Int. J. Anal. Appl. (2025), 23:139

International Journal of Analysis and Applications

On Tripolar Complex Fuzzy Sets and Their Application in Ordered Semigroups

Nuttapong Wattanasiripong[1], **Nuchanat Tiprachot**[2], **Somsak Lekkoksung**[3,*]

[1]*Division of Applied Mathematics, Faculty of Science and Technology, Valaya Alongkorn Rajabhat University under the Royal Patronage, Pathum Thani 13180, Thailand*

[2]*Mathematics Department (Secondary), Demonstration School of Khon Kaen University (Modindaeng), Khon Kaen University, Khon Kaen 40002, Thailand*

[3]*Division of Mathematics, Faculty of Engineering, Rajamangala University of Technology Isan, Khon Kaen Campus, Khon Kaen 40000, Thailand*

[*]*Corresponding author:* lekkoksung_somsak@hotmail.com

Abstract. The tripolar complex fuzzy set is a generalization of the tripolar fuzzy sets. In this paper, we introduce the notion of tripolar complex fuzzy sets in ordered semigroups. The concepts of tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (right, two-sided) ideals are introduced. Some algebraic properties of such tripolar complex fuzzy subsemigroups and their tripolar complex fuzzy ideals are studied. We characterize subsemigroup and left (resp., right, two-sided) ideals by using tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (resp., right, two-sided) ideals. Finally, we characterized intra-regular ordered semigroups in terms of tripolar complex fuzzy left ideals and tripolar complex fuzzy right ideals.

1. Introduction

The theory of fuzzy sets, which is the most appropriate theory for dealing with uncertainty, was first introduced by Zadeh [1] in 1965. After Zadeh's introduction of fuzzy sets, several researchers explored generalizations of these notions, leading to significant applications in various fields such as computer science, artificial intelligence, control engineering, robotics, automata theory, decision theory, finite state machines, graph theory, logic, operations research, and many branches of pure and applied mathematics.

The idea of a complex fuzzy set was first introduced by D. Ramot et al. [2,3]. The complex fuzzy set is a generalization of fuzzy sets. Y. B. Jun and X. L. Xin [4] were the first applied complex fuzzy

Received: Apr. 8, 2025.

2020 *Mathematics Subject Classification.* 06F05, 03E72, 08A72.

Key words and phrases. ordered semigroup; intra-regular ordered semigroups; complex fuzzy set; complex fuzzy left ideal; complex fuzzy right ideal; complex fuzzy two-sided ideal.

https://doi.org/10.28924/2291-8639-23-2025-139
ISSN: 2291-8639

set theory to BCK/BCI-algebras theory. R. Rasuli [5] applied complex fuzzy sets to Lie algebras and studied complex fuzzy ideals of Lie algebras with respect to t-norms. After that, many researcher have applied complex fuzzy sets theory to many structures of algebras (see [6–11]).

M. Murali Krishna Rao and B. Venkateswarlu [12] were first introduced the concept of tripolar fuzzy set theory. N. Wattanasiripong et al. applied the tripolar fuzzy set theory to the ordered semigroup theory (see [13–16]). The tripolar complex fuzzy set is a generalization of tripolar fuzzy sets. This present paper, we introduce the notion of tripolar complex fuzzy sets in ordered semigroups. The concepts of tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (right, two-sided) ideals are introduced. Some algebraic properties of such tripolar complex fuzzy subsemigroups and their tripolar complex fuzzy ideals are studied. We characterized subsemigroups and left (resp., right, two-sided) ideals by using tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (resp., right, two-sided) ideals. Finally, we characterized intra-regular ordered semigroups in terms of tripolar complex fuzzy left ideals and tripolar complex fuzzy right ideals.

2. Preliminary

In this section, we will recall the basic terms, and definitions from the ordered semigroup theory and the tripolar complex fuzzy set theory that we will use in this paper.

Let S be a nonempty set, and a binary operation "$\cdot$" on the set S. The structure $\langle S; \cdot \rangle$ is called *groupoid*. If the binary operation "$\cdot$" satisfied associative property, that is

$$x \cdot (y \cdot z) = (x \cdot y) \cdot z$$

for all $x, y, z \in S$, the groupoid $\langle S; \cdot \rangle$ is called *semigroup*.

A binary relation $\leq$ on S is called *partial relation on S* if it satisfies the following three properties: (1) reflexive, i.e. $a \leq a$ for all $a \in S$. (2) antisymmetric, i.e., if $a \leq b$ and $b \leq a$, then $a = b$ for all $a, b \in S$. (3) transitive, i.e., if $a \leq b$, and $b \leq c$, then $a \leq c$ for all $a, b, c \in S$. The structure $\langle S; \leq \rangle$ is called a *partially ordered set* if the relation $\leq$ is a partial relation on S.

Definition 2.1. *[17] The structure $\langle S; \cdot, \leq \rangle$ is called an ordered semigroup if the following conditions are satisfied:*

(1) *$\langle S; \cdot \rangle$ is a semigroup.*
(2) *$\langle S; \leq \rangle$ is a partially ordered set.*
(3) *For every $a, b, c \in S$ if $a \leq b$, then $a \cdot c \leq b \cdot c$ and $c \cdot a \leq c \cdot b$.*

For simplicity, we denoted an ordered semigroup $\langle S; \cdot, \leq \rangle$ by its carrier set as a bold letter $\mathbf{S}$, and if $a, b \in S$, we will instead of $a \cdot b$ by ab. Let A and B be two nonempty subsets of S. Then we define

$$AB := \{ab \ : \ a \in A \text{ and } b \in B\}.$$

Let $\mathbf{S}$ be an ordered semigroup. A nonempty subset A of S is called a *subsemigroup* of $\mathbf{S}$ [17] if $AA \subseteq A$.

Definition 2.2. *[17] Let* **S** *be an ordered semigroup. A nonempty subset A of S is called a left ideal of* **S** *if it satisfies*

(1) $SA \subseteq A$.
(2) *For $x, y \in S$, if $x \leq y$ and $y \in A$, then $x \in A$.*

Definition 2.3. *[17] Let* **S** *be an ordered semigroup. A nonempty subset A of S is called a right ideal of* **S** *if it satisfies*

(1) $AS \subseteq A$.
(2) *For $x, y \in S$, if $x \leq y$ and $y \in A$, then $x \in A$.*

A nonempty subset of S is called *two-sided ideal* or simply *ideal* if it is both a left and a right ideal of **S**.

A fuzzy subset (or fuzzy set) of a nonempty subset X is a mapping $f : X \to [0,1]$ from X to a unit closed interval (see [1]).

Definition 2.4. *Let X be a nonempty set. A tripolar complex fuzzy set f of X is an object having the form*

$$f := \left\{\left(x, f^+(x), f^*(x), f^-(x)\right) : x \in X,\ 0 \leq f^+(x) + f^*(x) \leq 1\right\},$$

such that $f^+(x) = \mu^+(x) + i\nu^+(x)$, $f^*(x) = \mu^*(x) + i\nu^*(x)$, and $f^-(x) = \mu^-(x) + i\nu^-(x)$ where $\mu^+, \nu^+, \mu^*, \nu^* : X \to [0,1]$, $\mu^-, \nu^- : X \to [-1,0]$, and $i = \sqrt{-1}$ such that $0 \leq f^+(x) + f^*(x) \leq 1$, it means that $0 \leq \mu^+(x) + \mu^*(x) \leq 1$ and $0 \leq \nu^+(x) + \nu^*(x) \leq 1$.

For the sake of simplicity, we will use the symbol $f = (f^+, f^*, f^-)$ for $f = \{(x, f^+(x), f^*(x), f^-(x)) : x \in X,\ 0 \leq f^+(x) + f^*(x) \leq 1\}$ and throughout the paper, we write $f = (f^+, f^*, f^-)$ instead of $f = (f^+ = \mu^+ + i\nu^+, f^* = \mu^* + i\nu^*, f^- = \mu^- + i\nu^-)$.

Remark 2.1. *It is easy to see that if $\nu^+(x) = 0, \nu^*(x) = 0$ and $\nu^-(x) = 0$ for all $x \in X$, then by Definition 2.4, the tripolar complex fuzzy set f become tripolar fuzzy set. This means that the tripolar complex fuzzy set is a generalization of tripolar fuzzy set.*

Example 2.1. *Let $H = \{a_1, a_2, a_3, a_4, a_5\}$ be the set of five employees in a company. We will characterize them according to four qualities in the form of tripolar complex fuzzy set, given in the follows:*

	Honesty	*Punctual*	*Communication*	*Hardworking*
a_1	0.6	0.5	0.8	1
a_2	1	0.8	0.5	0.4
a_3	0.5	1	1	0.8
a_4	0.8	0.5	1	0.7
a_5	1	0.5	0	0.6

We define tripolar complex fuzzy set $f = (f^+, f^*, f^-)$ as follows.

X	$f^+(x)$	$f^*(x)$	$f^-(x)$
a_1	$0.6 + i(0.5)$	$0.3 + i(0.5)$	$-0.2 + i(-0.5)$
a_2	$1 + i(0.8)$	$0 + i(0.1)$	$-1 + i(-0.3)$
a_3	$0.1 + i(1)$	$0.8 + i(0)$	$-0.1 + i(-0.5)$
a_4	$0.8 + i(0.3)$	$0.1 + i(0.3)$	$-1 + i(-0.2)$
a_5	$1 + i(0)$	$0 + i(0.8)$	$-0.7 + i(-0.4)$

We denoted by $TCF(X)$ the set of all tripolar complex fuzzy sets of X. Now we will give the relation, and the operation on $TCF(X)$ as follows.

Definition 2.5. *Let $f_1 = (f_1^+, f_1^*, f_1^-)$ and $f_2 = (f_2^+, f_2^*, f_2^-)$ be elements of $TCF(X)$. Then for each $x \in X$,*

(1) $f_1 \sqsubseteq f_2$ *if*

(1.1) $\mu_1^+(x) \leq \mu_2^+(x)$ *and* $\nu_1^+(x) \leq \nu_2^+(x)$.

(1.2) $\mu_1^*(x) \geq \mu_2^*(x)$ *and* $\nu_1^*(x) \geq \nu_2^*(x)$.

(1.3) $\mu_1^-(x) \geq \mu_2^-(x)$ *and* $\nu_1^-(x) \geq \nu_2^-(x)$.

(2) $f_1 \sqcap f_2 = (f_1^+ \cap f_2^+, f_1^* \cup f_2^*, f_1^- \cup f_2^-)$ *is an element of $TCF(X)$, and is defined by*

(2.1) $(f_1^+ \cap f_2^+)(x) := (\mu_1^+ \cap \mu_2^+)(x) + i(\nu_1^+ \cap \nu_2^+)(x)$ *where*

$$(\mu_1^+ \cap \mu_2^+)(x) := \min\{\mu_1^+(x), \mu_2^+(x)\} \text{ and } (\nu_1^+ \cap \nu_2^+)(x) := \min\{\nu_1^+(x), \nu_2^+(x)\}.$$

(2.2) $(f_1^* \cup f_2^*)(x) := (\mu_1^* \cup \mu_2^*)(x) + i(\nu_1^* \cup \nu_2^*)(x)$ *where*

$$(\mu_1^* \cup \mu_2^*)(x) := \max\{\mu_1^*(x), \mu_2^*(x)\} \text{ and } (\nu_1^* \cup \nu_2^*)(x) := \max\{\nu_1^*(x), \nu_2^*(x)\}.$$

(2.3) $(f_1^- \cup f_2^-)(x) := (\mu_1^- \cup \mu_2^-)(x) + i(\nu_1^- \cup \nu_2^-)(x)$ *where*

$$(\mu_1^- \cup \mu_2^-)(x) := \max\{\mu_1^-(x), \mu_2^-(x)\} \text{ and } (\nu_1^- \cup \nu_2^-)(x) := \max\{\nu_1^-(x), \nu_2^-(x)\}.$$

It is easy to see that the structure $\langle TCF(X); \sqsubseteq\rangle$ is a partially ordered set. Let $\mathbf{S}$ be an ordered semigroup and $a \in S$. We set

$$\mathbf{S}_a := \{(x, y) \in S \times S : a \leq xy \text{ for some } x, y \in S\}.$$

We now define operation on $TCF(S)$ as follows.

Definition 2.6. *Let $f_1 = (f_1^+, f_1^*, f_1^-)$ and $f_2 = (f_2^+, f_2^*, f_2^-)$ be elements of $TCF(S)$. The product of f_1 and f_2 is an element of $TCF(S)$, denoted by $f_1 \diamond f_2 := (f_1^+ \diamond f_2^+, f_1^* \diamond f_2^*, f_1^- \diamond f_2^-)$, and is defined as follows. For each $a \in S$,*

(1) $(f_1^+ \diamond f_2^+)(a) := (\mu_1^+ \diamond \mu_2^+)(a) + i(\nu_1^+ \diamond \nu_2^+)(a)$, *where*

$$\left(\mu_1^+ \diamond \mu_2^+\right)(a) := \begin{cases} \bigvee_{(x,y) \in \mathbf{S}_a} \{\min\{\mu_1^+(x), \mu_2^+(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\left(\nu_1^+ \diamond \nu_2^+\right)(a) := \begin{cases} \bigvee\limits_{(x,y)\in \mathbf{S}_a} \{\min\{\nu_1^+(x), \nu_2^+(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

(2) $(f_1^* \diamond f_2^*)(a) := (\mu_1^* \diamond \mu_2^*)(a) + i(\nu_1^* \diamond \nu_2^*)(a)$, *where*

$$\left(\mu_1^* \diamond \mu_2^*\right)(a) := \begin{cases} \bigwedge\limits_{(x,y)\in \mathbf{S}_a} \{\max\{\mu_1^*(x), \mu_2^*(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 1 & \text{otherwise,} \end{cases}$$

and

$$\left(\nu_1^* \diamond \nu_2^*\right)(a) := \begin{cases} \bigwedge\limits_{(x,y)\in \mathbf{S}_a} \{\max\{\nu_1^*(x), \nu_2^*(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 1 & \text{otherwise,} \end{cases}$$

(3) $(f_1^- \diamond f_2^-)(a) := (\mu_1^- \diamond \mu_2^-)(a) + i(\nu_1^- \diamond \nu_2^-)(a)$, *where*

$$\left(\mu_1^- \diamond \mu_2^-\right)(a) := \begin{cases} \bigwedge\limits_{(x,y)\in \mathbf{S}_a} \{\max\{\mu_1^-(x), \mu_2^-(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\left(\nu_1^- \diamond \nu_2^-\right)(a) := \begin{cases} \bigwedge\limits_{(x,y)\in \mathbf{S}_a} \{\max\{\nu_1^-(x), \nu_2^-(y)\}\} & \text{if } \mathbf{S}_a \neq \emptyset \\ 0 & \text{otherwise.} \end{cases}$$

Let f, g and h be elements of $TCF(S)$. If $f \sqsubseteq g$, then by the definitions of the relation $\sqsubseteq$, and the operation $\diamond$, it is easy to see that

$$f \diamond h \sqsubseteq g \diamond h, \text{ and } h \diamond f \sqsubseteq h \diamond g.$$

The structure $\langle TCF(S); \diamond, \sqsubseteq\rangle$ is an ordered semigroup, and its called a *tripolar complex fuzzy ordered semigroup*.

3. Main Results

In this main section, we introduce the concepts of tripolar complex fuzzy subsemigroups, tripolar complex fuzzy left (right, two-sided) ideals, and study algebraic properties of tripolar complex fuzzy subsemigroups, and their corresponding tripolar complex fuzzy ideals. The relations between subsemigroups (resp., left ideals, right ideals and two-sided ideals), and tripolar complex fuzzy subsemigroups (resp., tripolar complex fuzzy left ideals, tripolar complex fuzzy right ideals, tripolar complex fuzzy two-sided ideals) are investigated. Finally, we characterized intra-regular ordered semigroups in terms of tripolar complex fuzzy left ideals, and tripolar complex fuzzy right ideals.

Definition 3.1. *Let* $\mathbf{S}$ *be an ordered semigroup. A tripolar complex fuzzy set* $f = (f^+, f^*, f^-)$ *of* S *is called a tripolar complex fuzzy subsemigroup of* $\mathbf{S}$ *if the following conditions are satisfied. For each* $x, y \in S$,

(1) $\mu^+(xy) \geq \min\{\mu^+(x), \mu^+(y)\}$ *and* $\nu^+(xy) \geq \min\{\nu^+(x), \nu^+(y)\}$.
(2) $\mu^*(xy) \leq \max\{\mu^*(x), \mu^*(y)\}$ *and* $\nu^*(xy) \leq \max\{\nu^*(x), \nu^*(y)\}$.

(3) $\mu^-(xy) \leq \max\{\mu^-(x), \mu^-(y)\}$ *and* $\nu^-(xy) \leq \max\{\nu^-(x), \nu^-(y)\}$.

Example 3.1. *Let* $S = \{a, b, c, d, e, k\}$ *with the following operation "$*$", and the order "$\leq$" on* S *be defined as follows:*

$*$	a	b	c	d	e	k
a	a	a	a	d	a	a
b	a	b	b	d	b	b
c	a	b	c	d	e	e
d	a	a	d	d	d	d
e	a	b	c	d	e	e
k	a	b	c	d	e	k

and

$$\leq := \{(k, e)\} \cup \Delta_S,$$

where Δ_S *is the identity relation on* S. *It is easy to verify that* $\mathbf{S} = \langle S; *, \leq \rangle$ *is an ordered semigroup. We define tripolar complex fuzzy set* $f = (f^+, f^*, f^-)$ *as follows.*

S	$f^+(x)$	$f^*(x)$	$f^-(x)$
a	$0.4 + i(0.5)$	$0.3 + i(0.4)$	$-0.1 + i(-0.6)$
b	$0.4 + i(0.5)$	$0.3 + i(0.4)$	$-0.1 + i(-0.6)$
c	$0.4 + i(0.5)$	$0.3 + i(0.4)$	$-0.1 + i(-0.6)$
d	$0.6 + i(0.3)$	$0.1 + i(0.6)$	$-0.3 + i(-0.4)$
e	$0.6 + i(0.3)$	$0.1 + i(0.6)$	$-0.3 + i(-0.4)$
k	$0.6 + i(0.3)$	$0.1 + i(0.6)$	$-0.3 + i(-0.4)$

It is easy to see that f *is a tripolar complex fuzzy subsemigroup of* $\mathbf{S}$.

Definition 3.2. *Let* $\mathbf{S}$ *be an ordered semigroup. A tripolar complex fuzzy set* $f = (f^+, f^*, f^-)$ *of* S *is called a tripolar complex fuzzy left ideal of* $\mathbf{S}$ *if the following conditions are satisfied. For each* $x, y \in S$,

(1) $\mu^+(xy) \geq \mu^+(y)$ *and* $\nu^+(xy) \geq \nu^+(y)$.
(2) $\mu^*(xy) \leq \mu^*(y)$ *and* $\nu^*(xy) \leq \nu^*(y)$.
(3) $\mu^-(xy) \leq \mu^-(y)$ *and* $\nu^-(xy) \leq \nu^-(y)$.
(4) *If* $x \leq y$, *then*
  (4.1) $\mu^+(x) \geq \mu^+(y)$ *and* $\nu^+(x) \geq \nu^+(y)$.
  (4.2) $\mu^*(x) \leq \mu^*(y)$ *and* $\nu^*(x) \leq \nu^*(y)$.
  (4.3) $\mu^-(x) \leq \mu^-(y)$ *and* $\nu^-(x) \leq \nu^-(y)$.

Example 3.2. *Let $S = \{k, l, m, n\}$ with the following operation "$*$" and the order "$\preceq$" on S be defined as follows:*

$*$	k	l	m	n
k	k	k	k	k
l	k	k	k	k
m	k	k	l	l
n	k	k	l	k

and

$$\preceq := \{(k,n), (l,n), (m,n)\} \cup \Delta_S,$$

where Δ_S is the identity relation on S. It is easy to verify that $\mathbf{S} = \langle S; *, \preceq \rangle$ is an ordered semigroup. We define tripolar complex fuzzy set $f = (f^+, f^*, f^-)$ as follows.

S	$f^+(x)$	$f^*(x)$	$f^-(x)$
k	$0.4 + i(0.5)$	$0.2 + i(0.4)$	$-0.5 + i(-0.5)$
l	$0.3 + i(0.4)$	$0.3 + i(0.4)$	$-0.5 + i(-0.5)$
m	$0.2 + i(0.3)$	$0.5 + i(0.5)$	$-0.2 + i(-0.4)$
n	$0.2 + i(0.3)$	$0.5 + i(0.6)$	$-0.2 + i(-0.4)$

It is easy to see that f is a tripolar complex fuzzy left ideal of $\mathbf{S}$.

Definition 3.3. *Let $\mathbf{S}$ be an ordered semigroup. A tripolar complex fuzzy set $f = (f^+, f^*, f^-)$ of S is called a tripolar complex fuzzy right ideal of $\mathbf{S}$ if the following conditions are satisfied. For each $x, y \in S$,*

(1) $\mu^+(xy) \geq \mu^+(x)$ *and* $\nu^+(xy) \geq \nu^+(x)$.
(2) $\mu^*(xy) \leq \mu^*(x)$ *and* $\nu^*(xy) \leq \nu^*(x)$.
(3) $\mu^-(xy) \leq \mu^-(x)$ *and* $\nu^-(xy) \leq \nu^-(x)$.
(4) *If $x \preceq y$, then*
  (4.1) $\mu^+(x) \geq \mu^+(y)$ *and* $\nu^+(x) \geq \nu^+(y)$.
  (4.2) $\mu^*(x) \leq \mu^*(y)$ *and* $\nu^*(x) \leq \nu^*(y)$.
  (4.3) $\mu^-(x) \leq \mu^-(y)$ *and* $\nu^-(x) \leq \nu^-(y)$.

Example 3.3. *Consider the ordered semigroup $\mathbf{S} = \langle S; *, \preceq \rangle$ given in Example 3.2. We define a tripolar complex fuzzy set $f = (f^+, f^*, f^-)$ as follows.*

S	$f^+(x)$	$f^*(x)$	$f^-(x)$
k	$0.40 + i(0.55)$	$0.09 + i(0.18)$	$-0.40 + i(-0.55)$
l	$0.29 + i(0.45)$	$0.29 + i(0.28)$	$-0.29 + i(-0.45)$
m	$0.19 + i(0.42)$	$0.39 + i(0.35)$	$-0.19 + i(-0.42)$
n	$0.19 + i(0.42)$	$0.45 + i(0.45)$	$-0.19 + i(-0.42)$

It is easy to see that f is a tripolar fuzzy complex fuzzy right ideal of $\mathbf{S}$.

A tripolar complex fuzzy set f of S is called a *tripolar complex fuzzy two-sided ideal of* $\mathbf{S}$ (or *tripolar complex fuzzy ideal of* $\mathbf{S}$) if f is both a tripolar complex fuzzy left and a tripolar complex fuzzy right ideal of $\mathbf{S}$.

Example 3.4. *Consider the ordered semigroup $\mathbf{S} = \langle S; *, \leq \rangle$ given in Example 3.2. We define a tripolar complex fuzzy set $f = (f^+, f^*, f^-)$ as follows.*

S	$f^+(x)$	$f^*(x)$	$f^-(x)$
k	$0.65 + i(0.55)$	$0.35 + i(0.45)$	$-0.65 + i(-0.55)$
l	$0.35 + i(0.45)$	$0.35 + i(0.45)$	$-0.35 + i(-0.45)$
m	$0.35 + i(0.45)$	$0.35 + i(0.45)$	$-0.35 + i(-0.45)$
n	$0.35 + i(0.45)$	$0.65 + i(0.55)$	$-0.35 + i(-0.45)$

It is easy to see that f is a tripolar complex fuzzy ideal of $\mathbf{S}$.

Next propositions, we study the algebraic properties of tripolar complex fuzzy subsemigroups and their tripolar complex fuzzy ideals as follows.

Proposition 3.1. *Let $\mathbf{S}$ be an ordered semigroup and let f_1, f_2 be tripolar complex fuzzy subsemigroups of $\mathbf{S}$. Then $f_1 \sqcap f_2$ is a tripolar complex fuzzy subsemigroup of $\mathbf{S}$.*

Proof. Let $f_1 = (f_1^+, f_1^*, f_1^-)$, $f_2 = (f_2^+, f_2^*, f_2^-)$ be tripolar complex fuzzy subsemigroups of $\mathbf{S}$, and $x, y \in S$. Let us consider as follows,

$$\begin{aligned}
(\mu_1^+ \cap \mu_2^+)(xy) &= \min\{\mu_1^+(xy), \mu_2^+(xy)\} \\
&\geq \min\{\min\{\mu_1^+(x), \mu_1^+(y)\}, \min\{\mu_2^+)(x), \mu_2^+(y)\}\} \\
&= \min\{\min\{\mu_1^+(x), \mu_2^+(x)\}, \min\{\mu_1^+(y), \mu_2^+(y)\}\} \\
&= \min\{(\mu_1^+ \cap \mu_2^+)(x), (\mu_1^+ \cap \mu_2^+)(y)\},
\end{aligned}$$

and

$$\begin{aligned}
(\nu_1^+ \cap \nu_2^+)(xy) &= \min\{\nu_1^+(xy), \nu_2^+(xy)\} \\
&\geq \min\{\min\{\nu_1^+(x), \nu_1^+(y)\}, \min\{\nu_2^+(x), \nu_2^+(y)\}\} \\
&= \min\{\min\{\nu_1^+(x), \nu_2^+(x)\}, \min\{\nu_1^+(y), \nu_2^+(y)\}\} \\
&= \min\{(\nu_1^+ \cap \nu_2^+)(x), (\nu_1^+ \cap \nu_2^+)(y)\},
\end{aligned}$$

and

$$\begin{aligned}
(\mu_1^* \cup \mu_2^*)(xy) &= \max\{\mu_1^*(xy), \mu_2^*(xy)\} \\
&\leq \max\{\max\{\mu_1^*(x), \mu_1^*(y)\}, \max\{\mu_2^*(x), \mu_2^*(y)\}\} \\
&= \max\{\max\{\mu_1^*(x), \mu_2^*(x)\}, \max\{\mu_1^*(y), \mu_2^*(y)\}\} \\
&= \max\{(\mu_1^* \cup \mu_2^*)(x), (\mu_1^* \cup \mu_2^*)(y)\},
\end{aligned}$$

and

$$\begin{aligned}(\nu_1^* \cup \nu_2^*)(xy) &= \max\{\nu_1^*(xy), \nu_2^*(xy)\} \\ &\leq \max\{\max\{\nu_1^*(x), \nu_1^*(y)\}, \max\{\nu_2^*(x), \nu_2^*(y)\}\} \\ &= \max\{\max\{\nu_1^*(x), \nu_2^*(x)\}, \max\{\nu_1^*(y), \nu_2^*(y)\}\} \\ &= \max\{(\nu_1^* \cup \nu_2^*)(x), (\nu_1^* \cup \nu_2^*)(y)\},\end{aligned}$$

and

$$\begin{aligned}(\mu_1^- \cup \mu_2^-)(xy) &= \max\{\mu_1^-(xy), \mu_2^-(xy)\} \\ &\leq \max\{\max\{\mu_1^-(x), \mu_1^-(y)\}, \max\{\mu_2^-(x), \mu_2^-(y)\}\} \\ &= \max\{\max\{\mu_1^-(x), \mu_2^-(x)\}, \max\{\mu_1^-(y), \mu_2^-(y)\}\} \\ &= \max\{(\mu_1^- \cup \mu_2^-)(x), (\mu_1^- \cup \mu_2^-)(y)\},\end{aligned}$$

and

$$\begin{aligned}(\nu_1^* \cup \nu_2^*)(xy) &= \max\{\nu_1^*(xy), \nu_2^*(xy)\} \\ &\leq \max\{\max\{\nu_1^*(x), \nu_1^*(y)\}, \max\{\nu_2^*(x), \nu_2^*(y)\}\} \\ &= \max\{\max\{\nu_1^*(x), \nu_2^*(x)\}, \max\{\nu_1^*(y), \nu_2^*(y)\}\} \\ &= \max\{(\nu_1^* \cup \nu_2^*)(x), (\nu_1^* \cup \nu_2^*)(y)\}.\end{aligned}$$

Therefore $f_1 \sqcap f_2$ is a tripolar complex fuzzy subsemigroup of $\mathbf{S}$. □

Proposition 3.2. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* f_1, f_2 *be tripolar complex fuzzy left ideals of* $\mathbf{S}$. *Then* $f_1 \sqcap f_2$ *is a tripolar complex fuzzy left ideal of* $\mathbf{S}$.

Proof. Let $f_1 = (f_1^+, f_1^*, f_1^-)$ and $f_2 = (f_2^+, f_2^*, f_2^-)$ be tripolar complex fuzzy left ideals of $\mathbf{S}$, and $x, y \in S$. Let us consider as follows.

$$\begin{aligned}(\mu_1^+ \cap \mu_2^+)(xy) &= \min\{\mu_1^+(xy), \mu_2^+(xy)\} \\ &\geq \min\{\mu_1^+(y), \mu_2^+(y)\} \\ &= (\mu_1^+ \cap \mu_2^+)(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_1^+ \cap \nu_2^+)(xy) &= \min\{\nu_1^+(xy), \nu_2^+(xy)\} \\ &\geq \min\{\nu_1^+(y), \nu_2^+(y)\} \\ &= (\nu_1^+ \cap \nu_2^+)(y),\end{aligned}$$

and

$$\begin{aligned}(\mu_1^* \cup \mu_2^*)(xy) &= \max\{\mu_1^*(xy), \mu_2^*(xy)\} \\ &\leq \max\{\mu_1^*(y), \mu_2^*(y)\}\end{aligned}$$

$$= (\mu_1^* \cup \mu_2^*)(y),$$

and

$$\begin{aligned}(\nu_1^* \cup \nu_2^*)(xy) &= \max\{\nu_1^*(xy), \nu_2^*(xy)\}\\ &\leq \max\{\nu_1^*(y), \nu_2^*(y)\}\\ &= (\mu_1^* \cup \mu_2^*)(y),\end{aligned}$$

and

$$\begin{aligned}(\mu_1^- \cup \mu_2^-)(xy) &= \max\{\mu_1^-(xy), \mu_2^-(xy)\}\\ &\leq \max\{\mu_1^-(y), \mu_2^-(y)\}\\ &= (\mu_1^- \cup \mu_2^-)(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_1^- \cup \nu_2^-)(xy) &= \max\{\nu_1^-(xy), \nu_2^-(xy)\}\\ &\leq \max\{\nu_1^-(y), \nu_2^-(y)\}\\ &= (\nu_1^- \cup \nu_2^-)(y).\end{aligned}$$

Let $x, y \in S$ be such that $x \leq y$. Then, we obtain

$$\begin{aligned}(\mu_1^+ \cap \mu_2^+)(x) &= \min\{\mu_1^+(x), \mu_2^+(x)\}\\ &\geq \min\{\mu_1^+(y), \mu_2^+(y)\}\\ &= (\mu_1^+ \cap \mu_2^+)(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_1^+ \cap \nu_2^+)(x) &= \min\{\nu_1^+(x), \nu_2^+(x)\}\\ &\geq \min\{\nu_1^+(y), \nu_2^+(y)\}\\ &= (\nu_1^+ \cap \nu_2^+)(y),\end{aligned}$$

and

$$\begin{aligned}(\mu_1^* \cup \mu_2^*)(x) &= \max\{\mu_1^*(x), \mu_2^*(x)\}\\ &\leq \max\{\mu_1^*(y), \mu_2^*(y)\}\\ &= (\mu_1^* \cup \mu_2^*)(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_1^* \cup \nu_2^*)(x) &= \max\{\nu_1^*(x), \nu_2^*(x)\}\\ &\leq \max\{\nu_1^*(y), \nu_2^*(y)\}\\ &= (\nu_1^* \cup \nu_2^*)(y),\end{aligned}$$

and

$$\begin{aligned}(\mu_1^- \cup \mu_2^-)(x) &= \max\{\mu_1^-(x), \mu_2^-(x)\} \\ &\leq \max\{\mu_1^-(y), \mu_2^-(y)\} \\ &= (\mu_1^- \cup \mu_2^-)(y),\end{aligned}$$

and

$$\begin{aligned}(\nu_1^- \cup \nu_2^-)(x) &= \max\{\nu_1^-(x), \nu_2^-(x)\} \\ &\leq \max\{\nu_1^-(y), \nu_2^-(y)\} \\ &= (\nu_1^- \cup \nu_2^-)(y).\end{aligned}$$

Therefore $f_1 \sqcap f_2$ is a tripolar complex fuzzy left ideal of $\mathbf{S}$. □

Similar to Proposition 3.2, we have the following proposition.

Proposition 3.3. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* f_1, f_2 *be tripolar complex fuzzy right ideals of* $\mathbf{S}$*. Then* $f_1 \sqcap f_2$ *is a tripolar complex fuzzy right ideal of* $\mathbf{S}$*.*

Combining Proposition 3.2 and Proposition 3.3, we obtain the following corollary.

Corollary 3.1. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* f_1, f_2 *be tripolar complex fuzzy ideals of* $\mathbf{S}$*. Then* $f_1 \sqcap f_2$ *is a tripolar complex fuzzy ideal of* $\mathbf{S}$*.*

Let A be a subset of S. We define the tripolar complex fuzzy set $\chi_A = (\chi_A^+, \chi_A^*, \chi_A^-)$ such that $\chi_A^+ = \chi_A^+(\mu^+) + i\chi_A^+(\nu^+)$, $\chi_A^* = \chi_A^*(\mu^*) + i\chi_A^*(\nu^*)$ and $\chi_A^- = \chi_A^-(\mu^-) + i\chi_A^-(\nu^-)$ and, it called *characteristic tripolar complex fuzzy set* of a subset A of S and is defined as follows. For each $x \in S$,

$$\chi_A^+(\mu^+)(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise,} \end{cases} \qquad \chi_A^+(\nu^+)(x) = \begin{cases} 1 & \text{if } x \in A \\ 0 & \text{otherwise,} \end{cases}$$

and

$$\chi_A^*(\mu^*)(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise,} \end{cases} \qquad \chi_A^*(\nu^*)(x) = \begin{cases} 0 & \text{if } x \in A \\ 1 & \text{otherwise,} \end{cases}$$

and

$$\chi_A^-(\mu^-)(x) = \begin{cases} -1 & \text{if } x \in A \\ 0 & \text{otherwise,} \end{cases} \qquad \chi_A^-(\nu^-)(x) = \begin{cases} -1 & \text{if } x \in A \\ 0 & \text{otherwise.} \end{cases}$$

Remark 3.1. *Let* A *be a subset of* S*, and let* $\chi_A = (\chi_A^+, \chi_A^*, \chi_A^-)$ *be a tripolar complex fuzzy subset of* S*. Suppose that one of the following statements holds: For any* $x \in S$*,*

(1) $\chi_A^+(\mu^+)(x) = 1$ *and* $\chi_A^+(\nu^+)(x) = 1$*,*
(2) $\chi_A^*(\mu^*)(x) = 0$ *and* $\chi_A^*(\nu^*)(x) = 0$*,*
(3) $\chi_A^-(\mu^-)(x) = -1$ *and* $\chi_A^-(\nu^-)(x) = -1$*.*

Then $x \in A$*.*

We, now characterize subsemigroups, left (resp., right, two-sided) ideals by using tripolar complex fuzzy subsemigroups, tripolar complex fuzzy left (resp., right, two-side) ideals as the following theorems.

Theorem 3.1. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) *L is a left ideal of* **S**.
(2) $\chi_L = (\chi_L^+, \chi_L^*, \chi_L^-)$ *is a tripolar complex fuzzy left ideal of* **S**.

Proof. (1)⇒(2). Let L be a left ideal of **S**, and $x, y \in S$. We consider two cases. If $y \in L$, then since L is a left ideal of **S**, we have $xy \in L$, and then

$$\chi_L^+(\mu^+)(xy) = 1 = \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(xy) = 1 = \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(xy) = 0 = \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(xy) = 0 = \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(xy) = -1 = \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(xy) = -1 = \chi_L^-(\nu^-)(y).$$

If $y \notin L$, then we obtain

$$\chi_L^+(\mu^+)(xy) \geq 0 = \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(xy) \geq 0 = \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(xy) \leq 1 = \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(xy) \leq 1 = \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(xy) \leq 0 = \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(xy) \leq 0 = \chi_L^-(\nu^-)(y).$$

For any two cases, we obtain

$$\chi_L^+(\mu^+)(xy) \geq \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(xy) \geq \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(xy) \leq \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(xy) \leq \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(xy) \leq \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(xy) \leq \chi_L^-(\nu^-)(y).$$

Let $x, y \in S$ be such that $x \preceq y$. Then, if $y \in L$, then since L is a left ideal of **S**, we obtain $x \in L$ and then

$$\chi_L^+(\mu^+)(x) = 1 = \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(x) = 1 = \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(x) = 0 = \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(x) = 0 = \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(x) = -1 = \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(x) = -1 = \chi_L^-(\nu^-)(y).$$

If $y \notin L$, then we obtain

$$\chi_L^+(\mu^+)(x) \geq 0 = \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(x) \geq 0 = \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(x) \leq 1 = \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(x) \leq 1 = \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(x) \leq 0 = \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(x) \leq 0 = \chi_L^-(\nu^-)(y).$$

For any two cases, we obtain

$$\chi_L^+(\mu^+)(x) \geq \chi_L^+(\mu^+)(y),\ \chi_L^+(\nu^+)(x) \geq \chi_L^+(\nu^+)(y),$$

and

$$\chi_L^*(\mu^*)(x) \leq \chi_L^*(\mu^*)(y),\ \chi_L^*(\nu^*)(x) \leq \chi_L^*(\nu^*)(y),$$

and

$$\chi_L^-(\mu^-)(x) \leq \chi_L^-(\mu^-)(y),\ \chi_L^-(\nu^-)(x) \leq \chi_L^-(\nu^-)(y).$$

Therefore χ_L is a tripolar complex fuzzy left ideal of **S**.

(2)⇒(1). Let $x \in S$ and $y \in L$. Then, let us consider as follows.

$$1 \geq \chi_L^+(\mu^+)(xy) \geq \chi_L^+(\mu^+)(y) = 1.$$

It follows that $\chi_L^+(\mu^+)(xy) = 1$ and

$$1 \geq \chi_L^+(\nu^+)(xy) \geq \chi_L^+(\nu^+)(y) = 1.$$

It follows that $\chi_L^+(\nu^+)(xy) = 1$. By Remark 3.1, we have $xy \in L$.
Let $x, y \in S$ be such that $x \leq y$. If $y \in L$, then, let us consider as follows.

$$1 \geq \chi_L^+(\mu^+)(x) \geq \chi_L^+(\mu^+)(y) = 1.$$

It follows that $\chi_L^+(\mu^+)(x) = 1$ and

$$1 \geq \chi_L^+(\nu^+)(x) \geq \chi_L^+(\nu^+)(y) = 1.$$

It follows that $\chi_L^+(\nu^+)(x) = 1$. By Remark 3.1, we have $x \in L$. Therefore L is a left ideal of **S**. □

Similar to Theorem 3.1, we obtain the following theorem.

Theorem 3.2. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) *R is a right ideal of* **S**.
(2) $\chi_R = (\chi_R^+, \chi_R^*, \chi_R^-)$ *is a tripolar complex fuzzy right ideal of* **S**.

Combining Theorem 3.1 and Theorem 3.2, we have the following corollary.

Corollary 3.2. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent:*

(1) *I is an ideal of* **S**.
(2) $\chi_I = (\chi_I^+, \chi_I^*, \chi_I^-)$ *is a tripolar complex fuzzy ideal of* **S**.

Remark 3.2. *The characteristic tripolar complex fuzzy set of a subset A of S, $\chi_A = (\chi_A^+, \chi_A^*, \chi_A^-)$, in the case of $A = S$, we denoted by $\mathbf{1} = (1^+, 0^*, 0^-)$, and is defined by as follows.*

$$1^+(x) = 1(x) + i(1(x)),\ 0^*(x) = 0(x) + i(0(x)) \text{ and } 0^-(x) = -1(x) + i(-1(x)),$$

where $1(x) = 1$, $0(x) = 0$ and $-1(x) = -1$ for all $x \in S$.

We, now characterize tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (right, two-sided) ideals by using some properties of tripolar complex fuzzy subsemigroups, and tripolar complex fuzzy left (right, two-sided) ideals, respectively.

Theorem 3.3. *Let $\mathbf{S}$ be an ordered semigroup, and let $f = (f^+, f^*, f^-)$ be a tripolar complex fuzzy set of S. Then the following statements are equivalent.*

(1) *f is a tripolar complex fuzzy subsemigroup of $\mathbf{S}$.*
(2) *f satisfies that $f \diamond f \sqsubseteq f$.*

Proof. (1)⇒(2). Assume that (1) holds, and let $a \in S$. We consider two cases as follows. If $\mathbf{S}_a = \emptyset$, we obtain

$$(\mu^+ \diamond \mu^+)(a) = 0 \leq \mu^+(a), \text{ and } (\nu^+ \diamond \nu^+)(a) = 0 \leq \nu^+(a),$$

and

$$(\mu^* \diamond \mu^*)(a) = 1 \geq \mu^*(a), \text{ and } (\nu^* \diamond \nu^*)(a) = 1 \geq \nu^*(a),$$

and

$$(\mu^- \diamond \mu^-)(a) = 0 \geq \mu^-(a), \text{ and } (\nu^- \diamond \nu^-)(a) = 0 \geq \nu^*(a).$$

If $\mathbf{S}_a \neq \emptyset$, we obtain that

$$\begin{aligned}(\mu^+ \diamond \mu^+)(a) &= \bigvee_{(x,y)\in\mathbf{S}_a} \{\min\{\mu^+(x), \mu^+(y)\}\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\mu^+(xy)\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\mu^+(a)\} \\ &= \mu^+(a),\end{aligned}$$

and

$$\begin{aligned}(\nu^+ \diamond \nu^+)(a) &= \bigvee_{(x,y)\in\mathbf{S}_a} \{\min\{\nu^+(x), \nu^+(y)\}\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\nu^+(xy)\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\nu^+(a)\} \\ &= \nu^+(a),\end{aligned}$$

and

$$\begin{aligned}(\mu^* \diamond \mu^*)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{\mu^*(x), \mu^*(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^*(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^*(a)\} \\ &= \mu^*(a),\end{aligned}$$

and

$$\begin{aligned}(\nu^* \diamond \nu^*)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{\nu^*(x), \nu^*(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^*(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^*(a)\} \\ &= \nu^*(a),\end{aligned}$$

and

$$\begin{aligned}(\mu^- \diamond \mu^-)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{\mu^-(x), \mu^-(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^-(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^-(a)\} \\ &= \mu^-(a),\end{aligned}$$

and

$$\begin{aligned}(\nu^- \diamond \nu^-)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{\nu^-(x), \nu^-(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^-(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^-(a)\} \\ &= \nu^-(a).\end{aligned}$$

For any two cases, we have $f \diamond f \sqsubseteq f$.

(2)⇒(1). Assume that (2) holds, and let $a, b \in S$. Then we obtain

$$\mu^+(ab) \geq (\mu^+ \diamond \mu^+)(ab)$$

$$\begin{aligned} &= \bigvee_{(x,y)\in \mathbf{S}_{ab}} \{\min\{\mu^+(x), \mu^+(y)\}\} \\ &\geq \min\{\mu^+(a), \mu^+(b)\}, \end{aligned}$$

and

$$\begin{aligned} \nu^+(ab) &\geq (\nu^+ \diamond \nu^+)(ab) \\ &= \bigvee_{(x,y)\in \mathbf{S}_{ab}} \{\min\{\nu^+(x), \nu^+(y)\}\} \\ &\geq \min\{\nu^+(a), \nu^+(b)\}, \end{aligned}$$

and

$$\begin{aligned} \mu^*(ab) &\leq (\mu^* \diamond \mu^*)(ab) \\ &= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{\mu^*(x), \mu^*(y)\}\} \\ &\leq \max\{\mu^*(a), \mu^*(b)\}, \end{aligned}$$

and

$$\begin{aligned} \nu^*(ab) &\leq (\nu^* \diamond \nu^*)(ab) \\ &= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{\nu^*(x), \nu^*(y)\}\} \\ &\leq \max\{\nu^*(a), \nu^*(b)\}, \end{aligned}$$

and

$$\begin{aligned} \mu^-(ab) &\leq (\mu^- \diamond \mu^-)(ab) \\ &= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{\mu^-(x), \mu^-(y)\}\} \\ &\leq \max\{\mu^-(a), \mu^-(b)\}, \end{aligned}$$

and

$$\begin{aligned} \nu^-(ab) &\leq (\nu^- \diamond \nu^-)(ab) \\ &= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{\nu^-(x), \nu^-(y)\}\} \\ &\leq \max\{\nu^-(a), \nu^-(b)\}. \end{aligned}$$

Therefore f is a tripolar complex fuzzy subsemigroup of **S**. □

Theorem 3.4. *Let* **S** *be an ordered semigroup, and let $f = (f^+, f^*, f^-)$ be a tripolar complex fuzzy set of S. Then the following statements are equivalent.*

(1) *f is a tripolar complex fuzzy left ideal of* **S**.
(2) *f satisfies that*

(2.1) *For each $x, y \in S$, if $x \leq y$, then*

(2.1.1) $\mu^+(x) \geq \mu^+(y)$ *and* $\nu^+(x) \geq \nu^+(y)$.

(2.1.2) $\mu^*(x) \leq \mu^*(y)$ *and* $\nu^*(x) \leq \nu^*(y)$.

(2.1.3) $\mu^-(x) \leq \mu^-(y)$ *and* $\nu^-(x) \leq \nu^-(y)$.

(2.2) $\mathbf{1} \diamond f \sqsubseteq f$.

Proof. (1)⇒(2). Assume that (1) holds, and then (2.1) is clear. We will prove (2.2) holds. Let $a \in S$. We consider two cases as follows. If $\mathbf{S}_a = \emptyset$, we obtain

- $(1 \diamond \mu^+)(a) = 0 \leq \mu^+(a)$ and $(1 \diamond \nu^+)(a) = 0 \leq \nu^+(a)$.
- $(0 \diamond \mu^*)(a) = 1 \geq \mu^*(a)$ and $(0 \diamond \nu^*)(a) = 1 \geq \nu^*(a)$.
- $(-1 \diamond \mu^-)(a) = 0 \geq \mu^-(a)$ and $(-1 \diamond \nu^-)(a) = 0 \geq \nu^-(a)$.

If $\mathbf{S}_a \neq \emptyset$, then we obtain that

$$\begin{aligned}(1 \diamond \mu^+)(a) &= \bigvee_{(x,y)\in\mathbf{S}_a} \{\min\{1(x), \mu^+(y)\}\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\mu^+(xy)\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\mu^+(a)\} \\ &= \mu^+(a),\end{aligned}$$

and

$$\begin{aligned}(1 \diamond \nu^+)(a) &= \bigvee_{(x,y)\in\mathbf{S}_a} \{\min\{1(x), \nu^+(y)\}\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\nu^+(xy)\} \\ &\leq \bigvee_{(x,y)\in\mathbf{S}_a} \{\nu^+(a)\} \\ &= \nu^+(a),\end{aligned}$$

and

$$\begin{aligned}(0 \diamond \mu^*)(a) &= \bigwedge_{(x,y)\in\mathbf{S}_a} \{\max\{0(x), \mu^*(y)\}\} \\ &\geq \bigwedge_{(x,y)\in\mathbf{S}_a} \{\mu^*(xy)\} \\ &\geq \bigwedge_{(x,y)\in\mathbf{S}_a} \{\mu^*(a)\} \\ &= \mu^*(a),\end{aligned}$$

and

$$\begin{aligned}(0 \diamond \nu^*)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{0(x), \nu^*(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^*(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^*(a)\} \\ &= \nu^*(a),\end{aligned}$$

and

$$\begin{aligned}(-1 \diamond \mu^-)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{-1(x), \mu^-(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^-(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\mu^-(a)\} \\ &= \mu^-(a),\end{aligned}$$

and

$$\begin{aligned}(-1 \diamond \nu^-)(a) &= \bigwedge_{(x,y)\in \mathbf{S}_a} \{\max\{-1(x), \nu^-(y)\}\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^-(xy)\} \\ &\geq \bigwedge_{(x,y)\in \mathbf{S}_a} \{\nu^-(a)\} \\ &= \nu^-(a).\end{aligned}$$

For any two cases, we obtain that $\mathbf{1} \diamond f \sqsubseteq f$.

(2)⇒(1). Assume that (2) holds, and $a, b \in S$. Let us consider as follows.

$$\begin{aligned}\mu^+(ab) &\geq (1 \diamond \mu^+)(ab) \\ &= \bigvee_{(x,y)\in \mathbf{S}_{ab}} \{\min\{1(x), \mu^+(y)\}\} \\ &\geq \min\{1(a), \mu^+(b)\} \\ &= \mu^+(b),\end{aligned}$$

and

$$\begin{aligned}\nu^+(ab) &\geq (1 \diamond \nu^+)(ab) \\ &= \bigvee_{(x,y)\in \mathbf{S}_{ab}} \{\min\{1(x), \nu^+(y)\}\}\end{aligned}$$

$$
\begin{aligned}
&\geq \min\{1(a), \nu^{+}(b)\} \\
&= \nu^{+}(b),
\end{aligned}
$$

and

$$
\begin{aligned}
\mu^{*}(ab) &\leq (0 \diamond \mu^{*})(ab) \\
&= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{0(x), \mu^{*}(y)\}\} \\
&\leq \max\{0(a), \mu^{+}(b)\} \\
&= \mu^{*}(b),
\end{aligned}
$$

and

$$
\begin{aligned}
\nu^{*}(ab) &\leq (0 \diamond \nu^{*})(ab) \\
&= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{0(x), \nu^{*}(y)\}\} \\
&\leq \max\{0(a), \nu^{*}(b)\} \\
&= \nu^{*}(b),
\end{aligned}
$$

and

$$
\begin{aligned}
\mu^{-}(ab) &\leq (-1 \diamond \mu^{-})(ab) \\
&= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{-1(x), \mu^{-}(y)\}\} \\
&\leq \max\{-1(a), \mu^{-}(b)\} \\
&= \mu^{-}(b),
\end{aligned}
$$

and

$$
\begin{aligned}
\nu^{-}(ab) &\leq (-1 \diamond \nu^{*})(ab) \\
&= \bigwedge_{(x,y)\in \mathbf{S}_{ab}} \{\max\{-1(x), \nu^{-}(y)\}\} \\
&\leq \max\{-1(a), \nu^{-}(b)\} \\
&= \nu^{-}(b).
\end{aligned}
$$

By hypothesis (2.1), it follows that f is a tripolar complex fuzzy left ideal of $\mathbf{S}$. □

Similar to Theorem 3.4, we obtain the following theorem.

Theorem 3.5. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* $f = (f^{+}, f^{*}, f^{-})$ *be a tripolar complex fuzzy set of* S*. Then the following statements are equivalent.*

(1) f *is a tripolar complex fuzzy right ideal of* $\mathbf{S}$*.*

(2) f *satisfies that*

(2.1) *For each* $x, y \in S$*, if* $x \leq y$*, then*

(2.1.1) $\mu^+(x) \geq \mu^+(y)$ *and* $\nu^+(x) \geq \nu^+(y)$.
(2.1.2) $\mu^*(x) \leq \mu^*(y)$ *and* $\nu^*(x) \leq \nu^*(y)$.
(2.1.3) $\mu^-(x) \leq \mu^-(y)$ *and* $\nu^-(x) \leq \nu^-(y)$.
(2.2) $f \diamond \mathbf{1} \sqsubseteq f$.

Combining Theorem 3.4 and Theorem 3.5, we have the following corollary.

Corollary 3.3. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* $f = (f^+, f^*, f^-)$ *be a tripolar complex fuzzy set of* S. *Then the following statements are equivalent.*

(1) f *is a tripolar complex fuzzy ideal of* $\mathbf{S}$.
(2) f *satisfies that*
(2.1) *For each* $x, y \in S$, *if* $x \leq y$, *then*
(2.1.1) $\mu^+(x) \geq \mu^+(y)$ *and* $\nu^+(x) \geq \nu^+(y)$.
(2.1.2) $\mu^*(x) \leq \mu^*(y)$ *and* $\nu^*(x) \leq \nu^*(y)$.
(2.1.3) $\mu^-(x) \leq \mu^-(y)$ *and* $\nu^-(x) \leq \nu^-(y)$.
(2.2) $\mathbf{1} \diamond f \sqsubseteq f$ *and* $f \diamond \mathbf{1} \sqsubseteq f$.

Lemma 3.1. *Let* $\mathbf{S}$ *be an ordered semigroup, and let* A, B *be subsets of* S. *Then for each conditions are hold.*

(1) $\chi_A = \chi_B$ *if and only if* $A = B$.
(2) $\chi_A \sqcap \chi_B = \chi_{A \cap B}$.
(3) $\chi_{(AB]} = \chi_A \diamond \chi_B$.

Proof. We will give a proof only (3), the rest, it is easy to verifies. Let A, B be subsets of S and $x \in S$. If $x \notin (AB]$, then $x \npreceq ab$ for all $a \in A$ and $b \in B$ and it also means that $\mathbf{S}_x = \emptyset$, and we obtain

$$\chi^+_{(AB]}(\mu^+)(x) = 0 = \left(\chi^+_A(\mu^+) \diamond \chi^+_B(\mu^+)\right)(x),$$

and

$$\chi^+_{(AB]}(\nu^+)(x) = 0 = \left(\chi^+_A(\nu^+) \diamond \chi^+_B(\nu^+)\right)(x),$$

and

$$\chi^*_{(AB]}(\mu^*)(x) = 1 = \left(\chi^*_A(\mu^*) \diamond \chi^*_B(\mu^*)\right)(x),$$

and

$$\chi^*_{(AB]}(\nu^*)(x) = 1 = \left(\chi^*_A(\nu^*) \diamond \chi^*_B(\nu^*)\right)(x),$$

and

$$\chi^-_{(AB]}(\mu^-)(x) = 0 = \left(\chi^-_A(\mu^-) \diamond \chi^-_B(\mu^-)\right)(x),$$

and

$$\chi^-_{(AB]}(\nu^-)(x) = 0 = \left(\chi^-_A(\nu^-) \diamond \chi^-_B(\nu^-)\right)(x).$$

If $x \in (AB]$, then there exist $a \in A$ and $b \in B$ such that $x \preceq ab$, and then $\mathbf{S}_x \neq \emptyset$, we obtain

$$\begin{aligned} 1 &= \min\{\chi_A^+(\mu^+)(a), \chi_B^+(\mu^+)(b)\} \\ &\leq \bigvee_{(u,v)\in\mathbf{S}_x} \{\min\{\chi_A^+(\mu^+)(u), \chi_A^+(\mu^+)(v)\}\} \\ &= \left(\chi_A^+(\mu^+) \diamond \chi_B^+(\mu^+)\right)(x) \\ &\leq 1. \end{aligned}$$

It means that $(\chi_A^+(\mu^+) \diamond \chi_B^+(\mu^+))(x) = 1 = \chi_{(AB]}^+(\mu^+)(x)$, and

$$\begin{aligned} 1 &= \min\{\chi_A^+(\nu^+)(a), \chi_B^+(\nu^+)(b)\} \\ &\leq \bigvee_{(u,v)\in\mathbf{S}_x} \{\min\{\chi_A^+(\nu^+)(u), \chi_A^+(\nu^+)(v)\}\} \\ &= \left(\chi_A^+(\nu^+) \diamond \chi_B^+(\nu^+)\right)(x) \\ &\leq 1. \end{aligned}$$

It means that $(\chi_A^+(\nu^+) \diamond \chi_B^+(\nu^+))(x) = 1 = \chi_{(AB]}^+(\nu^+)(x)$, and

$$\begin{aligned} 0 &= \max\{\chi_A^*(\mu^*)(a), \chi_B^*(\mu^*)(b)\} \\ &\geq \bigwedge_{(u,v)\in\mathbf{S}_x} \{\max\{\chi_A^*(\mu^*)(u), \chi_A^*(\mu^*)(v)\}\} \\ &= \left(\chi_A^*(\mu^*) \diamond \chi_B^*(\mu^*)\right)(x) \\ &\geq 0. \end{aligned}$$

It means that $(\chi_A^*(\mu^*) \diamond \chi_B^*(\mu^*))(x) = 0 = \chi_{(AB]}^*(\mu^*)(x)$, and

$$\begin{aligned} 0 &= \max\{\chi_A^*(\nu^*)(a), \chi_B^*(\nu^*)(b)\} \\ &\geq \bigwedge_{(u,v)\in\mathbf{S}_x} \{\min\{\chi_A^*(\nu^*)(u), \chi_A^*(\nu^*)(v)\}\} \\ &= \left(\chi_A^*(\nu^*) \diamond \chi_B^*(\nu^*)\right)(x) \\ &\geq 0. \end{aligned}$$

It means that $(\chi_A^*(\nu^*) \diamond \chi_B^*(\nu^*))(x) = 0 = \chi_{(AB]}^*(\nu^*)(x)$, and

$$\begin{aligned} -1 &= \max\{\chi_A^-(\mu^-)(a), \chi_B^-(\mu^-)(b)\} \\ &\geq \bigwedge_{(u,v)\in\mathbf{S}_x} \{\max\{\chi_A^-(\mu^-)(u), \chi_A^-(\mu^-)(v)\}\} \\ &= \left(\chi_A^-(\mu^-) \diamond \chi_B^-(\mu^-)\right)(x) \\ &\geq -1. \end{aligned}$$

It means that $(\chi_A^-(\mu^-) \diamond \chi_B^-(\mu^-))(x) = -1 = \chi_{(AB]}^-(\mu^-)(x)$, and

$$\begin{aligned} -1 &= \max\{\chi_A^-(\nu^-)(a), \chi_B^-(\nu^-)(b)\} \\ &\geq \bigwedge_{(u,v)\in \mathbf{S}_x} \{\min\{\chi_A^-(\nu^-)(u), \chi_A^-(\nu^-)(v)\}\} \\ &= \left(\chi_A^-(\nu^-) \diamond \chi_B^-(\nu^-)\right)(x) \\ &\geq -1. \end{aligned}$$

It means that $(\chi_A^-(\nu^-) \diamond \chi_B^-(\nu^-))(x) = -1 = \chi_{(AB]}^-(\nu^-)(x)$. For any two cases, we obtain $\chi_{(AB]} = \chi_A \diamond \chi_B$. □

Let **S** be an ordered semigroup. Then **S** is called *intra-regular* if for each $a \in S$ there exist $x, y \in S$ such that $a \leq xa^2y$.

Lemma 3.2. *[18] Let* **S** *be an ordered semigroup. Then the following statements are equivalent.*

(1) **S** *is intra-regular.*
(2) $R \cap L \subseteq (LR]$ *for every left ideal* L *of* **S** *and every right ideal* R *of* **S**.

Finally of this present paper, we characterization of intra-regular ordered semigroups by using tripolar complex fuzzy left ideals and tripolar complex fuzzy right ideals as follows.

Theorem 3.6. *Let* **S** *be an ordered semigroup. Then the following statements are equivalent.*

(1) **S** *is intra-regular.*
(2) $f_1 \sqcap f_2 \sqsubseteq f_2 \diamond f_1$ *for every tripolar complex fuzzy left ideal* f_2 *of* **S** *and every tripolar complex fuzzy right ideal* f_1 *of* **S**.

Proof. (1)⇒(2). Assume that (1) holds. Let $f_1 = (f_1^+, f_1^*, f_1^-)$ and $f_2 = (f_2^+, f_2^*, f_2^-)$ be a tripolar complex fuzzy right ideal of **S** and a tripolar complex fuzzy left ideal of **S**, respectively, and $a \in S$. Since **S** is intra-regular, there exist $x, y \in S$ such that $a \leq xa^2y = (xa)(ay)$, and then $\mathbf{S}_a \neq \emptyset$. We obtain that

$$\begin{aligned} (\mu_2^+ \diamond \mu_1^+)(a) &= \bigvee_{(u,v)\in \mathbf{S}_a} \{\min\{\mu_2^+(u), \mu_1^+(v)\}\} \\ &\geq \min\{\mu_2^+(xa), \mu_1^+(ay)\} \\ &\geq \min\{\mu_2^+(a), \mu_1^+(a)\} \\ &= (\mu_2^+ \cap \mu_1^+)(a), \end{aligned}$$

and

$$\begin{aligned} (\nu_2^+ \diamond \nu_1^+)(a) &= \bigvee_{(u,v)\in \mathbf{S}_a} \{\min\{\nu_2^+(u), \nu_1^+(v)\}\} \\ &\geq \min\{\nu_2^+(xa), \nu_1^+(ay)\} \\ &\geq \min\{\nu_2^+(a), \nu_1^+(a)\} \\ &= (\nu_2^+ \cap \nu_1^+)(a), \end{aligned}$$

and

$$\begin{aligned}(\mu_2^* \diamond \mu_1^*)(a) &= \bigwedge_{(u,v)\in \mathbf{S}_a} \{\max\{\mu_2^*(u), \mu_1^*(v)\}\} \\ &\leq \max\{\mu_2^*(xa), \mu_1^*(ay)\} \\ &\leq \max\{\mu_2^*(a), \mu_1^*(a)\} \\ &= (\mu_2^* \cup \mu_1^*)(a),\end{aligned}$$

and

$$\begin{aligned}(\nu_2^* \diamond \nu_1^*)(a) &= \bigwedge_{(u,v)\in \mathbf{S}_a} \{\max\{\nu_2^*(u), \nu_1^*(v)\}\} \\ &\leq \max\{\nu_2^*(xa), \nu_1^*(ay)\} \\ &\leq \max\{\nu_2^*(a), \nu_1^*(a)\} \\ &= (\nu_2^* \cup \nu_1^*)(a),\end{aligned}$$

and

$$\begin{aligned}(\mu_2^- \diamond \mu_1^-)(a) &= \bigwedge_{(u,v)\in \mathbf{S}_a} \{\max\{\mu_2^-(u), \mu_1^-(v)\}\} \\ &\leq \max\{\mu_2^-(xa), \mu_1^-(ay)\} \\ &\leq \max\{\mu_2^-(a), \mu_1^-(a)\} \\ &= (\mu_2^- \cup \mu_1^-)(a),\end{aligned}$$

and

$$\begin{aligned}(\nu_2^- \diamond \nu_1^-)(a) &= \bigwedge_{(u,v)\in \mathbf{S}_a} \{\max\{\nu_2^-(u), \nu_1^-(v)\}\} \\ &\leq \max\{\nu_2^-(xa), \nu_1^-(ay)\} \\ &\leq \max\{\nu_2^-(a), \nu_1^-(a)\} \\ &= (\nu_2^- \cup \nu_1^-)(a).\end{aligned}$$

Therefore (2) holds.

(2)⇒(1). Assume that (2) holds. Let L and R be a left ideal of $\mathbf{S}$, and a right ideal of $\mathbf{S}$, respectively. By Theorem 3.1 and Theorem 3.2, we obtain χ_L and χ_R is a tripolar complex fuzzy left ideal of $\mathbf{S}$, and a tripolar complex fuzzy right ideal of $\mathbf{S}$, respectively. By hypothesis, and Lemma 3.1, we have

$$\begin{aligned}\chi_{R\cap L} &= \chi_R \sqcap \chi_L \\ &\sqsubseteq \chi_L \diamond \chi_R \\ &= \chi_{(LR]}.\end{aligned}$$

By Lemma 3.1, we obtain $R \cap L \subseteq (LR]$, and by Lemma 3.2, we have that **S** is an intra-regular ordered semigroup. □

4. Conclusions

This present paper, we have presented the notion of tripolar complex fuzzy sets in ordered semigroups. The concepts of tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (right, two-sided) ideals are introduced. Moreover, we studied some algebraic properties of tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (right, two-sided) ideals. We also characterized subsemigroups and left (resp., right, two-sided) ideals by using tripolar complex fuzzy subsemigroups and tripolar complex fuzzy left (resp., right, two-sided) ideals. Finally, we used some properties of tripolar complex fuzzy left and tripolar complex fuzzy right ideals to characterized intra-regular ordered semigroups. In the future our work, we will characterization of some class of ordered semigroups by using other tripolar complex fuzzy ideals. These findings may be extended to study more general algebraic structures or to develop decision-making models under uncertainty.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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