

Advancements in Fuzzy Translations: Extensions and Contractions in IUP-Algebras

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Abstract. In this paper, we present three different fuzzy translations of a fuzzy set in IUP-algebras. The ideas of fuzzy α -translation of μ of type I, fuzzy β -translation of μ of type II, and fuzzy γ -translation of μ of type III are introduced for any fuzzy set μ in an IUP-algebra. Their basic properties are looked into, and some useful examples are discussed. The concepts of prime fuzzy sets and weakly prime fuzzy sets in IUP-algebras are also studied. Moreover, we discuss the concepts of fuzzy extensions and fuzzy contractions of a fuzzy set in IUP-algebras.

1. INTRODUCTION

The concept of fuzzy sets was first introduced by Zadeh [44] as a means of handling uncertainty and imprecision inherent in many real-world problems. Unlike classical set theory, where an element either belongs or does not belong to a set, fuzzy set theory allows for degrees of membership, represented by values in the unit interval $[0, 1]$. This innovation has led to significant advancements in various fields, including artificial intelligence, decision-making, and mathematical logic [12, 44]. Since its inception, fuzzy set theory has undergone extensive development, resulting in numerous generalizations such as intuitionistic fuzzy sets [7], type-2 fuzzy sets [25], and neutrosophic sets [37], all of which aim to enhance its applicability in different domains.

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One notable extension of fuzzy set theory is the study of fuzzy translations, which has been explored across various algebraic structures. Abu and Chandramouleeswaran [6] introduced the notions of fuzzy translations, extensions, and multiplications of fuzzy β -ideals of β -algebras. Senapati et al. [32] introduced the concepts of fuzzy translations, extensions, and multiplications of fuzzy H-ideals in BCK/BCI-algebras. Priya and Ramachandran [29] introduced the concepts of fuzzy α -translations, extensions, and multiplications of fuzzy PS-algebras. Areej [2] introduced the concepts of fuzzy translations, multiplications, and magnified translations of KU-algebras. Areej and Esraa [15] introduced the notion of α -translations of intuitionistic fuzzy AT-ideals and intuitionistic fuzzy AT-subalgebras. Senthilkumar et al. [36] introduced the ideas of anti-intuitionistic fuzzy extensions and multiplications of anti-intuitionistic fuzzy T-ideals in subtraction BCK/BCI-algebras. Chandramouleeswaran et al. [8] introduced the notion of fuzzy translations on BF-algebras. Kaviyarasu et al. [19] introduced the concepts of intuitionistic fuzzy translations and multiplications on INK-algebras. Senapati et al. [35] introduced the ideas of intuitionistic fuzzy translations, extensions, and multiplications of intuitionistic fuzzy subalgebras in BG-algebras. Nisha et al. [11] explored interval-valued fuzzy translations and multiplications in Z-algebras. Anitha and Seethalakshmi [5] introduced the concept of magnified translations of fuzzy hemirings and fuzzy normal hemirings. Senapati et al. [33] introduced the concept of intuitionistic fuzzy translations to intuitionistic fuzzy H-ideals in BCK/BCI-algebras. Jun [17] discussed fuzzy translations, extensions, and multiplications of fuzzy ideals in BCK/BCI-algebras. Lekkoksung [24] introduced the concept of fuzzy magnified translations of fuzzy bi-ideals in ternary hemirings. Alghamdi et al. [1] introduced the concepts of translations, extensions, and multiplications of bipolar fuzzy BCK-submodules. Altairy and Bashammakh [4] investigated the relationships between generalized fuzzy translations, multiplications, and magnified translations of BCK-submodules. Abu and Chandramouleeswaran [6] introduced the notions of fuzzy translations, extensions, and multiplications of fuzzy β -ideals of β -algebras. Dutta et al. [13] introduced the concept of translations and multiplications of cubic subalgebras and cubic ideals of BCK/BCI-algebras. Halimah [3] introduced the concepts of fuzzy translations and multiplications of BRK-algebras. Hameed and Abed [14] discussed fuzzy β, α -magnified translations and extensions of fuzzy KUS-subalgebras in KUS-algebras. Kyoung et al. [23] discussed fuzzy translations, (normalized, maximal) extensions, and multiplications of fuzzy subalgebras in BCK/BCI-algebras. Min [18] discussed fuzzy α -translations, (normalized, maximal) extensions, and multiplications of fuzzy hyper BCK-subalgebras in hyper BCK-algebras. Mohsin et al. [20] introduced the idea of intuitionistic fuzzy translations to intuitionistic fuzzy subalgebras and ideals over G-algebras. Mohsin et al. [21] introduced the concepts of intuitionistic fuzzy α -translations, multiplications, and magnified $\beta\alpha$ -translations in PS-algebras. Prasanna et al. [28] defined ω -fuzzy BP-subalgebras and explained the idea of ω -fuzzy translations and multiplications in BP-algebras. Prasanna et al. [27] defined ω -fuzzy BH-subalgebras and explained the idea of ω -fuzzy translations and multiplications in BH-algebras. Priya and Ramachandran [29] introduced the concepts of fuzzy

translations and multiplications on PS-algebras. Rashma and Sobha [30] introduced the concepts of intuitionistic fuzzy translations and multiplications on intuitionistic fuzzy ideals in BG-algebras. Mostafa et al. [26] discussed fuzzy α -translation and (normalized, maximal) S-extension of fuzzy KUS-subalgebras in KUS-algebras. Senapati et al. [34] introduced the concepts of intuitionistic fuzzy translations, extensions, and multiplications to intuitionistic fuzzy subalgebras and ideals in BCK/BCI-algebras. Senapati et al. [35] introduced the concepts of intuitionistic fuzzy translations, extensions, and multiplications to intuitionistic fuzzy subalgebras in BG-algebras. Senapati et al. [32] introduced the concepts of fuzzy translations, extensions, and multiplications to fuzzy H-ideals in BCK/BCI-algebras. Senapati [31] introduced the concept of intuitionistic fuzzy translations to intuitionistic fuzzy subalgebras and ideals in B-algebras. Fuzzy translations provide a framework for transforming fuzzy sets while preserving essential algebraic properties, making them instrumental in both theoretical research and practical applications.

The concept of IUP-algebras was first introduced by Iampan et al. in 2022 [16] as a novel algebraic structure characterized by four primary subsets: IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals. Their pioneering work not only laid the theoretical foundations of IUP-algebras but also highlighted their potential as a fertile ground for mathematical research. By establishing fundamental properties and exploring their initial applications, Iampan et al. opened new avenues for the study of algebraic structures. Building on this foundation, research in IUP-algebras has rapidly expanded, addressing both theoretical advancements and practical implications. Chanmanee et al. [10] contributed significantly to this growing body of knowledge in 2023 by exploring the direct product of infinite families of IUP-algebras. Their study introduced the notion of weak direct products and developed essential results concerning (anti-)IUP-homomorphisms, deepening the structural understanding of IUP-algebras and providing crucial tools for further exploration. Chanmanee et al. [9] extended these ideas by investigating direct products for infinite families of IUP-algebras, thereby formalizing the concept of DIUP-algebras and proposing the innovative framework of weak direct product DIUP-algebras, enhancing the structural depth of the theory. In 2024, Kuntama et al. [22] integrated fuzzy set (FS) theory with IUP-algebras, introducing constructs such as fuzzy IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals. Their meticulous analysis of these fuzzy subsets not only expanded the applicability of IUP-algebras but also bridged the gap between algebraic structures and fuzzy logic, enabling new interpretations and applications. Further enriching the framework, Suayngam et al. [43] extended the theory of IUP-algebras by incorporating intuitionistic fuzzy sets (IFSs). Their introduction of intuitionistic fuzzy IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals created hybrid structures that offered fresh perspectives and avenues for research. This innovative integration demonstrated the flexibility of IUP-algebras in accommodating advanced fuzzy logic concepts, thereby broadening their potential applications. Building upon these developments, Suayngam et al. [40] expanded the framework by incorporating neutrosophic sets (NSs), proposing neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals. These

advancements provided comprehensive conditions for NSs to conform to these structures, alongside an analysis of their interactions with level subsets, thereby illustrating the adaptability of IUP-algebras in managing uncertainty. Further theoretical depth was added by their subsequent work on intuitionistic neutrosophic sets (INSs) [38], which introduced intuitionistic neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals. This framework integrated the advantages of both intuitionistic and neutrosophic paradigms, enabling finer granularity in the handling of indeterminacy and hesitancy within algebraic structures. The proposed conditions and illustrative examples underscored the strength of IUP-algebras as a robust model for generalized uncertainty. Additionally, Suayngam et al. [41] advanced the field by applying Fermatean fuzzy sets (FFSs) to IUP-algebras. Their work focused on Fermatean fuzzy IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals, further expanding the algebra's theoretical landscape and underscoring its versatility in integrating sophisticated fuzzy systems. Extending this line of inquiry, Suayngam et al. [42] explored the application of Pythagorean fuzzy sets (PFSs) to IUP-algebras, presenting Pythagorean fuzzy IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals. The analysis revealed that Pythagorean fuzzy IUP-ideals and subalgebras serve as generalizations of Pythagorean fuzzy strong IUP-ideals within IUP-algebras, with the latter constrained to constant PFSs. In a parallel development, their most recent research further explored Pythagorean neutrosophic sets (PNSs) within the IUP-algebraic context [39]. By formulating and analyzing Pythagorean neutrosophic IUP-subalgebras, IUP-ideals, IUP-filters, and strong IUP-ideals, the authors synthesized both Pythagorean and neutrosophic reasoning to model a broader spectrum of uncertainty. This dual generalization demonstrated the rich expressiveness of the IUP framework and introduced novel algebraic properties aligned with real-world imprecision. This ongoing progression in the study of IUP-algebras reflects their significance as a dynamic and evolving algebraic framework. With contributions ranging from direct product constructions to fuzzy, intuitionistic, neutrosophic, and hybrid fuzzy-neutrosophic extensions, IUP-algebras continue to inspire innovative research, offering a robust platform for mathematical exploration and interdisciplinary applications.

Motivated by the extensive body of literature on fuzzy translations and their increasing relevance in algebraic structures, we aim to further explore their role in IUP-algebras. In this paper, we present three different fuzzy translations of a fuzzy set in IUP-algebras. The ideas of fuzzy α -translation of μ of type I, fuzzy β -translation of μ of type II, and fuzzy γ -translation of μ of type III are introduced for any fuzzy set μ in an IUP-algebra. Their basic properties are examined, and several illustrative examples are provided. Furthermore, we investigate the concepts of prime fuzzy sets and weakly prime fuzzy sets in IUP-algebras. Additionally, we discuss the notions of fuzzy extensions and fuzzy contractions of a fuzzy set in IUP-algebras, further enriching the theoretical understanding of these transformations.

2. PRELIMINARIES

IUP-algebras, introduced by Iampan et al. [16], represent a significant development in the study of algebraic structures, particularly within the domain of non-classical logic and fuzzy algebra. Defined by a distinctive axiomatic framework, IUP-algebras encompass four fundamental subsets: IUP-subalgebras, IUP-filters, IUP-ideals, and strong IUP-ideals. These subsets form the structural foundation of IUP-algebras, providing a versatile and rigorous framework for analyzing algebraic operations and logical relationships.

The defining characteristics of IUP-algebras, including their closure under binary operations and their contributions to algebraic completeness, underscore their importance in advancing mathematical theory. Their intrinsic properties have enabled researchers to explore the intersection between algebraic constructs and fuzzy logic, extending their applicability across various disciplines. This section establishes the foundational principles of IUP-algebras, revisiting their core definitions, fundamental examples, and essential properties to provide a comprehensive basis for the study of fuzzy translations within this algebraic framework.

Definition 2.1. [16] An algebra $X = (X; \cdot, 0)$ of type $(2, 0)$ is called an IUP-algebra, where X is a non-empty set, $\cdot$ is a binary operation on X , and 0 is the constant of X if it satisfies the following axioms:

$$(\forall x \in X)(0 \cdot x = x), \quad (\text{IUP-1})$$

$$(\forall x \in X)(x \cdot x = 0), \quad (\text{IUP-2})$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot (x \cdot z) = y \cdot z). \quad (\text{IUP-3})$$

For convenience, we refer to X as an IUP-algebra $X = (X; \cdot, 0)$ until otherwise specified.

Example 2.1. Let $X = \{0, 1, 2, 3, 4, 5\}$ be a set with the Cayley table as follows:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	4	0	3	1	5	2
2	2	5	0	4	3	1
3	5	4	1	0	2	3
4	1	3	5	2	0	4
5	3	2	4	5	1	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra.

In X , the following assertions are valid (see [16]).

$$(\forall x, y \in X)((x \cdot 0) \cdot (x \cdot y) = y), \quad (2.1)$$

$$(\forall x \in X)((x \cdot 0) \cdot (x \cdot 0) = 0), \quad (2.2)$$

$$(\forall x, y \in X)((x \cdot y) \cdot 0 = y \cdot x), \quad (2.3)$$

$$(\forall x \in X)((x \cdot 0) \cdot 0 = x), \quad (2.4)$$

$$(\forall x, y \in X)(x \cdot ((x \cdot 0) \cdot y) = y), \quad (2.5)$$

$$(\forall x, y \in X)((x \cdot 0) \cdot y \cdot x = y \cdot 0), \quad (2.6)$$

$$(\forall x, y, z \in X)(x \cdot y = x \cdot z \Leftrightarrow y = z), \quad (2.7)$$

$$(\forall x, y \in X)(x \cdot y = 0 \Leftrightarrow x = y), \quad (2.8)$$

$$(\forall x \in X)(x \cdot 0 = 0 \Leftrightarrow x = 0), \quad (2.9)$$

$$(\forall x, y, z \in X)(y \cdot x = z \cdot x \Leftrightarrow y = z), \quad (2.10)$$

$$(\forall x, y \in X)(x \cdot y = y \Rightarrow x = 0), \quad (2.11)$$

$$(\forall x, y, z \in X)((x \cdot y) \cdot 0 = (z \cdot y) \cdot (z \cdot x)), \quad (2.12)$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (z \cdot x) \cdot (z \cdot y) = 0), \quad (2.13)$$

$$(\forall x, y, z \in X)(x \cdot y = 0 \Leftrightarrow (x \cdot z) \cdot (y \cdot z) = 0), \quad (2.14)$$

$$\text{the right and the left cancellation laws hold.} \quad (2.15)$$

Definition 2.2. [16] A non-empty subset S of X is called

(i) an IUP-subalgebra of X if it satisfies the following condition:

$$(\forall x, y \in S)(x \cdot y \in S), \quad (2.16)$$

(ii) an IUP-filter of X if it satisfies the following conditions:

$$\text{the constant } 0 \text{ of } X \text{ is in } S, \quad (2.17)$$

$$(\forall x, y \in X)(x \cdot y \in S, x \in S \Rightarrow y \in S), \quad (2.18)$$

(iii) an IUP-ideal of X if it satisfies the condition (2.17) and the following condition:

$$(\forall x, y, z \in X)(x \cdot (y \cdot z) \in S, y \in S \Rightarrow x \cdot z \in S), \quad (2.19)$$

(iv) a strong IUP-ideal of X if it satisfies the following condition:

$$(\forall x, y \in X)(y \in S \Rightarrow x \cdot y \in S). \quad (2.20)$$

From axiom (IUP-2), we have the following remark:

Remark 2.1. Every IUP-subalgebra of X satisfies (2.17).

According to Iampan et al. [16], the concept of an IUP-filter generalizes both IUP-ideals and IUP-subalgebras, while these two structures in turn generalize the notion of a strong IUP-ideal. This establishes a natural hierarchy among the fundamental subsets of IUP-algebras. Notably, the definition of a strong IUP-ideal is particularly restrictive: a subset can only qualify as a strong IUP-ideal if it coincides exactly with the entire IUP-algebra X . In other words, no proper subset of X satisfies the stringent conditions required to be a strong IUP-ideal. This property positions strong IUP-ideals at the base of the structural hierarchy, serving as a degenerate or boundary case within the broader theory. The relationships among these subsets—ranging from the most specific

(strong IUP-ideals) to the most general (IUP-filters)—are summarized in the diagram shown in Figure 1, providing a visual map of their logical and structural inclusions.

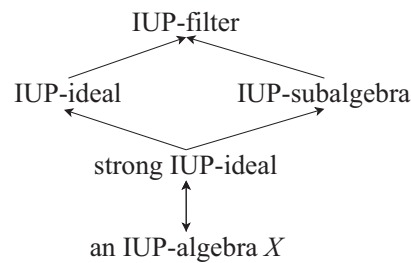


FIGURE 1. Special subsets of IUP-algebras

Definition 2.3. A nonempty subset B of X is said to be prime of X if it satisfies the following property:

$$(\forall x, y \in X)(x \cdot y \in B \Rightarrow x \in B \text{ or } y \in B) \quad (2.21)$$

Definition 2.4. A nonempty subset B of X is said to be weakly prime of X if it satisfies the following property:

$$(\forall x, y \in X, x \neq y)(x \cdot y \in B \Rightarrow x \in B \text{ or } y \in B) \quad (2.22)$$

Definition 2.5. [44] A fuzzy set in a nonempty set X (or a fuzzy subset of X) is an arbitrary function $f : X \rightarrow [0, 1]$ where $[0, 1]$ is the unit segment of the real line.

Definition 2.6. [44] The inclusion $\subseteq$ is defined by setting, for any fuzzy sets μ_1 and μ_2 in X ,

$$\mu_1 \subseteq \mu_2 \Leftrightarrow \mu_1(x) \leq \mu_2(x), \forall x \in X. \quad (2.23)$$

We say that μ_2 is a fuzzy extension of μ_1 , and μ_1 is a fuzzy contraction of μ_2 .

In the study of algebraic structures, particularly those incorporating fuzzy logic principles, the classification of subsets plays a crucial role in understanding their structural properties and interactions. Among these classifications, fuzzy IUP-subalgebras, fuzzy IUP-filters, fuzzy IUP-ideals, and fuzzy strong IUP-ideals serve as fundamental constructs that extend classical algebraic concepts into the realm of uncertainty and graded membership. These fuzzy subsets allow for a more nuanced examination of IUP-algebras by incorporating degrees of truth rather than binary inclusion.

The following definition formalizes these fuzzy subsets, providing rigorous mathematical criteria for their characterization. By establishing the necessary conditions for a fuzzy set to qualify as a fuzzy IUP-subalgebra, fuzzy IUP-filter, fuzzy IUP-ideal, or fuzzy strong IUP-ideal, this definition enables a structured approach to analyzing fuzzy algebraic systems. The properties of these subsets further facilitate the exploration of their algebraic relationships, forming the basis for subsequent discussions on fuzzy translations and their impact on IUP-algebras.

Definition 2.7. [22] A fuzzy set f in X is called

(i) a fuzzy IUP-subalgebra of X if it satisfies the following condition:

$$(\forall x, y \in X)(f(x \cdot y) \geq \min\{f(x), f(y)\}), \quad (2.24)$$

(ii) a fuzzy IUP-filter of X if it satisfies the following conditions:

$$(\forall x \in X)(f(0) \geq f(x)), \quad (2.25)$$

$$(\forall x, y \in X)(f(y) \geq \min\{f(x \cdot y), f(x)\}), \quad (2.26)$$

(iii) a fuzzy IUP-ideal of X if it satisfies the condition (2.25) and the following condition:

$$(\forall x, y, z \in X)(f(x \cdot z) \geq \min\{f(x \cdot (y \cdot z)), f(y)\}), \quad (2.27)$$

(iv) a fuzzy strong IUP-ideal of X if it satisfies the following condition:

$$(\forall x, y \in X)(f(x \cdot y) \geq f(y)). \quad (2.28)$$

Theorem 2.1. [22] Fuzzy strong IUP-ideals and constant fuzzy sets of X coincide.

3. FUZZY (α, I) -TRANSLATIONS

Fuzzy translations play a critical role in extending the theoretical foundations of fuzzy algebraic structures. In particular, the concept of fuzzy (α, I) -translations provides a mechanism for modifying fuzzy sets while preserving essential algebraic properties. These transformations enable the systematic alteration of membership values, facilitating deeper analysis of fuzzy subsets within IUP-algebras. This section introduces the definition of fuzzy (α, I) -translations and examines their fundamental properties. Theoretical results, supported by illustrative examples, highlight their significance in the broader framework of fuzzy algebra.

For any fuzzy set μ in X , we let $\dagger = 1 - \sup\{\mu(x) \mid x \in X\}$.

Definition 3.1. Let μ be a fuzzy set in X and let $\alpha \in [0, \dagger]$. A mapping $\mu_\alpha^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger(x) = \mu(x) + \alpha, \forall x \in X, \quad (3.1)$$

is said to be a fuzzy α -translation of μ of type I or, in short, a fuzzy (α, I) -translation of μ .

Example 3.1. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	0	5	4	3	2
2	4	5	0	1	2	3
3	3	2	1	0	5	4
4	2	3	4	5	0	1
5	5	4	3	2	1	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define a mapping $\mu : X \rightarrow [0, 1]$ as follows:

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.8 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 \end{pmatrix}.$$

Hence, μ is a fuzzy IUP-subalgebra of X . Then $\dagger = 1 - \sup\{0.8, 0.1, 0.2, 0.1, 0.2, 0.1\} = 1 - 0.8 = 0.2$. Let $\alpha = 0.1 \in [0, 0.2]$. Then the mapping $\mu_\alpha^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.2 & 0.3 & 0.2 & 0.3 & 0.2 \end{pmatrix}.$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-subalgebra of X .

Theorem 3.1. If μ is a fuzzy IUP-subalgebra of X , then the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-subalgebra of X for all $\alpha \in [0, \dagger]$.

Proof. Assume that μ is a fuzzy IUP-subalgebra of X . Let $x, y \in X$. Then

$$\begin{aligned} \mu_\alpha^\dagger(x \cdot y) &= \mu(x \cdot y) + \alpha \\ &\geq \min\{\mu(x), \mu(y)\} + \alpha \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \min\{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\}. \end{aligned}$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-subalgebra of X . □

Theorem 3.2. If there exists $\alpha \in [0, \dagger]$ such that the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-subalgebra of X , then μ is a fuzzy IUP-subalgebra of X .

Proof. Assume that $\mu_\alpha^\dagger$ is a fuzzy IUP-subalgebra of X for $\alpha \in [0, \dagger]$. Let $x, y \in X$. Then

$$\begin{aligned} \mu(x \cdot y) + \alpha &= \mu_\alpha^\dagger(x \cdot y) \\ &\geq \min\{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\} \\ &= \min\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \min\{\mu(x), \mu(y)\} + \alpha. \end{aligned}$$

So, $\mu(x \cdot y) \geq \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy IUP-subalgebra of X . □

Example 3.2. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	5	0	4	3	2	1
2	3	4	0	1	5	2
3	2	3	5	0	1	4
4	4	2	1	5	0	3
5	1	5	3	4	2	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define a mapping $\mu : X \rightarrow [0, 1]$ as follows:

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.1 & 0.1 & 0.1 & 0.3 & 0.1 \end{pmatrix}.$$

Hence, μ is a fuzzy IUP-filter of X . Then $\dagger = 1 - \sup\{0.9, 0.1, 0.1, 0.1, 0.3, 0.1\} = 1 - 0.9 = 0.1$. Let $\alpha = 0.05 \in [0, 0.1]$. Then the mapping $\mu_\alpha^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.95 & 0.15 & 0.15 & 0.15 & 0.35 & 0.15 \end{pmatrix}.$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-filter of X .

Example 3.3. [16] Let $\mathbb{R}^*$ be the set of all nonzero real numbers. Then $(\mathbb{R}^*; \cdot, 1)$ is an IUP-algebra, where $\cdot$ is the binary operation on $\mathbb{R}^*$ defined by $x \cdot y = \frac{y}{x}$ for all $x, y \in \mathbb{R}^*$. Let $S = \{x \in \mathbb{R}^* \mid x \geq 1\}$. Then S is an IUP-ideal and an IUP-filter of $\mathbb{R}^*$ but it is not an IUP-subalgebra of $\mathbb{R}^*$. Define a fuzzy set μ in $\mathbb{R}^*$ as follows:

$$\mu(x) = \begin{cases} 0.8 & \text{if } x \in S, \\ 0.3 & \text{otherwise.} \end{cases}$$

Then μ is a fuzzy IUP-filter of $\mathbb{R}^*$. Thus, $\dagger = 1 - \sup\{0.8, 0.3\} = 1 - 0.8 = 0.2$. Let $\alpha = 0.15 \in [0, 0.2]$. Then the mapping $\mu_\alpha^\dagger : \mathbb{R}^* \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger(x) = \begin{cases} 0.95 & \text{if } x \in S, \\ 0.45 & \text{otherwise.} \end{cases}$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-filter of $\mathbb{R}^*$.

Theorem 3.3. If μ is a fuzzy IUP-filter of X , then the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-filter of X for all $\alpha \in [0, \dagger]$.

Proof. Assume that μ is a fuzzy IUP-filter of X . Let $x, y \in X$. Then

$$\begin{aligned} \mu_\alpha^\dagger(0) &= \mu(0) + \alpha \\ &\geq \mu(x) + \alpha \\ &= \mu_\alpha^\dagger(x) \end{aligned}$$

and

$$\begin{aligned} \mu_\alpha^\dagger(y) &= \mu(y) + \alpha \\ &\geq \min\{\mu(x \cdot y), \mu(x)\} + \alpha \\ &= \min\{\mu(x \cdot y) + \alpha, \mu(x) + \alpha\} \\ &= \min\{\mu_\alpha^\dagger(x \cdot y), \mu_\alpha^\dagger(x)\}. \end{aligned}$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-filter of X . □

Theorem 3.4. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-filter of X , then μ is a fuzzy IUP-filter of X .*

Proof. Assume that $\mu_\alpha^\dagger$ is a fuzzy IUP-filter of X for $\alpha \in [0, \dagger]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(0) + \alpha &= \mu_\alpha^\dagger(0) \\ &\geq \mu_\alpha^\dagger(x) \\ &= \mu(x) + \alpha\end{aligned}$$

and

$$\begin{aligned}\mu(y) + \alpha &= \mu_\alpha^\dagger(y) \\ &\geq \min\{\mu_\alpha^\dagger(x \cdot y), \mu_\alpha^\dagger(x)\} \\ &= \min\{\mu(x \cdot y) + \alpha, \mu(x) + \alpha\} \\ &= \min\{\mu(x \cdot y), \mu(x)\} + \alpha.\end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(y) \geq \min\{\mu(x \cdot y), \mu(x)\}$. Hence, μ is a fuzzy IUP-filter of X . $\square$

Example 3.4. *Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:*

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	5	0	4	3	2	1
2	2	4	0	5	1	3
3	3	2	1	0	5	4
4	4	3	1	1	0	2
5	1	5	3	4	2	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define a mapping $\mu : X \rightarrow [0, 1]$ as follows:

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.7 & 0.3 & 0.2 & 0.2 & 0.2 & 0.3 \end{pmatrix}.$$

Hence, μ is a fuzzy IUP-ideal of X . Then $\dagger = 1 - \sup\{0.7, 0.3, 0.2, 0.2, 0.2, 0.3\} = 1 - 0.7 = 0.3$. Let $\alpha = 0.2 \in [0, 0.3]$. Then the mapping $\mu_\alpha^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.9 & 0.5 & 0.4 & 0.4 & 0.4 & 0.5 \end{pmatrix}.$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-ideal of X .

Example 3.5. [16] From Example 3.3, define a fuzzy set μ in $\mathbb{R}^*$ as follows:

$$\mu(x) = \begin{cases} 0.6 & \text{if } x \in S, \\ 0.3 & \text{otherwise.} \end{cases}$$

Then μ is a fuzzy IUP-ideal of $\mathbb{R}^*$. Thus, $\dagger = 1 - \sup\{0.6, 0.3\} = 1 - 0.6 = 0.4$. Let $\alpha = 0.25 \in [0, 0.4]$. Then the mapping $\mu_\alpha^\dagger : \mathbb{R}^* \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger(x) = \begin{cases} 0.85 & \text{if } x \in S, \\ 0.55 & \text{otherwise.} \end{cases}$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-ideal of $\mathbb{R}^*$.

Theorem 3.5. If μ is a fuzzy IUP-ideal of X , then the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-ideal of X for all $\alpha \in [0, \dagger]$.

Proof. Assume that μ is a fuzzy IUP-ideal of X . Let $x, y, z \in X$. Then

$$\begin{aligned} \mu_\alpha^\dagger(0) &= \mu(0) + \alpha \\ &\geq \mu(x) + \alpha \\ &= \mu_\alpha^\dagger(x) \end{aligned}$$

and

$$\begin{aligned} \mu_\alpha^\dagger(x \cdot z) &= \mu(x \cdot z) + \alpha \\ &\geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} + \alpha \\ &= \min\{\mu(x \cdot (y \cdot z)) + \alpha, \mu(y) + \alpha\} \\ &= \min\{\mu_\alpha^\dagger(x \cdot (y \cdot z)), \mu_\alpha^\dagger(y)\}. \end{aligned}$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy IUP-ideal of X . □

Theorem 3.6. If there exists $\alpha \in [0, \dagger]$ such that the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-ideal of X , then μ is a fuzzy IUP-ideal of X .

Proof. Assume that $\mu_\alpha^\dagger$ is a fuzzy IUP-ideal of X for $\alpha \in [0, \dagger]$. Let $x, y, z \in X$. Then

$$\begin{aligned} \mu(0) + \alpha &= \mu_\alpha^\dagger(0) \\ &\geq \mu_\alpha^\dagger(x) \\ &= \mu(x) + \alpha \end{aligned}$$

and

$$\begin{aligned} \mu(x \cdot z) + \alpha &= \mu_\alpha^\dagger(x \cdot z) \\ &\geq \min\{\mu_\alpha^\dagger(x \cdot (y \cdot z)), \mu_\alpha^\dagger(y)\} \\ &= \min\{\mu(x \cdot (y \cdot z)) + \alpha, \mu(y) + \alpha\} \\ &= \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} + \alpha. \end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(x \cdot z) \geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\}$. Hence, μ is a fuzzy IUP-ideal of X . □

Example 3.6. Let $X = \{0, 1, 2, 3, 4, 5\}$ with the following Cayley table:

$\cdot$	0	1	2	3	4	5
0	0	1	2	3	4	5
1	5	0	3	4	2	1
2	4	3	0	5	1	3
3	3	2	1	0	5	4
4	2	4	5	1	0	3
5	1	5	4	2	3	0

Then $X = (X; \cdot, 0)$ is an IUP-algebra. We define a mapping $\mu : X \rightarrow [0, 1]$ as follows:

$$\mu = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.5 & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \end{pmatrix}.$$

Hence, μ is a fuzzy strong IUP-ideal of X . Then $\dagger = 1 - \sup\{0.5, 0.5, 0.5, 0.5, 0.5, 0.5\} = 1 - 0.5 = 0.5$.

Let $\alpha = 0.3 \in [0, 0.5]$. Then the mapping $\mu_\alpha^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\alpha^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.8 & 0.8 & 0.8 & 0.8 & 0.8 & 0.8 \end{pmatrix}.$$

Hence, $\mu_\alpha^\dagger$ is a fuzzy strong IUP-ideal of X .

Theorem 3.7. If μ is a fuzzy strong IUP-ideal of X , then the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy strong IUP-ideal of X for all $\alpha \in [0, \dagger]$.

Proof. It is straightforward by Theorem 2.1. □

Theorem 3.8. If there exists $\alpha \in [0, \dagger]$ such that the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a fuzzy strong IUP-ideal of X , then μ is a fuzzy strong IUP-ideal of X .

Proof. It is straightforward by Theorem 2.1. □

Definition 3.2. A fuzzy set f in X is said to be prime in X if it satisfies the following property:

$$(\forall x, y \in X)(f(x \cdot y) \leq \max\{f(x), f(y)\}) \quad (3.2)$$

Theorem 3.9. If μ is a prime fuzzy set in X , then the fuzzy $(\alpha, \dagger)$ -translation $\mu_\alpha^\dagger$ of μ is a prime fuzzy set in X for all $\alpha \in [0, \dagger]$.

Proof. Assume that μ is a prime fuzzy set in X . Let $x, y \in X$. Then

$$\begin{aligned} \mu_\alpha^\dagger(x \cdot y) &= \mu(x \cdot y) + \alpha \\ &\leq \max\{\mu(x), \mu(y)\} + \alpha \\ &= \max\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \max\{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\}. \end{aligned}$$

Hence, $\mu_\alpha^\dagger$ is a prime fuzzy set in X . □

Theorem 3.10. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a prime fuzzy set in X , then μ is a prime fuzzy set in X .*

Proof. Assume that $\mu_\alpha^\dagger$ is a prime fuzzy set in X for $\alpha \in [0, \dagger]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(x \cdot y) + \alpha &= \mu_\alpha^\dagger(x \cdot y) \\ &\leq \max\{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\} \\ &= \max\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \max\{\mu(x), \mu(y)\} + \alpha.\end{aligned}$$

So, $\mu(x \cdot y) \leq \max\{\mu(x), \mu(y)\}$. Hence, μ is a prime fuzzy set in X . $\square$

Theorem 3.11. *If μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\alpha \in [0, \dagger]$.*

Proof. It follows from Theorem 3.1 (resp., Theorem 3.3, Theorem 3.5, Theorem 3.7) and Theorem 3.9. $\square$

Theorem 3.12. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 3.2 (resp., Theorem 3.4, Theorem 3.6, Theorem 3.8) and Theorem 3.10. $\square$

Theorem 3.13. *If μ is a weakly prime fuzzy set in X , then the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a weakly prime fuzzy set in X for all $\alpha \in [0, \dagger]$.*

Proof. Assume that μ is a prime fuzzy set in X . For any $\alpha \in [0, \dagger]$ and for all $x, y \in X$ with $x \neq y$, we have

$$\begin{aligned}\mu_\alpha^\dagger(x \cdot y) &= \mu(x \cdot y) + \alpha \\ &\leq \max\{\mu(x), \mu(y)\} + \alpha \\ &= \max\{\mu(x) + \alpha, \mu(y) + \alpha\} \\ &= \max\{\mu_\alpha^\dagger(x), \mu_\alpha^\dagger(y)\}.\end{aligned}$$

Hence, $\mu_\alpha^\dagger$ is a weakly prime fuzzy set in X . $\square$

Definition 3.3. *A fuzzy set f in X is said to be weakly prime in X if it satisfies the following property:*

$$(\forall x, y \in X, x \neq y)(f(x \cdot y) \leq \max\{f(x), f(y)\}) \quad (3.3)$$

Theorem 3.14. *If μ is a weakly prime fuzzy set in X , then the fuzzy (α, I) -translation $\mu_\alpha^\dagger$ of μ is a weakly prime fuzzy set in X for all $\alpha \in [0, \dagger]$.*

Proof. The proof is similar to Theorem 3.9, so we will omit it. $\square$

Theorem 3.15. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation μ_α^+ of μ is a weakly prime fuzzy set in X , then μ is a weakly prime fuzzy set in X .*

Proof. The proof is similar to Theorem 3.10, so we will omit it. $\square$

Theorem 3.16. *If μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, I) -translation μ_α^+ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\alpha \in [0, \dagger]$.*

Proof. It follows from Theorem 3.1 (resp., Theorem 3.3, Theorem 3.5, Theorem 3.7) and Theorem 3.14. $\square$

Theorem 3.17. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation μ_α^+ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 3.2 (resp., Theorem 3.4, Theorem 3.6, Theorem 3.8) and Theorem 3.15. $\square$

Note 3.1. *If μ is a fuzzy set in X and $\alpha \in [0, \dagger]$, then $\mu_\alpha^+(x) = \mu(x) + \alpha \geq \mu(x), \forall x \in X$. Hence, the fuzzy (α, I) -translation μ_α^+ of μ is a fuzzy extension of μ for all $\alpha \in [0, \dagger]$.*

Lemma 3.1. *Let μ and ν be fuzzy sets in X . If $\nu \supseteq \mu_\beta^+$ for $\beta \in [0, \dagger]$, there exists $\alpha \in [0, \dagger]$ with $\alpha \geq \beta$ such that $\nu \supseteq \mu_\alpha^+ \supseteq \mu_\beta^+$.*

Proof. Assume that $\nu \supseteq \mu_\beta^+$ for $\beta \in [0, \dagger]$. Then $\nu(x) \geq \mu_\beta^+(x), \forall x \in X$. Putting $\alpha = \beta + \inf_{x \in X} \{\nu(x) - \mu_\beta^+(x)\}$. Thus,

$$\begin{aligned} \inf_{x \in X} \{\nu(x) - \mu_\beta^+(x)\} &= \inf_{x \in X} \{\nu(x) - (\mu(x) + \beta)\} \\ &\leq \inf_{x \in X} \{1 - (\mu(x) + \beta)\} \\ &= 1 + \inf_{x \in X} \{-\mu(x) - \beta\} \\ &= 1 + \inf_{x \in X} \{-\mu(x)\} - \beta \\ &= 1 - \sup_{x \in X} \{\mu(x)\} - \beta \\ &= \dagger - \beta, \end{aligned}$$

so $\alpha = \beta + \inf_{x \in X} \{\nu(x) - \mu_\beta^+(x)\} \leq \beta + \dagger - \beta = \dagger$. Thus, $\alpha \in [0, \dagger]$ and $\alpha \geq \beta$, so $\mu_\alpha^+ \supseteq \mu_\beta^+$. Now, for all $x \in X$, we have

$$\begin{aligned} \mu_\alpha^+(x) &= \mu(x) + \alpha \\ &= \mu(x) + \beta + \inf_{x \in X} \{\nu(x) - \mu_\beta^+(x)\} \end{aligned}$$

$$\begin{aligned}
&= \mu_{\beta}^{\dagger}(x) + \inf_{x \in X} \{v(x) - \mu_{\beta}^{\dagger}(x)\} \\
&\leq \mu_{\beta}^{\dagger}(x) + v(x) - \mu_{\beta}^{\dagger}(x) \\
&= v(x),
\end{aligned}$$

so $v \supseteq \mu_{\alpha}^{\dagger}$. Hence, $v \supseteq \mu_{\alpha}^{\dagger} \supseteq \mu_{\beta}^{\dagger}$ for some $\alpha \in [0, \dagger]$ with $\alpha \geq \beta$. $\square$

Definition 3.4. Let μ_1 and μ_2 be two fuzzy sets in X and $\mu_1 \subseteq \mu_2$. If μ_1 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ_2 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , and we say that μ_2 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension of μ_1 .

Theorem 3.18. If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, I) -translation $\mu_{\alpha}^{\dagger}$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension of μ for all $\alpha \in [0, \dagger]$.

Proof. It follows from Theorem 3.1 (resp., Theorem 3.3, Theorem 3.5, Theorem 3.7) and Note 3.1. $\square$

Theorem 3.19. If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, I) -translation $\mu_{\alpha}^{\dagger}$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension of the fuzzy (β, I) -translation $\mu_{\beta}^{\dagger}$ of μ for all $\alpha, \beta \in [0, \dagger]$ with $\alpha \geq \beta$.

Proof. It follows from Theorem 3.1 (resp., Theorem 3.3, Theorem 3.5, Theorem 3.7). $\square$

Theorem 3.20. Let μ be a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X and $\beta \in [0, \dagger]$. For every fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension v of the fuzzy (β, I) -translation $\mu_{\beta}^{\dagger}$ of μ , there exists $\alpha \in [0, \dagger]$ with $\alpha \geq \beta$ such that v is the fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension of the fuzzy (α, I) -translation $\mu_{\alpha}^{\dagger}$ of μ .

Proof. It follows from Theorem 3.1 (resp., Theorem 3.3, Theorem 3.5, Theorem 3.7) and Lemma 3.1. $\square$

4. FUZZY (β, II) -TRANSLATIONS

While fuzzy (α, I) -translations focus on increasing the membership values of fuzzy sets, fuzzy (β, II) -translations serve as a complementary transformation by reducing these values under specific conditions. Such modifications are beneficial for studying contraction properties within IUP-algebras, as they provide insight into how fuzzy sets evolve under algebraic operations. This section explores the formal definition of fuzzy (β, II) -translations, establishes key theoretical results, and presents examples demonstrating their applications in fuzzy IUP-subalgebras, fuzzy IUP-filters, and fuzzy IUP-ideals.

For any fuzzy set $\mu \in X$, we let $\ddagger = \inf\{\mu(x) \mid x \in X\}$.

Definition 4.1. Let μ be a fuzzy set in X and let $\beta \in [0, \dagger]$. A mapping $\mu_\beta^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\beta^\dagger(x) = \mu(x) - \beta, \forall x \in X, \quad (4.1)$$

is said to be a fuzzy β -translation of μ of type II or, in short, a fuzzy (β, II) -translation of μ .

Example 4.1. From Example 3.1, we have $\dagger = \inf\{0.8, 0.1, 0.2\} = 0.1$. Let $\beta = 0.02 \in [0, 0.1]$. Then the mapping $\mu_\beta^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\beta^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.78 & 0.08 & 0.18 & 0.08 & 0.18 & 0.08 \end{pmatrix}.$$

Hence, $\mu_\beta^\dagger$ is a fuzzy IUP-subalgebra of X .

Theorem 4.1. If μ is a fuzzy IUP-subalgebra of X , then the fuzzy (β, II) -translation $\mu_\beta^\dagger$ of μ is a fuzzy IUP-subalgebra of X for all $\beta \in [0, \dagger]$.

Proof. Assume that μ is a fuzzy IUP-subalgebra of X . Let $x, y \in X$. Then

$$\begin{aligned} \mu_\beta^\dagger(x \cdot y) &= \mu(x \cdot y) - \beta \\ &\geq \min\{\mu(x), \mu(y)\} - \beta \\ &= \min\{\mu(x) - \beta, \mu(y) - \beta\} \\ &= \min\{\mu_\beta^\dagger(x), \mu_\beta^\dagger(y)\}. \end{aligned}$$

Hence, $\mu_\beta^\dagger$ is a fuzzy IUP-subalgebra of X . $\square$

Theorem 4.2. If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, II) -translation $\mu_\beta^\dagger$ of μ is a fuzzy IUP-subalgebra of X , then μ is a fuzzy IUP-subalgebra of X .

Proof. Assume that $\mu_\beta^\dagger$ is a fuzzy IUP-subalgebra of X for $\beta \in [0, \dagger]$. Let $x, y \in X$. Then

$$\begin{aligned} \mu(x \cdot y) - \beta &= \mu_\beta^\dagger(x \cdot y) \\ &\geq \min\{\mu_\beta^\dagger(x), \mu_\beta^\dagger(y)\} \\ &= \min\{\mu(x) - \beta, \mu(y) - \beta\} \\ &= \min\{\mu(x), \mu(y)\} - \beta. \end{aligned}$$

So, $\mu(x \cdot y) \geq \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy IUP-subalgebra of X . $\square$

Example 4.2. From Example 3.2, we have $\dagger = \inf\{0.9, 0.1, 0.3\} = 0.1$. Let $\beta = 0.05 \in [0, 0.1]$. Then the mapping $\mu_\beta^\dagger : X \rightarrow [0, 1]$ defined by

$$\mu_\beta^\dagger = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.85 & 0.05 & 0.05 & 0.05 & 0.25 & 0.05 \end{pmatrix}.$$

Hence, $\mu_\beta^\dagger$ is a fuzzy IUP-filter of X .

Example 4.3. From Example 3.3, we have $\ddagger = \inf\{0.8, 0.3\} = 0.3$. Let $\beta = 0.15 \in [0, 0.3]$. Then the mapping $\mu_{\beta}^{\ddagger} : \mathbb{R}^* \rightarrow [0, 1]$ defined by

$$\mu_{\beta}^{\ddagger}(x) = \begin{cases} 0.65 & \text{if } x \in S, \\ 0.15 & \text{otherwise.} \end{cases}$$

Hence, $\mu_{\beta}^{\ddagger}$ is a fuzzy IUP-filter of $\mathbb{R}^*$.

Theorem 4.3. If μ is a fuzzy IUP-filter of X , then the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a fuzzy IUP-filter of X for all $\beta \in [0, \ddagger]$.

Proof. Assume that μ is a fuzzy IUP-filter of X . Let $x, y \in X$. Then

$$\begin{aligned} \mu_{\beta}^{\ddagger}(0) &= \mu(0) - \beta \\ &\geq \mu(x) - \beta \\ &= \mu_{\beta}^{\ddagger}(x) \end{aligned}$$

and

$$\begin{aligned} \mu_{\beta}^{\ddagger}(y) &= \mu(y) - \beta \\ &\geq \min\{\mu(x \cdot y), \mu(x)\} - \beta \\ &= \min\{\mu(x \cdot y) - \beta, \mu(x) - \beta\} \\ &= \min\{\mu_{\beta}^{\ddagger}(x \cdot y), \mu_{\beta}^{\ddagger}(x)\}. \end{aligned}$$

Hence, $\mu_{\beta}^{\ddagger}$ is a fuzzy IUP-filter of X . □

Theorem 4.4. If there exists $\beta \in [0, \ddagger]$ such that the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a fuzzy IUP-filter of X , then μ is a fuzzy IUP-filter of X .

Proof. Assume that $\mu_{\beta}^{\ddagger}$ is a fuzzy IUP-filter of X for $\beta \in [0, \ddagger]$. Let $x, y \in X$. Then

$$\begin{aligned} \mu(0) - \beta &= \mu_{\beta}^{\ddagger}(0) \\ &\geq \mu_{\beta}^{\ddagger}(x) \\ &= \mu(x) - \beta \end{aligned}$$

and

$$\begin{aligned} \mu(y) - \beta &= \mu_{\beta}^{\ddagger}(y) \\ &\geq \min\{\mu_{\beta}^{\ddagger}(x \cdot y), \mu_{\beta}^{\ddagger}(x)\} \\ &= \min\{\mu(x \cdot y) - \beta, \mu(x) - \beta\} \\ &= \min\{\mu(x \cdot y), \mu(x)\} - \beta. \end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(y) \geq \min\{\mu(x \cdot y), \mu(x)\}$. Hence, μ is a fuzzy IUP-filter of X . □

Example 4.4. From Example 3.4, we have $\dagger = \inf\{0.7, 0.3, 0.2\} = 0.2$. Let $\beta = 0.1 \in [0, 0.1]$. Then the mapping $\mu_{\beta}^{\dagger} : X \rightarrow [0, 1]$ defined by

$$\mu_{\beta}^{\dagger} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.6 & 0.2 & 0.1 & 0.1 & 0.1 & 0.2 \end{pmatrix}.$$

Hence, $\mu_{\beta}^{\dagger}$ is a fuzzy IUP-ideal of X .

Example 4.5. From Example 3.5, we have $\dagger = \inf\{0.6, 0.3\} = 0.3$. Let $\beta = 0.25 \in [0, 0.3]$. Then the mapping $\mu_{\beta}^{\dagger} : \mathbb{R}^* \rightarrow [0, 1]$ defined by

$$\mu_{\beta}^{\dagger}(x) = \begin{cases} 0.35 & \text{if } x \in S, \\ 0.05 & \text{otherwise.} \end{cases}$$

Hence, $\mu_{\beta}^{\dagger}$ is a fuzzy IUP-ideal of $\mathbb{R}^*$.

Theorem 4.5. If μ is a fuzzy IUP-ideal of X , then the fuzzy (β, Π) -translation $\mu_{\beta}^{\dagger}$ of μ is a fuzzy IUP-ideal of X for all $\beta \in [0, \dagger]$.

Proof. Assume that $\mu_{\beta}^{\dagger}$ is a fuzzy IUP-ideal of X for $\beta \in [0, \dagger]$. Let $x, y, z \in X$. Then

$$\begin{aligned} \mu_{\beta}^{\dagger}(0) &= \mu(0) - \beta \\ &\geq \mu(x) - \beta \\ &= \mu_{\beta}^{\dagger}(x) \end{aligned}$$

and

$$\begin{aligned} \mu_{\beta}^{\dagger}(x \cdot z) &= \mu(x \cdot z) - \beta \\ &\geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} - \beta \\ &= \min\{\mu(x \cdot (y \cdot z)) - \beta, \mu(y) - \beta\} \\ &= \min\{\mu_{\beta}^{\dagger}(x \cdot (y \cdot z)), \mu_{\beta}^{\dagger}(y)\}. \end{aligned}$$

Hence, $\mu_{\beta}^{\dagger}$ is a fuzzy IUP-ideal of X . □

Theorem 4.6. If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, Π) -translation $\mu_{\beta}^{\dagger}$ of μ is a fuzzy IUP-ideal of X , then μ is a fuzzy IUP-ideal of X .

Proof. Assume that $\mu_{\beta}^{\dagger}$ is a fuzzy IUP-ideal of X for $\beta \in [0, \dagger]$. Let $x, y, z \in X$. Then

$$\begin{aligned} \mu(0) - \beta &= \mu_{\beta}^{\dagger}(0) \\ &\geq \mu_{\beta}^{\dagger}(x) \\ &= \mu(x) - \beta \end{aligned}$$

and

$$\begin{aligned}
 \mu(x \cdot z) - \beta &= \mu_{\beta}^{\ddagger}(x \cdot z) \\
 &\geq \min\{\mu_{\beta}^{\ddagger}(x \cdot (y \cdot z)), \mu_{\beta}^{\ddagger}(y)\} \\
 &= \min\{\mu(x \cdot (y \cdot z)) - \beta, \mu(y) - \beta\} \\
 &= \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} - \beta.
 \end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(x \cdot z) \geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\}$. Hence, μ is a fuzzy IUP-ideal of X . $\square$

Example 4.6. From Example 3.6, we have $\ddagger = \inf\{0.5\} = 0.5$. Let $\beta = 0.3 \in [0, 0.5]$. Then the mapping $\mu_{\beta}^{\ddagger} : X \rightarrow [0, 1]$ defined by

$$\mu_{\beta}^{\ddagger} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.2 & 0.2 & 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}.$$

Hence, $\mu_{\beta}^{\ddagger}$ is a fuzzy strong IUP-ideal of X .

Theorem 4.7. If μ is a fuzzy strong IUP-ideal of X , then the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a fuzzy strong IUP-ideal of X for all $\beta \in [0, \ddagger]$.

Proof. It is straightforward by Theorem 2.1. $\square$

Theorem 4.8. If there exists $\beta \in [0, \ddagger]$ such that the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a fuzzy strong IUP-ideal of X , then μ is a fuzzy strong IUP-ideal of X .

Proof. It is straightforward by Theorem 2.1. $\square$

Theorem 4.9. If μ is a prime fuzzy set in X , then the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a prime fuzzy set in X for all $\beta \in [0, \ddagger]$.

Proof. Assume that μ is a prime fuzzy set in X . Let $x, y \in X$. Then

$$\begin{aligned}
 \mu_{\alpha}^{\ddagger}(x \cdot y) &= \mu(x \cdot y) - \beta \\
 &\leq \max\{\mu(x), \mu(y)\} - \beta \\
 &= \max\{\mu(x) - \beta, \mu(y) - \beta\} \\
 &= \max\{\mu_{\beta}^{\ddagger}(x), \mu_{\beta}^{\ddagger}(y)\}.
 \end{aligned}$$

Hence, $\mu_{\beta}^{\ddagger}$ is a prime fuzzy set in X . $\square$

Theorem 4.10. If there exists $\beta \in [0, \ddagger]$ such that the fuzzy (β, Π) -translation $\mu_{\beta}^{\ddagger}$ of μ is a prime fuzzy set in X , then μ is a prime fuzzy set in X .

Proof. Assume that $\mu_\beta^\dagger$ is a prime fuzzy set in X for $\beta \in [0, \dagger]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(x \cdot y) - \beta &= \mu_\beta^\dagger(x \cdot y) \\ &\leq \max\{\mu_\beta^\dagger(x), \mu_\beta^\dagger(y)\} \\ &= \max\{\mu(x) - \beta, \mu(y) - \beta\} \\ &= \max\{\mu(x), \mu(y)\} - \beta.\end{aligned}$$

So, $\mu(x \cdot y) \leq \max\{\mu(x), \mu(y)\}$. Hence, μ is a prime fuzzy set in X . $\square$

Theorem 4.11. *If μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\beta \in [0, \dagger]$.*

Proof. It follows from Theorem 4.1 (resp., Theorem 4.3, Theorem 4.5, Theorem 4.7) and Theorem 4.9. $\square$

Theorem 4.12. *If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 4.2 (resp., Theorem 4.4, Theorem 4.6, Theorem 4.8) and Theorem 4.10. $\square$

Theorem 4.13. *If μ is a weakly prime fuzzy set in X , then the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a weakly prime fuzzy set in X for all $\beta \in [0, \dagger]$.*

Proof. The proof is similar to Theorem 4.9, so we will omit it. $\square$

Theorem 4.14. *If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a weakly prime fuzzy set in X , then μ is a weakly prime fuzzy set in X .*

Proof. The proof is similar to Theorem 4.10, so we will omit it. $\square$

Theorem 4.15. *If μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\beta \in [0, \dagger]$.*

Proof. It follows from Theorem 4.1 (resp., Theorem 4.3, Theorem 4.5, Theorem 4.7) and Theorem 4.13. $\square$

Theorem 4.16. *If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 4.2 (resp., Theorem 4.4, Theorem 4.6, Theorem 4.8) and Theorem 4.14. $\square$

Note 4.1. If μ is a fuzzy set in X and $\beta \in [0, \frac{1}{2}]$, then $\mu_\beta^\dagger(x) = \mu(x) - \beta \leq \mu(x), \forall x \in X$. Hence, the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a fuzzy contraction of μ for all $\beta \in [0, \frac{1}{2}]$.

Lemma 4.1. Let μ and ν be fuzzy sets in X . If $\nu \subseteq \mu_\beta^\dagger$ for $\beta \in [0, \frac{1}{2}]$, there exists $\alpha \in [0, \frac{1}{2}]$ with $\alpha \geq \beta$ such that $\nu \subseteq \mu_\alpha^\dagger \subseteq \mu_\beta^\dagger$.

Proof. Assume that $\nu \subseteq \mu_\beta^\dagger$ for $\beta \in [0, \frac{1}{2}]$. Then $\nu(x) \leq \mu_\beta^\dagger(x), \forall x \in X$. Putting $\alpha = \beta + \inf_{x \in X} \{\mu_\beta^\dagger(x) - \nu(x)\}$. Then

$$\begin{aligned} \inf_{x \in X} \{\mu_\beta^\dagger(x) - \nu(x)\} &= \inf_{x \in X} \{(\mu(x) - \beta) - \nu(x)\} \\ &\leq \inf_{x \in X} \{\mu(x) - \beta\} \\ &= \inf_{x \in X} \{\mu(x)\} - \beta \\ &= \frac{1}{2} - \beta, \end{aligned}$$

so $\alpha = \beta + \inf_{x \in X} \{\mu_\beta^\dagger(x) - \nu(x)\} \leq \beta + \frac{1}{2} - \beta = \frac{1}{2}$. Thus $\alpha \in [0, \frac{1}{2}]$ and $\alpha \geq \beta$, so $\mu_\alpha^\dagger \subseteq \mu_\beta^\dagger$. Now, for all $x \in X$, we have

$$\begin{aligned} \mu_\alpha^\dagger(x) &= \mu(x) - \alpha \\ &= \mu(x) - (\beta + \inf_{x \in X} \{\mu_\beta^\dagger(x) - \nu(x)\}) \\ &= \mu(x) - \beta - \inf_{x \in X} \{\mu_\beta^\dagger(x) - \nu(x)\} \\ &= \mu_\beta^\dagger(x) + \sup_{x \in X} \{\nu(x) - \mu_\beta^\dagger(x)\} \\ &\geq \mu_\beta^\dagger(x) + \nu(x) - \mu_\beta^\dagger(x) \\ &= \nu(x), \end{aligned}$$

so $\nu \subseteq \mu_\alpha^\dagger$. Hence, $\nu \subseteq \mu_\alpha^\dagger \subseteq \mu_\beta^\dagger$ for some $\alpha \in [0, \frac{1}{2}]$ with $\alpha \geq \beta$. $\square$

Definition 4.2. Let μ_1 and μ_2 be two fuzzy sets in X and $\mu_1 \subseteq \mu_2$. If μ_2 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ_1 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , and we say that μ_1 is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of μ_2 .

Theorem 4.17. If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of μ for all $\beta \in [0, \frac{1}{2}]$.

Proof. It follows from Theorem 4.1 (resp., Theorem 4.3, Theorem 4.5, Theorem 4.7) and Note 4.1. $\square$

Theorem 4.18. If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, Π) -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ for all $\alpha, \beta \in [0, \frac{1}{2}]$ with $\alpha \geq \beta$.

Proof. It follows from Theorem 4.1 (resp., Theorem 4.3, Theorem 4.5, Theorem 4.7). $\square$

Theorem 4.19. *Let μ be a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X and $\beta \in [0, \dagger]$. For every fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction ν of the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ , there exists $\alpha \in [0, \dagger]$ with $\alpha \geq \beta$ such that ν is the fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of the fuzzy α -translation $\mu_\alpha^\dagger$ of μ .*

Proof. It follows from Theorem 4.1 (resp., Theorem 4.3, Theorem 4.5, Theorem 4.7) and Lemma 4.1. $\square$

Theorem 4.20. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, Π) -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X for all $\beta \in [0, \dagger]$.*

Proof. It follows from Theorems 3.2 and 4.1 (resp., Theorems 3.4 and 4.3, Theorems 3.6 and 4.5, Theorems 3.8 and 4.7). $\square$

Theorem 4.21. *If there exists $\beta \in [0, \dagger]$ such that the fuzzy (β, Π) -translation $\mu_\beta^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, Π) -translation $\mu_\alpha^\dagger$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X for all $\alpha \in [0, \dagger]$.*

Proof. It follows from Theorems 4.2 and 3.1 (resp., Theorems 4.4 and 3.3, Theorems 4.6 and 3.5, Theorems 4.8 and 3.7). $\square$

5. FUZZY (γ, III) -TRANSLATIONS

Beyond the conventional expansion and contraction approaches, fuzzy (γ, III) -translations introduce a more intricate transformation that allows for dynamic adjustments in fuzzy set structures. These transformations account for both increasing and decreasing membership values based on predefined algebraic rules, making them a versatile tool for modeling complex fuzzy interactions in IUP-algebras. This section defines fuzzy (γ, III) -translations, investigates their algebraic properties, and discusses their implications for the study of fuzzy IUP-structures. Examples and theorems are provided to solidify the theoretical framework and practical applications of these translations.

Definition 5.1. *Let μ be a fuzzy set $\in X$ and let $\gamma \in [0, 1]$. A mapping $\mu_\gamma^\mp : X \rightarrow [0, 1]$ defined by*

$$\mu_\gamma^\mp(x) = \mu(x) \times \gamma, \forall x \in X, \quad (5.1)$$

is said to be a fuzzy γ -translation of μ of type III or, in short, a fuzzy (γ, III) -translation of μ .

Example 5.1. *From Example 3.1, let $\gamma = 0.02 \in [0, 1]$. Then the mapping $\mu_\gamma^\mp : X \rightarrow [0, 1]$ defined by*

$$\mu_\gamma^\mp = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.16 & 0.02 & 0.04 & 0.02 & 0.04 & 0.02 \end{pmatrix}.$$

Hence, $\mu_\gamma^\mp$ is a fuzzy IUP-subalgebra of X .

Theorem 5.1. If μ is a fuzzy IUP-subalgebra of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra of X for all $\gamma \in [0, 1]$.

Proof. Assume that μ is a fuzzy IUP-subalgebra of X . Let $x, y \in X$. Then

$$\begin{aligned}\mu_\gamma^\mp(x \cdot y) &= \mu(x \cdot y) \times \gamma \\ &\geq \min\{\mu(x), \mu(y)\} \times \gamma \\ &= \min\{\mu(x) \times \gamma, \mu(y) \times \gamma\} \\ &= \min\{\mu_\gamma^\mp(x), \mu_\gamma^\mp(y)\}.\end{aligned}$$

Hence, $\mu_\gamma^\mp$ is a fuzzy IUP-subalgebra of X . □

Theorem 5.2. If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra of X , then μ is a fuzzy IUP-subalgebra of X .

Proof. Assume that $\mu_\gamma^\mp$ is a fuzzy IUP-subalgebra of X for $\gamma \in (0, 1]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(x \cdot y) \times \gamma &= \mu_\gamma^\mp(x \cdot y) \\ &\geq \min\{\mu_\gamma^\mp(x), \mu_\gamma^\mp(y)\} \\ &= \min\{\mu(x) \times \gamma, \mu(y) \times \gamma\} \\ &= \min\{\mu(x), \mu(y)\} \times \gamma.\end{aligned}$$

So, $\mu(x \cdot y) \geq \min\{\mu(x), \mu(y)\}$. Hence, μ is a fuzzy IUP-subalgebra of X . □

Example 5.2. From Example 3.2, let $\gamma = 0.2 \in [0, 1]$. Then the mapping $\mu_\gamma^\mp : X \rightarrow [0, 1]$ defined by

$$\mu_\gamma^\mp = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.18 & 0.02 & 0.02 & 0.02 & 0.06 & 0.02 \end{pmatrix}.$$

Hence, $\mu_\gamma^\mp$ is a fuzzy IUP-filter of X .

Example 5.3. From Example 3.3, let $\gamma = 0.4 \in [0, 1]$. Then the mapping $\mu_\gamma^\mp : \mathbb{R}^* \rightarrow [0, 1]$ defined by

$$\mu_\gamma^\mp(x) = \begin{cases} 0.32 & \text{if } x \in S, \\ 0.12 & \text{otherwise.} \end{cases}$$

Hence, $\mu_\gamma^\mp$ is a fuzzy IUP-filter of $\mathbb{R}^*$.

Theorem 5.3. If μ is a fuzzy IUP-filter of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-filter of X for all $\gamma \in [0, 1]$.

Proof. Assume that μ is a fuzzy IUP-filter of X . Let $x, y \in X$. Then

$$\begin{aligned}\mu_\gamma^\mp(0) &= \mu(0) \times \gamma \\ &\geq \mu(x) \times \gamma \\ &= \mu_\gamma^\mp(x)\end{aligned}$$

and

$$\begin{aligned}\mu_{\gamma}^{\mp}(y) &= \mu(y) \times \gamma \\ &\geq \min\{\mu(x \cdot y), \mu(x)\} \times \gamma \\ &= \min\{\mu(x \cdot y) \times \gamma, \mu(x) \times \gamma\} \\ &= \min\{\mu_{\gamma}^{\mp}(x \cdot y), \mu_{\gamma}^{\mp}(x)\}.\end{aligned}$$

Hence, $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-filter of X . □

Theorem 5.4. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_{\gamma}^{\mp}$ of μ is a fuzzy IUP-filter of X , then μ is a fuzzy IUP-filter of X .*

Proof. Assume that $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-filter of X for $\gamma \in (0, 1]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(0) \times \gamma &= \mu_{\gamma}^{\mp}(0) \\ &\geq \mu_{\gamma}^{\mp}(x) \\ &= \mu(x) \times \gamma\end{aligned}$$

and

$$\begin{aligned}\mu(y) \times \gamma &= \mu_{\gamma}^{\mp}(y) \\ &\geq \min\{\mu_{\gamma}^{\mp}(x \cdot y), \mu_{\gamma}^{\mp}(x)\} \\ &= \min\{\mu(x \cdot y) \times \gamma, \mu(x) \times \gamma\} \\ &= \min\{\mu(x \cdot y), \mu(x)\} \times \gamma.\end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(y) \geq \min\{\mu(x \cdot y), \mu(x)\}$. Hence, μ is a fuzzy IUP-filter of X . □

Example 5.4. *From Example 3.4, let $\gamma = 0.2 \in [0, 1]$. Then the mapping $\mu_{\gamma}^{\mp} : X \rightarrow [0, 1]$ defined by*

$$\mu_{\gamma}^{\mp} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.14 & 0.06 & 0.04 & 0.4 & 0.4 & 0.6 \end{pmatrix}.$$

Hence, $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-ideal of X .

Example 5.5. *From Example 3.5, let $\gamma = 0.4 \in [0, 1]$. Then the mapping $\mu_{\gamma}^{\mp} : \mathbb{R}^* \rightarrow [0, 1]$ defined by*

$$\mu_{\gamma}^{\mp}(x) = \begin{cases} 0.24 & \text{if } x \in S, \\ 0.12 & \text{otherwise.} \end{cases}$$

Hence, $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-ideal of $\mathbb{R}^$.*

Theorem 5.5. *If μ is a fuzzy IUP-ideal of X , then the fuzzy (γ, III) -translation $\mu_{\gamma}^{\mp}$ of μ is a fuzzy IUP-ideal of X for all $\gamma \in [0, 1]$.*

Proof. Assume that μ is a fuzzy IUP-ideal of X . Let $x, y, z \in X$, we have

$$\begin{aligned}\mu_{\gamma}^{\mp}(0) &= \mu(0) \times \gamma \\ &\geq \mu(x) \times \gamma \\ &= \mu_{\gamma}^{\mp}(x)\end{aligned}$$

and

$$\begin{aligned}\mu_{\gamma}^{\mp}(x \cdot z) &= \mu(x \cdot z) \times \gamma \\ &\geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} \times \gamma \\ &= \min\{\mu(x \cdot (y \cdot z)) \times \gamma, \mu(y) \times \gamma\} \\ &= \min\{\mu_{\gamma}^{\mp}(x \cdot (y \cdot z)), \mu_{\gamma}^{\mp}(y)\}.\end{aligned}$$

Hence, $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-ideal of X . □

Theorem 5.6. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_{\gamma}^{\mp}$ of μ is a fuzzy IUP-ideal of X , then μ is a fuzzy IUP-ideal of X .*

Proof. Assume that $\mu_{\gamma}^{\mp}$ is a fuzzy IUP-ideal of X for $\gamma \in (0, 1]$. Let $x, y, z \in X$. Then

$$\begin{aligned}\mu(0) \times \gamma &= \mu_{\gamma}^{\mp}(0) \\ &\geq \mu_{\gamma}^{\mp}(x) \\ &= \mu(x) \times \gamma\end{aligned}$$

and

$$\begin{aligned}\mu(x \cdot z) \times \gamma &= \mu_{\gamma}^{\mp}(x \cdot z) \\ &\geq \min\{\mu_{\gamma}^{\mp}(x \cdot (y \cdot z)), \mu_{\gamma}^{\mp}(y)\} \\ &= \min\{\mu(x \cdot (y \cdot z)) \times \gamma, \mu(y) \times \gamma\} \\ &= \min\{\mu(x \cdot (y \cdot z)), \mu(y)\} \times \gamma.\end{aligned}$$

So, $\mu(0) \geq \mu(x)$ and $\mu(x \cdot z) \geq \min\{\mu(x \cdot (y \cdot z)), \mu(y)\}$. Hence, μ is a fuzzy IUP-ideal of X . □

Example 5.6. *From Example 3.6, let $\gamma = 0.3 \in [0, 0.5]$. Then the mapping $\mu_{\gamma}^{\mp} : X \rightarrow [0, 1]$ defined by*

$$\mu_{\gamma}^{\mp} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 \\ 0.15 & 0.15 & 0.15 & 0.15 & 0.15 & 0.15 \end{pmatrix}.$$

Hence, $\mu_{\gamma}^{\mp}$ is a fuzzy strong IUP-ideal of X .

Theorem 5.7. *If μ is a fuzzy strong IUP-ideal of X , then the fuzzy (γ, III) -translation $\mu_{\gamma}^{\mp}$ of μ is a fuzzy strong IUP-ideal of X for all $\gamma \in [0, 1]$.*

Proof. It is straightforward by Theorem 2.1. □

Theorem 5.8. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_{\gamma}^{\mp}$ of μ is a fuzzy strong IUP-ideal of X , then μ is a fuzzy strong IUP-ideal of X .*

Proof. It is straightforward by Theorem 2.1. $\square$

Theorem 5.9. *If μ is a prime fuzzy set in X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a prime fuzzy set in X for all $\gamma \in [0, 1]$.*

Proof. Assume that μ is a prime fuzzy set in X . Let $x, y \in X$. Then

$$\begin{aligned}\mu_\gamma^\mp(x \cdot y) &= \mu(x \cdot y) \times \gamma \\ &\leq \max\{\mu(x), \mu(y)\} \times \gamma \\ &= \max\{\mu(x) \times \gamma, \mu(y) \times \gamma\} \\ &= \max\{\mu_\gamma^\mp(x), \mu_\gamma^\mp(y)\}.\end{aligned}$$

Hence, $\mu_\gamma^\mp$ is a prime fuzzy set in X . $\square$

Theorem 5.10. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a prime fuzzy set in X , then μ is a prime fuzzy set in X .*

Proof. Assume that $\mu_\gamma^\mp$ is a prime fuzzy set in X for $\gamma \in (0, 1]$. Let $x, y \in X$. Then

$$\begin{aligned}\mu(x \cdot y) \times \gamma &= \mu_\gamma^\mp(x \cdot y) \\ &\leq \max\{\mu_\gamma^\mp(x), \mu_\gamma^\mp(y)\} \\ &= \max\{\mu(x) \times \gamma, \mu(y) \times \gamma\} \\ &= \max\{\mu(x), \mu(y)\} \times \gamma.\end{aligned}$$

So, $\mu(x \cdot y) \leq \max\{\mu(x), \mu(y)\}$. Hence, μ is a prime fuzzy set in X . $\square$

Theorem 5.11. *If μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\gamma \in [0, \mp]$.*

Proof. It follows from Theorem 5.1 (resp., Theorem 5.3, Theorem 5.5, Theorem 5.7) and Theorem 5.9. $\square$

Theorem 5.12. *If there exists $\gamma \in [0, \mp]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 5.2 (resp., Theorem 5.4, Theorem 5.6, Theorem 5.8) and Theorem 5.10. $\square$

Theorem 5.13. *If μ is a weakly prime fuzzy set in X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a weakly prime fuzzy set in X for all $\gamma \in [0, 1]$.*

Proof. The proof is similar to Theorem 5.9, so we will omit it. $\square$

Theorem 5.14. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a weakly prime fuzzy set in X , then μ is a weakly prime fuzzy set in X .*

Proof. The proof is similar to Theorem 5.10, so we will omit it. $\square$

Theorem 5.15. *If μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X all $\gamma \in [0, 1]$.*

Proof. It follows from Theorem 5.1 (resp., Theorem 5.3, Theorem 5.5, Theorem 5.7) and Theorem 5.13. $\square$

Theorem 5.16. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then μ is a weakly prime fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X .*

Proof. It follows from Theorem 5.2 (resp., Theorem 5.4, Theorem 5.6, Theorem 5.8) and Theorem 5.14. $\square$

Note 5.1. *If μ is a fuzzy set in X and $\gamma \in [0, 1]$, then $\mu_\gamma^\mp(x) = \mu(x) \times \gamma \leq \mu(x), \forall x \in X$. Hence, the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy contraction of μ for all $\gamma \in [0, 1]$.*

Lemma 5.1. *Let μ and ν be fuzzy sets in X . If $\nu \supseteq \mu_\gamma^\mp$ for $\gamma \in [0, 1]$, there exists $\alpha \in [0, 1]$ with $\alpha \geq \gamma$ such that $\nu \supseteq \mu_\alpha^\mp \supseteq \mu_\gamma^\mp$.*

Proof. Assume that $\nu \supseteq \mu_\gamma^\mp$ for $\gamma \in [0, 1]$. Then $\nu(x) \geq \mu_\gamma^\mp(x)$ for all $x \in X$. Putting $\alpha = \gamma + \inf_{x \in X}(\{\nu(x) - \mu_\gamma^\mp(x)\})$. Now, for all $x \in X$, we have

$$\begin{aligned}
 \mu_\alpha^\mp(x) &= \mu(x) \times \alpha \\
 &= \mu(x) \times (\gamma + \inf_{x \in X}(\{\nu(x) - \mu_\gamma^\mp(x)\})) \\
 &= (\mu(x) \times \gamma) + (\mu(x) \times \inf_{x \in X}(\{\nu(x) - \mu_\gamma^\mp(x)\})) \\
 &\leq \mu_\gamma^\mp(x) + (\mu(x) \times (\nu(x) - \mu_\gamma^\mp(x))) \\
 &= \mu_\gamma^\mp(x) + (\mu(x) \times \nu(x)) - (\mu(x) \times \mu_\gamma^\mp(x)) \\
 &= (\mu_\gamma^\mp(x) \times (1 - \mu(x))) + (\mu(x) \times \nu(x)) \\
 &\leq (\nu(x) \times (1 - \mu(x))) + (\mu(x) \times \nu(x)) \\
 &= \nu(x) \times (1 - \mu(x) + \mu(x)) \\
 &= \nu(x) \times 1 \\
 &= \nu(x),
 \end{aligned}$$

so $\nu \supseteq \mu_\alpha^\mp(x)$. Hence, $\nu \supseteq \mu_\alpha^\mp \supseteq \mu_\gamma^\mp$ for some $\alpha \in [0, 1]$ with $\alpha \geq \gamma$. $\square$

Theorem 5.17. *If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of μ for all $\gamma \in [0, 1]$.*

Proof. It follows from Theorem 5.1 (resp., Theorem 5.3, Theorem 5.5, Theorem 5.7) and Note 5.1. $\square$

Theorem 5.18. *If μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) contraction of the fuzzy (α, III) -translation $\mu_\alpha^\mp$ of μ for all $\alpha, \gamma \in [0, 1]$ with $\alpha \geq \gamma$.*

Proof. It follows from Theorem 5.1 (resp., Theorem 5.3, Theorem 5.5, Theorem 5.7). $\square$

Theorem 5.19. *Let μ be a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X and $\gamma \in [0, 1]$. For every fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension ν of the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ , there exists $\alpha \in [0, 1]$ with $\alpha \geq \gamma$ such that ν is the fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) extension of the fuzzy (α, III) -translation $\mu_\alpha^\mp$ of μ .*

Proof. It follows from Theorem 5.1 (resp., Theorem 5.3, Theorem 5.5, Theorem 5.7) and Lemma 5.1. $\square$

Theorem 5.20. *If there exists $\alpha \in [0, \dagger]$ such that the fuzzy (α, I) -translation μ_α^+ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X for all $\gamma \in [0, 1]$.*

Proof. It follows from Theorems 3.2 and 5.1 (resp., Theorems 3.4 and 5.3, Theorems 3.6 and 5.5, Theorems 3.8 and 5.7). $\square$

Theorem 5.21. *If there exists $\gamma \in (0, 1]$ such that the fuzzy (γ, III) -translation $\mu_\gamma^\mp$ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X , then the fuzzy (α, I) -translation μ_α^+ of μ is a fuzzy IUP-subalgebra (resp., fuzzy IUP-filter, fuzzy IUP-ideal, fuzzy strong IUP-ideal) of X for all $\alpha \in [0, \dagger]$.*

Proof. It follows from Theorems 5.2 and 3.1 (resp., Theorems 5.4 and 3.3, Theorems 5.6 and 3.5, Theorems 5.8 and 3.7). $\square$

6. CONCLUSION

In this study, we have introduced and analyzed three types of fuzzy translations—fuzzy (α, I) -translation, fuzzy (β, II) -translation, and fuzzy (γ, III) -translation—within the framework of IUP-algebras. We have established their fundamental properties and provided illustrative examples to demonstrate their applicability. Moreover, we have explored the interactions between fuzzy

translations and prime as well as weakly prime fuzzy sets, extending our analysis to fuzzy extensions and contractions. Our findings contribute to the growing body of knowledge on fuzzy algebraic structures and provide a foundation for further research on the role of fuzzy translations in advanced mathematical frameworks. Future investigations may focus on extending these concepts to other algebraic systems or exploring their potential applications in computational and decision-making processes involving uncertainty and imprecision.

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