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A New Approach of Possibility Single-Valued Neutrosophic Set and Its Application in Decision-Making Environment

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Abstract. The single-valued neutrosophic set (SVNS) is a widely known model for dealing with uncertain, conflicting, and indeterminate information. In practice, the SVNSs are very useful tools to be used in solving multi-criteria decision-making (MCDM), but in the process of processing by the three functions of SVNS the evaluation process for this handling disappears. To overcome this deficiency, we present in this work a new approach called possibility single-valued neutrosophic set (PSVNS) that differs from previous approaches. The implementation of this proposed approach in this work is based on giving each of the three functions in SVNS a fuzzy degree ranging between 0 and 1. As a result, firstly, the elementary notion of possibility single neutrosophic set is proposed, and some of its primary properties, i.e., subset, null set, absolute set, and complement are explored, as well as some numerical examples that explain the mechanism of the obtained results. Secondly, the basic set-theoretic operations i.e., such as extended union, the intersection of two PSVNSs, and the complement operation of PSVNS, as well as some relevant properties, are investigated, and numerical examples are provided to illustrate the mechanism behind these results. Lastly, the similarity measure between two PSVNSs is characterized with the help of an example. This technique of similarity measure is successfully used in decision-making to choose the appropriate college.

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1. Introduction

Classical mathematical methods struggle to address complicated problems in everyday fields such as economics, medical sciences, engineering, and management sciences due to the inherent uncertainty in the graphical environments of these fields. To manage this shortage, Zadeh [1] found the notion of fuzzy sets (FSs) as a mathematical structure that contains a membership foundation called true membership and is denoted as follows μ_A. The degree of truth in FS emphasizes the degree of belonging or membership to a certain particular object (element) from the initial universal set space X. In 1986, Atanassov [2]realized that the degree of belonging in FS was no longer sufficient, so he resorted to adding another degree called the degree of non-belonging, so that the mathematical structure that combines the two degrees of belonging and non-belonging is called intuitionistic fuzzy sets (IFSs).

Both FS and IFS have been a great inspiration to researchers, which has led them to present many research achievements in all areas of life. Because science is constantly advancing, the scientist Smarandache [3] appeared in 1998 and presented the neutrosophic set (NS) theory by adding a new aspect to the aspect of correctness and disagreement, which is the neutrality principle to cover human thinking. Mathematically, this theory consists of three functions, the starting point of which is the universal set and the rest of each of which is the closed set 0 and 1. Because of its integration with the previous tools, NS is considered a good tool for dealing with problems that contain inaccurate, uncertain, and conflicting data. Thus, many research works related to this theory have emerged, for instance: Yang et al [4] lighted a single valued neutrosophic set (SVNS) to facilitate application in real applications. Chai et al. [5] proposed several similarity measures for SVNSs and employed them in pattern recognition and medical diagnosis problems. Majumdar et al. [6] introduced the measures of distance between two SVNSs and studied their properties. Bolturk and Kahraman [7] introduced a novel Analytic Hierarchy Process (AHP) method for the design-making process with interval-valued neutrosophic sets (IVNS). Al-Quran et al. [8,9] employ the NS methods with some aggregation techniques. Hazaymeh and Bataihah [10,11]studied some topological methods on SVNS. Maji [12] proposed the notion of a neutrosophic soft set (NSS) by merging both soft sets (SSs) and NS. Deli and Broumi [13] studied the relationship between two NSSs and their applications. Ali et al. [14] defined bipolar property on NSS and used this method in design making. Al-Qudah et al. [15,16] showed some mathematical structures on NSSs and employed them in solving some real-life applications. In addition, many researchers[17-21] have employed these techniques to deal with issues characterised by ambiguity, uncertainty, and obscurity.

On the other hand, Probability theory plays an important role in indicating the probability of an event occurring or indicating the user's confidence in the effectiveness of the assessment. From the human thinking side, which drives the decision-making process, probability theory evaluates outcomes and decisions according to well-studied criteria. Therefore, it is essential to employ this theory in the fuzzy structures mentioned above. The concept of the possibility fuzzy soft sets (PFSSs) was first introduced by Alkhazaleh et al. [22] when they gives each overall PFSSs structure has a fuzzy probability score ranging from 0 to 1, where this score expresses the degree of satisfaction with this PFSSs evaluation. Based on this idea and following this trend, many works have emerged through applying this idea to many vague concepts. Among these works are: Selvachandran and Salleh [23] applied this idea to intuitionistic fuzzy soft sets. Karaaslan [24] introduced concept of possibility neutrosophic soft set and defined some related properties. Romdhini et al. [25] explored similarity measures of possibility interval-valued fuzzy soft sets and possibility interval-valued fuzzy hypersoft sets, and they showed how these tools can be applied in solving DM problems. Al-Qudah and Al-Sharqi [15] pointed out the similarity measures of possibility interval-valued neutrosophic soft and checked it with some applications. Al-Hijjawi and Alkhazaleh [26] introduced concept of Possibility Neutrosophic Hypersoft Set (PNHSS) and their operations and discussed similarity measure between two PNHSSs. In addition to many research works in this field, including [27-33].

From the above and through a comprehensive study of these research works, we note that the degree of probability is related to the overall mathematical structure of the concept, given that this concept consists of more than one belonging function. Accordingly, to overcome this gap, we will present in this work a new organization of the concept presented in PSVNS by linking or distributing the degree of fuzzy probability to all three mathematical components of SVNS. Therefore, this work will provide greater freedom for the decision maker to set a probability score (evaluation) for each of the SVNS scores, and thus the decision taken will become more credible and reliable. The principal contributions in this work are described as follows:

(1) The new view of concepts in PSVNS has been introduced, where every SVN membership has a fuzzy possibility grade.
(2) The comprehension investigation of the set-theoretic operations of PSVNS is proposed, which is necessary for understanding the suggested idea.
(3) An algorithm is proposed based on a similarity measure between two PSVNSs to choose a suitable real solution.

This manuscript is designed as follows: The definitions and features of SNS are covered in Section 2. The definitions and basic properties of PSVNS are introduced in Section 3. The MADM problem based on the similarity measure between two PSVNSs is proposed in Section 4.

2. Preliminaries

All authors have read and approved the final version of the manuscript. The authors contributed equally to this work.

Definition 2.1. *[4] Let $\boldsymbol{V}$ be a universe then the SVNS structure given as following:*

$$\ddot{O}=\left\{\left\langle \varepsilon,\ddot{T}_{\ddot{O}}\left(\varepsilon\right),\ddot{I}_{\ddot{O}}\left(\varepsilon\right),\ddot{F}_{\ddot{O}}\left(\varepsilon\right)\right\rangle|\varepsilon\in\boldsymbol{V}\right\}$$

is called SVNS where the three memberships functions: true $\widehat{\mathcal{T}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$*,indeterminacy* $\widehat{\mathcal{I}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$*,falsity* $\widehat{\mathcal{F}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$ *all of them for component* $(\ddot{v})$ *in* $\boldsymbol{U}$ *with stander condition* $0\leq \widehat{\mathcal{T}}_{\Theta}(\ddot{v})+\widehat{\mathcal{I}}_{\Theta}(\ddot{v})+\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\leq 3$.

Definition 2.2. *[4] The following structure defined on* $\boldsymbol{U}$

$$\tilde{\Theta}=\left\{\left\langle \ddot{v},\widehat{\mathcal{T}}_{\Theta}(\ddot{v}),\widehat{\mathcal{I}}_{\Theta}(\ddot{v}),\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\right\rangle|\ddot{v}\in U\right\}$$

is called SVNS where the three memberships functions: true $\widehat{\mathcal{T}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$*,indeterminacy* $\widehat{\mathcal{I}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$*,falsity* $\widehat{\mathcal{F}}_{\Theta}(\ddot{v})$*:*$\boldsymbol{U}\rightarrow[0,1]$ *all of them for component* $(\ddot{v})$ *in* $\boldsymbol{U}$ *with stander condition* $0\leq \widehat{\mathcal{T}}_{\Theta}(\ddot{v})+\widehat{\mathcal{I}}_{\Theta}(\ddot{v})+\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\leq 3$.

Definition 2.3. *[4] Assume that*
$\ddot{O}_1=\left\{\left\langle \varepsilon,\ddot{T}_{\ddot{O}_1}(\varepsilon),\ddot{I}_{\ddot{O}_1}(\varepsilon),\ddot{F}_{\ddot{O}_1}(\varepsilon)\right\rangle|\varepsilon\in V\right\}$ *and* $\ddot{O}_2=\left\{\left\langle \varepsilon,\ddot{T}_{\ddot{O}_2}(\varepsilon),\ddot{I}_{\ddot{O}_2}(\varepsilon),\ddot{F}_{\ddot{O}_2}(\varepsilon)\right\rangle|\varepsilon\in V\right\}$ *be two SVNSs on* $\boldsymbol{V}$*. Then we know the basic operations as follows:*

(i.) $\ddot{O}_1$ *is submerged in* $\ddot{O}_2$ *and denotes as* $\ddot{O}_1\subseteq\ddot{O}_2$ *if*

$$\ddot{T}_{\ddot{O}_1}(\varepsilon)\leq\ddot{T}_{\ddot{O}_2}(\varepsilon),\ddot{I}_{\ddot{O}_1}(\varepsilon)\geq\ddot{I}_{\ddot{O}_2}(\varepsilon),\ddot{F}_{\ddot{O}_1}(\varepsilon)\geq\ddot{F}_{\ddot{O}_2}(\varepsilon)$$

(ii.) $\ddot{O}_1$ *is equal in* $\ddot{O}_2$ *and denotes as* $\ddot{O}_1=\ddot{O}_2$ *if*

$$\ddot{T}_{\ddot{O}_1}(\varepsilon)=\ddot{T}_{\ddot{O}_2}(\varepsilon),\ddot{I}_{\ddot{O}_1}(\varepsilon)=\ddot{I}_{\ddot{O}_2}(\varepsilon),\ddot{F}_{\ddot{O}_1}(\varepsilon)=\ddot{F}_{\ddot{O}_2}(\varepsilon)$$

(iii.) $\ddot{O}_1$ *union* $\ddot{O}_2$ *denotes as* $\ddot{O}_1\cup\ddot{O}_2$ *given as following*

$$\ddot{O}_1\cup\ddot{O}_2=\begin{Bmatrix}\max\left(\ddot{T}_{\ddot{O}_1}(\varepsilon),\ddot{T}_{\ddot{O}_2}(\varepsilon)\right)\\ \min\left(\ddot{I}_{\ddot{O}_1}(\varepsilon),\ddot{I}_{\ddot{O}_2}(\varepsilon)\right)\\ \min\left(\ddot{F}_{\ddot{O}_1}(\varepsilon),\ddot{F}_{\ddot{O}_2}(\varepsilon)\right)\end{Bmatrix}$$

(iv.) $\ddot{O}_1$ *intersection* $\ddot{O}_2$ *denotes as* $\ddot{O}_1\cap\ddot{O}_2$ *given as following*

$$\ddot{O}_1\cap\ddot{O}_2=\begin{Bmatrix}\min\left(\ddot{T}_{\ddot{O}_1}(\varepsilon),\ddot{T}_{\ddot{O}_2}(\varepsilon)\right)\\ \max\left(\ddot{I}_{\ddot{O}_1}(\varepsilon),\ddot{I}_{\ddot{O}_2}(\varepsilon)\right)\\ \max\left(\ddot{F}_{\ddot{O}_1}(\varepsilon),\ddot{F}_{\ddot{O}_2}(\varepsilon)\right)\end{Bmatrix}$$

(iiv.) The complement of $\ddot{O}_1$ *denotes as* $\ddot{O}_1^c$ *and given as following:*

$$\ddot{O}_1^c=\begin{Bmatrix}\ddot{T}_{\ddot{O}_1^c}(\varepsilon)=\ddot{F}_{\ddot{O}_1}(\varepsilon)\\ \ddot{I}_{\ddot{O}_1^c}(\varepsilon)=1-\ddot{I}_{\ddot{O}_1}(\varepsilon)\\ \ddot{F}_{\ddot{O}_1^c}(\varepsilon)=\ddot{T}_{\ddot{O}_1}(\varepsilon)\end{Bmatrix}$$

3. Possibility single-valued neutrosophic environment

In this section, we will introduce PSVNS as a new approach. We also present the basic operations related to this concept.

Definition 3.1. *Let*
$\boldsymbol{V}$=$\{\ddot{\varepsilon}_1, \ddot{\varepsilon}_2, \ddot{\varepsilon}_3, ..., \ddot{\varepsilon}_m\}$. *Then the mathematical structure of PSVNS is given as follows:*

$$\ddot{O}_p = \left\{\left\langle \varepsilon_m, \left(p_{\ddot{T}_{\ddot{O}}}\ddot{T}_{\ddot{O}}\right)(\varepsilon_m), \left(p_{\ddot{I}_{\ddot{O}}}\ddot{I}_{\ddot{O}}\right)(\varepsilon_m), \left(p_{\ddot{F}_{\ddot{O}}}\ddot{F}_{\ddot{O}}\right)(\varepsilon_m)\right\rangle | \varepsilon_m \in V\right\}$$

where
$\left(p_{\ddot{T}_{\ddot{O}}}\ddot{T}_{\ddot{O}}\right)(\varepsilon_m) : V \to [0,1]$ *is the product of the probability degree with the truth-neutrosophic function.* $\left(p_{\ddot{I}_{\ddot{O}}}\ddot{I}_{\ddot{O}}\right)(\varepsilon_m) : V \to [0,1]$ *is the product of the probability dgree with the non-neutrality-neutrosophic function and* $\left(p_{\ddot{F}_{\ddot{O}}}\ddot{F}_{\ddot{O}}\right)(\varepsilon_m) : V \to [0,1]$ *is the product of the probability dgree with the non-truth-neutrosophic, such that there is stander condution given as following* $0 \leq \left(p_{\ddot{T}_{\ddot{O}}}\ddot{T}_{\ddot{O}}\right)(\varepsilon_m) + \left(p_{\ddot{I}_{\ddot{O}}}\ddot{I}_{\ddot{O}}\right)(\varepsilon_m) + \left(p_{\ddot{F}_{\ddot{O}}}\ddot{F}_{\ddot{O}}\right)(\varepsilon_m) \leq 3$.

To explain the main definition above, we present the following numerical example.

Example 3.1. *Let* $\boldsymbol{V}$=$\{\ddot{\varepsilon}_1, \ddot{\varepsilon}_2, \ddot{\varepsilon}_3\}$. *Then a PSVNS* $\ddot{O}_p$ *on* $\boldsymbol{V}$ *given as following:*

$$\ddot{O}_p = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (0.4\,(0.7)\,, 0.6\,(0.5)\,, 0.8\,(0.4))\rangle \\ \langle \ddot{\varepsilon}_2, (0.6\,(0.3)\,, 0.3\,(0.2)\,, 0.5\,(0.4))\rangle \\ \langle \ddot{\varepsilon}_3, (0.7\,(0.4)\,, 0.9\,(0.4)\,, 0.6\,(0.2))\rangle \end{Bmatrix}$$

Definition 3.2. *A PSVNS* $\tilde{\Theta}_p = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})}\widehat{\mathcal{T}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})}\widehat{\mathcal{I}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})}\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\right\rangle | \ddot{v} \in U\right\}$ *on* $\boldsymbol{U}$ *is side to Null-PSVNS and given as following*

$$\tilde{\Theta}_p^{\varnothing} = \left\{\left\langle \ddot{v}, p_{\widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})}\widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v}), p_{\widehat{I}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})}\widehat{I}_{\tilde{\Theta}^{\varnothing}}(\ddot{v}), p_{\widehat{F}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})}\widehat{F}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})\right\rangle | \ddot{v} \in U\right\}$$

Where
$p_{\widehat{\mathcal{T}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{T}}_{\Theta^{\varnothing}}(\ddot{v}) = 0, p_{\widehat{\mathcal{I}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{I}}_{\Theta^{\varnothing}}(\ddot{v}) = 0$ *and* $p_{\widehat{\mathcal{F}}_{\Theta^{\varnothing}}(\ddot{v})} = 0, \widehat{\mathcal{F}}_{\Theta^{\varnothing}}(\ddot{v}) = 0$

Example 3.2. *Assume that* $\ddot{O}_p$ *given as following*

$$\ddot{O}_p = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (0\,(0)\,, 0\,(0)\,, 0\,(0))\rangle \\ \langle \ddot{\varepsilon}_2, (0\,(0)\,, 0\,(0)\,, 0\,(0))\rangle \\ \langle \ddot{\varepsilon}_3, (0\,(0)\,, 0\,(0)\,, 0\,(0))\rangle \end{Bmatrix}$$

Then the $\ddot{O}_p$ here named Null-PSVNS.

Definition 3.3. *A PSVNS* $\tilde{\Theta}_p = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})}\widehat{\mathcal{T}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})}\widehat{\mathcal{I}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})}\widehat{\mathcal{F}}_{\Theta}(\ddot{v})\right\rangle | \ddot{v} \in U\right\}$ *on* $\boldsymbol{U}$ *is side to Absolute-PSVNS and given as following*

$$\tilde{\Theta}_p^{U} = \left\{\left\langle \ddot{v}, p_{\widehat{T}_{\tilde{\Theta}^{\varnothing}}(\ddot{v})}\widehat{T}_{\tilde{\Theta}^{U}}(\ddot{v}), p_{\widehat{I}_{\tilde{\Theta}^{U}}(\ddot{v})}\widehat{I}_{\tilde{\Theta}^{U}}(\ddot{v}), p_{\widehat{F}_{\tilde{\Theta}^{U}}(\ddot{v})}\widehat{F}_{\tilde{\Theta}^{U}}(\ddot{v})\right\rangle | \ddot{v} \in U\right\}$$

Where
$p_{\widehat{\mathcal{T}}_{\Theta^U}(\ddot{v})} = 1, \widehat{\mathcal{T}}_{\Theta^U}(\ddot{v}) = 1, p_{\widehat{\mathcal{I}}_{\Theta^U}(\ddot{v})} = 1, \widehat{\mathcal{I}}_{\Theta^U}(\ddot{v}) = 1$ *and* $p_{\widehat{\mathcal{F}}_{\Theta^U}(\ddot{v})} = 1, \widehat{\mathcal{F}}_{\Theta^U}(\ddot{v}) = 1$

Example 3.3. *Assume that $\ddot{O}_p$ given as following*

$$\ddot{O}_p = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (1(1), 1(1), 1(1)) \rangle \\ \langle \ddot{\varepsilon}_2, (1(1), 1(1), 1(1)) \rangle \\ \langle \ddot{\varepsilon}_3, (1(1), 1(1), 1(1)) \rangle \end{Bmatrix}$$

Then the $\ddot{O}_p$ here named Absolute-PSVNS.

Definition 3.4. *We say that the* $\tilde{\Theta}^1_p = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *is subset of* $\tilde{\Theta}^2_p = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *and denotes as* $\tilde{\Theta}^1_p \leq \tilde{\Theta}^2_p$ *if :*
$p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}) \leq p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})$, $p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}) \geq p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})$, $p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \geq p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})$.

Example 3.4. *Assume that $\ddot{O}^1_p$ and $\ddot{O}^2_p$ given as following*

$$\ddot{O}^1_p = \left\{ \langle \ddot{\varepsilon}_1, (0.3(0.5), 0.4(0.6), 0.5(0.4)) \rangle \right\}$$

and

$$\ddot{O}^2_p = \left\{ \langle \ddot{\varepsilon}_2, (0.4(0.6), 0.2(0.1), 0.3(0.4)) \rangle \right\}$$

Then here $\tilde{\Theta}^1_p \leq \tilde{\Theta}^2_p$.

Definition 3.5. *We say that the* $\tilde{\Theta}^1_p = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *is equal of* $\tilde{\Theta}^2_p = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *and denotes as* $\tilde{\Theta}^1_p = \tilde{\Theta}^2_p$ *if :*
$p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}) = p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})$, $p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}) = p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})$, $p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) = p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})$.

Example 3.5. *Assume that $\ddot{O}^1_p$ and $\ddot{O}^2_p$ given as following*

$$\ddot{O}^1_p = \left\{ \langle \ddot{\varepsilon}_1, (0.3(0.5), 0.4(0.6), 0.5(0.4)) \rangle \right\}$$

and

$$\ddot{O}^2_p = \left\{ \langle \ddot{\varepsilon}_2, (0.3(0.5), 0.4(0.6), 0.5(0.4)) \rangle \right\}$$

Then here $\tilde{\Theta}^1_p = \tilde{\Theta}^2_p$.

Definition 3.6. *The complement of PSVNS* $\tilde{\Theta}_p = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *is denotes as* $\tilde{\Theta}_p^c = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^c}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^c}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^c}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$.
Where
$p_{\widehat{\mathcal{T}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^c}(\ddot{v}) = p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} . \widehat{\mathcal{F}}_{\Theta}(\ddot{v})$, $p_{\widehat{\mathcal{I}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^c}(\ddot{v}) = \left(1 - p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})}\right) . \left(1 - \widehat{\mathcal{I}}_{\Theta}(\ddot{v})\right)$, $p_{\widehat{\mathcal{F}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^c}(\ddot{v}) = p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} . \widehat{\mathcal{T}}_{\Theta}(\ddot{v})$.

Example 3.6. *Assume that* $\ddot{O}_p$ *given as following*

$$\ddot{O}_p = \left\{ \langle \ddot{\varepsilon}_1, (0.3\,(0.5)\,, 0.4\,(0.6)\,, 0.5\,(0.4)) \rangle \right\}$$

then the complement of $\ddot{O}_p$

$$\ddot{O}_p^c = \left\{ \langle \ddot{\varepsilon}_2, (0.5\,(0.4)\,, 0.6\,(0.4)\,, 0.3\,(0.5)) \rangle \right\}$$

Proposition 3.1. *For PSVNS* $\tilde{\Theta}_p$ *then* $\left(\left(\tilde{\Theta}_p\right)^c\right)^c = \tilde{\Theta}_p$.

Proof. This proof depends on the definition 3.6.
Firstly take $\tilde{\Theta}_p = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ and take

$$\tilde{\Theta}_p^c = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^c}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^c}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^c}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^c}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}.$$

$$= \left\{\left\langle \ddot{v}, p_{\widehat{F}_{\Theta}(\ddot{v})} \widehat{F}_{\Theta}(\ddot{v}), \left(1 - p_{\widehat{I}_{\Theta}(\ddot{v})}\right)\left(1 - \widehat{I}_{\Theta}(\ddot{v})\right), p_{\widehat{T}_{\Theta}(\ddot{v})} \widehat{T}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

Now take $\left(\left(\tilde{\Theta}_p\right)^c\right)^c$

$$\left(\tilde{\Theta}_p^c\right)^c = \left\{\left\langle \ddot{v}, p_{\widehat{F}_{\Theta^c}(\ddot{v})} \widehat{F}_{\Theta^c}(\ddot{v}), \left(1 - p_{\widehat{I}_{\Theta^c}(\ddot{v})}\right)\left(1 - \widehat{I}_{\Theta^c}(\ddot{v})\right), p_{\widehat{T}_{\Theta^c}(\ddot{v})} \widehat{T}_{\Theta^c}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$
$$= \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), \left(1 - \left(1 - p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})}\right)\right)\left(1 - \left(1 - \widehat{\mathcal{I}}_{\Theta}(\ddot{v})\right)\right), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$
$$= \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$$

$= \tilde{\Theta}_p$ is a PSVNS.

□

Definition 3.7. *The union between two PSVNSs*
$\tilde{\Theta}_p^1 = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *and*
$\tilde{\Theta}_p^2 = \left\{\left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *given as following:*

$$\tilde{\Theta}_p^1 \bigcup \tilde{\Theta}_p^2 = \left\{ \langle \ddot{v},\ \max\left[p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v}), p_{\widehat{T}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}) \right], \right.$$
$$\left. \min\left[p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}) \right], \min\left[p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}), p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right] \right\rangle \right\}.$$

Example 3.7. *Assume that the following two PSVNSs*

$$\tilde{\Theta}_p^1 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.3\,(.8)\,, .9\,(.6)\,, .2\,(.7)) \rangle \\ \langle \ddot{\varepsilon}_2, (.7\,(.1)\,, .8\,(.6)\,, .3\,(.9)) \rangle \\ \langle \ddot{\varepsilon}_3, (.6\,(.1)\,, .6\,(.7)\,, .5\,(.6)) \rangle \end{Bmatrix}$$

and

$$\tilde{\Theta}_p^2 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.1\,(.5)\,, .3\,(.4)\,, .5\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_2, (.2\,(.2)\,, .6\,(.3)\,, .7\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_3, (.4\,(.6)\,, .1\,(.5)\,, .3\,(.7)) \rangle \end{Bmatrix}$$

Then

$$\tilde{\Theta}_p^1 \bigcup \tilde{\Theta}_p^2 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.3\,(.8)\,, .3\,(.4)\,, .2\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_2, (.7\,(.2)\,, .6\,(.3)\,, .3\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_3, (.6\,(.6)\,, .1\,(.5)\,, .3\,(.6)) \rangle \end{Bmatrix}$$

Proposition 3.2. *Let* $\ddot{O}_p^1 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}} \ddot{T}_{\ddot{O}^1} \right)(\varepsilon)\,, \left(p_{\ddot{I}_{\ddot{O}^1}} \ddot{I}_{\ddot{O}^1} \right)(\varepsilon)\,, \left(p_{\ddot{F}_{\ddot{O}^1}} \ddot{F}_{\ddot{O}^1} \right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$
$\ddot{O}_p^2 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}} \ddot{T}_{\ddot{O}^2} \right)(\varepsilon)\,, \left(p_{\ddot{I}_{\ddot{O}^2}} \ddot{I}_{\ddot{O}^2} \right)(\varepsilon)\,, \left(p_{\ddot{F}_{\ddot{O}^2}} \ddot{F}_{\ddot{O}^2} \right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *and*
$\ddot{O}_p^3 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^3}} \ddot{T}_{\ddot{O}^2} \right)(\varepsilon)\,, \left(p_{\ddot{I}_{\ddot{O}^3}} \ddot{I}_{\ddot{O}^3} \right)(\varepsilon)\,, \left(p_{\ddot{F}_{\ddot{O}^3}} \ddot{F}_{\ddot{O}^3} \right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *be three PSVNSs on* $\boldsymbol{V}$. *Then a following points are satisfied:*
(i.) $\ddot{O}_p^1 \cup \ddot{O}_p^{\phi} = \ddot{O}_p^{\phi} \cup \ddot{O}_p^1 = \ddot{O}_p^1$
(ii.) $\ddot{O}_p^1 \cup \ddot{O}_p^V = \ddot{O}_p^V \cup \ddot{O}_p^1 = \ddot{O}_p^V$
(iii.) $\ddot{O}_p^1 \cup \ddot{O}_p^2 = \ddot{O}_p^2 \cup \ddot{O}_p^1$
(iv.) $\ddot{O}_p^1 \cup \left(\ddot{O}_p^2 \cup \ddot{O}_p^3 \right) = \left(\ddot{O}_p^1 \cup \ddot{O}_p^2 \right) \cup \ddot{O}_p^3$

Proof. Direct depending on the definition 3.7 □

Definition 3.8. *The intersection between two PSVNSs*
$\tilde{\Theta}_p^1 = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})\,, p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})\,, p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *and*
$\tilde{\Theta}_p^2 = \left\{ \left\langle \ddot{v}, p_{\widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v})\,, p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})\,, p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right\rangle | \ddot{v} \in U \right\}$ *given as following:*

$$\tilde{\Theta}_p^1 \bigcap \tilde{\Theta}_p^2 = \left\{ \langle \ddot{v},\ \min\left[p_{\widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^1}(\ddot{v})\,, p_{\widehat{T}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{T}}_{\Theta^2}(\ddot{v}) \right], \right.$$
$$\left. \max\left[p_{\widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^1}(\ddot{v})\,, p_{\widehat{\mathcal{I}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{I}}_{\Theta^2}(\ddot{v}) \right], \max\left[p_{\widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^1}(\ddot{v})\,, p_{\widehat{\mathcal{F}}_{\Theta^2}(\ddot{v})} \widehat{\mathcal{F}}_{\Theta^2}(\ddot{v}) \right] \right\rangle \right\}.$$

Example 3.8. *Assume that the following two PSVNSs*

$$\tilde{\Theta}_p^1 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.3\,(.8)\,, .9\,(.6)\,, .2\,(.7)) \rangle \\ \langle \ddot{\varepsilon}_2, (.7\,(.1)\,, .8\,(.6)\,, .3\,(.9)) \rangle \\ \langle \ddot{\varepsilon}_3, (.6\,(.1)\,, .6\,(.7)\,, .5\,(.6)) \rangle \end{Bmatrix}$$

and

$$\tilde{\Theta}_p^2 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.1\,(.5)\,, .3\,(.4)\,, .5\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_2, (.2\,(.2)\,, .6\,(.3)\,, .7\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_3, (.4\,(.6)\,, .1\,(.5)\,, .3\,(.7)) \rangle \end{Bmatrix}$$

Then

$$\tilde{\Theta}_p^1 \bigcap \tilde{\Theta}_p^2 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.1\,(.5)\,, .9\,(.6)\,, .5\,(.7)) \rangle \\ \langle \ddot{\varepsilon}_2, (.2\,(.2)\,, .8\,(.6)\,, .7\,(.9)) \rangle \\ \langle \ddot{\varepsilon}_3, (.4\,(.1)\,, .6\,(.7)\,, .5\,(.7)) \rangle \end{Bmatrix}$$

Proposition 3.3. *Let* $\ddot{O}_p^1 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}} \ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1}} \ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1}} \ddot{F}_{\ddot{O}^1}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$
$\ddot{O}_p^2 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}} \ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2}} \ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2}} \ddot{F}_{\ddot{O}^2}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *and*
$\ddot{O}_p^3 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^3}} \ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^3}} \ddot{I}_{\ddot{O}^3}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^3}} \ddot{F}_{\ddot{O}^3}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *be three PSVNSs on* ***V****. Then a following points are satisfied:*
(i.) $\ddot{O}_p^1 \cap \ddot{O}_p^\phi = \ddot{O}_p^\phi \cap \ddot{O}_p^1 = \ddot{O}_p^\phi$
(ii.) $\ddot{O}_p^1 \cap \ddot{O}_p^V = \ddot{O}_p^V \cap \ddot{O}_p^1 = \ddot{O}_p^1$
(iii.) $\ddot{O}_p^1 \cap \ddot{O}_p^2 = \ddot{O}_p^2 \cap \ddot{O}_p^1$
(iv.) $\ddot{O}_p^1 \cap \left(\ddot{O}_p^2 \cap \ddot{O}_p^3\right) = \left(\ddot{O}_p^1 \cap \ddot{O}_p^2\right) \cap \ddot{O}_p^3$

Proof. Direct depending on the definition 3.8 □

Proposition 3.4. *Let* $\ddot{O}_p^1 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}} \ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1}} \ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1}} \ddot{F}_{\ddot{O}^1}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *and*
$\ddot{O}_p^2 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}} \ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2}} \ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2}} \ddot{F}_{\ddot{O}^2}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *be two PSVNSs on* ***V****. Then a following points are satisfied the De Morgan's laws:*
(i.) $\left(\ddot{O}_p^1 \cup \ddot{O}_p^2\right)^c = \left(\ddot{O}_p^1\right)^c \cap \left(\ddot{O}_p^2\right)^c$.
(ii.) $\left(\ddot{O}_p^1 \cap \ddot{O}_p^2\right)^c = \left(\ddot{O}_p^1\right)^c \cup \left(\ddot{O}_p^2\right)^c$

Proof. Direct depending on the definition 3.7 and 3.8.

□

Proposition 3.5. *Let* $\ddot{O}_p^1 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}} \ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1}} \ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1}} \ddot{F}_{\ddot{O}^1}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$
$\ddot{O}_p^2 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}} \ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2}} \ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2}} \ddot{F}_{\ddot{O}^2}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *and*
$\ddot{O}_p^3 = \left\{ \left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^3}} \ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^3}} \ddot{I}_{\ddot{O}^3}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^3}} \ddot{F}_{\ddot{O}^3}\right)(\varepsilon) \right\rangle | \varepsilon \in V \right\}$ *be three PSVNSs on* ***V****. Then a following points are satisfied:*
(i.) $\ddot{O}_p^1 \cup \left(\ddot{O}_p^2 \cap \ddot{O}_p^3\right) = \left(\ddot{O}_p^1 \cup \ddot{O}_p^2\right) \cap \left(\ddot{O}_p^1 \cup \ddot{O}_p^3\right)$.
(ii.) $\ddot{O}_p^1 \cap \left(\ddot{O}_p^2 \cup \ddot{O}_p^3\right) = \left(\ddot{O}_p^1 \cap \ddot{O}_p^2\right) \cup \left(\ddot{O}_p^1 \cap \ddot{O}_p^3\right)$.

Proof. (i.) Take $\ddot{O}_p^1 \cup \left(\ddot{O}_p^2 \cap \ddot{O}_p^3\right)$ and for all $\varepsilon \in \mathbf{V}$, Then
$\mathcal{W}_{\left(\ddot{O}_p^1 \cup \left(\ddot{O}_p^2 \cap \ddot{O}_p^3\right)\right)}(\varepsilon) = \cup \left\{ W_{\ddot{O}_p^1(\varepsilon)}, W_{\left(\ddot{O}_p^2(\varepsilon) \cap \ddot{O}_p^3(\varepsilon)\right)} \right\}$

$$= \cup\left\{W_{\ddot{O}^1_p(\varepsilon)}, \cap\left(W_{\left(\ddot{O}^2_p(\varepsilon)\right)}, W_{\ddot{O}^3_p(\varepsilon)}\right)\right\}$$
$$= \left\{\left\langle \varepsilon, \max\left(\ddot{T}_{\ddot{O}^1_p(\varepsilon)}, \min\left(\ddot{T}_{\ddot{O}^2_p(\varepsilon)}, \ddot{T}_{\ddot{O}^3_p(\varepsilon)}\right)\right),\ \min\left(\ddot{I}_{\ddot{O}^1_p(\varepsilon)}, max\left(\ddot{I}_{\ddot{O}^2_p(\varepsilon)}, \ddot{I}_{\ddot{O}^3_p(\varepsilon)}\right)\right),\ \min\left(\ddot{F}_{\ddot{O}^1_p(\varepsilon)}, max\left(\ddot{F}_{\ddot{O}^2_p(\varepsilon)}, \ddot{F}_{\ddot{O}^3_p(\varepsilon)}\right)\right)\right\rangle\right\}$$
$$= \left\{\left\langle \varepsilon, \min\left(, \max\left(\ddot{T}_{\ddot{O}^1_p(\varepsilon)}, \ddot{T}_{\ddot{O}^2_p(\varepsilon)}\right), \max\left(\ddot{T}_{\ddot{O}^1_p(\varepsilon)}, \ddot{T}_{\ddot{O}^3_p(\varepsilon)}\right)\right),\right.$$
$$\max\left(\min\left(\ddot{I}_{\ddot{O}^1_p(\varepsilon)}, \ddot{I}_{\ddot{O}^2_p(\varepsilon)}\right), \min\left(\ddot{I}_{\ddot{O}^1_p(\varepsilon)}, \ddot{I}_{\ddot{O}^3_p(\varepsilon)}\right)\right),$$
$$\left.\max\left(\min\left(\ddot{F}_{\ddot{O}^1_p(\varepsilon)}, \ddot{F}_{\ddot{O}^2_p(\varepsilon)}\right), \min\left(\ddot{F}_{\ddot{O}^1_p(\varepsilon)}, \ddot{F}_{\ddot{O}^3_p(\varepsilon)}\right)\right)\right\rangle\right\}$$
$$= \cap\left(\cup\left(W_{\left(\ddot{O}^1_p(\varepsilon)\cup\ddot{O}^2_p(\varepsilon)\right)}, W_{\left(\ddot{O}^1_p(\varepsilon)\cup\ddot{O}^3_p(\varepsilon)\right)}\right)\right)$$
$$= W_{\left(\ddot{O}^1_p(\varepsilon)\cup\ddot{O}^2_p(\varepsilon)\right)\cap\left(\ddot{O}^1_p(\varepsilon)\cup\ddot{O}^3_p(\varepsilon)\right)}$$
$$= \left(\ddot{O}^1_p \cup \ddot{O}^2_p\right) \cap \left(\ddot{O}^1_p \cup \ddot{O}^3_p\right).$$

The Proof for the second (ii) is similar to the first (i).

□

Definition 3.9. *Let* $\ddot{O}^1_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}}\ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1}}\ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1}}\ddot{F}_{\ddot{O}^1}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *and* $\ddot{O}^2_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}}\ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2}}\ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2}}\ddot{F}_{\ddot{O}^2}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *be two PSVNSs on* $\boldsymbol{V}$. *Then the 'OR' product between PSVNSs* $\ddot{O}^1_p$ *and* $\ddot{O}^2_p$ *denotes as* $\ddot{O}^1_p \vee \ddot{O}^2_p$ *and given by following formalh:*

$$\ddot{O}^1_p \vee \ddot{O}^2_p = \left\{\left\langle \varepsilon, \left(\left(p_{\ddot{T}_{\ddot{O}^1}}\ddot{T}_{\ddot{O}^1}\right) \vee \left(p_{\ddot{T}_{\ddot{O}^2}}\ddot{T}_{\ddot{O}^2}\right)\right)(\varepsilon), \left(\left(p_{\ddot{I}_{\ddot{O}^1}}\ddot{I}_{\ddot{O}^1}\right) \wedge \left(p_{\ddot{I}_{\ddot{O}^2}}\ddot{I}_{\ddot{O}^2}\right)\right)(\varepsilon),\right.$$
$$\left.\left(\left(p_{\ddot{F}_{\ddot{O}^1}}\ddot{F}_{\ddot{O}^1}\right) \wedge \left(p_{\ddot{F}_{\ddot{O}^2}}\ddot{F}_{\ddot{O}^2}\right)\right)(\varepsilon)\right\rangle\right\}.$$

Definition 3.10. *Let* $\ddot{O}^1_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1}}\ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1}}\ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1}}\ddot{F}_{\ddot{O}^1}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *and* $\ddot{O}^2_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2}}\ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2}}\ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2}}\ddot{F}_{\ddot{O}^2}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *be two PSVNSs on* $\boldsymbol{V}$. *Then the 'AND' product between PSVNSs* $\ddot{O}^1_p$ *and* $\ddot{O}^2_p$ *denotes as* $\ddot{O}^1_p \wedge \ddot{O}^2_p$ *and given by following formalh:*

$$\ddot{O}^1_p \wedge \ddot{O}^2_p = \left\{\left\langle \varepsilon, \left(\left(p_{\ddot{T}_{\ddot{O}^1}}\ddot{T}_{\ddot{O}^1}\right) \wedge \left(p_{\ddot{T}_{\ddot{O}^2}}\ddot{T}_{\ddot{O}^2}\right)\right)(\varepsilon), \left(\left(p_{\ddot{I}_{\ddot{O}^1}}\ddot{I}_{\ddot{O}^1}\right) \vee \left(p_{\ddot{I}_{\ddot{O}^2}}\ddot{I}_{\ddot{O}^2}\right)\right)(\varepsilon),\right.$$
$$\left.\left(\left(p_{\ddot{F}_{\ddot{O}^1}}\ddot{F}_{\ddot{O}^1}\right) \vee \left(p_{\ddot{F}_{\ddot{O}^2}}\ddot{F}_{\ddot{O}^2}\right)\right)(\varepsilon)\right\rangle\right\}.$$

4. Noval similarity measures between PSVN-sets

In this section, we will explore the idea of a similarity measure between PSVNSs, which is organized as follows.

Definition 4.1. *Suppose that* $\ddot{O}^1_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^1_p}}\ddot{T}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^1_p}}\ddot{I}_{\ddot{O}^1}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^1_p}}\ddot{F}_{\ddot{O}^1}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *and* $\ddot{O}^2_p = \left\{\left\langle \varepsilon, \left(p_{\ddot{T}_{\ddot{O}^2_p}}\ddot{T}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{I}_{\ddot{O}^2_p}}\ddot{I}_{\ddot{O}^2}\right)(\varepsilon), \left(p_{\ddot{F}_{\ddot{O}^2_p}}\ddot{F}_{\ddot{O}^2}\right)(\varepsilon)\right\rangle | \varepsilon \in V\right\}$ *be two PSVNSs on* $\boldsymbol{V}$. *Then the similarity measure (SM) between two component PSVNSs* $\ddot{O}^1_p$ *and* $\ddot{O}^2_p$ *is indicate by* $SM\left(\ddot{O}^1_p, \ddot{O}^2_p\right)$ *and given as the following formalh:*

$$SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{A_1\left(\ddot{O}_p^1, \ddot{O}_p^2\right) + A_2\left(\ddot{O}_p^1, \ddot{O}_p^2\right) + A_3\left(\ddot{O}_p^1, \ddot{O}_p^2\right)}{3}$$

Where

$$A_1\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{\sum_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)}{\sum_{i=1}^{n}\left[1 - \sqrt{\left(\left[1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right].\left[1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right]\right)}\right]}$$

$$A_2\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{\sum_{i=1}^{n}\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)\left(p_{\ddot{I}_{\ddot{O}_p^2}}\ddot{I}_{\ddot{O}^2}(\varepsilon_i)\right)}{\sum_{i=1}^{n}\left[1 - \sqrt{\left(\left[(1-\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right].\left[1-\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right]\right)}\right]}$$

$$A_3\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \sqrt{1 - \frac{\sum_{i=1}^{n}\left|\left(p_{\ddot{F}_{\ddot{O}_p^1}}\ddot{F}_{\ddot{O}^1}(\varepsilon_i)\right)^2 - \left(p_{\ddot{F}_{\ddot{O}_p^2}}\ddot{F}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right|}{\sum_{i=1}^{n} 1 + \left[\left(p_{\ddot{F}_{\ddot{O}_p^1}}\ddot{F}_{\ddot{O}^1}(\varepsilon_i)\right)^2.\left(p_{\ddot{F}_{\ddot{O}_p^2}}\ddot{F}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right]}}$$

Proposition 4.1. *Assume that $\ddot{O}_p^1, \ddot{O}_p^2$ and $\ddot{O}_p^3$ be three PSVNSs on $\boldsymbol{V}$. Then a following points are satisfied*

(i.) $SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = SM\left(\ddot{O}_p^2, \ddot{O}_p^1\right)$.
(ii.) $0 \leq SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) \leq 1$.
(iii.) If $\ddot{O}_p^1 = \ddot{O}_p^2$ *then* $SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = 1$.
(vi.) $\ddot{O}_p^1 \subseteq \ddot{O}_p^2 \subseteq \ddot{O}_p^3$ *then* $SM\left(\ddot{O}_p^1, \ddot{O}_p^3\right) \leq SM\left(\ddot{O}_p^2, \ddot{O}_p^3\right)$.
(vii.) $\ddot{O}_p^1 \cap \ddot{O}_p^2 = \phi$ *then* $SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = 0$.

Proof. The proofs (i), (ii) and (v) are clear. Now we will proof (iii).
Suppose that $\ddot{O}_p^1 = \ddot{O}_p^2$ and T.P $SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = 1$ implies
That mean we have
$p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i) = p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)$, $p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i) = p_{\ddot{I}_{\ddot{O}_p^2}}\ddot{I}_{\ddot{O}^2}(\varepsilon_i)$, and $p_{\ddot{F}_{\ddot{O}_p^1}}\ddot{F}_{\ddot{O}^1}(\varepsilon_i) = p_{\ddot{F}_{\ddot{O}_p^2}}\ddot{F}_{\ddot{O}^2}(\varepsilon_i)$

$$A_1\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{\sum_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2}{\sum_{i=1}^{n}\left(1-\left[1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right]\right)} = \frac{\sum_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2}{\sum_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2} = 1$$

$$A_2\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{\sum_{i=1}^{n}\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)^2}{\sum_{i=1}^{n}\left(1-\left[1-\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right]\right)} = \frac{\sum_{i=1}^{n}\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)^2}{\sum_{i=1}^{n}\left(p_{\ddot{I}_{\ddot{O}_p^1}}\ddot{I}_{\ddot{O}^1}(\varepsilon_i)\right)^2} = 1$$

$$A_3\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \sqrt{1-0} = 1$$

Thus, we get

$$SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{1+1+1}{3} = 1$$

(vi) Clearly, $p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i).p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i) \le p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i).p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)$
implies that

$$\sum_{i=1}^{n} p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i).p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i) \le \sum_{i=1}^{n} p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i).p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)$$

$\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2 \le \left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2$ implies that $-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2 \ge -\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2$

and

$$\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right) \ge \left(1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)$$

$$\sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)} \ge \sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right)}$$

$$1-\sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)} \le 1-\sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right)}$$

and

$$\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)} \le$$
$$\sum_{i=1}^{n} 1-\sqrt{\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)}$$

from above, we get

$$\frac{\sum\limits_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)}{\sum\limits_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^1}}\ddot{T}_{\ddot{O}^1}(\varepsilon_i)\right)^2\right)\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)\right)}\right)} \le$$
$$\frac{\sum\limits_{i=1}^{n}\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)}{\sum\limits_{i=1}^{n}\left(1-\sqrt{\left(\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^2}}\ddot{T}_{\ddot{O}^2}(\varepsilon_i)\right)^2\right).\left(1-\left(p_{\ddot{T}_{\ddot{O}_p^3}}\ddot{T}_{\ddot{O}^3}(\varepsilon_i)\right)^2\right)\right)}\right)}$$

The rest of the terms can be treated in the same way, and thus we get:

$SM\left(\ddot{O}_p^1, \ddot{O}_p^3\right) \le SM\left(\ddot{O}_p^2, \ddot{O}_p^3\right).$ □

Example 4.1. *Assume that the following two PSVNSs*

$$\tilde{\Theta}_p^1 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.3\,(.8)\,, .9\,(.6)\,, .2\,(.7)) \rangle \\ \langle \ddot{\varepsilon}_2, (.7\,(.1)\,, .8\,(.6)\,, .3\,(.9)) \rangle \\ \langle \ddot{\varepsilon}_3, (.6\,(.1)\,, .6\,(.7)\,, .5\,(.6)) \rangle \end{Bmatrix}$$

and

$$\tilde{\Theta}_p^2 = \begin{Bmatrix} \langle \ddot{\varepsilon}_1, (.1\,(.5)\,, .3\,(.4)\,, .5\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_2, (.2\,(.2)\,, .6\,(.3)\,, .7\,(.3)) \rangle \\ \langle \ddot{\varepsilon}_3, (.4\,(.6)\,, .1\,(.5)\,, .3\,(.7)) \rangle \end{Bmatrix}$$

Then, the similarity measure between the two PSVNSs calculate as following:

$$A_1\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{(.3\times.8)(.1\times.5)+(.7\times.1)(.2\times.2)+(.6\times.1)(.4\times.6)}{1-\left(\sqrt{(0.9424)(0.9975)}\right)+1-\left(\sqrt{(0.9951)(0.9984)}\right)+1-\left(\sqrt{(0.9964)(0.9424)}\right)} = \frac{0.0292}{0.09792} = 0.2982$$

$$A_2\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{(.9\times.6)(.3\times.4)+(.8\times.6)(.6\times.3)+(.6\times.7)(.1\times.5)}{1-\left(\sqrt{(0.8354)(0.6438)}\right)+1-\left(\sqrt{(0.9632)(0.9972)}\right)+1-\left(\sqrt{(0.8598)(0.9972)}\right)} = \frac{0.1722}{0.5082} = 0.3388$$

$$A_3\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \sqrt{1 - \frac{\left|(.2\times.7)^2-(.5\times.3)^2\right|+\left|(.3\times.9)^2-(.7\times.3)^2\right|+\left|(.5\times.6)^2-(.3\times.7)^2\right|}{1+\left((.2\times.7)^2\times(.5\times.3)^2+(.3\times.9)^2\times(.7\times.3)^2+(.5\times.6)^2\times(.3\times.7)^2\right)}} = \sqrt{1-\frac{0.0972}{1.0076}} = 0.9505$$

Then,

$$SM\left(\ddot{O}_p^1, \ddot{O}_p^2\right) = \frac{0.2982 + 0.3388 + 0.9505}{3} = 0.5292$$

.

Therefor, the similarity degree between the two PSVNSs $\ddot{O}_p^1, \ddot{O}_p^2$ *is 0.5292.*

5. Application

In various fields of life, such as education, politics, management, and economics, users face difficulties in making decisions among a number of available options. For example, in the field of education, there is confusion or hesitation in choosing the best academic program among many. Accordingly, numerous studies have been conducted on the influential criteria that parents rely on when choosing educational programs for their newly graduated children from middle school. The influencing factors can be divided into the academic reputation of the university campus, academic quality, the financial cost of the academic program, and employment opportunities. To clarify this context, we will present a scenario in this section that illustrates this problem, and we will employ our proposed concept in this work to solve it.

Example 5.1. *Alex recently completed his high school education with a GPA of 95, which qualified him to enter many available colleges. Here, Alex entered an internal conflict about choosing the appropriate college. In order to help him, the family sat down and began discussing the options that Alex was considering in order to help him choose according to specific criteria. Therefore, in order to solve this problem and gives their helps to Alex they are using our proposed concept, we assume that these colleges are as follows:* $\varepsilon_1 = College\ of\ Medicine$, $\varepsilon_2 = College\ of\ Dentistry$ *and* $\varepsilon_3 = College\ of\ Artificial\ Intelligence$. *As for* *the criteria, they are as follows:* $\gamma_1 = Quality$, $\gamma_2 = Studycost$, *and* $\gamma_3 = Job\ opportunities$. *Therefor the following PSVNSs shows the numerical analysis in line with our proposed concept of the family's opinions and suggestions according to the choices and criteria mentioned above.*

$$\ddot{O}_p^1(\varepsilon_1) = \begin{Bmatrix} \langle \gamma_1, (.3)\,.8, (.9)\,.6, (.1)\,.4 \rangle \\ \langle \gamma_2, (.5)\,.2, (.6)\,.3, (.7)\,.2 \rangle \\ \langle \gamma_3, (.2)\,.6, (.4)\,.9, (.6)\,.7 \rangle \end{Bmatrix}$$

$$\ddot{O}_p^2(\varepsilon_2) = \begin{Bmatrix} \langle \gamma_1, (.1)\,.4, (.7)\,.2, (.8)\,.3 \rangle \\ \langle \gamma_2, (.5)\,.2, (.8)\,.4, (.2)\,.1 \rangle \\ \langle \gamma_3, (.2)\,.6, (.8)\,.7, (.4)\,.3 \rangle \end{Bmatrix}$$

$$\ddot{O}_p^3(\varepsilon_3) = \begin{Bmatrix} \langle \gamma_1, (.6)\,.8, (.9)\,.1, (.2)\,.1 \rangle \\ \langle \gamma_2, (.8)\,.2, (.4)\,.5, (.3)\,.6 \rangle \\ \langle \gamma_3, (.2)\,.4, (.2)\,.7, (.6)\,.8 \rangle \end{Bmatrix}$$

Now we work on finding the SM between each of the three models $\ddot{O}_p^1$, $\ddot{O}_p^2$ and $\ddot{O}_p^3$ shown above and the standard solution shown in $\ddot{O}_p^4$:

$$\ddot{O}_p^4(\varepsilon_i) = \begin{Bmatrix} \langle \gamma_1, (1)\,1, (1)\,1, (1)\,1 \rangle \\ \langle \gamma_2, (1)\,1, (1)\,1, (1)\,1 \rangle \\ \langle \gamma_3, (1)\,1, (1)\,1, (1)\,1 \rangle \end{Bmatrix}$$

Using the methods shown in Example 4.1, we obtain:
$SM\left(\ddot{O}_p^1, \ddot{O}_p^4\right) = 0,463$, $SM\left(\ddot{O}_p^2, \ddot{O}_p^4\right) = 0,628$, $SM\left(\ddot{O}_p^3, \ddot{O}_p^4\right) = 0,842$

Therefore, the relative choice is $\varepsilon_3 = College\ of\ Artificial\ Intelligence$.

6. Conclusion

The primary goal of this study is to present a possibility single neutrosophic set (PSVNS) as a new methodology that differs from current methods to solve the phenomena related to decision-making that involve uncertainty issues. As a result, the elementary notion of possibility single neutrosophic set is proposed, and some of its primary properties, i.e., subset, null set, absolute set, and complement, are explored, as well as some numerical examples that explain the mechanism of the obtained results. Secondly, the basic set-theoretic operations i.e., such as extended union, the intersection of two PSVNSs, and the complement operation of PSVNS, as well as some relevant properties, are investigated, and numerical examples are provided to illustrate the mechanism behind these results. Lastly, the above discussion has an important point in influence on the similarity measure; therefore, the similarity measure between two PSVNSs is characterized with the help of an example. This technique of similarity measure is successfully used in decision-making to judge the appropriate college according to specific criteria.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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