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Approximating Solutions of Resolvents of Monotone Operators and Convex Functions in Hadamard Spaces

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Abstract. In this paper, we study the products of finitely many resolvents of monotone operators and convex functions in the settings of Hadamard space. We propose an iterative method for finding products of finitely many resolvents of monotone operators, convex functions and fixed points of k-strictly pseudocontractive mappings. A strong convergence result of our proposed algorithm was established without imposing any strict conditions on our operators. We provide some consequences of our result and display a numerical example to illustrate the performance of our result. Our result complements and extends some related results in the literature.

1. Introduction

Let X be a Hadamard space and let Z be a nonempty, closed and convex subset of X. A point $x \in Z$ is said to be a fixed point of a nonlinear mapping $U : X \to X$, if $x = Ux$. We denote by $F(U)$, the set of all fixed points of U. The approximation of fixed points of nonlinear operators and different optimization problems has been of great importance in the field of nonlinear functional analysis due to its extensive applications in diverse mathematical problems such as game theory, signal processing, fuzzy theory, inverse problems and many others, (see [4, 11, 16, 20–22] and the references contained in). It is one of most flourishing areas of research in mathematics that has engaged prosperous development and has been a major source of attraction for researchers working in this direction.

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The theory of fixed point plays a crucial role in approximating other optimization problems. It has been seen in the literature that optimization problems such as monotone inclusion, equilibrium, variational inequalities and convex minimization problems can be solved using the fixed point approach. The minimization problem (in short, MP) is to find $x \in X$ such that

$$h(x) = \min_{y \in X} h(y), \tag{1.1}$$

where $h : X \to (-\infty, \infty]$ is a proper, convex and lower semi-continuous function. The Moreau-Yosida resolvent $R^h_\lambda : X \to X$ of a proper, convex and lower semi-continuous function h in X is defined as follows:

$$R^h_\lambda(x) = \min_{y \in X}\left[h(y) + \frac{1}{2\lambda}d^2(y,x)\right], \ \forall\, x \in X,\ \lambda > 0. \tag{1.2}$$

It is known that R^h_λ which is the resolvent of a lower semi-continuous function h in X is well defined and non-expansive for $\lambda > 0$ (see [6]). Several iterative methods have been employed to approximate solution of (1.2). The Proximal Point Algorithm (PPA) is known to be one of the most effective method for solving MP (1.2). This method was introduced by Martinet [17] in 1970 and further developed by Rockerfellar [24] in the settings of real Hilbert space H as follows: Find $x^1 \in H$ such that

$$x^{k+1} = \min_{y \in X}\left(h(y) + \frac{1}{2\lambda^k}\|y - x^k\|^2\right), \tag{1.3}$$

where $\lambda^k > 0$ for all $k \geq 1$. It was established in [24] that if h has a minimizer in H and $\sum_{k=1}^{\infty} \lambda^k = \infty$, then the sequence $\{x^k\}$ generated iteratively by (1.3) converges weakly to a minimizer of h. In order to establish a strong convergence result, Kamimura and Takahasi [15] combined the PPA together with the Halpern's algorithm for approximating the solution of (1.2) as follows:

$$\begin{cases} u, x^0 \in H \\ x^{k+1} = \alpha^k u + (1-\alpha^k) R^k_\lambda x^k. \end{cases} \tag{1.4}$$

The PPA was later introduced and studied in $CAT(0)$ spaces by Bačak [5] for approximating a solution of MP (1.2), using the following iterative method: For arbitrary $x^1 \in X$, the sequence $\{x^k\}$ is defined by

$$x^{k+1} = \min_{y \in X}\left(h(y) + \frac{1}{2\lambda^k}d^2(y, x^k)\right), \tag{1.5}$$

where $\lambda^k > 0$ for all $k \geq 1$. Bačak [5] established that $\{x^k\}$ Δ-converges to a minimizer of h under the condition that h has a minimizer in X and $\sum_{k=1}^{\infty} \lambda^k = \infty$. In 2017, Suparatulatorn *et al.* [26] proposed the following Halpern iterative method together with PPA for solving MP (1.2) and fixed point of a nonexpansive mapping T as follows:

$$\begin{cases} u, x^1 \in X \\ y^k = \min_{y \in X}\left(h(y) + \frac{1}{2\lambda^k} d^2(y, x^k)\right) \\ x^{k+1} = \alpha^k u + (1-\alpha^k) T y^k. \end{cases} \tag{1.6}$$

They proved a strong convergence result using the following conditions:

(i) $\sum_{k=1}^{\infty} |\lambda^k - \lambda^{k+1}| < \infty$,

(ii) $\sum_{k=1}^{\infty} |\alpha^k - \alpha^{k+1}| < \infty$,

(iii) $\lim_{k\to\infty} \alpha^k = 0$ and $\sum_{k=1}^{\infty} \alpha^k = \infty$.

For more iterative methods for solving MP (1.2), readers should consult ([1,2,4,9,12] and the references therein).

Let X be a complete $CAT(0)$ space with dual X^*. A multivalued operator $B : X \to 2^{X^*}$ with domain $Dom(B) = \{x \in X : Bx = \emptyset\}$ is monotone (see [10]) if and only if for all $x, y \in Dom(B),\ x^* \in Bx,\ y^* \in By$, we have

$$\langle x^* - y^*, \overrightarrow{yx} \rangle \geq 0. \tag{1.7}$$

The resolvent of the operator B of order $\mu > 0$ is the multivalued operator $J^B_\mu : X \to 2^X$ defined by

$$J^B_\mu := \{z \in X : [\frac{1}{\mu}\overrightarrow{zx} \in Bz]\}. \tag{1.8}$$

For $\mu > 0$, the operator B satisfies the range conditions if $Dom(J^B_\mu) \in X$ (see [10]). The theory of monotone operators plays a crucial role in analysis and numerical solutions of convex minimization problems. It is a valuable tool used in the study of gradient and subdifferential mappings and other mappings that appear in many problems, such as optimization, equilibrium or in variational inequality problems. In the case of real Hilbert spaces, the problem of finding zeros of monotone operators has been investigated by several authors (see [3,4,11,13,15,18,25]). One of the most important problems in monotone operator theory is the problem of finding a zero of a monotone operator for which is defined by

$$\text{find } x \in Dom(B) : 0 \in Bx, \tag{1.9}$$

and sometimes called Monotone Inclusion Problem (MIP) with solution set $B^{-1}(0)$. In the setting of real Hilbert spaces, Martinet [17] and Rockellar [24] defined the PPA for solving MIP as follows:

$$0 \in B(x^{k+1}) + \mu^k (x^{k+1} - x^k), k = 0, 1, 2, \cdots \tag{1.10}$$

where $\{\mu^k\}$ is a sequence of positive real numbers. Rockfellar [24] established that the sequence $\{x^k\}$ generated by the PPA converges weakly to a zero of monotone operator B provided that $\mu^k \geq 0$ for each $k \geq 1$. In 2017 Ranjbar and Khatibzadeli [23] extended the study of monotone operators and their resolvents from the setting of real Hilbert spaces to $CAT(0)$ spaces. They proposed the following iterative method to approximate solution of (1.9) and established a strong convergence result as follows:

$$\begin{cases} u, x^0 \in X \\ x^{k+1} = \beta^k u \oplus (1-\beta^k) J^B_{\mu^k} x^k. \end{cases} \tag{1.11}$$

where $\mu^k \in (0,\infty)$ and $\{\beta^k\} \subset (0,1)$. Since the inception of (1.9) in $CAT(0)$ space, several authors have considered solving (1.9) in the setting of a Hadamard space. In 2019, Okeke and Izuchukwu [19] introduced a Halpern method to approximate the solution of the resolvents of monotone operators and convex functions due to its numerous applications in real life problems. They established a strong convergence result for approximating the solution of the composition of (1.5) and (1.7) using the following iterative method:

$$\begin{cases} u, x^0 \in X \\ y^k = J^B_\mu \circ R^h_\lambda (x^k) \\ x^{k+1} = \beta^k u \oplus (1-\beta^k) T y^k,\ k \geq 0 \end{cases} \tag{1.12}$$

where $\lambda, \mu \in (0,\infty)$ and $\{\beta^k\} \subset (0,1)$.
Our contributions is stated as follows:

(1) We extend the results of [24] and [15] from Hilbert spaces to Hadamard spaces.
(2) The results in [5,19,23,26] are special cases of the result discussed in this manuscript as we considered a finitely many resolvents of monotone operators and convex functions.
(3) We considered a k-strictly pseudocontractive mapping which generalizes the nonexpansive mapping considered in [19,26].

Spurred by the result of ([1,3,8,18,19,23,25]), we introduce a Halpern method together with the PPA to approximate solutions of products of finitely many resolvents of monotone operators and convex functions in the setting of Hadamard spaces. Using our proposed iterative method, we establish a strong convergence theorem for approximating solutions of finite family of $k-$ strictly pseudocontrative mappings, finitely many resolvents of monotone operators and convex functions. Lastly, we state some consequences of our main result and display some numerical example to show the behaviour of our main result.

2. Preliminaries

In this section we recall some results and definitions that will be needed in our main results.

Definition 2.1. *Let X be a Hadamard space. A nonlinear mapping U is said to be:*

(1) *a contraction, if there exists $k \in (0,1)$ such that*

$$d(Ux, Uy) \leq k d(x,y), \forall x, y \in X,$$

if $k = 1$, then U is called nonexpansive.

(2) *k-strictly pseudo-contractive, if $p \in F(U)$ and $k \in (0,1)$, then*

$$d^2(Ux,p) \leq d^2(x,p) + kd^2(x,Ux),\ \forall\ x \in X.$$

Let X be a Hadamard space. A mapping $h : X \to (-\infty,\infty)$ is said to be convex, if

$$h(\lambda x \oplus (1-\lambda)y) \leq \lambda h(x) + (1-\lambda)h(y)\ \forall x,y \in X,\ \lambda \in (0,1).$$

h is proper, if $Dom(h)$ is nonempty. The mapping $h : Dom(h) \to (-\infty,\infty]$ is lower semicontinuous at a point $x \in Dom$, if

$$h(x) \leq \liminf_{n\to\infty} h(x_n). \tag{2.1}$$

Lemma 2.1. *[27] Let X be a CAT(0) space, $\{v_1, v_2, ..., v_m\} \subset X$ and $\{\beta_1, \beta_2, ...\beta_m\} \subset (0,1)$ with $\sum_{j=0}^{m} \beta_j = 1$. Then*

$$d^2\left(\sum_{j=1}^{m} \beta_j v_j, x\right) \leq \sum_{j=1}^{m} \beta_j d^2(v_j,x) - \sum_{j,k=1, j\neq k}^{m} \beta_j\beta_k d^2(v_j,v_k).$$

Lemma 2.2. *[28] Let X be a Hadamard space . Then for all $v,x,y,z \in X$ and $t \in [0,1]$, we have*
(1) $d\left(tx \oplus (1-t)y, z\right) \leq td(x,z) + (1-t)d(y,z)$,
(2) $d^2\left(tx \oplus (1-t)y, z\right) \leq td^2(y,z) - t(1-t)d^2(x,z)$,
(3) $d^2\left(z, tx \oplus (1-t)y\right) \leq t^2d^2(z,x) + (1-t)d^2(z,y) + 2t(1-t)\langle\overrightarrow{zx}, \overrightarrow{zy}\rangle$.

Lemma 2.3. *[28] Every bounded sequence in a Hadamard space has a Δ-convergence subsequence.*

Definition 2.2. *Let C be a nonempty,closed and convex subset of a Hadamard space X. A mapping $U : C \to C$ is said to be Δ-demiclosed at 0, if for any bounded sequence $\{x^k\} \subset X$ such that $\Delta - \lim_{k\to\infty} x^k = x$ and $\lim_{k\to\infty} d\left(x^k, Ux^k\right) = 0$, then $x = Ux$.*

Lemma 2.4. *[30] Let C be a nonempty, closed and convex subset of a Hadamard space X and $U : X \to X$ be a σ-strictly pseudocontractive mapping, then $I - U$ is demiclosed at 0.*

Remark 2.1. *[13] Suppose X is a Hadamard space and J_μ^B is the resolvent of the monotone operator $B : X \to 2^{X^*}$ of order $\mu > 0$ then*

$$d^2\left(x, J_\mu^B y\right) + d^2\left(J_\mu^B y, y\right) \leq d^2\left(x,y\right),$$

for all $x \in B^{-1}(0)$ and $y \in Dom(J_\mu^B)$.

Lemma 2.5. *[32] Let X be a Hadamard space and $h : X \to (-\infty.\infty]$ be a proper convex and lower semi-continuous function. Then for all $x, y \in X$ and $\lambda > 0$, we have*

$$\frac{1}{2\lambda}d^2\left(R_\lambda^h u, y\right) - \frac{1}{2\lambda}d^2\left(u,y\right) + \frac{1}{2\lambda}d^2\left(u, R_\lambda^h u\right) + h\left(R_\lambda^h u\right) \leq h(y).$$

Lemma 2.6. *[13] Let X be a Hadamard space and X^* be its dual space. For each $j = 1, 2, ..., m$, let $h_j : X \to (-\infty, \infty]$, $j = 1, 2, ..., N$ be a proper,convex and lower semi-continuous functions. For each $j = 1, 2, \cdots, N$, let $B_j : X \to 2^{X^*}$ be a multivalued monotone mappings that satisfy the range conditions. Then*

$$F\left(\Delta_\lambda^{h_m} \circ \psi_\mu^{B_N}\right) = \left(\bigcap_{j=1}^{N} B_j^{-1}(0)\right) \bigcap \left(\bigcap_{j=1}^{m} \arg\min_{y \in X} h_j(y)\right),$$

where $\Delta_\lambda^m = R_\lambda^{h_m} \circ R_\lambda^{h_{m-1}} \circ \cdots \circ R_\lambda^{h_2} \circ R_\lambda^{h_1}, \Delta_\lambda^0 = I, \psi_\mu^N = J_\mu^{B_N} \circ J_\mu^{B_{N-1}} \circ \cdots \circ J_\mu^{B_2} \circ J_\mu^{B_1}, \psi_\mu^0 = I, \mu, \lambda \in (0, \infty)$.

Lemma 2.7. *[13] Let X be a Hadamard space, $\{x^k\}$ be a sequence in X and $x \in X$. Then sequence $\{x^k\}$ Δ-converges to x if and only if $\limsup_{k\to\infty} \langle \overrightarrow{x^k x}, \overrightarrow{yx} \rangle \leq 0, \forall y \in X$.*

Lemma 2.8. *[31] Let $\{g^k\}$ be a sequence of nonnegative real numbers, $\{r^k\}$ be a sequence of real numbers in $(0, 1)$ such that $\sum_{k=1}^{\infty} r^k = \infty$ and $\{v^k\}$ be a sequence of real numbers. Assume that*

$$g^{k+1} \leq \left(1 - r^k\right) g^k + r^k v^k, \forall n \geq 1,$$

if $\limsup_{n\to\infty} v^{k_n} \geq 0$ for every subsequence $\{g^{k_n}\}$ of $\{g^k\}$ satisfying the condition:

$$\limsup_{n\to\infty} \left(g^{k_n} - g^{k_{n+1}}\right) \leq 0,$$

then $\lim_{k\to\infty} g^k = 0$.

3. Main Results

Theorem 3.1. *Let X be a Hadamard space and X^* be its dual space. Let $B_j : X \to (-\infty, \infty]$, $j = 1, 2, \cdots, m$ be a proper,convex and lower semi-continuous functions. Let U_j, $j = 1, 2, \cdots, m$ be a finite family of σ_j-strictly pseudocontractive mappings for some $0 < \sigma_j < 1$ and $h_j : X \to 2^{X^*}$, $j = 1, 2, \cdots, N$ be a multivalued monotone mappings that satisfy the range conditions. Assume that $\Omega = \bigcap_{j=1}^{m} \left(F(U_j) \bigcap \arg\min_{y\in X} B_j(y)\right) \bigcap \left(\bigcap_{j=1}^{N} h_j^{-1}(0)\right)$ is nonempty; For arbitrary $x^1, u \in X$, let the sequence $\{x^k\}$ be generated iteratively by*

$$\begin{cases} w^k = \left(1 - \alpha^k\right) x^k \oplus \alpha^k u \\ u^k = \Delta_\lambda^m \circ \psi_\mu^N(w^k) \\ x^{k+1} = \beta^{k,0} u^k \oplus \sum_{j=1}^{m} \oplus \beta^{k,j} U_j u^k \end{cases} \tag{3.1}$$

where $\beta^{k,0} \subset [a, b]$ for some $a, b \in (\sigma_j, 1)$, $\alpha^k \in (0, 1)$ satisfying $\sum_{j=0}^{m} \beta^{k,j} = 1$ and $\Delta_\lambda^m = R_\lambda^{h_m} \circ R_\lambda^{h_{m-1}} \circ \cdots \circ R_\lambda^{h_2} \circ R_\lambda^{h_1}, \Delta_\lambda^0 = I, \psi_\mu^N = J_\mu^{B_N} \circ J_\mu^{B_{N-1}} \circ \cdots \circ J_\mu^{B_2} \circ J_\mu^{B_1}, \psi_\mu^0 = I, \mu, \lambda \in (0, \infty)$, satisfying the following condition:

(1) $\lim_{k\to\infty} \alpha^k = 0$, *and* $\sum_{k=1}^{\infty} \alpha^k = \infty$.
Then $\{x^k\}$ *converges strongly to* $p = P_\Omega u$, *where* P_Ω *is the metric projection of* X.

Proof. Let $p \in \Omega$, then we have from (3.1) and Lemma 2.8 that

$$\begin{aligned}
d^2\left(x^{k+1}, p\right) =& d^2\left(\beta^{k,0}u^k \oplus \sum_{j=1}^{m} \oplus \beta^{k,j} U_j u^k, p\right) \\
&\le \beta^{k,0} d^2\left(u^k, p\right) + \sum_{j=1}^{m} \oplus \beta^{k,j} d^2\left(U_j u^k, p\right) - \beta^{k,0} \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \\
&\le \beta^{k,0} d^2\left(u^k, p\right) + \sum_{j=1}^{m} \left(d^2\left(u^k, p\right) + \sigma_j d^2\left(u^k, U_j u^k\right)\right) - \beta^{k,0} \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \\
&= d^2\left(u^k, p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \qquad (3.2) \\
&= d^2\left(\Delta_\lambda^{h_m} \psi_\mu^{B_N}(w^k), p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \\
&\vdots \\
&\le d^2\left(\psi_\mu^{B_N} w^k, p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \\
&\le d^2\left(\psi_\mu^{B_{N-1}} w^k, p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \\
&\vdots \\
&\le d^2\left(w^k, p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right) \qquad (3.3) \\
&\le d^2\left(w^k, p\right) \qquad (3.4) \\
&\le \left(1 - \alpha^k\right) d^2\left(x^k, p\right) + \alpha^k d^2\left(u, p\right) \\
&\le \max\{d^2\left(x^k, p\right), d^2\left(u, p\right)\}. \qquad (3.5)
\end{aligned}$$

Hence $\{x^k\}$ is bounded, consequently $\{w^k\}$ and $\{u^k\}$ are also bounded.
Furthermore, using Lemma 2.8 and (3.4), we obtain

$$\begin{aligned}
& d^2\left(x^{k+1}, p\right) \\
\le & d^2\left(w^k, p\right) - \left(\beta^{k,0} - \sigma_j\right) \sum_{j=1}^{m} \beta^{k,j} d^2\left(u^k, U_j u^k\right)
\end{aligned}$$

$$\leq (\alpha^k)^2 d^2(u,p) + \left(1-\alpha^k\right)^2 d^2\left(x^k,p\right) + 2\alpha^k\left(1-\alpha^k\right)\langle\overrightarrow{up},\overrightarrow{x^kp}\rangle - \left(\beta^{k,0}-\sigma_j\right)\sum_{j=1}^{m}\beta^{k,j}d^2\left(u^k,U_ju^k\right) \quad (3.6)$$

$$\leq \left(1-\alpha^k\right)^2 d^2\left(x^k,p\right) + \alpha^k\left[\alpha^k d^2(u,p) + 2\left(1-\alpha^k\right)\langle\overrightarrow{up},\overrightarrow{x^kp}\rangle\right]$$

$$= \left(1-\alpha^k\right) d^2\left(x^k,p\right) + \alpha^k\Theta^k, \quad (3.7)$$

where $\Theta^k = \alpha^k d(u,p) + 2\left(1-\alpha^k\right)\langle\overrightarrow{up},\overrightarrow{x^kp}\rangle$. From Lemma 2.8, it suffices that

$$\limsup_{n\to\infty}\left(d^2\left(x^{k_n},p\right) - d^2\left(x^{k_{n+1}},p\right)\right) \leq 0. \quad (3.8)$$

To establish this, suppose $\{d(x^{k_n},p)\}$ is a subsequence of $\{d\left(x^k,p\right)\}$, then

$$\limsup_{n\to\infty}\left(d^2\left(x^{k_n},p\right) - d^2\left(x^{k_n+1},p\right)\right) = \limsup_{n\to\infty}\left(d\left(x^{k_n},p\right) - d\left(x^{k_n+1},p\right)\right)\left(d\left(x^{k_n},p\right) + d\left(x^{k_n+1},p\right)\right)$$
$$\leq 0.$$

Now from (3.6) and (3.8) and condition (1) of (3.1), we set

$$\begin{aligned}
&\limsup_{n\to\infty}\left(\left(\beta^{k_n,0}-\sigma_j\right)\sum_{j=1}^{m}\beta^{k_n,j}d^2\left(u^{k_n},U_ju^{k_n}\right)\right)\\
&\leq \limsup_{n\to\infty}\left((\alpha^{k_n})^2 d^2(u,p) + \left(1-\alpha^{k_n}\right)d^2\left(x^{k_n},p\right) - d^2\left(x^{k_{n+1}},p\right)\right)\\
&\quad + \limsup_{n\to\infty}\left(2\alpha^{k_n}\left(1-\alpha^{k_n}\right)\langle\overrightarrow{up},\overrightarrow{x^{k_n}p}\rangle\right)\\
&= \limsup_{n\to\infty}\left(d^2\left(x^{k_n},p\right) - d^2\left(x^{k_{n+1}},p\right)\right)\\
&= -\liminf_{n\to\infty}\left(d^2\left(x^{k_{n+1}},p\right) - d^2\left(x^{k_n},p\right)\right)\\
&\leq 0.
\end{aligned} \quad (3.9)$$

Using the condition on $\beta^{k,0}$, we obtain that

$$\lim_{n\to\infty} d\left(u^{k_n},U_ju^{k_n}\right) = 0. \quad (3.10)$$

It is obvious from Algorithm 3.1 and condition (1) of Algorithm (3.1) that

$$d\left(x^{k_n+1},u^{k_n}\right) \leq \beta^{k,0}d\left(u^k,u^k\right) + \sum_{j=1}^{m}\beta^{k,j}d\left(U_ju^{k_n},u^{k_n}\right) \to 0, n\to\infty, \quad (3.11)$$

and

$$d\left(w^{k_n},x^{k_n}\right) \leq \alpha^{k_n}d\left(u,x^{k_n}\right) \to 0, n\to\infty. \quad (3.12)$$

Using Remark 2.1 and (3.12), we get

$$\begin{aligned}
d^2\left(u^k, \Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^k\right) &= d^2\left(R_\lambda^{h_m}(\Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^k), \Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^k\right)\\
&\le d^2\left(\Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^k, p\right) - d^2\left(R_\lambda^{h_m}(\Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^k, p\right)\\
&\vdots\\
&\le d^2\left(w^k, p\right) - d^2\left(x^{k+1}, p\right)\\
&\le d^2\left(w^k, x^k\right) + d^2\left(x^k, p\right) + 2d\left(w^k, x^k\right)d\left(x^k, p\right) - d^2\left(x^{k+1}, p\right)\\
&= d^2\left(w^k, x^k\right) + 2d\left(w^k, x^k\right)d^2\left(x^k, p\right) + \left[d^2\left(x^k, p\right) - d^2\left(x^{k+1}, p\right)\right] \to 0, n \to \infty.
\end{aligned} \tag{3.13}$$

Thus

$$\lim_{n\to\infty} d^2\left(u^{k_n}, \Delta_\lambda^{h_{m-1}}\psi_\mu^{B_N}w^{k_n}\right) = 0. \tag{3.14}$$

Following the same argument as in (3.14) and applying Remark 2.1, we get

$$\begin{aligned}
&d^2\left(\Delta_\lambda^{m-1}\psi_\mu^{N}w^{k_n}, \Delta_\lambda^{m-2}\psi_\mu^{N}w^{k_n}\right)\\
&\le d^2\left(\Delta_\lambda^{m-2}\psi_\mu^{N}w^{k_n}, p\right) - d^2\left(\Delta_\lambda^{m-1}\psi_\mu^{N}w^{k_n}, p\right)\\
&\vdots\\
&\le d^2\left(w^{k_n}, p\right) - d^2\left(u^{k_n}, p\right)\\
&\le d^2\left(w^{k_n}, p\right) - d^2\left(x^{k_n+1}, p\right)\\
&\le d^2\left(w^{k_n}, x^{k_n}\right) + 2d\left(w^{k_n}, x^{k_n}\right)d\left(x^{k_n}, p\right) + \left[d^2\left(x^{k_n}, p\right) - d^2\left(x^{k_n+1}, p\right)\right] \to 0, n\\
&\to \infty.
\end{aligned} \tag{3.15}$$

Thus by continuing the process in the same manner, we obtain that

$$\lim_{n\to\infty} d^2\left(\Delta_\lambda^{h_{m-2}}\psi_\mu^{B_N}w^{k_n}, \Delta_\lambda^{h_{m-2}}\psi_\mu^{B_N}w^{k_n}\right) = \cdots = \lim_{n\to\infty} d^2\left(\Delta_\lambda^{h_1}\psi_\mu^{B_N}w^{k_n}, \psi_\mu^{B_N}w^{k_n}\right) = 0. \tag{3.16}$$

From Lemma 2.5, we have
$\frac{1}{2\mu}d^2\left(\psi_\mu^{B_N}w^{k_n}, p\right) - \frac{1}{2\mu}d^2\left(\psi_\mu^{B_{N-1}}w^{k_n}, p\right) + \frac{1}{2\mu}d^2\left(\psi_\mu^{B_{N-1}}w^{k_n}, \psi_\mu^{B_N}p\right) + h(\psi_\mu^{B_N}w^{k_n}) \le h(p).$

Since $h(p) \le h(\psi_\mu^{B_N}w^{k_n})$, we have

$$\begin{aligned}
d^2\left(\psi_\mu^{B_N}w^{k_n}, \psi_\mu^{B_{N-1}}w^{k_n}\right) &\le d^2\left(\psi_\mu^{B_{N-1}}w^{k_n}, p\right) - d^2\left(\psi_\mu^{B_N}w^{k_n}, p\right)\\
&\vdots\\
&\le d^2\left(w^{k_n}, p\right) - d^2\left(u^{k_n}, p\right)\\
&\le d^2\left(w^{k_n}, p\right) - d^2\left(x^{k_n+1}, p\right)\\
&\le d^2\left(w^{k_n}, x^{k_n}\right) + 2d\left(w^{k_n}, x^{k_n}\right)d\left(x^{k_n}, p\right) + \left[d^2\left(x^{k_n}, p\right) - d^2\left(x^{k_n+1}, p\right)\right] \to 0, n \to \infty.
\end{aligned} \tag{3.17}$$

Following the same approach as in (3.17), we can prove that

$$\lim_{n\to\infty} d^2\left(\psi_\mu^{B_{N-1}} w^{k_n}, \psi_\mu^{B_{N-2}} w^{k_n}\right) = \cdots = \lim_{n\to\infty} d^2\left(\psi_\mu^{B_2} w^{k_n}, \psi_\mu^{B_1} w^{k_n}\right) = d^2\left(\psi_\mu^{B_1} w^{k_n}, w^{k_n}\right) = 0. \tag{3.18}$$

Hence

$$\begin{aligned} d\left(u^{k_n}, w^{k_n}\right) \le\, & d\left(u^{k_n}, \Delta_\lambda^{h_{m-1}} \psi_\mu^{B_N} w^{k_n}\right) + d\left(\Delta_\lambda^{h_{m-1}} \psi_\mu^{B_N} w^{k_n}, \Delta_\lambda^{h_{m-2}} \psi_\mu^{B_N} w^{k_n}\right) + \cdots + d\left(\Delta_\lambda^{h_1} \psi_\mu^{B_N} w^{k_n}, \psi_\mu^{B_N} w^{k_n}\right) \\ & + d\left(\psi_\mu^{B_N} w^{k_n}, \psi_\mu^{B_{N-1}} w^{k_n}\right) + d\left(\psi_\mu^{B_{N-1}} w^{k_n}, \psi_\mu^{B_{N-2}} w^{k_n}\right) + \cdots + d\left(\psi_\mu^{B_1} w^{k_n}, w^{k_n}\right), \end{aligned}$$

which implies from (3.13)-(3.18) that

$$\lim_{n\to\infty} d^2\left(u^{k_n}, w^{k_n}\right) = 0. \tag{3.19}$$

Using (3.13) and (3.19), we obtain that

$$\lim_{n\to\infty} d^2\left(u^{k_n}, x^{k_n}\right) = 0. \tag{3.20}$$

Also, from (3.14) and (3.20), we get

$$\lim_{n\to\infty} d^2\left(x^{k_n+1}, x^{k_n}\right) = 0. \tag{3.21}$$

since $\{x^{k_n}\}$ is bounded, then from Definition 2.2, there exists a subsequence $\{x^{k_{nj}}\}$ of $\{x^{k_n}\}$ such that $\Delta - \lim_{j\to\infty}\{x^{k_{nj}}\} = q$. Also, using (3.13) and (3.21), there exist a subsequence $\{w^{k_{nj}}\}$ of $\{w^{k_n}\}$ and $\{u^{k_{nj}}\}$ of $\{u^{k_n}\}$ such that $\Delta - \lim_{j\to\infty}\{w^{k_{nj}}\} = q$ and $\Delta - \lim_{j\to\infty}\{u^{k_{nj}}\} = q$ respectively. Using (3.11) and Definition (2.2), we obtain that $q \in \bigcap_{j=1}^{m} F(U_j)$. Also, since $\Delta_\lambda^m \circ \psi_\mu^N$ is nonexpansive. Thus it follows from (3.20) and Lemma (2.6) that

$q \in F\left(\Delta_\lambda^m \circ \psi_\mu^N\right) = \left(\bigcap_{j=1}^{N} B_j^{-1}(0)\right) \bigcap \left(\bigcap_{j=1}^{m} \arg\min_{y\in X} h_j(y)\right)$. Hence, we conclude that $q \in \Omega$.

Now, for arbitrary $u \in X$, we obtain from Lemma (2,10), that

$$\limsup_{j\to\infty} \langle \overrightarrow{up}, \overrightarrow{x^k p} \rangle \le 0,$$

which implies from (3.7) that

$$\limsup_{n\to\infty} \left(\alpha^{k_n} d^2(u,p) + 2(1-\alpha^{k_n}) \langle \overrightarrow{up}, \overrightarrow{x^k p} \rangle\right). \tag{3.22}$$

Thus $\limsup_{n\to\infty} \Theta^{k_n} \le 0$. By substituting (3.22) into (3.4) and applying Lemma 2.8, we have that $d\left(x^{k_n}, p\right) = 0$. Hence $\{x^k\} \to p \in \Omega$. □

The following results can be obtained as consequences of our main results.

By setting $m = N = 1$ in Theorem 3.1 we have the following result:

Corollary 3.1. *Let X be a Hadamard space and X^* be its dual space. Let $h : X \to (-\infty, \infty]$ be a proper,convex and lower semi-continuous functions. Let U_j be σ-strictly pseudocontractive mappings and $B : X \to 2^{X^*}$ be a multivalued monotone mappings that satisfy the range conditions. Assume that*

$\Omega = \bigcap_{j=1}^{m} \Big(F(U_j)\Big) \bigcap B^{-1}(0) \bigcap \Big(\arg\min_{y\in X} h(y)\Big)$ *is nonempty; For arbitrary* $x^1, u \in X$, *let the sequence* $\{x^k\}$ *be generated iteratively by*

$$\begin{cases} w^k = \left(1-\alpha^k\right)x^k \oplus \alpha^k u \\ u^k = \Delta_\lambda \circ \psi_\mu(w^k) \\ x^{k+1} = \beta^{k,0}u^k \oplus \sum_{j=1}^{m} \oplus \beta^{k,j}U_j u^k, \end{cases} \tag{3.23}$$

where $\beta^{k,0} \subset [a,b]$ *for some* $a, b \in (\sigma, 1)$, $\alpha^k \in (0,1)$ *satisfying* $\sum_{j=0}^{m} \beta^{k,j} = 1$ *and satisfying the following condition:*

(1) $\lim_{k\to\infty} \alpha^k = 0$, *and* $\sum_{k=1}^{\infty} \alpha^k = \infty$.

Then $\{x^k\}$ *converges strongly to* $p = P_\Omega u$, *where* P_Ω *is the metric projection of X.*

Setting $\Delta_\lambda^m = I$ we get the following result:

Corollary 3.2. *Let X be a Hadamard space and* X^* *be its dual space. Let* $U_j, j = 1, 2, , , m$ *be a finite family of* σ_j*-strictly pseudocontractive mappings for some* $0 < \sigma_j < 1$ *and* $B_j : X \to 2^{X^*}, j = 1, 2, , , N$ *be a multivalued monotone mappings that satisfy the range conditions. Assume that* $\Omega = \bigcap_{j=1}^{m} F(U_j) \bigcap \bigcap_{j=1}^{N} B_j^{-1}(0)$ *is nonempty; For arbitrary* $x^1, u \in X$, *let the sequence* $\{x^k\}$ *be generated iteratively by*

$$\begin{cases} w^k = \left(1-\alpha^k\right)x^k \oplus \alpha^k u \\ u^k = \psi_\mu^N(w^k) \\ x^{k+1} = \beta^{k,0}u^k \oplus \sum_{j=1}^{m} \oplus \beta^{k,j}U_j u^k, \end{cases} \tag{3.24}$$

where $\beta^{k,0} \subset [a,b]$ *for some* $a, b \in (\sigma_j, 1)$, $\alpha^k \in (0,1)$ *satisfying* $\sum_{j=0}^{m} \beta^{k,j} = 1$ *and* $\psi_\mu^N = J_\mu^{B_N} \circ J_\mu^{B_{N-1}} \circ \cdots \circ J_\mu^{B_2} \circ J_\mu^{B_1}, \psi_\mu^0 = I, \mu \in (0,\infty)$, *satisfying the following condition:*

(1) $\lim_{k\to\infty} \alpha^k = 0$, *and* $\sum_{k=1}^{\infty} \alpha^k = \infty$.

Then $\{x^k\}$ *converges strongly to* $p = P_\Omega u$, *where* P_Ω *is the metric projection of X.*

4. Numerical Example.

In this section, we present a numerical example of Theorem 3.1 to illustrate the performance of our main result. Let $X = \mathbb{R}^2$ be endowed with the Euclidean $\|.\|_2$. Then, for $j = 1$, we define $B_j : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$B_1(x) = (x^{(1)} - 2x^{(2)}, 2x^{(1)} + x^{(2)}).$$

Clearly, B_1 is a monotone operator.

Hence, we compute the resolvent of B_1 as follows:

$$J_\mu^{B_1}(x) = \left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} \mu & -2\mu \\ 2\mu & \mu \end{bmatrix} \right)^{-1} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix}$$
$$= \frac{1}{1+2\mu+5\mu^2} \begin{bmatrix} 1+\mu & 2\mu \\ -2\mu & 1+\mu \end{bmatrix} \begin{bmatrix} x^{(1)} \\ x^{(2)} \end{bmatrix},$$

which implies that

$$J_\mu^{B_1}(x) = \left(\frac{(1+\mu)x^{(1)} + 2\mu x^{(2)}}{1+2\mu+5\mu^2}, \frac{(1+\mu)x^{(2)} - 2\mu x^{(1)}}{1+2\mu+5\mu^2} \right).$$

Thus, for $j = 2, 3$, we define $B_j : \mathbb{R}^2 \to \mathbb{R}^2$ by

$$B_2(x) = (x^{(1)} - x^{(2)}, x^{(1)} + x^{(2)}),\ B_3(x) = (x^{(2)}, -x^{(1)}).$$

Thus, by similar argument as above, we obtain that

$$J_\mu^{B_2}(x) = \left(\frac{(1+\mu)x^{(1)} + \mu x^{(2)}}{1+2\mu+2\mu^2}, \frac{(1+\mu)x^{(2)} - \mu x^{(1)}}{1+2\mu+2\mu^2} \right).$$

and

$$J_\mu^{B_3}(x) = \left(\frac{x^{(1)} - \mu x^{(2)}}{1+\mu^2}, \frac{x^{(2)} + \mu x^{(1)}}{1+\mu^2} \right).$$

Now, define $h_1 : \mathbb{R}^2 \to (-\infty, +\infty]$ be defined by $h_1(x_1, x_2) = 100((x_2+1) - (x_1+1))^2 + x_1^2$. Then h_1 is a proper, convex and lower semi-continuous function in $(\mathbb{R}^2, d)$. We also define $h_j : \mathbb{R}^2 \to (-\infty, +\infty]$ by $h_j(x_1, x_2) = 70jx_1^2, j = 2, 3$. Thus h_j is a proper, convex and lower semi-continuous, (see [9]). Let $U_j(x_1, x_2) = (\frac{x_1}{2}j, \frac{x_2}{3}j),\ j = 1,\ \forall\ k = \frac{1}{2}$. Thus, U is strictly pseudo-contractive. Now, take $\alpha^k = \frac{1}{k+3}, \lambda = 0.1\ ,\ \mu = 0.75,\ \beta^{k,0} = \frac{k+1}{10k+4},\ \beta^{k,j} = \frac{(j+1)k+1}{2(5k+2)},\ \forall\ k \geq 1,\ j = 1, 2, 3.$

Case 1: $u = [-0.7\ 1.2]$ and $x_1 = [-1.1\ 0.1]$;

Case 2: $u = [2.1\ 1.2]$ and $x_1 = [0.8\ 2.0]$;

Case 3: $u = [-2.1\ 1.2]$ and $x_1 = [0.8\ \ -2]$;

Case 4: $u = [-3\ \ -5]$ and $x_1 = [-1\ \ -1]$.

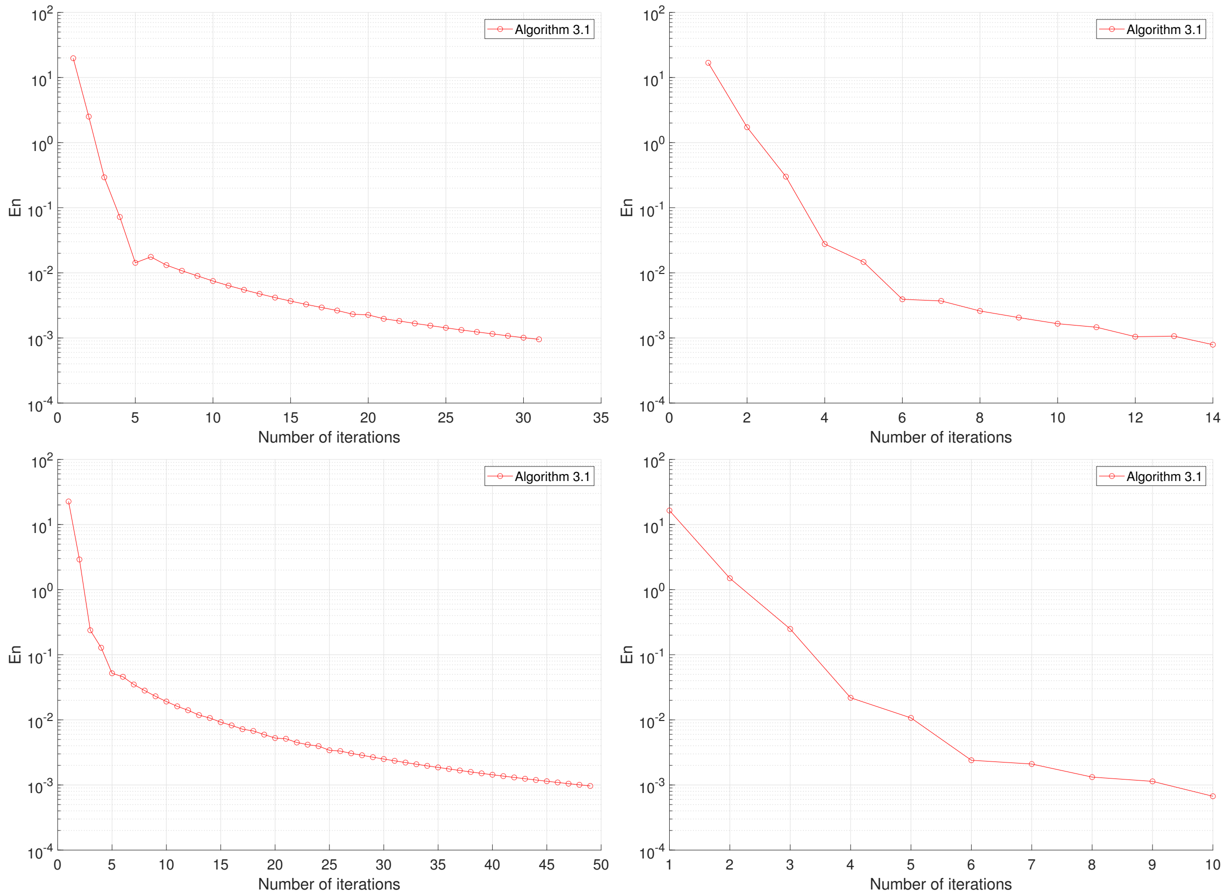

Figure 1. Top left: Case 1, Top right: Case 2, Bottom left: Case 3, Bottom right: Case 4.

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