho^-{}_\gamma(\nu_i,\nu_j), \rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{pmatrix}\right\} \epsilon\Omega,$$

$$\left\{\nu_k, \begin{pmatrix} [\mu^-{}_\alpha(\nu_k), \mu^+{}_\alpha(\nu_k)] e^{[\rho^-{}_\alpha(\nu_k), \rho^+{}_\alpha(\nu_k)]2\pi i}, \\ [\mu^-{}_\beta(\nu_k), \mu^+{}_\beta(\nu_k)] e^{[\rho^-{}_\beta(\nu_k), \rho^+{}_\beta(\nu_k)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_k), \mu^+{}_\gamma(\nu_k)] e^{[\rho^-{}_\gamma(\nu_k), \rho^+{}_\gamma(\nu_k)]2\pi i} \end{pmatrix}\right\} \epsilon\ \Omega^{\nu_j}, \text{ implies}$$

$$\left\{(\nu_k, \nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_k, \nu_j), \mu^+{}_\alpha(\nu_k, \nu_j)] e^{[\rho^-{}_\alpha(\nu_k,\nu_j), \rho^+{}_\alpha(\nu_k,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_k, \nu_j), \mu^+{}_\beta(\nu_k, \nu_j)] e^{[\rho^-{}_\beta(\nu_k,\nu_j), \rho^+{}_\beta(\nu_k,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_k, \nu_j), \mu^+{}_\gamma(\nu_k, \nu_j)] e^{[\rho^-{}_\gamma(\nu_k,\nu_j), \rho^+{}_\gamma(\nu_k,\nu_j)]2\pi i} \end{pmatrix}\right\} \epsilon\Omega. \tag{16}$$

As,

$$\left\{(\nu_i, \nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_i, \nu_j), \mu^+{}_\alpha(\nu_i, \nu_j)] e^{[\rho^-{}_\alpha(\nu_i,\nu_j), \rho^+{}_\alpha(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_i, \nu_j), \mu^+{}_\beta(\nu_i, \nu_j)] e^{[\rho^-{}_\beta(\nu_i,\nu_j), \rho^+{}_\beta(\nu_i,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_i, \nu_j), \mu^+{}_\gamma(\nu_i, \nu_j)] e^{[\rho^-{}_\gamma(\nu_i,\nu_j), \rho^+{}_\gamma(\nu_i,\nu_j)]2\pi i} \end{pmatrix}\right\} \epsilon\Omega, \text{ implies}$$

$$\left\{(\nu_j, \nu_i), \begin{pmatrix} [\mu^-{}_\alpha(\nu_j, \nu_i), \mu^+{}_\alpha(\nu_j, \nu_i)] e^{[\rho^-{}_\alpha(\nu_j,\nu_i), \rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\beta(\nu_j, \nu_i), \mu^+{}_\beta(\nu_j, \nu_i)] e^{[\rho^-{}_\beta(\nu_j,\nu_i), \rho^+{}_\beta(\nu_j,\nu_i)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_j, \nu_i), \mu^+{}_\gamma(\nu_j, \nu_i)] e^{[\rho^-{}_\gamma(\nu_j,\nu_i) \rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i} \end{pmatrix}\right\} \epsilon\Omega, \tag{17}$$

Because Ω is IVCN symmetric relation.

Now,

$$\left\{(\nu_k, \nu_j), \begin{pmatrix} [\mu^-{}_\alpha(\nu_k, \nu_j), \mu^+{}_\alpha(\nu_k, \nu_j)] e^{[\rho^-{}_\alpha(\nu_k,\nu_j), \rho^+{}_\alpha(\nu_k,\nu_j)]2\pi i}, \\ [\mu^-{}_\beta(\nu_k, \nu_j), \mu^+{}_\beta(\nu_k, \nu_j)] e^{[\rho^-{}_\beta(\nu_k,\nu_j), \rho^+{}_\beta(\nu_k,\nu_j)]2\pi i}, \\ [\mu^-{}_\gamma(\nu_k, \nu_j), \mu^+{}_\gamma(\nu_k, \nu_j)] e^{[\rho^-{}_\gamma(\nu_k,\nu_j), \rho^+{}_\gamma(\nu_k,\nu_j)]2\pi i} \end{pmatrix}\right\} \epsilon\Omega,$$

$$\left\{(\nu_j,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_j,\nu_i),\mu^+{}_\alpha(\nu_j,\nu_i)]e^{[\rho^-{}_\alpha(\nu_j,\nu_i),\rho^+{}_\alpha(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_j,\nu_i),\mu^+{}_\beta(\nu_j,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_j,\nu_i),\rho^+{}_\beta(\nu_j,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_j,\nu_i),\mu^+{}_\gamma(\nu_j,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_j,\nu_i)\rho^+{}_\gamma(\nu_j,\nu_i)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega, \text{ implies}$$

$$\left\{(\nu_k,\nu_i),\begin{pmatrix}[\mu^-{}_\alpha(\nu_k,\nu_i),\mu^+{}_\alpha(\nu_k,\nu_i)]e^{[\rho^-{}_\alpha(\nu_k,\nu_i),\rho^+{}_\alpha(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\beta(\nu_k,\nu_i),\mu^+{}_\beta(\nu_k,\nu_i)]\,e^{[\rho^-{}_\beta(\nu_k,\nu_i),\rho^+{}_\beta(\nu_k,\nu_i)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k,\nu_i),\mu^+{}_\gamma(\nu_k,\nu_i)]\,e^{[\rho^-{}_\gamma(\nu_k,\nu_i),\rho^+{}_\gamma(\nu_k,\nu_i)]2\pi i}\end{pmatrix}\right\}\epsilon\Omega, \tag{18}$$

Because Ω is an IVCN transitive relation

$$\left\{\nu_k,\begin{pmatrix}[\mu^-{}_\alpha(\nu_k),\mu^+{}_\alpha(\nu_k)]e^{[\rho^-{}_\alpha(\nu_k),\rho^+{}_\alpha(\nu_k)]2\pi i},\\ [\mu^-{}_\beta(\nu_k),\mu^+{}_\beta(\nu_k)]\,e^{[\rho^-{}_\beta(\nu_k),\rho^+{}_\beta(\nu_k)]2\pi i},\\ [\mu^-{}_\gamma(\nu_k),\mu^+{}_\gamma(\nu_k)]\,e^{[\rho^-{}_\gamma(\nu_k),\rho^+{}_\gamma(\nu_k)]2\pi i}\end{pmatrix}\right\}\epsilon\ \Omega^{\nu_i}.$$

Therefore,

$$\Omega^{\nu_j}\subseteq\Omega^{\nu_i} \tag{19}$$

By equation (15), (19) we have $\Omega^{\nu_i}=\Omega^{\nu_j}$. □

4. Complex Neutrosophic Soft Topological Space and their Properties

Definition 4.1. Let X be an initial universe and E be a set of parameters. Let P(X) denote the set of all complex neutrosophic sets of X. A neutrosophic soft set $\tilde{F}_E$ over X is a set defined by a set valued function $\tilde{F}$ representing a mapping $\tilde{F}$: E → P (X) where $\tilde{F}$ is called approximate function of $\tilde{F}_E$. In other words, $\tilde{F}_E$ is a parameterized family of some elements of the set P (X) and therefore it can be written as a set of ordered pairs,

$$\tilde{F}_E=\{(e,\langle x,T_{\tilde{F}(e)}(x),I_{\tilde{F}(e)}(x),F_{\tilde{F}(e)}(x)\rangle : x\in X) : e\in E\}$$

where $T_{\tilde{F}(e)}(x),I_{\tilde{F}(e)}(x),F_{\tilde{F}(e)}(x)\in[0,1]$ called the truth-membership, indeterminacy-membership, falsity-membership function of $\tilde{F}$(e), respectively. Since supremum of each T, I, F is 1 so the inequality $0\le T_{\tilde{F}(e)}(x)+I_{\tilde{F}(e)}(x)+F_{\tilde{F}(e)}(x)\le 3$ is obvious. Throughout this work, NSS(X_E) refers to the class of all neutrosophic soft sets over X.

Definition 4.2. Let $\tilde{F}_E\in$ CNSS(X_E). The complement of $\tilde{F}_E$ is denoted by $\tilde{F}_E^c$ and is defined by:

$$\tilde{F}_E^c=\{(e,\langle x,F_{\tilde{F}(e)}(x),1-I_{\tilde{F}(e)}(x),T_{\tilde{F}(e)}(x)\rangle : x\in X) : e\in E\}$$

Obvious that, $(\tilde{F}_E^c)^c=\tilde{F}_E$.

Definition 4.3. Let $\tilde{F}_E$, $\tilde{G}_E\in$ CNSS(X_E). Then $\tilde{F}_E$ is a subset of $\tilde{G}_E$, denoted by $\tilde{F}_E\subseteq\tilde{G}_E$. If for $\forall e\in E,\forall x\in X$;

1. $T_{\tilde{F}(e)}(x)\le T_{\tilde{G}(e)}(x)$
2. $I_{\tilde{F}(e)}(x)\le I_{\tilde{G}(e)}(x)$
3. $F_{\tilde{F}(e)}(x)\ge F_{\tilde{G}(e)}(x)$.

$\tilde{F}_E$ equals to $\tilde{G}_E$ if $\tilde{F}_E \subseteq \tilde{G}_E$ and $\tilde{G}_E \subseteq \tilde{F}_E$. It is denoted by $\tilde{F}_E = \tilde{G}_E$.

Definition 4.4. Let $\tilde{F}_E, \tilde{G}_E \in$ CNSS(X_E). Then, their union is denoted by $\tilde{F}_E \cup \tilde{G}_E = \tilde{H}_E$ and is defined by:

$$\tilde{H}_E = \{(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x)\rangle : x \in X) : e \in E\}$$

$were, T_{\tilde{H}(e)}(x) = \max\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}, I_{\tilde{H}(e)}(x) = \max\{I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x)\}, F_{\tilde{H}(e)}(x) =$
$\min\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$

Definition 4.5. Let $\tilde{F}_E, \tilde{G}_E \in$ CNSS(X_E). Then, their intersection is denoted by $\tilde{F}_E \cap \tilde{G}_E = \tilde{H}_E$ and is defined by:

$$\tilde{H}_E = \{(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x)\rangle : x \in X) : e \in E\}$$

$where, T_{\tilde{H}(e)}(x) = \min\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}, I_{\tilde{H}(e)}(x) = \min\{I_{\tilde{F}(e)}(x), I_{\tilde{G}(e)}(x)\}, F_{\tilde{H}(e)}(x) =$
$\max\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$

Definition 4.6. Let $\tilde{F}_E, \tilde{G}_E \in$ CNSS(X_E). Then, $\tilde{F}_E$ difference $\tilde{G}_E$ operation on them is denoted by,

$$\tilde{F}_E \backslash \tilde{G}_E = \tilde{H}_E$$

and is defined by $\tilde{H}_E = \tilde{F}_E \cap \tilde{G}_E^c$ as follows:

$$\tilde{H}_E = \{(e, \langle x, T_{\tilde{H}(e)}(x), I_{\tilde{H}(e)}(x), F_{\tilde{H}(e)}(x)\rangle : x \in X) : e \in E\}$$

$where, T_{\tilde{H}(e)}(x) = \min\{T_{\tilde{F}(e)}(x), T_{\tilde{G}(e)}(x)\}, I_{\tilde{H}(e)}(x) = \min\{I_{\tilde{F}(e)}(x), 1 - I_{\tilde{G}(e)}(x)\}, F_{\tilde{H}(e)}(x) =$
$\max\{F_{\tilde{F}(e)}(x), F_{\tilde{G}(e)}(x)\}$

Definition 4.7. $\tilde{F}_E \in$ CNSS(X_E). Then, $\tilde{F}_E$ is said to be,

I. a null neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 0, I_{\tilde{F}(e)}(x) = 0, F_{\tilde{F}(e)}(x) = 1 : \forall e \in E . \forall x \in X$. It is denoted by 0_{XE}.

2. an absolute complex neutrosophic soft set if $T_{\tilde{F}(e)}(x) = 1, I_{\tilde{F}(e)}(x) = 1, F_{\tilde{F}(e)}(x) = 0 : \forall e \in E . \forall x \in X$. It is denoted by 1_{XE}.

Clearly, $0^c_{XE} = 1_{XE}$ and 1^c_{XE}.

Definition 4.8. Let CNSS(X,E) be the family of all complex neutrosophic soft sets over the universe set X and $\tau^{CNSS} \in CNSS(X, E)$. Then τ^{CCNSS} is said to be complex neutrosophic soft topology on X if

1. $0_{(X,E)}$ and $1_{(X,E)}$ belongs to τ^{CCNSS}
2. the union of any number of complex neutrosophic soft sets in τ^{CNSS} belongs to τ^{CNSS}
3. the intersection of finite number of complex neutrosophic soft sets in τ^{CNSS} belongs to τ^{CNSS}.

Then (X, τ^{CNSS}, E) is said to be a complex neutrosophic soft topological space over X. Each members of τ^{CNSS} is said to be complex neutrosophic soft open set.

Definition 4.9. Let (X, τ^{CNSS}, E) be a complex neutrosophic soft topological space over X and $(\tilde{F}, E)$ be a complex neutrosophic soft set over X. Then $(\tilde{F}, E)$ is said to be complex neutrosophic soft closed set iff its complement is a complex neutrosophic soft open set.

Definition 4.10. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and $(\tilde{F},E) \in CNSS\,(X,E)$ be a complex neutrosophic soft set. Then, the complex neutrosophic soft interior of $(\tilde{F},E)$, denoted $(\tilde{F},E)$, is defined as the complex neutrosophic soft union of all complex neutrosophic soft open subsets of $(\tilde{F},E)$.

Clearly, $(\tilde{F},E)^{\circ}$ is the biggest complex neutrosophic soft open set that is contained by $(\tilde{F},E)$.

Definition 4.11. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and $(\tilde{F},E) \in CNSS\,(X,E)$

be a complex neutrosophic soft set. Then, the complex neutrosophic soft closure of $(\tilde{F},E)$, denoted $\overline{(\tilde{F},E)}$, is defined as the complex neutrosophic soft intersection of all complex neutrosophic soft closed supersets of $(\tilde{F},E)$.

Clearly, $\overline{(\tilde{F},E)}$ is the smallest complex neutrosophic soft closed set that containing $(\tilde{F},E)$.

5. Topological Properties in Complex Neutrosophic Soft Frameworks

Definition 5.1. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and $(\tilde{F},E) \in C N$SS(X,E) be a complex neutrosophic soft set over X. If $(\tilde{F},E) = \overline{(\tilde{F},E)} \cap \overline{((\tilde{F},E)^{c})}$, then Fr $(\tilde{F},E)$ is said to be boundary of the complex neutrosophic soft set $(\tilde{F},E)$.

Theorem 5.2. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and $(\widetilde{F_1},E)$, $(\widetilde{F_2},E) \in$ CNSS(X,E).

Then,

1. $(\widetilde{F_1},E)^{0} = (\widetilde{F_1},E) \backslash Fr(\widetilde{F_1},E)$,
2. $\overline{(\widetilde{F_1},E)} = (\widetilde{F_1},E) \cup Fr(\widetilde{F_1},E)$,
3. Fr$\left((\widetilde{F_1},E) \cup (\widetilde{F_2},E)\right) \subseteq Fr(\widetilde{F_1},E) \cup Fr(\widetilde{F_2},E)$,
4. Fr$\left((\widetilde{F_1},E)^{c}\right) = Fr(\widetilde{F_1},E)$,
5. $1_{(X,E)} = (\widetilde{F_1},E)^{0} \cup Fr\,(\widetilde{F_1},E) \cup \left(1_{(X,E)}(\widetilde{F_1},E)\right)^{0}$,
6. Fr $\left((\widetilde{F_1},E)\right) \subseteq Fr(\widetilde{F_1},E)$,
7. Fr $((\widetilde{F_1},E)^{\circ}) \subseteq Fr(\widetilde{F_1},E)$,
8. $(\widetilde{F_1},E)$ is a complex neutrosophic soft open set $\leftrightarrow$ Fr$(\widetilde{F_1},E) = \overline{(\widetilde{F_1},E)} \backslash (\widetilde{F_1},E)$,
9. $(\widetilde{F_1},E)$ is a complex neutrosophic soft open set $\leftrightarrow$ Fr$(\widetilde{F_1},E) = (\widetilde{F_1},E) \backslash (\widetilde{F_1},E)^{\circ}$.

Proof. 1. $(\widetilde{F_1},E) \backslash Fr(\widetilde{F_1},E) = (\widetilde{F_1},E) \backslash \left(\overline{(\widetilde{F_1},E)} \cap \left(\overline{1_{(X,E)} \backslash (\widetilde{F_1},E)}\right)\right) = (\widetilde{F_1},E) \cap \left(\overline{(\widetilde{F_1},E)} \cap \right.$

$\left.\left(\overline{1_{(X,E)}(\widetilde{F_1},E)}\right)\right)^{c} = (\widetilde{F_1},E) \cap \left(\overline{(\widetilde{F_1},E)}\right)^{c} \cup\ (\widetilde{F_1},E) \cap \left(\overline{1_{(X,E)} \backslash (\widetilde{F_1},E)}\right)^{c} = \left((\widetilde{F_1},E) \backslash \overline{(\widetilde{F_1},E)}\right) \cup$

$\left((\widetilde{F_1},E)\setminus\left(\left(1_{(X,E)}\setminus(\widetilde{F_1},E)\right)\right)\right) = (\widetilde{F_1},E)\setminus\left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}\right) = (\widetilde{F_1},E) \cap ((\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}))^c = (\widetilde{F_1},E) \cap (\widetilde{F_1},E)^{\circ} = (\widetilde{F_1},E)^{\circ}$.

2. It is clear.

3. $\text{Fr}\left((\widetilde{F_1},E) \cup (\widetilde{F_2},E)\right) = \left(\overline{(\widetilde{F_1},E) \cup (\widetilde{F_2},E)}\right) \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E) \cup (\widetilde{F_2},E)}\right) = \left(\overline{(\widetilde{F_1},E)} \cap \overline{(\widetilde{F_1},E)}\right) \cap \left(\left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}\right) \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_2},E)}\right)\right) \subseteq \left(\overline{(\widetilde{F_1},E)} \cap \overline{(\widetilde{F_1},E)}\right) \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}\right) \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}\right) \subseteq \left(\overline{(\widetilde{F_1},E)} \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_1},E)}\right)\right) \cup \left(\overline{(\widetilde{F_2},E)} \cap \left(\overline{1_{(X,E)}\setminus(\widetilde{F_2},E)}\right)\right) = Fr(\widetilde{F_1},E) \cup Fr(\widetilde{F_2},E)$

4. $\text{Fr}\,((\widetilde{F_1},E)^c) = \overline{((\widetilde{F_1},E)^c)} \cap \overline{((\widetilde{F_1},E)^c)^c} = \overline{((\widetilde{F_1},E)^c)} \cap \overline{(\widetilde{F_1},E)} = Fr(\widetilde{F_1},E)$

5. It is clear.

6. $\text{Fr}\left(\overline{(\widetilde{F_1},E)}\right) = \overline{\left(\overline{(\widetilde{F_1},E)}\right)} \cap \overline{\overline{((\widetilde{F_1},E)^c)}} \subseteq \overline{(\widetilde{F_1},E)} \cap \overline{((\widetilde{F_1},E)^c)} = \text{Fr}(\widetilde{F_1},E)$

7. It is clear.

8. Suppose that $(\widetilde{F_1},E)$ is a complex neutrosophic soft open Then $(\widetilde{F_1},E)^c$ is a complex neutrosophic soft closed set and $\overline{((\widetilde{F_1},E)^c)} = (\widetilde{F_1},E)^c$ .In here,

$\text{Fr}(\widetilde{F_1},E) = \overline{(\widetilde{F_1},E)} \cap \overline{((\widetilde{F_1},E)^c)} = \overline{(\widetilde{F_1},E)} \cap (\widetilde{F_1},E)^c = \overline{(\widetilde{F_1},E)}\setminus(\widetilde{F_1},E)$.

From the condition -1,

$(\widetilde{F_1},E)^{\circ} = (\widetilde{F_1},E)\setminus Fr(\widetilde{F_1},E) = (\widetilde{F_1},E)\setminus\left(\overline{(\widetilde{F_1},E)}\setminus(\widetilde{F_1},E)\right) = (\widetilde{F_1},E) \cap (\overline{(\widetilde{F_1},E)} \cap (\widetilde{F_1},E)^c)^c = (\widetilde{F_1},E) \cap (\overline{(\widetilde{F_1},E)})^c \cup ((\widetilde{F_1},E)^c)^c) = (\widetilde{F_1},E) \cap \overline{((\widetilde{F_1},E)^c)} \cup \left((\widetilde{F_1},E) \cap (\widetilde{F_1},E)\right) = ((\widetilde{F_1},E) \cap \left(\overline{(\widetilde{F_1},E)})^c\right) \cup (\widetilde{F_1},E) = (\widetilde{F_1},E)$

That is $(\widetilde{F_1},E)$ is a complex neutrosophic soft open set.

Definition 5.3. Let (X, τ^{CNSS}, E) be a complex neutrosophic soft topological space over X and $(\tilde{F},E) \in CNSS(X,E)$.

A) $(\tilde{F},E)$ is a said to be complex neutrosophic soft dense set in (X, τ^{CNSS}, E) if $\overline{(\tilde{F},E)} = 1_{(X,E)}$.

b) $(\tilde{F},E)$ is a said to be a complex neutrosophic soft co-dense set in (X, τ^{CNSS}, E) if $\left(\overline{1_{(X,E)}\setminus(\tilde{F},E)}\right) = 1_{(X,E)}$.

c) $(\tilde{F},E)$ is a said to be a complex neutrosophic soft not-dense set in (X, τ^{CNSS}, E) if $\overline{(\tilde{F},E)}$ is a complex neutrosophic soft dense set over (X, τ^{CNSS}, E).

Theorem 5.4. Let (X, τ^{CNSS}, E) be a complex neutrosophic soft topological space over X and $(\tilde{F},E) \in$ NSS(X,E). Then,

1. $(\tilde{F},E)$ is a complex neutrosophic soft dense set in (X, τ^{CNSS}, E) iff $(\tilde{F},E) \cap (\widetilde{U},E) \neq 0_{(X,E)}$ for each $0_{(X,E)} \neq (\widetilde{U},E) \in \tau^{CNSS}$,

2. $\left(\tilde{F}, E\right)$ is a complex neutrosophic soft co-dense set in (X, τ^{CNSS},E) iff $(1_{(X,E)} \setminus \left(\tilde{F}, E\right)) \cap \left(\widetilde{U}, E\right) \neq 0_{(X,E)}$ for each $0_{(X,E)} \neq \left(\widetilde{U}, E\right) \in \tau^{CNSS}$,

3. $\left(\tilde{F}, E\right)$ is a complex neutrosophic soft not-dense set in any part of (X, τ^{CNSS},E) iff there is a complex neutrosophic soft open $\left(\widetilde{U}, E\right) \in \tau^{CNSS}$ $such\ that\ \left(\tilde{V}, E\right) \cap \left(\tilde{F}, E\right) = 0_{(X,E)}\ and\ 0_{(X,E)} \neq \left(\tilde{V}, E\right) \subseteq \left(\widetilde{U}, E\right)\ for\ each\ 0_{(X,E)} \neq \left(\widetilde{U}, E\right) \in \tau^{CNSS}$.

Proof. Trivial.

Definition 5.5. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and B^{CNSS} be a sub-family of τ^{CNSS}. B^{CNSS} is said to be a complex neutrosophic soft basis for the complex neutrosophic soft topology τ^{CNSS} if every element of τ^{CNSS} can be written as the complex neutrosophic soft union of elements of B^{CNSS}.

Theorem 5.6. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and B^{CNSS} be a complex neutrosophic soft basis for τ^{CNSS} . Then, τ^{CNSS} equals to the collection of all complex neutrosophic soft unions of elements of B^{CNSS}.

Proof. This is easily seen from the definition of complex neutrosophic soft basis.

Theorem 5.7. Let (X, τ^{CNSS},E) be a complex neutrosophic soft topological space over X and B^{CNSS} be a sub-family of τ^{CNSS}.

1. The family B^{CNSS} is a complex neutrosophic soft basis of the complex neutrosophic soft topology τ^{CNSS} iff there exist a complex neutrosophic soft set $\left(\tilde{B}, E\right) \in B^{CNSS}$ such that $x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in (\tilde{B}, \mathrm{E}) \subseteq \left(\tilde{F}, E\right)\ for\ each\ \left(\tilde{F}, \mathrm{E}\right) \in \tau^{CNSS} and\ x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right)$.

2. If the family $B^{CNSS} = \{(\widetilde{F_1}, E)\}_{i \in I}$ is a complex neutrosophic soft basis for τ^{CNSS} , then there exist a complex neutrosophic soft set $\left(\widetilde{F_1}, E\right) \in B^{CNSS}$ such that $x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\widetilde{B_{i3}}, E\right) \subseteq \left(\tilde{B}_{i1}, E\right) \cap \left(\tilde{B}_{i2}, E\right)\ for\ each\ \left(\tilde{B}_{i1}, E\right), \left(\tilde{B}_{i2}, E\right) \in B^{CNSS}$ and each $x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{B}_{i1}, E\right) \cap \left(\tilde{B}_{i2}, E\right)$.

Proof. 1 $\rightarrow$ Suppose that B^{CNSS} is a complex neutrosophic soft basis the complex neutrosophic soft topology $\tau^{CNSS} \subseteq \left(\tilde{F}, \mathrm{E}\right) \in \tau^{CNSS}\ and\ \left(\tilde{B}, \mathrm{E}\right)\ for\ x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right)$. Then $\left(\tilde{F}, \mathrm{E}\right) = \cup \left(\tilde{B}, \mathrm{E}\right)$. Therefore $x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{B}, \mathrm{E}\right) \subseteq \left(\tilde{F}, \mathrm{E}\right)\ from\ x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right) = \left(\tilde{B}, \mathrm{E}\right) \in B^{CNSS}$

$$\bigcup_{\left(\tilde{B}, \mathrm{E}\right) \in B^{CNSS}} \left(\tilde{B}, \mathrm{E}\right)\ for\ x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{B}, \mathrm{E}\right) \subseteq \left(\tilde{F}, \mathrm{E}\right).$$

Suppose that the condition of theorem to be provided Then,

$$\left(\tilde{F}, \mathrm{E}\right) = \bigcup_{x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right)} \left\{x^{e}_{(\text{ɥ},\text{ş},\text{ķ})}\right\} \subseteq \bigcup_{x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right)} \left\{x^{e}_{(\text{ɥ},\text{ş},\text{ķ})}\right\} \subseteq \left(\tilde{F}, \mathrm{E}\right) \rightarrow \left(\tilde{F}, \mathrm{E}\right) = \bigcup_{x^{e}_{(\text{ɥ},\text{ş},\text{ķ})} \in \left(\tilde{F}, \mathrm{E}\right)} \left(\tilde{B}, \mathrm{E}\right).$$

That is B^{CNSS} is a complex neutrosophic soft basis for τ^{CNSS}.

2. Let $(\tilde{B}_{i1},E),(\tilde{B}_{i2},E) \in B^{CNSS}$ and $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_{i1},E) \cap (\tilde{B}_{i2},E)$. $Since$ $(\tilde{B}_{i1},E) \cap (\tilde{B}_{i2},E)$ is a complex neutrosophic soft open set and B^{CNSS} is a complex neutrosophic soft basis for τ^{CNSS}, $then$

$(\tilde{B}_{i1},E) \cap (\tilde{B}_{i2},E) = \cup_j(\tilde{B}_i,E) \to x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_{i1},E) \cap (\tilde{B}_{i2},E) = \cup_j(\tilde{B}_i,E) \to \exists(\tilde{B}_{i1},E), x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_{i3},E) \subseteq (\tilde{B}_{i1},E) \cap (\tilde{B}_{i2},E)$. □

Theorem 5.8. Let τ_1^{CNSS} and τ_2^{CNSS} be two complex neutrosophic soft topologies over X generated by the complex neutrosophic soft bases B_1^{CNSS} and B_2^{CNSS} , respectively. Then $\tau_1^{CNSS} \subseteq \tau_2^{CNSS}$ iff for each $x^e_{(ꞕ,ȿ,ꝁ)} \in CNSS(X,E)$ and for each $(\tilde{B}_1,E) \in B_1^{CNSS}$ containing $x^e_{(ꞕ,ȿ,ꝁ)}$ there exists $(\tilde{B}_2,E) \in B_2^{CNSS}$ $such\ that$ $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_2,E) \subseteq (\tilde{B}_1,E)$.

Proof. Suppose that $\tau_1^{CNSS} \subseteq \tau_2^{CNSS}$ and $x^e_{(ꞕ,ȿ,ꝁ)} \in CNSS(X,E), (\tilde{B}_1,E) \in B_1^{CNSS}$ such that $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_1,E)$. Since B_1^{CNSS} is a complex neutrosophic soft basis for complex neutrosophic soft topology τ_1^{CNSS} over X, then $B_1^{CNSS} \subseteq \tau_1^{CNSS} \to x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_1,E) \in B_2^{CNSS} \subseteq \tau_1^{CNSS}$ $i.e,$ $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_1,E) \in \tau_2^{CNSS}$. Since B_2^{CNSS} is a complex neutrosophic soft basis for τ_2^{CNSS}, so for $(\tilde{B}_2,E) \in B_2^{CNSS}$ we have $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_2,E) \subseteq (\tilde{B}_1,E)$.

Conversely, assume that the hypothesis holds. Let $(\tilde{F},E) \in \tau_1^{CNSS}$. Since B_1^{CNSS} is a complex neutrosophic soft basis for complex neutrosophic soft topology τ_1^{CNSS}, then for $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{F},E)$ there exist $(\tilde{B}_1,E) \in B_1^{CNSS}$ such that $x^e_{(ꞕ,ȿ,ꝁ)} \in (\tilde{B}_1,E) \subseteq (\tilde{F}, \mathrm{E})$. No by hypothesis, there exist $(\tilde{B}_2 , \mathrm{E}) \in B_2^{CNSS}$ such that $(\tilde{B}_2 , \mathrm{E}) \subseteq (\tilde{B}_1 , \mathrm{E}) \Rightarrow (\tilde{B}_2 , \mathrm{E}) \subseteq (\tilde{B}_1 , \mathrm{E}) \subseteq (\tilde{F}, \mathrm{E}) \Rightarrow (\tilde{B}_2 , \mathrm{E}) \subseteq (\tilde{F}, \mathrm{E}) \Rightarrow (\tilde{F}, \mathrm{E}) \in \tau_2^{CNSS}$.This show that $\tau_1^{CNSS} \subseteq \tau_2^{CNSS}$.

Theorem 5.9. Let $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ be a neutrosophjci soft topological space over X and $(\tilde{F}, \mathrm{E}) \in$ NSS (X;E). Then the collection

$$\tau^{CNSS}_{(\tilde{F},E)} = \{(\tilde{F},E) \cap (\tilde{F}_i,E) : (\tilde{F}_i,E) \in \tau^{CNSS}\ for\ i \in I\}$$

is a complex neutrosophic soft topology on $(\tilde{F}, \mathrm{E})$ and $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ is a complex neutrosophic soft topological space.

Proof. Since $0_{(\tilde{X},\mathrm{E})} \cap (\tilde{F},\mathrm{E}) = 0_{(F,\mathrm{E})}$ and $1_{(X,E)} \cap (\tilde{F},\mathrm{E}) = (\tilde{F},\mathrm{E})$, Then $0_{(\tilde{F},\mathrm{E})}$ and $(\tilde{F},\mathrm{E}) \in \tau^{CNSS}_{(\tilde{F},\mathrm{E})}$ Moreover,

$$\bigcap_{i=1}^{n}\left((\tilde{F}_i,\mathrm{E}) \cap (\tilde{F},\mathrm{E})\right) = \left(\bigcap_{i=1}^{n}(\tilde{F}_i,E)\right) \cap (\tilde{F},\mathrm{E})$$

and

$$\bigcup_{i\in I}\left((\tilde{F}_i,\mathrm{E}) \cap (\tilde{F},\mathrm{E})\right) = \left(\bigcup_{i\in I}(\tilde{F}_i,E)\right) \cup (\tilde{F},\mathrm{E})$$

for $\tau^{CNSS} = \{(\tilde{F}_i,\mathrm{E}) : i \in I\}$. Therefore $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$ is a complex neutrosophic soft topology over $(\tilde{F},\mathrm{E})$.

Definition 5.10. Let $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ be a complex neutrosophic soft topological space over X and $\left(\tilde{F},\mathrm{E}\right) \in \mathrm{NSS}\ (\mathrm{X},\mathrm{E})$.Then the collection

$$\tau^{CNSS}_{(\tilde{F},E)} = \left\{\left(\tilde{F},E\right) \cap \left(\tilde{F}_i,E\right) :\ \left(\tilde{F}_i,E\right) \in\ \tau^{CNSS}\, for\ i\ \in I\right\}$$

is called a complex neutrosophic soft subspace topology on $(\tilde{F},\mathrm{E})$ and $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ is called a complex neutrosophic soft topological subspace of $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$.

Theorem 5.11. Let $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ be a complex neutrosophic soft topological space over of $\left(\tilde{F},E\right), \left(\tilde{K},E\right) \in CNSS\ (X,E)$.

1. If B^{CNSS} is a complex neutrosophic soft base for $\tau^{CNSS}, then\ B^{CNSS}_{(\tilde{F},\mathrm{E})} = \left\{\left(\tilde{B},E\right) \cap \left(\tilde{F},E\right) : \left(\tilde{B},E\right) \in B^{CNSS}\right\}$ is a complex neutrosophic soft base for the complex neutrosophic soft sub-topology $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$,
2. If $\left(\tilde{G},E\right)$ is a complex neutrosophic soft set in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}\ and\ \left(\tilde{F},E\right)$ is a complex neutrosophic soft closed set in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$,then $\left(\tilde{F},E\right)$ is a complex neutrosophic soft closed in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$,
3. Let $\left(\tilde{F},E\right) \subseteq \left(\tilde{F},E\right). If\ \overline{\left(\tilde{G},E\right)}$ is the complex neutrosophic soft closure (X, τ^{CNSS}, E),then $\overline{\left(\tilde{G},E\right)} \cap \left(\tilde{F},E\right)$ is a complex neutrosophic soft closure in $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$.

Proof. 1. Since B^{CNSS} is a complex neutrosophic soft base for τ^{CNSS} so for arbitrary $\left(\tilde{U},E\right) \in \tau^{CNSS}$, we have $\left(\tilde{U},E\right) = \bigcup_{(\tilde{B},E)\in B^{CNSS}} \left(\tilde{B},E\right)$.In case,

$$\left(\tilde{U},E\right) \cap \left(\tilde{F},E\right) = \left(\bigcup_{(\tilde{B},E)\in B^{CNSS}} \left(\tilde{B},E\right)\right) \cap \left(\tilde{F},E\right) = \bigcup_{(\tilde{B},E)\in B^{CNSS}} \left(\left(\tilde{B},E\right) \cap \left(\tilde{F},E\right)\right)$$

for $\left(\tilde{U},E\right) \cap \left(\tilde{F},E\right) \in \tau^{CNSS}_{(\tilde{F},\mathrm{E})}$.Since arbitrary member $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$ can be expressed as the union of members of $B^{CNSS}_{(\tilde{F},\mathrm{E})}$.

2. we first show that if $\left(\tilde{G},E\right)$ is a complex neutrosophic soft closed set in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$ then there exist a closed set $\left(\tilde{V},E\right) \subseteq \left(\tilde{K},E\right)\ i.e, \left(\tilde{V},E\right) \notin \tau^{CNSS}\ such\ that \left(\tilde{G},E\right) = \left(\tilde{V},E\right) \cap \left(\tilde{F},E\right)$.

Let $\left(\tilde{G},E\right)$ be a closed in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$. Then $(\tilde{G}_i,E)^c$ is a complex neutrosophic soft open set in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$ i.e, $(\tilde{G}_i,E)^c$ can be put as $(\tilde{G},E)^c = \left(\tilde{U},E\right) \cap \left(\tilde{F},E\right) for \left(\tilde{U},E\right) \in \tau^{CNSS} \Longrightarrow ((\tilde{G},E)^c)^c = \left(\tilde{F},E\right) \cap (\left(\tilde{U},E\right) \cap \left(\tilde{F},E\right))^c = (\tilde{U},E)^c \cap \left(\tilde{F},E\right)$.Here $(\tilde{U},E)^c \notin \tau^{CNSS}$ i.e, $(\tilde{U},E)^c$ is a closed in τ^{CNSS}.So here acts as $\left(\tilde{V},E\right) \subseteq \left(\tilde{K},E\right)$.

Conversely, suppose that $\left(\tilde{G},E\right) = \left(\tilde{V},E\right) \cap \left(\tilde{F},E\right)$ where $\left(\tilde{F},E\right) \subseteq \left(\tilde{K},E\right)$ and $\left(\tilde{V},E\right)$ is closed in $\tau^{CNSS}_{(K,\mathrm{E})}$.

Clearly, $(\tilde{V},E)^c \in \tau^{CNSS}$ so that $(\tilde{V},E)^c \cap \left(\tilde{F},E\right) \in \tau^{CNSS}_{(K,\mathrm{E})}$.Now,

$(\tilde{V},E)^c \cap \left(\tilde{F},E\right) = \left(\left(\tilde{K},E\right) \backslash \left(\tilde{V},E\right)\right) \cap \left(\tilde{F},E\right) = \left(\left(\tilde{K},E\right) \cap \left(\tilde{F},E\right)\right) \backslash \left(\left(\tilde{V},E\right) \cap \left(\tilde{F},E\right)\right) = \left(\tilde{F},E\right) \backslash \left(\tilde{G},E\right)$.This implies $\left(\tilde{F},E\right) \backslash \left(\tilde{G},E\right)$ is a complex neutrosophic soft set in $\left(\tilde{F},E\right) i,e., \left(\tilde{G},E\right)$ is a neutrosophic soft closed set in $\tau^{CNSS}_{(K,\mathrm{E})}$. $\overline{(\tilde{G},E)} = \cap\{(\tilde{G}_i,E) : (\tilde{G}_i,E)\ is\ closed\ and\ (\tilde{G}_i,E) \supseteq (\tilde{G},E)\}$ is the complex neutrosophic soft closure of $(\tilde{G},E)$ and so $\overline{(\tilde{G},E)}$ is a complex neutrosophic soft closed set. Now, $\overline{(\tilde{G},E)} \cap (\tilde{F},E) = \cap\{(\tilde{G}_i,E) : (\tilde{G}_i,E)\ is\ closed\ and\ (\tilde{G}_i,E) \supseteq (\tilde{G},E)\} \cap (\tilde{F},E) = \cap\left((\tilde{G}_i,E) \cap (\tilde{F},E)\right)$.Since each $(\tilde{G}_i,E)$is closed then each $(\tilde{G}_i,E) \cap (\tilde{F},E)$ is closed in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$. Now $(G,E) \subseteq (\tilde{G}_i,E)\ and\ (G,E) \subseteq (\tilde{F},E)$.So $\left((\tilde{G},E) \cap (\tilde{F},E)\right) \subseteq \left((\tilde{G}_i,E) \cap (\tilde{F},E)\right) \Rightarrow (\tilde{G},E) \subseteq (\tilde{G}_i,E) \cap (\tilde{F},E)$.Therefore, $\overline{(\tilde{G},E)} \cap (\tilde{F},E) = \cap\left\{\left((\tilde{G}_i,E) \cap (\tilde{F},E)\right): (\tilde{G}_i,E) \cap (\tilde{F},E)\right.$ is closed and $\left.\left((\tilde{G}_i,E) \cap (\tilde{F},E)\right) \supseteq (\tilde{G}_i,E)\right\}$. Thus $\overline{(\tilde{G},E)} \cap (\tilde{F},E)$ is a complex neutrosophic soft closure of $(\tilde{G},E)$ in $\tau^{CNSS}_{(\tilde{F},\mathrm{E})}$.

□

Theorem 5.12. Let $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$ be a complex neutrosophic soft subspace of a complex neutrosophic soft topological space $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$ over X. If $(\tilde{F},E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E)iff $(\tilde{F}_1,E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E).

Proof. Suppose that $(\tilde{F},E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E) such that a complex neutrosophic soft subset $(\tilde{F}_1,E)$ of $(\tilde{F},E)$ is open set in $\left(X_{(\tilde{F},\mathrm{E})}, \tau^{CNSS}_{(\tilde{F},\mathrm{E})}, E\right)$.Then $(\tilde{F}_1,E) \in \tau^{CNSS}_{(\tilde{F},\mathrm{E})}\ and\ so\ (\tilde{F}_1,E) = (\tilde{U},E) \cap (\tilde{F},E)$ for $(\tilde{U},E) \in \tau^{CNSS}$.But $(\tilde{F}_1,E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E) as $(\tilde{U},E)$ and $(\tilde{F},E)$ both are complex neutrosophic soft open set in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$.

Conversely, assume that $(\tilde{F}_1,E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E) when $(\tilde{F},E)$ is a complex neutrosophic soft open set in (X, τ^{CNSS}, E) and $(\tilde{F}_1,E) \subseteq (\tilde{F},E)$. Then $(\tilde{F}_1,E) \in \tau^{CNSS}$.But $(\tilde{F}_1,E) \cap (\tilde{F},E) = (\tilde{F}_1,E)$ and so $(\tilde{F}_1,E)$ is a complex neutrosophic soft set in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$.Therefore, the first part is proved.□

Theorem 5.13. Let $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$ be a complex neutrosophic soft subspace of a complex neutrosophic soft topological space $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$ over X. If $(\tilde{K},E)$ is a complex neutrosophic soft closed set in (X, τ^{CNSS}, E),then a complex neutrosophic soft set $(\tilde{K}_1,E) \subseteq (\tilde{K},E)$ is a complex neutrosophic soft closed set in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)\ iff\ (\tilde{K}_1,E)$ is a complex neutrosophic soft closed set in (X, τ^{CNSS}, E).

Proof. Suppose that $(\tilde{K},E)$ is a complex neutrosophic soft closed set (X, τ^{CNSS}, E) such that a complex neutrosophic soft subset $(\tilde{K}_1,E)\ or (\tilde{K},E)$ is a complex neutrosophic soft closed set in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$. Since $(\tilde{K}_1,E)$ is closed in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$ and so $(\tilde{K}_1,E) = (\tilde{V},E) \cap (\tilde{K},E)\ for$ $(\tilde{V},E)$ being complex neutrosophic soft closed set in (X, τ^{CNSS}, E).But $(\tilde{K}_1,E)$ is a complex neutrosophic soft closed set in (X, τ^{CNSS}, E) as $(\tilde{V}, E)$ and $(\tilde{K}, E)$ both are complex neutrosophic soft closed sets in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$.

Conversely, assume that $(\tilde{F}_1, E)$ is a complex neutrosophic soft open set in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$when $(\tilde{K}, E)$ is complex neutrosophic soft closed set in (X, τ^{CNSS}, E) and $(\tilde{K}_1, E) \subseteq (\tilde{K}, E)$.Then $(\tilde{K}_1, E) \cap (\tilde{K}, E) = (\tilde{K}_1, E)$ and so $(\tilde{K}_1, E)$ is a complex neutrosophic soft closed in $\left(X_{(\tilde{K},\mathrm{E})}, \tau^{CNSS}_{(\tilde{K},\mathrm{E})}, E\right)$. Hence the first part is proved.

6. Application

In this section, the application about the performance of network is defined. We studied the existing research, in the field of networking and identify commonly discussed factors and challenges affecting network performance. Interval valued complex neutrosophic fuzzy relations (IVCNFRs) used to establish the performance of networking with different factors. Also, to simulate a network's behavior in a controlled setting, a virtual model of the network is created. This makes it possible to experiment with different parameters and see how they affect performance. Tools for network simulation, like NS-3 or Cisco Packet Tracer, allow various scenarios to be tested. Performance of network depends on the different factors that affect the network.

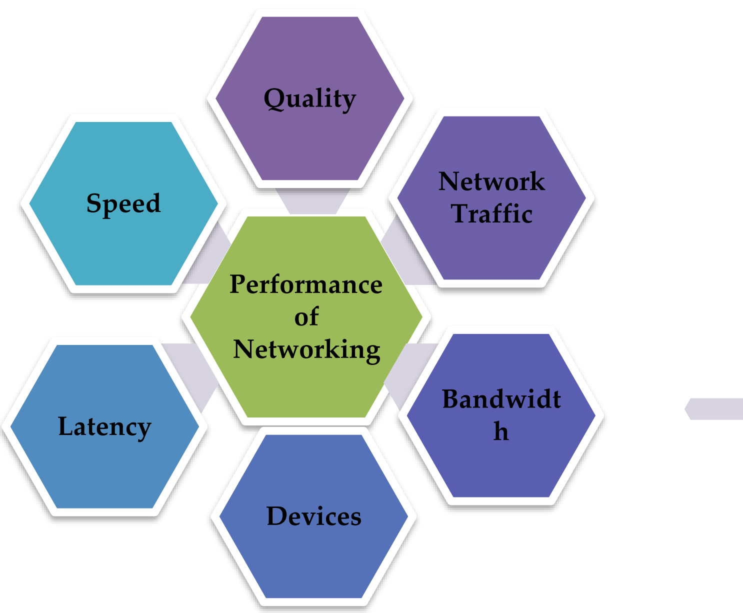

Figure 1. Performance of network.

4.1. Performance of Networking

The process of transporting and exchanging the data among different nodes with a shared medium in an information system is known as networking. The performance of a network describes the measurement of the quality of a network considered by the user. Different techniques are used to measure the performance of a network, depending on the nature and design of the network. The complex and dynamic nature of computer networks is reflected in the statement "network performance/status depends on different factors". Networks are complex systems made up of many interconnected parts, and a wide range of factors affect how they function. There are different factors that affect the performance of a network. These factors are as follows:

1) Bandwidth
2) Number of devices
3) Network traffic
4) Latency

Normal network transmission

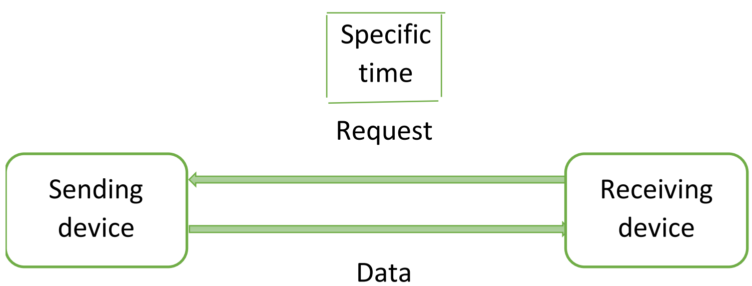

Figure 2. Normal network transmission.

The maximum amount of data transmitted in a transmission medium with some time period is known as bandwidth. With the use of grater bandwidth more devices can connect at once. But it may affect the speed of overall network. Small amount of data with bandwidth in more devices can easily and quickly transfer but large amount of data takes time to transfer using transmission medium. Devices are an important part of any network. Data is transmitted through bandwidth from one device to other. Large amount of data can transfer through the small packets otherwise data loss or data errors occur. These packets can combine before receiving device. This all manage through network traffic. Latency is the time that takes some data to transfer. Usually, buffering is caused by latency.

Network with high latency

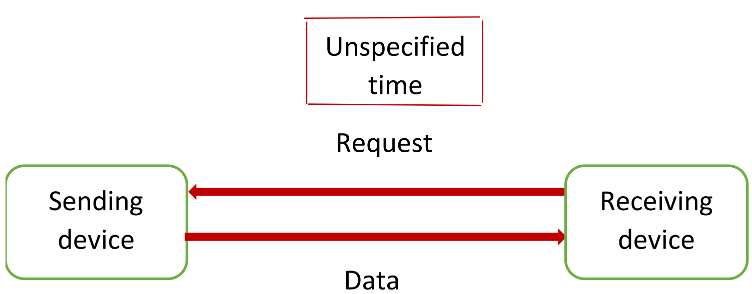

Figure 3. Effect of latency on network.

IVCNRs are used to examine the performance of networking on Bandwidth, Number of devices, Network traffic, and Latency. Modeling problems by using the idea of IVCNSs and IVCNRs will not only formulate the effects of one factor to other but also defines the grades of membership, abstains and non-membership. Let B, D, NT and L symbolize the bandwidth, number of devices, network traffic and latency, respectively. So, the set of the factors is given as

$$\mathbb{F}=\left\{\begin{matrix}\left(B,\left([0.4,0.8]e^{2\pi i[0.2,0.5]}\right),\left([0.1,0.6]e^{2\pi i[0.1,0.5]}\right),\left([0,0.3]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left(D,\left([0.3,0.6]e^{2\pi i[0.2,0.6]}\right),\left([0,0.3]e^{2\pi i[0.3,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.1,0.6]}\right)\right),\\ \left(NT,\left([0.2,0.7]e^{2\pi i[0.1,0.5]}\right),\left([0,0.5]e^{2\pi i[0,0.6]}\right),\left([0.1,0.5]e^{2\pi i[0.1,0.4]}\right)\right),\\ \left(L,\left([0.4,0.7]e^{2\pi i[0,0.4]}\right),\left([0.2,0.4]e^{2\pi i[0.5,0.7]}\right),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right)\right)\end{matrix}\right\}$$

The relation Ω of CP is given below

$$\Omega=\left\{\begin{matrix}\left((B,D),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right),\left([0,0.3]e^{2\pi i[0.1,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left((B,NT),\left([0.2,0.7]e^{2\pi i[0.1,0.5]}\right),\left([0,0.5]e^{2\pi i[0,0.5]}\right),\left([0.1,0.5]e^{2\pi i[0.1,0.4]}\right)\right),\\ \left((B,L),\left([0.4,0.7]e^{2\pi i[0,0.4]}\right),\left([0.1,0.4]e^{2\pi i[0.1,0.5]}\right),\left([0.3,0.6]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left((D,B),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right),\left([0,0.3]e^{2\pi i[0.1,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left((D,NT),\left([0.2,0.6]e^{2\pi i[0.1,0.5]}\right),\left([0,0.3]e^{2\pi i[0,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.1,0.6]}\right)\right),\\ \left((D,L),\left([0.3,0.6]e^{2\pi i[0,0.4]}\right),\left([0,0.3]e^{2\pi i[0.3,0.5]}\right),\left([0.3,0.6]e^{2\pi i[0.2,0.6]}\right)\right),\\ \left((NT,B),\left([0.2,0.7]e^{2\pi i[0.1,0.5]}\right),\left([0,0.5]e^{2\pi i[0,0.5]}\right),\left([0.1,0.5]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left((NT,D),\left([0.2,0.6]e^{2\pi i[0.1,0.5]}\right),\left([0,0.3]e^{2\pi i[0,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.1,0.6]}\right)\right),\\ \left((NT,L),\left([0.2,0.7]e^{2\pi i[0,0.4]}\right),\left([0,0.4]e^{2\pi i[0,0.6]}\right),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right)\right),\\ \left((L,B),\left([0.4,0.7]e^{2\pi i[0,0.4]}\right),\left([0.1,0.4]e^{2\pi i[0.1,0.5]}\right),\left([0.3,0.6]e^{2\pi i[0.3,0.6]}\right)\right),\\ \left((L,NT),\left([0.2,0.7]e^{2\pi i[0,0.4]}\right),\left([0,0.4]e^{2\pi i[0,0.6]}\right),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right)\right),\\ \left((L,D),\left([0.3,0.6]e^{2\pi i[0,0.4]}\right),\left([0,0.3]e^{2\pi i[0.3,0.5]}\right),\left([0.3,0.6]e^{2\pi i[0.2,0.6]}\right)\right)\end{matrix}\right\}$$

The event $\left((B,D),\left([0.3,0.6]e^{2\pi i[0.2,0.5]}\right),\left([0,0.3]e^{2\pi i[0.1,0.5]}\right),\left([0.2,0.5]e^{2\pi i[0.3,0.6]}\right)\right)$ describes the membership grade from 0.3 to 0.6 with better-quality bandwidth that provides better transfer rate if the number of devices is limited with some time interval. Absence of bandwidth from 0 to 0.3 provides no effect on the number of devices with the time interval. Also, non-membership from 0.2 to 0.5 describes the low-quality bandwidth that creates difficulties in transferring data and information through connected devices with particular time. The membership defines the good performance of different relations, non-membership discusses the bad or low-quality network and abstains provides no effect or neutral effects on the performance of network using different factors.

7. Comparative Analysis

In this section, we describe the comparison between different presented and existing methods like FR, IVFR, IVIFR, IVCq-ROFR, IVNR with IVCNR was described. The idea of IVCNSs and IVCNRs are incredible of all above methods and ideas to manipulate the fuzziness. With the use of these methods in the performance of network one can manage the quality and speed of the overall network by tackle any problem with using grades (membership, non-membership and abstinence). In these sets we discuss about the membership grade, abstinence and non-membership grades for finding a solution of any problem. Construction of IVCNR is made out of the intervals of amplitude term and phase term, which permit it to project the situations with phase modification and periodicity. This application discusses the performance of networking over different factors. The performance of one factor on the other effect the whole network performance. 7.1. FS vs IVCNS FS

defines the membership grades for any problem, but IVCNS is an improved method rather than FS. IVCNS determine not only membership grade of any problem it also defines the non-membership grade and abstains for the purpose of problem solving.

7.2. IVFS vs IVCNS

IVFS discusses the conception of intervals in membership grads. The intervals break down the single value of membership into the lower and upper value of membership grades. But IVFS have limitations that it only discusses about membership grades. Since IVCNS defines the good performance, neutral and bad performance of one factor to the other in intervals. So, IVCNS is a better conception than IVFS.

7.3. IVIFS vs IVCNS

IVIFS describes the formulation of intervals in the membership and non-membership grades. IVIFS discusses the performance of different factors in the term of good and bad effects. On the other hand, IVCNS have no limitations of good and bad effects it is step forward to good, bad and neutral values so that the performance of one factor to the other specifies these effects.

7.4. IVCq-ROFS vs IVCNS

IVCNS is step forward than IVCq-ROFS. IVCq-ROFS determines only the complex intervals of membership and non-membership grades for fixing any problem. But IVCNS provides the membership grade and non-membership grade and neutral values for performing any task to do.

7.5. IVNS vs IVCNS

IVCNS is one step higher than IVNS. IVNS provides the intervals of membership grade, abstains and non-membership grades whereas IVCNS provides the complex intervals of membership grade, abstains and non-membership grades.

Sets	Membership	Non-membership	Abstains	Multi-dimensional	Remarks
FS	Yes	No	No	No	Contains single value of membership
IVFS	Yes	No	No	No	Contains interval of single value
IVIFS	Yes	Yes	No	No	Intervals of membership and non-membership.
IVCq-ROFS	Yes	Yes	No	Yes	Is provides multidimensional values in intervals.
IVNS	Yes	Yes	Yes	No	Having good, neutral and bad influences.
IVCNS	Yes	Yes	Yes	Yes	It consists of multidimensional complex intervals.

Table 1. Comparison of FS, IVFS, IVIFS, IVCq-ROFS, IVNS and IVCNS.

8. Conclusions

The major purpose of this study was to represent the conception of IVCNRs that is a new concept. The IVCNSs and IVCNRs are ranging in three complex valued mappings of membership, abstinence, and non-membership grades in a complex plane. Each degree consists of two terms that are discuss as real and imaginary parts. Also, each term consists of the interval from lower value to high value. The real term known as amplitude term and imaginary term is called phase term of the degrees. Moreover, the types of IVCNRs also define with examples. IVCNRs used to produce better and useful results. IVCNRs used to implement the factor affecting the performance of network. In continuation this study introduced and investigates the structure of complex neutrosophic soft topological spaces. The foundational definitions of complex neutrosophic soft topology, open and closed sets, interior, closure, and boundary are formally established. The study also explores the concept of complex neutrosophic soft bases and subspace topologies, along with criteria for basis generation and topological refinement. Several theorems elucidate the relationships among topological constructs and operations such as union, intersection, and complementation under complex neutrosophic soft conditions. We apply pervious methods on these problems and collect some results. But through this method, the required results achieved more reliable than the previous methods. So, the proposed method is the best method for modeling uncertain complexities in the required results. Some applications are also given that can be applied in our day to day life.

The application provides important results by using the proposed method. IVCNRs is the best method for the purpose of improving the quality and speed of the computer network. A comparative study is having been established between proposed methodology to existing methods. Future research can explore extending IVCNRs to multi-criteria decision-making and integrating them with machine learning for dynamic network optimization. Further studies may also focus on developing software tools, applying IVCNRs in other fields like healthcare and finance, and expanding the theoretical framework of complex neutrosophic soft topologies. Empirical validation and comparative analysis with existing models will enhance the practical reliability of the proposed approach.

Author Contributions: All authors equally contributed.

Conflicts of Interest: The author(s) declare(s) that there are no conflicts of interest regarding the publication of this paper.

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