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On Neutrosophic e-Compactness

Wadei AL-Omeri*

Department of Mathematics, Faculty of Science, Jadara University, Irbid, Jordan

**Corresponding author:* wadeimoon1@hotmail.com

Abstract. In the exploration of Neutrosophic fine spaces, this article investigates and study a novel concept known as Neutrosophic fine open sets (N_fOS). After giving the fundamental concepts of Neutrosophic fine open sets (N_fOS) in topological spaces, we present the properties of these sets, the study obtained and analyzes both Neutrosophic fine open and closed sets within the context of Neutrosophic fine spaces. The article establishes fundamental definitions, accompanied by illustrative real time example, to provide a comprehensive understanding of the newly introduced sets. Furthermore, the exploration extends to defining and examining key concepts such as Neutrosophic fine continuity, Neutrosophic fine irresoluteness, and Neutrosophic fine irresolute homeomorphism. This progression aims to contribute to the broader comprehension and application of Neutrosophic fine spaces in topological contexts.

1. Historical background

Neutrosophic system was defined by Smarandache at the beginning of 20th century In various recent papers, has laid the foundation for a whole family of new mathematical theories generalizing both their fuzzy and classical counterparts, and have wide range of real applications for the fields of decision making,Medicine, Applied Mathematic,Information Systems, Computer Science. F. Smarandache modified the concepts of intuitionistic fuzzy sets and different styles of sets to obtained neutrosophic sets (NSs for short) [13].

F. Smarandache and A. Al Shumrani obtained the concept of neutrosophic topology on the non-general and standard interval [14,17]. Several authors was extended this principle with many applications (see [4,5,16,19,27–30,32]). Recently, Alomari and Smarandache [26,27] introduce and discussed the concepts of continuity in neutrosophic topology.

S. Saha [21] introduced δ-open sets in topological spaces. In 2021, Vadivel and John Sunda [4] defined δ-open sets in a neutrosophic topological space. E. Ekici [12] introduced and discussed the

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notion of e-open sets in a general topology. Several authors was extended and studied this principle with many Topological space [24].Seenivasan and Kamala [23] in 2014 introduced fuzzy e-open sets in a topological space along with fuzzy e-continuity. Vadivel et al. [6] studied Neutrosophic e-open sets in Neutrosophic topological space.

In [8] Vadivel, introduced the notions of Neutrosophic e-open sets and Neutrosophic e-continuity. Throughout this paper, we define and study the concept of Neutrosophic e-compactness. This concept is stronger than the notion of Neutrosophic compactness . Also we investigate the behavior of Neutrosophic e-compactness under several types of Neutrosophic continuous functions in addition specialized some of their basic properties. Finally, we define a Neutrosophic locally e-compactness and give some results on it.

Definition 1.1. *[1] Let $\mathcal{Z}$ be a non-empty set. A neutrosophic set (NS for short) $\tilde{R}$ is an object having the form $\tilde{R} = \{\langle r, \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$, where $\gamma_{\tilde{R}}(r)$, $\sigma_{\tilde{R}}(r)$, $\mu_{\tilde{R}}(r)$, and the degree of non-membership (namely $\gamma_{\tilde{R}}(r)$), the degree of indeterminacy (namely $\sigma_{\tilde{R}}(r)$), and the degree of membership function (namely $\mu_{\tilde{R}}(r)$), of each element $r \in \mathcal{Z}$ to the set $\tilde{R}$.*

Since our main purpose is to construct the tools for developing neutrosophic set and neutrosophic topology, we must introduce the Neutrosophic sets (*NSs*) 0_N and 1_N [1] in $\mathcal{Z}$ are introduced as follows:

$1 - 0_N$ can be defined as four types:

(1) $0_N = \{\langle r, 0, 1, 0\rangle : r \in \mathcal{Z}\}$,
(2) $0_N = \{\langle r, 0, 0, 0\rangle : r \in \mathcal{Z}\}$,
(3) $0_N = \{\langle r, 0, 0, 1\rangle : r \in \mathcal{Z}\}$,
(4) $0_N = \{\langle r, 0, 1, 1\rangle : r \in \mathcal{Z}\}$.

2- 1_N can be defined as four types:

(1) $1_N = \{\langle r, 1, 1, 1\rangle : r \in \mathcal{Z}\}$,
(2) $1_N = \{\langle r, 1, 1, 0\rangle : r \in \mathcal{Z}\}$,
(3) $1_N = \{\langle r, 1, 0, 0\rangle : r \in \mathcal{Z}\}$,
(4) $1_N = \{\langle r, 1, 0, 1\rangle : r \in \mathcal{Z}\}$.

Definition 1.2. *[1] Let $\{A_j : j \in J\}$ be a arbitrary family of NSS in $\mathcal{Z}$, then*

(1) *Intersection and Union $\cap A_j, \cup A_j$ may be defined as follows:*
-T_1: $\cap A_j = \langle r, \underset{j\in J}{\wedge}\mu_{Aj}(r), \underset{j\in J}{\wedge}\sigma_{Aj}(r), \underset{j\in J}{\vee}\gamma_{Aj}(r)\rangle$
-T_2: $\cap A_j = \langle r, \underset{j\in J}{\wedge}\mu_{Aj}(r), \underset{j\in J}{\vee}\sigma_{Aj}(r), \underset{j\in J}{\vee}\gamma_{Aj}(r)\rangle$.
-T_1: $\cup A_j = \langle r, \underset{j\in J}{\vee}\mu_{Aj}(r), \underset{j\in J}{\vee}\sigma_{Aj}(r), \underset{j\in J}{\wedge}\gamma_{Aj}(r)\rangle$
-T_2: $\cup A_j = \langle r, \underset{j\in J}{\vee}\mu_{Aj}(r), \underset{j\in J}{\wedge}\sigma_{Aj}(r), \underset{j\in J}{\wedge}\gamma_{Aj}(r)\rangle$

Definition 1.3. *Let $\tilde{R} = \langle \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\rangle$ be an NS on $\mathcal{Z}$. [10] The complement of the set $\tilde{R}(C(\tilde{R}), for\ short)$ may be defined as follows:*

(1) $C(\tilde{R}) = \{\langle r, 1-\mu_{\tilde{R}}(r), 1-\gamma_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$,
(2) $C(\tilde{R}) = \{\langle r, \gamma_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \mu_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$,
(3) $C(\tilde{R}) = \{\langle r, \gamma_{\tilde{R}}(r), 1-\sigma_{\tilde{R}}(r), \mu_{\tilde{R}}(r)\rangle : r \in \mathcal{Z}\}$.

Definition 1.4. *[1] Let X be a non-empty set, and GNSS H and K in the form $H = \{r, \mu_H(r), \sigma_H(r), \gamma_H(r)\}$, $K = \{r, \mu_K(r), \sigma_K(r), \gamma_K(r)\}$, then we may consider two possible definitions for subsets $(H \subseteq K)$ $(H \subseteq K)$ may be defined as*

(1) T_1*:* $H \subseteq K \Leftrightarrow \mu_H(r) \le \mu_K(r), \sigma_H(r) \le \sigma_K(r)$*, and* $\gamma_H(r) \ge \gamma_K(r)$.
(2) T_2*:* $H \subseteq K \Leftrightarrow \mu_H(r) \le \mu_K(r), \sigma_H(r) \ge \sigma_K(r)$*, and* $\gamma_H(r) \ge \gamma_K(r)$.

Definition 1.5. *[15] A neutrosophic topology (NT for short) and a non empty set $\mathcal{Z}$ is a family $\mathcal{T}$ of neutrosophic subsets of $\mathcal{Z}$ satisfying the following axioms*

(1) $0_N, 1_N \in \mathcal{T}$.
(2) $H_1 \cap H_2 \in \mathcal{T}$ *for any* $H_1, H_2 \in \mathcal{T}$.
(3) $\cup H_i \in \mathcal{T}, \forall \{H_i | j \in J\} \subseteq \mathcal{T}$.

The pair $(\mathcal{Z}, \mathcal{T})$ is called a neutrosophic topological space (briefly NTS).

Definition 1.6. *[1] Let $\tilde{R} = \{\mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\}$ be a neutrosophic open sets (briefly NROs) and $B = \{\mu_B(r), \sigma_B(r), \gamma_B(r)\}$ a neutrosophic set on a neutrosophic topological space $(\mathcal{Z}, \mathcal{T})$. Then*

(1) *$\tilde{R}$ is called neutrosophic regular open iff $\tilde{R} = NInt(NCl(\tilde{R}))$.*
(2) *The complement of neutrosophic regular open (NROs) is neutrosophic regular closed (briefly NRCs).*

Definition 1.7. *[7] Let $(\mathcal{Z}, \mathcal{T})$ be NTs on $\mathcal{Z}$ and A be an Ns on $\mathcal{Z}$. A set A is said to be a Neutrosophic*

(1) *δ-interior of A (for short, $N\delta Int(A)$) is defined by*
$N\delta Int(A) = \bigcup\{K : K \subseteq A, K \text{ is a NROs} \in \mathcal{Z}\}$.
(2) *δ-closure of A (for short, $N\delta Cl(A)$) is defined by*
$N\delta Cl(A) = \bigcap\{L : A \subseteq L, L \text{ is a NRCs} \in \mathcal{Z}\}$.

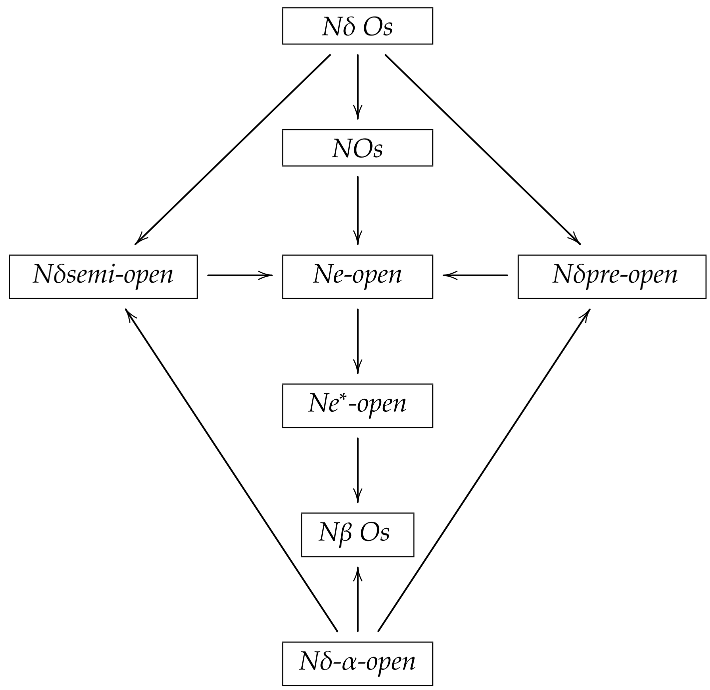
Definition 1.8. *[7] Let $(\mathcal{Z}, \mathcal{T})$ be NTs on $\mathcal{Z}$ and A be an Ns on $\mathcal{Z}$. A set A is said to be a*

(1) *Neutrosophic δ-open set (briefly, NδOs) if $A = N\delta Int(A)$.*
(2) *Neutrosophic δ-pre open set (briefly, Nδ POs) if $A \subseteq NInt(N\delta Cl(A))$.*
(3) *Neutrosophic δ-semi open set (briefly, N δ SOs) if $A \subset NCl(N\delta Int(A))$.*
(4) *Neutrosophic δ-α-open (or) a-open set (briefly, NδαOs (or) NaOs) if $A \subseteq NInt(NCl(N\delta Int(A)))$.*
(5) *Neutrosophic e^*-open set (briefly, Ne^*Os) if $A \subseteq NCl(NInt(N\delta Cl(A)))$.*
(6) *Neutrosophic β-open set (briefly, NβOs) if $A \subseteq NCl(NInt(NCl(A)))$.*

Definition 1.9. *[6] Let $(\mathcal{Z}, \mathcal{T})$ be NTs on $\mathcal{Z}$ and A be an Ns on $\mathcal{Z}$. A set A is said to be a*

(1) *Neutrosophic e-open set (briefly, NeOs) if $A \subseteq NCl(N\delta Int(A)) \cup NInt(N\delta Cl(A))$,*
(2) *Neutrosophic e-closed set (briefly, NeOs) if $A \supseteq NCl(N\delta Int(A)) \cap NInt(N\delta Cl(A))$.*

Remark 1.1. *From the above definition and some types of NOS's,we have the following diagram:*



Definition 1.10. *[6] Let $\tilde{R}$ be an NS and $(\mathcal{Z},\mathcal{T})$ an NT where $\tilde{R} = \{r, \mu_{\tilde{R}}(r), \sigma_{\tilde{R}}(r), \gamma_{\tilde{R}}(r)\}$. Then,*

(1) $NCL_e(\tilde{R}) = \bigcap\{H : H$ *is an NeCS in* $\mathcal{Z}$ *and* $\tilde{R} \subseteq H\}$.
(2) $NInt_e(\tilde{R}) = \bigcup\{W : W$ *is an NeOS in* $\mathcal{Z}$ *and* $W \subseteq \tilde{R}\}$.

It is clear that $\tilde{R}$ is an NeCS (NeOS) in $\mathcal{Z}$ iff $\tilde{R} = Cl_e(\tilde{R})(\tilde{R} = Int_e(\tilde{R}))$.

Definition 1.11. *[26] Let $(\mathcal{Z},\Gamma)$ be a neutrosophic topological space and $x_{r,t,s}$ be a neutrosophic point in $\mathcal{Z}$. A neutrosophic set S of $\mathcal{Z}$ is called a neutrosophic neighbourhood if there exists a neutrosophic open set $x_{r,t,s}$ in $\mathcal{Z}$ such that $p_\epsilon \in x_{r,t,s} \leq S$.*

Definition 1.12. *Let $(\mathcal{X},\mathcal{T}_1)$, $(\mathcal{Y},\mathcal{T}_2)$ be two NTSs, and let $f : \mathcal{X} \longrightarrow \mathcal{Y}$ be a function, then*

(1) *If $\tilde{A} = \{\mu_{\tilde{A}}(A), \sigma_{\tilde{A}}(A), \gamma_{\tilde{A}}(A)\}$ be a neutrosophic sets (briefly Ns) in $\mathcal{Y}$ then the preimage of $\tilde{A}$ under f, denoted by $f^{-1}(\tilde{A})$ is a Ns in $\mathcal{X}$ defined by $f^{-1}(\tilde{A}) = \{f^{-1}(\mu_{\tilde{A}}(A)), f^{-1}(\sigma_{\tilde{A}}(A)), f^{-1}(\gamma_{\tilde{A}}(A))\}$.*
(2) *If $\tilde{B} = \{\mu_{\tilde{B}}(B), \sigma_{\tilde{B}}(B), \gamma_{\tilde{B}}(B)\}$ be a neutrosophic sets (briefly Ns) in $\mathcal{X}$ then the image of $\tilde{B}$ under f, denoted by $f(\tilde{B})$ is a Ns in $\mathcal{Y}$ defined by $f(\tilde{B}) = \{f(\mu_{\tilde{B}}(B)), f(\sigma_{\tilde{B}}(B)), f(\gamma_{\tilde{B}}(B))\}$.*

Definition 1.13. *[20] Let $\mathcal{N}(Z)$ be the set of all neutrosophic sets over Z. A NP $x_{r,t,s} \in \mathcal{N}(Z)$ is said to be quasi-coincident with a NS $B \in \mathcal{N}(Z)$ or $x_{r,t,s} \in \mathcal{N}(Z)$ quasi-coincides with a NS $B \in \mathcal{N}(Z)$, denoted by $x_{r,t,s}qB$, iff $r > \mu_{\tilde{A}^c}(x)$ or $t < \sigma_{\tilde{A}^c}(x)$ or $s < \gamma_{\tilde{A}^c}(x)$, i.e., $r > \gamma_{\tilde{A}}(x)$ or $t < 1_N - \sigma_{\tilde{A}}(x)$ or $s < \mu_{\tilde{A}}(x)$.*

A NS B is said to be quasi-coincident with a NS A at $x \in Z$ or B quasi-coincides with A at $x \in Z$, denoted by BqA at x, iff $\mu_{\tilde{B}}(x) > \mu_{\tilde{A}^c}(x)$ or $\sigma_{\tilde{B}}(x) < \sigma_{\tilde{B}^c}(x)$ or $\gamma_{\tilde{B}}(x) < \gamma_{\tilde{A}^c}(x)$. Now B quasi-coincides with A or B is quasi-coincident with A, denoted by BqA, iff A quasi-coincides with A at some point $x \in Z$. Thus B quasi-coincides with A or B is quasi-coincident with A iff there exists an element $x \in Z$ such that

$\mu_{\tilde{B}}(x) > \mu_{\tilde{A}^c}(x)$ or $\sigma_{\tilde{B}}(x) < \sigma_{\tilde{A}^c}(x)$ or $\gamma_{\tilde{B}}(x) < \gamma_{\tilde{A}^c}(x)$, i.e., $\gamma_{\tilde{B}}(x) > \gamma_{\tilde{A}}(x)$ or $\sigma_{\tilde{B}}(x) < 1_N - \sigma_{\tilde{A}}(x)$ or $\gamma_{\tilde{B}}(x) < \mu_{\tilde{A}}(x)$. If the NP $x_{r,t,s}$ is not quasi-coincident with a NS B, we shall denote it by $x_{r,t,s}\hat{q}B$.

A NS N is called ϵ-nbd of $x_{r,t,s}$ if there exists an NOS Q in $\mathcal{N}(Z)$ such that $x_{r,t,s} \in Q \leq N$.

Similarly if the NS B is not quasi-coincident with the NS A, we shall denote it by $B\hat{q}A$. The set of all the points in Z, at which BqA, will be denoted by $B\mho A$, i.e., $B\mho A = \{x \in Z : BqA\}$at x.

Definition 1.14. *[11] Let $(\mathcal{Z}, \Gamma)$ be a NTS. A collection $\{H_\upsilon : \upsilon \in \Lambda\}$ of neutrosophic closed sets (NCs) of $\mathcal{Z}$ is said to have the finite intersection property (briefly, FIP) if every finite sub-collection $\{H_{\upsilon_j} : j = 1, 2, ..., n\}$ of $\{H_\upsilon : \upsilon \in \Lambda\}$ satisfies the condition $\bigcap_{j=1}^{k} H_{\upsilon_j} \neq 0$, where Λ is the index set.*

Definition 1.15. *[11] Let $(\mathcal{Z}, \Gamma)$ be a NTS and $S \in N(\mathcal{Z})$. A collection $G = \{H_\upsilon : \upsilon \in \Lambda\}$ of neutrosophic open sets of $\mathcal{Z}$ is said a neutrosophic open cover (briefly, NOC) of S if $S \subseteq \vee_{\upsilon \in \Lambda} H_\upsilon$. Then said G covers S. In general, G is said to be an NOC of $\mathcal{Z}$ if $S = \vee_{\upsilon \in \Lambda} H_\upsilon$.*

Let G be a NOC of the NS S and $G' \subseteq G$. Then G' is said a neutrosophic open subcover (briefly, NOSC) of G if G' covers S. A NOC of S is said to be finite (resp. countable) if it consists of a finite (resp. countable) number of neutrosophic open sets.

Definition 1.16. *[11] A neutrosophic set S in an NTS $(\mathcal{Z}, \Gamma)$ is called to be a neutrosophic compact (N-compact, for short) set if every NOC of S has a finite NOSC. In particular, the space $\mathcal{Z}$ is said to be a neutrosophic compact space if every NOC of $\mathcal{Z}$ has a finite NOSC.*

2. Neutrosophic e-compactness

Definition 2.1. *A neutrosophic topology $(\mathcal{Z}, \Gamma)$ (NT, for short) is said to be neutrosophic locally-compact (briefly, N-L-compact) if for every NP $x_{r,t,s} \in \mathcal{N}(Z)$ there is a N-nbd S of $x_{r,t,s}$ such that $x_{r,t,s} \in S$ and S is a neutrosophic-compact relative to $\mathcal{Z}$.*

Definition 2.2. *Let $(\mathcal{Z}, \Gamma)$ be a NTS on $\mathcal{Z}$ and $x_{r,t,s}$ a NP in $\mathcal{Z}$. A NS S is called ϵ-e-nbd (ϵeq-nbd) of $x_{r,t,s}$ if there exists a NeOS $Q \in \mathcal{Z}$ such that $x_{r,t,s} \in Q \leq S$ ($x_{r,t,s}qQ \leq S$).*

Definition 2.3. *A neutrosophic topology $(\mathcal{Z}, \Gamma)$ (NT, for short) is said a Nδ S-compact(resp. NδP-compact, Nδ-α-compact, Ne*-compact, Nβ-compact) iff every family of Nδ SOs (resp. NδPOs, Nδ δ-αOs, Ne*Os, NβOs) cover of $\mathcal{Z}$ has a finite subcover.*

Definition 2.4. *Let $(\mathcal{Z}, \Gamma)$ be a NTS and $S \in N(\mathcal{Z})$. A collection $G = \{H_\upsilon : \upsilon \in \Lambda\}$ of neutrosophic e-open sets of $\mathcal{Z}$ is said a neutrosophic e-open cover (briefly, NeOC) of S if $S \subseteq \vee_{\upsilon \in \Lambda} H_\upsilon$. Then said G covers S. In general, G is said to be an NEOC of $\mathcal{Z}$ if $S = \vee_{\upsilon \in \Lambda} H_\upsilon$.*

Let G be a NeOC of the NS S and $G' \subseteq G$. Then G' is said a neutrosophic e-open subcover (briefly, NeOSC) of G if G' covers S.

A NOC of S is said to be finite (resp. countable) if it consists of a finite (resp. countable) number of neutrosophic e-open sets.

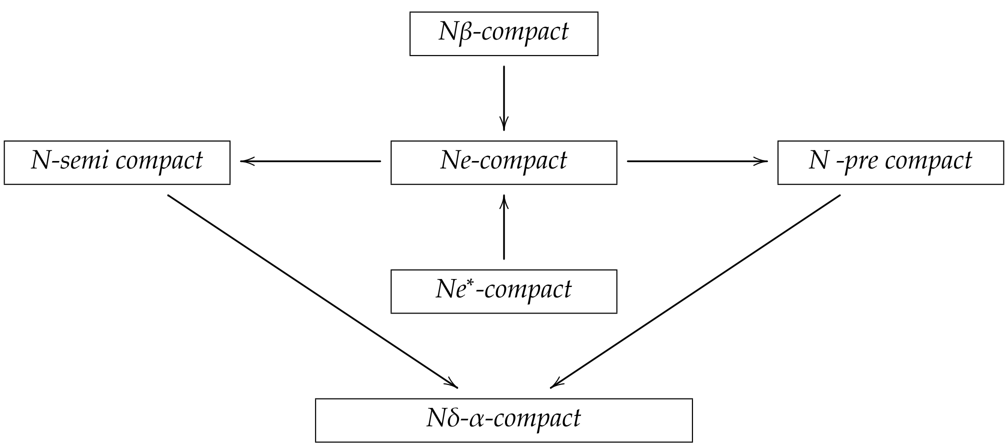
Definition 2.5. *A neutrosophic topology* $(\mathcal{Z}, \Gamma)$ *is called neutrosophic e-compact (briefly, Ne-compact) iff every NeOSs cover of* $\mathcal{Z}$ *has a finite subcover. Also* $(\mathcal{Z}, \Gamma)$ *is called Ne-compact relative to* $\mathcal{Z}$ *iff every NeOSs cover of* $\mathcal{Z}$ *has a finite subcover.*

Example 2.1. *Consider the NTS* $(\mathcal{Z}, \Gamma)$, *where* $\mathcal{Z} = \{x, y\}$,
$H_n = \left\langle z, \left(\frac{x}{\frac{n}{n+1}}, \frac{y}{\frac{n+1}{n+2}}\right), \left(\frac{x}{\frac{1}{n+2}}, \frac{y}{\frac{1}{n+3}}\right), \left(\frac{x}{\frac{1}{n+3}}, \frac{y}{\frac{1}{n+2}}\right)\right\rangle$,
and $\Gamma = \{0, 1\} \cup \{H_n : n \in N\}$. *Note that* $\bigcup_{n \in N} H_n$ *is a open cover for* $\mathcal{Z}$,*but this cover has no finite subcover. Consider*
$H_1 = \left\langle z, \left(\frac{x}{0.5}, \frac{y}{0.6}\right), \left(\frac{x}{0.25}, \frac{y}{0.3}\right), \left(\frac{x}{0.4}, \frac{y}{0.35}\right)\right\rangle$
$H_2 = \left\langle z, \left(\frac{x}{0.75}, \frac{y}{0.6}\right), \left(\frac{x}{0.2}, \frac{y}{0.25}\right), \left(\frac{x}{0.4}, \frac{y}{0.35}\right)\right\rangle$
$H_3 = \left\langle z, \left(\frac{x}{0.8}, \frac{y}{0.75}\right), \left(\frac{x}{0.2}, \frac{y}{0.16}\right), \left(\frac{x}{0.2}, \frac{y}{0.25}\right)\right\rangle$

and observe that $H_1 \cup H_2 \cup H_3$. *So, for any finite sub-collection* $\{H_{nj} : j \in \Lambda, where\, \Lambda\, is\ a\ finite\ subset\ of N\}$, $\bigcup_{nj} \in \Lambda H_{nj} = H_w = 1_N$, *where* $w = max\{nj : ni \in \Lambda\}$.*Therefore NTS* $(\mathcal{Z}, \Gamma)$ *is not compact.*

From the above definitions of compactness the relations in the following diagram is clear,

Remark 2.1. *From the above definition and some types of NOS's,we have the following diagram:*



Theorem 2.1. *A neutrosophic topology* $(\mathcal{Z}, \Gamma)$ *is Ne-compact iff every family* $H = \{H_v : v \in \Lambda\}$,*where* $H_v = \{r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r)\}\ v \in \Lambda$ *of NeCS in* $\mathcal{Z}$ *having the FIP, we have* $\bigwedge_{v \in \Lambda} H_v \neq 0_N$.

Proof. Let $\{r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r)\} : v \in \Lambda$ be a family of NeCS in $\mathcal{Z}$ which satisfies the finite intersection property (FIP). To show that $\bigwedge_{v \in \Lambda}\{r, \mu_{H_v}(r)$
$, \sigma_{H_v}(r), \gamma_{H_v}(r) : v \in \Lambda\} \neq 0_N$, now let $\bigwedge_{v \in \Lambda}\{r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r) : v \in \Lambda\} \neq 0_N$. Then we have $\bigvee \overline{\{r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r) : v \in \Lambda\}} = 1_N$. Then$\{r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r)\} : v \in \Lambda$ is a NeOS cover of $\mathcal{Z}$. Since $\mathcal{Z}$ is Ne-compact, there exist a finite subfamily $\{\ll r, \mu_{H_v}(r), \sigma_{H_v}(r), \gamma_{H_v}(r) \gg : v = 1, 2, ..., n\}$ such that $\bigvee_{v=1}^{n} \overline{H}_v = 1_N$ and so $\bigwedge_{v=1}^{n} H_v = 0_N$ which is a contradiction with the FIP of the family. Hence $\bigwedge_{v=1}^{n} H_v \neq 0_N$.

Conversely, let $H = \{H_v : v \in \Lambda\}$ be a NeOSs cover of $\mathcal{Z}$, and suppose that $\mathcal{Z}$ is not Ne-compact. Then there is no finite subfamily of H cover of $\mathcal{Z}$. Now, $\bigvee_{v=1}^{n} H_v \neq 1_N$ implies $\bigwedge_{v=1}^{n} H_v \neq 0_N$. Since the family $H = \{H_v : v \in \Lambda\}$ satisfies the FIP. Therefore $\bigwedge_{v=1}^{n} \overline{H}_v \neq 0_N$ so that $\bigwedge_{v=1}^{n} H_v \neq 1$ which is a contradiction and then $\mathcal{Z}$ is a Ne-compact. □

Theorem 2.2. *Let S is a Ns of a neutrosophic topology $(\mathcal{Z}, \Gamma)$ is Ne-compact relative to $\mathcal{Z}$ iff every family $Q = \{Q_v : v \in \Lambda\}$, where $Q_v = \{r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r) : v \in \Lambda\}$ of Ns in $\mathcal{Z}$ having the finite intersection property (FIP) such that $\bigwedge_{v=1}^{n} Q_v q S$, then we have $\bigwedge_{Q_v \in Q} NCl_e(Q_v) \wedge S \neq 0_N$.*

Proof. Suppose $S = \{r, \mu_S(r), \sigma_S(r), \gamma_S(r)\}$ not to be Ne-compact relative to space $\mathcal{Z}$, so that there exists a NOS cover K of $\mathcal{Z}$ such that K has no finite subcover H . Hence $\bigvee_{w_v \in H} w_v(r) \ll S(r)$, for some $r \in S$, (i.e. $\mu_S(r) \ll \bigvee_{w_v \in H} \mu_{w_v}$, $\sigma_S \gg \bigwedge_{w_v \in H} \sigma_{w_v}$ and $\gamma_S \gg \bigwedge_{w_v \in H} \gamma_{w_v}$. Therefore $\bigvee_{w_v \in H} \overline{w}_v(r) \gg \overline{S}(r) \geq 0_N$ and so that $H = \{\overline{w}_v : w_v \in K\}$ has the (FIP) and $\bigwedge_{w_v \in H} \overline{w}_v(r) q S$, (now, suppose $w_v = \{r, \mu_{w_v}(r), \sigma_{w_v}(r), \gamma_{w_v}(r)\}$, then $\bigwedge \overline{w}_v = \{r, \wedge\mu_{w_v}(r), \vee\sigma_{w_v}(r), \vee\gamma_{w_v}(r)\}$ and for the reason that $\mu_S(r) \gg \bigvee \mu_{w_v}(r)$, hence $\bigwedge_{w_v \in H} \overline{w}_v(r) q S$. By assumption $\bigwedge_{w_v \in H} NCl_e(\overline{w}_v) \wedge S \neq 0_N$ and then $\bigwedge_{w_v \in H} \overline{w}_v \wedge S \neq 0_N$. Hence for some $r \in S$ $\bigwedge_{w_v \in H} \overline{w}_v \gg 0_N$ this implies $\bigvee_{w_v \in H} \overline{w}_v \ll 1_N$, which is a contradiction. Then S is Ne-compact relative to $\mathcal{Z}$.

Conversely, suppose that there exists a family Q of NS having the FIP such that $\bigwedge_{v=1}^{n} Q_v q S$ and $\bigwedge_{Q_v \in Q} NCl_e(Q_v) \wedge S = 0_N$. Hence for each $r \in S$, $(\bigwedge_{Q_v \in Q} NCl_e(Q_v)(r)) = 0_N$ and then $(\bigvee_{Q_v \in Q} \overline{NCl_e(Q_v)})(r) = 1_N$. Then $K = \{\overline{NCl_e(Q_v)} : Q_v \in Q\}$ is an NeOS cover of S. Since S is a Ne-compact to $\mathcal{Z}$, hence there exists a finite subcover, now we say $\{\overline{NCl_e(Q_1)}, \overline{NCl_e(Q_2)}, ..., \overline{NCl_e(Q_n)}\}$ s.t $\bigvee_{v=1}^{n} \overline{NCl_e(Q_v)(r)} \geq S(r)$ for each $r \in S$. Then $\bigwedge_{v=1}^{n} NCl_e(Q_v)(r) \leq \overline{S}(r)$ for each $r \in S$, (i.e., $\mu_{NCl_e(Q_v)} \leq \mu_S$, $\sigma_{NCl_e(Q_v)} \geq \sigma(S)$ and $\gamma_{NCl_e(Q_v)} \geq \gamma(S)$) therefore $\bigwedge_{v=1}^{n} NCl_e(Q_v) \hat{q} S(r)$, a contradiction. Then we got the result. □

Theorem 2.3. *Every NeCS of a Ne-compact space $(\mathcal{Z}, \Gamma)$ is Ne-compact relative to $\mathcal{Z}$.*

Proof. Let $Q = \{r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r) : v \in \Lambda\}$ be a family of NS having the finite intersection property (FIP) and $Sq \bigwedge_{Q_v \in Q_0}$ holds for each finite sub-collection Q_0 of Q and a NeCS S. Consider $Q^* = S \bigvee Q$. For each finite sub-collection Q_0^* of Q, if $S \notin Q_0^*$, hence $\bigwedge_{Q_v \in Q_0^*} Q_v \neq 0_N$. Now if $S \in Q_0^*$ and since $Sq \bigwedge Q_v$ such that $Q_v \in Q_0^*$ and $Q_v \notin S$, which implies $\bigwedge_{Q_v \in Q_0^*} Q_v \neq 0_N$. Since Q^* is a family of NS having the FIP. Hence $\mathcal{Z}$ is Ne-compact, then $\bigwedge_{Q_v \in Q^*} Q_v \neq 0_N$ therefore $\bigwedge_{Q_v \in Q^*} Cl_e(Q_v) \neq 0_N$, this implies $\bigwedge_{Q_v \in Q} Cl_e(Q_v) \bigwedge S = \bigwedge_{Q_v \in Q} Cl_e(Q_v) \bigwedge Cl_(S) \neq 0_N$. Then by Theorem 2.2, S is a Ne-compact. □

Theorem 2.4. *If L is a neutrosophic e-closed crisp set in $\mathcal{Z}$ and S is an Ne-compact relative to $\mathcal{Z}$, then $S \vee L$ is a Ne-compact relative to $\mathcal{Z}$.*

Proof. Let $S = \{r, \mu_S(r), \sigma_S(r),_S(r) : v \in \Lambda\}$ and $L = \{r, \mu_L(r), \sigma_L(r), \gamma_L(r) : v \in \Lambda\}$. Let $Q = \{Q_v : v \in \Lambda\}$, where $Q_v = \{r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r) : v \in \Lambda\}$ is a NeOS cover of $S \wedge L$, i.e., $S \wedge L \leq \bigvee_{Q_v \in Q} Q_v$. Which implies that $(\mu_S \wedge \mu_L)(r) \leq \bigvee_{Q_v \in Q} \mu(Q_v)(r)$, $(\overline{\sigma_S \vee \sigma_L})(r) \leq \bigvee_{Q_v \in Q} \overline{\sigma_{(Q_v)}}(r)$ and $(\overline{\gamma_S \vee \gamma_L})(r) \geq \bigvee_{Q_v \in Q} \overline{\gamma_{(Q_v)}}(r)$, for every $r \in S \wedge L$. Since $\overline{L} = \{r, \gamma_L(r), \sigma_L(r), \mu_L(r) : v \in \Lambda\}$ is a neutrosophic e-open crisp set in $\mathcal{Z}$ and $\overline{L} \vee (\bigvee_{Q_v \in Q} Q_v) = \langle r, \bigvee_{Q_v \in Q} \mu_{(Q_v)} \vee \gamma_L, \bigwedge_{Q_v \in Q} \sigma_{(Q_v)} \vee \sigma_L, \bigwedge_{Q_v \in Q} \gamma_{(Q_v)} \vee \mu_L \rangle$, then $\overline{L} \vee Q$ is a NeOS cover of S, (since $S \leq \overline{L} \vee (S \wedge L)$, hence $\underline{\mu_S \leq \gamma_L \vee (\mu_S \wedge \mu_L)} \leq \sigma_L \vee (\bigvee_{Q_v \in Q} \mu_{(Q_v)}$, $\sigma_S \geq (\sigma_S \vee \sigma_L) \wedge \sigma_B$ and $\gamma_S \geq (\gamma_S \vee \gamma_L) \wedge \mu_B$, then $\overline{\sigma}_S \geq \overline{(\sigma_S \vee \sigma_L) \wedge \sigma_B} = \overline{(\sigma_S \vee \sigma_L)} \vee \overline{\sigma}_B \leq \bigvee_{Q_v \in Q} \overline{\sigma}_{(Q_v)} \vee \overline{\mu}_L = \bigvee_{Q_v \in Q}(Q_v \wedge \mu_L)$. By assumption S is a Ne-compact relative to $\mathcal{Z}$, this implies there exists a finite subcover of S, this mean there exists Q_v, $(v = 1, 2, ..., n)$ such that $S \leq \bigvee_v^n Q_v \vee \overline{L}$. Since S is a neutrosophic e-open crisp set, hence $S \wedge L \leq \bigvee_v^n Q_v$. Therefore $S \wedge L$ is an Ne-compact relative to $\mathcal{Z}$. □

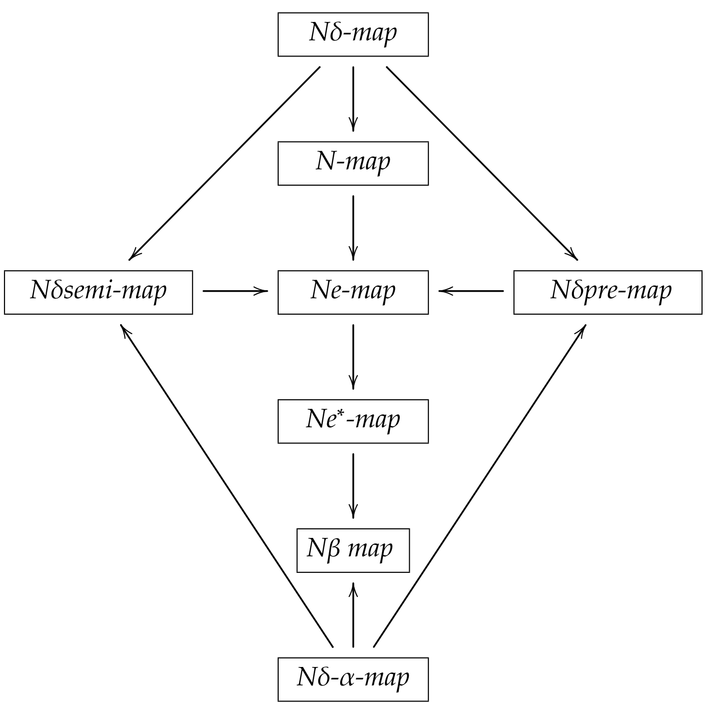
Theorem 2.5. *Let $\{Q_v : v = 1, 2, ..., n\}$ be a finite family of Ne-compact subsets of neutrosophic topological space $(\mathcal{Z}, \Gamma)$, then $\bigvee_v^n Q_v$ is Ne-compact relative to $\mathcal{Z}$.*

Proof. Let $V = \{V_l : l \in L\}$, where $V_l = \{r, \mu_{V_l}(r), \sigma_{V_l}(r), \gamma_{V_l}(r) : v \in \Lambda\}$ be a NeOS cover of $\bigvee_v^n Q_v$, this implies V is a NeOS cover of Q_v, for any $(v = 1, 2, ..., n)$. Hence for any $(v = 1, 2, ..., n)$, there exists a finite subset L_v of L such that $Q_v \leq \bigvee_{L \in L_v} \vee V_L$. Then, $\bigvee_{v=1}^n Q_v \leq \bigvee_{L \in L_v} \vee V_L$, where $\bigvee_{v=1}^n L_v$ is a finite subset of L. which implies that $\bigvee_{v=1}^n Q_v$ is Ne-compact relative to $\mathcal{Z}$. □

3. Neutrosophic e-compactness and functions

Definition 3.1. *[4] Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). A map and $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is called neutrosophic (resp. δ, δS, δP, e, β and e*) open map(briefly, NO(resp. NδO,NδSO,NδPO, NeO, NβO and Nse*O)) if the image of each Nsos in $(\mathcal{Z}, \Gamma_1)$ is a Nos (resp.NOs,NδSOs,NδPOs, NeOs, NβOs and Ne*Os) in $(\mathcal{R}, \Gamma_2)$.*

Remark 3.1. *From the above definition and some types of NOS's,we have the following diagram:*



Definition 3.2. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs) and $g : \mathcal{Z} \longrightarrow \mathcal{R}$ a function. Then g is called,*

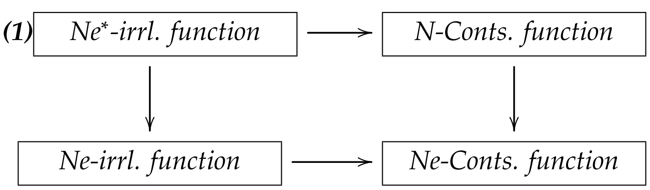
(1) *Neutrosophic continuous [6] (briefly, N-conts.) if and only if the preimage of every NOS in $\mathcal{R}$ is a NOS in $\mathcal{Z}$.*
(2) *Neutrosophic e-continuous [6] (briefly, Ne-conts.) if and only if the preimage of every NOS in $\mathcal{R}$ is a NeOS in $\mathcal{Z}$.*
(3) *Neutrosophic e-irresolute [6] (briefly, Ne-irrl.) if and only if the preimage of every NeOS in $\mathcal{R}$ is a NeOS in $\mathcal{Z}$.*
(4) *Neutrosophic open [25] (resp. e -open) (briefly, NO (resp. NeO)) if and only if the image of every NOS in $\mathcal{R}$ is a NeOS (resp. NeOS) in in $\mathcal{Z}$.*

Definition 3.3. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs) and $g : \mathcal{Z} \longrightarrow \mathcal{R}$ a function. Then g is said to be,*

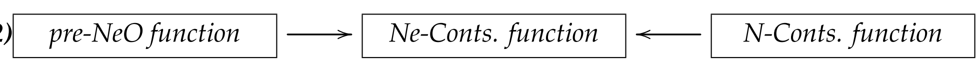
(1) *Pre-Neutrosophic e-open (briefly, pre-NeO) if and only if the image of every NeOS in $\mathcal{R}$ is a NeOS in $\mathcal{Z}$.*
(2) *Neutrosophic e^*-irresolute (briefly, Ne^*-irrl.) if and only if the preimage of every NeOS in $\mathcal{R}$ is a NeOS in $\mathcal{Z}$.*

Remark 3.2. *From the above definitions none of these implications is reversible in the following two diagrams:*

(1)



(2)



Theorem 3.1. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs) and $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is a Ne-irresolute surjection function. If $(\mathcal{Z}, \Gamma_1)$ is Ne-compact, then $(\mathcal{R}, \Gamma_2)$ is Ne-compact.*

Proof. Let $Q = \{Q_v : v \in \Lambda\}$, where $Q_v = \{r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r) : v \in \Lambda\}$, be a NeOS cover of $\mathcal{R}$. Hence from the Ne-irrl. of g, which implies that $S = \{g^{-1}(Q_v) : v \in \Lambda\}$ is a NeOS cover of $\mathcal{Z}$. Since $(\mathcal{Z}, \Gamma_1)$ is Ne-compact, then there exist $\{Q_v : v = 1, 2, ..., n\}$ such that $\bigvee_{v=1}^{n} g^{-1}(Q_v) = 1_N$. Hence $g(\bigvee_{v=1}^{n} g^{-1}(Q_v)) = \bigvee_{v=1}^{n} gg^{-1}(Q_v) = \bigvee_{v=1}^{n} Q_v = g(1_N) = 1_N$. This implies that $(\mathcal{R}, \Gamma_2)$ is Ne-compact. □

Corollary 3.1. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is Ne-conts. function. surjection and $\mathcal{Z}$ is a Nβ-compact then $\mathcal{R}$ is a N-compact.*

Proof. From Remark 2.1 since every Nβ-compact is Ne-compact, then the proof is obvious. □

Theorem 3.2. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is Ne-conts. function. surjection and $\mathcal{Z}$ is a Ne-compact then $\mathcal{R}$ is a N-compact.*

Proof. The proof is similar pattern to Theorem 3.1. □

Corollary 3.2. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is Ne-conts. function. If S is a Ne-compact relative to $(\mathcal{Z}, \Gamma_1)$, therefore $g(S)$ is a Ne-compact in $(\mathcal{R}, \Gamma_2)$.*

Proof. From Remark 3.2 the proof is obvious. □

Corollary 3.3. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g : \mathcal{Z} \longrightarrow \mathcal{R}$ is Ne-irrl. function. If S is a Ne-compact relative to $(\mathcal{Z}, \Gamma_1)$, therefore $g(S)$ is a Ne-compact in $(\mathcal{R}, \Gamma_2)$.*

Proof. Let $\{\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r)\rangle : v \in \Lambda\}$ be any neutrosophic e-open cover of $g(S)$. Therefore

$$g(S) \subseteq \bigcup\Big(\{\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r)\rangle : v \in \Lambda\}\Big).$$

From the above relation

$$S \subseteq g^{-1}\Big(\bigcup\{\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r)\rangle : v \in \Lambda\}\Big),$$

follows that

$$S \subseteq \bigcup\{g^{-1}(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r))\rangle : v \in \Lambda\},$$

so $\{g^{-1}(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r)\rangle) : v \in \Lambda\}$ is a neutrosophic e-open cover of S. Since S is neutrosophic e-compact, there exists a finite subcover $\{g^{-1}(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r)\rangle) : v = 1, 2, ..., n\}$. Then

$$S \subseteq \bigcup\{g^{-1}(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r))\rangle : v = 1, 2, ..., n\}.$$

Then

$$\begin{aligned} g(S) &\subseteq g\Big(\bigcup\{g^{-1}(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r))\rangle : v = 1, 2, ..., n\}\Big) \\ &= \bigcup\{g(g^{-1}\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r))\rangle : v = 1, 2, ..., n\} \\ &= \bigcup\{(\langle r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r))\rangle : v = 1, 2, ..., n\}. \end{aligned}$$

so $g(S)$ is neutrosophic e-compact.

□

Theorem 3.3. *Let $(\mathcal{Z}, \Gamma_1)$, $(\mathcal{R}, \Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g : \mathcal{Z} \longrightarrow \mathcal{R}$ a pre-NeO bijective function. If $(\mathcal{R}, \Gamma_2)$ is Ne-compact, this implies $(\mathcal{Z}, \Gamma_1)$ is Ne-compact too.*

Proof. Let $Q = \{Q_v : v \in \Lambda\}$, where $Q_v = \{r, \mu_{Q_v}(r), \sigma_{Q_v}(r), \gamma_{Q_v}(r) : v \in \Lambda\}$, be a NeOSs cover of $\mathcal{Z}$. Since g is pre-NeO function, hence $\{g(Q_v) : v \in \Lambda\}$ is a NOS's cover of $\mathcal{Z}$. Since $\mathcal{R}$ is Ne-compact, therefore there exist a finite subfamily $\{g(Q_v) : v = 1, 2, ..., n\}$ such that $\bigvee_{v=1}^{n} g(Q_v) = 1_N$. Since g is bijection function, we obtain $1 = g^{-1}(1) = g^{-1}(\bigvee_{v=1}^{n} g(Q_v) = g^{-1}g(\bigvee_{v=1}^{n}(Q_v) = \bigvee_{v=1}^{n} Q_v$. Then $(\mathcal{Z}, \Gamma_1)$ is Ne-compact. □

Corollary 3.4. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a pre-NeO bijective function. If H is a Ne-compact relative to $(\mathcal{R},\Gamma_2)$, hence $g^{-1}(H)$ is a Ne-compact relative to $(\mathcal{R},\Gamma_2)$.*

Proof. The proof is obvious. □

Theorem 3.4. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NeO bijective function. If H is a Ne-compact relative to $(\mathcal{R},\Gamma_2)$, hence $g^{-1}(H)$ is a Ne-compact relative to $(\mathcal{R},\Gamma_2)$.*

Proof. Let $\{Q_v : v\in\Lambda\}$, where $Q_v=\{r,\mu_{Q_v}(r),\sigma_{Q_v}(r),\gamma_{Q_v}(r) : v\in\Lambda\}$, be a NeOSs cover of $\mathcal{Z}$. Since g is pre-NeO function, hence $\{g(Q_v) : v\in\Lambda\}$ is a NOS's cover of $g^{-1}(H)$. Hence $\{g(Q_v) : v\in\Lambda\}$ is a NeOS cover of H. Since H is a Ne-compact relative to $(\mathcal{R},\Gamma_2)$, there is a finite subset v_0 of Λ such that $\{g(Q_v) : v\in\Lambda_0\}$ is a NeOS cover of H. Since, g is bijection function, $g^{-1}(g(Q_v))=Q_v$ is a NOS, thus $\{Q_v : v\in\Lambda_0\}$ is a cover of $g^{-1}(H)$. Then $g^{-1}(H)$ is a N-compact relation to $(\mathcal{Z},\Gamma_1)$. □

4. Neutrosophic locally e-compactness

Definition 4.1. *For any NTS $(\mathcal{Z},\Gamma)$ is called neutrosophic locally e-compact (briefly, NLe-compact) if for each NP $x_{r,t,s}\in\mathcal{Z}$ there is a ϵ-nbd S of $x_{r,t,s}$ such that $S(x_{r,t,s})=1_N$ and S is a Ne-compact relative to $\mathcal{Z}$.*

Remark 4.1. *From above Definition and Definition 2.1 it clear that,*

(1) *Every Ne-compact is NLe-compact, but the converse not need to be true, in general.*
(2) *Every NLe-compact is NL-compact but the converse not need to be true, as you see in the following example.*

Example 4.1. *Suppose $(\mathcal{Z},\Gamma)$ be an infinite neutrosophic discrete topological space, then $\mathcal{Z}$ is NLe-compact but not Ne-compact.*

Example 4.2. *Let $\Lambda=[0_N,1_N]$ and $Q_v=\{r,\mu_{Q_v}(r),\sigma_{Q_v}(r),\gamma_{Q_v}(r) : v\in\Lambda\}$, where $Q_v(r)=\frac{1}{4}$, $\forall r\in\Lambda$ and $\mu_{Q_v}(r)=0_N$, $\forall r\in\Lambda$. Then the family $\Psi=\{0_N,1_N,Q\}$ is a NT on Λ. It is easily to check that (Λ,Ψ) is NL-compact but not NLe-compact.*

Theorem 4.1. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NO surjection function. If g is Ne-cont. and $(\mathcal{Z},\Gamma_1)$ is a NLe-compact, hence $(\mathcal{R},\Gamma_2)$ is a NLe-compact.*

Proof. Let $r\in\mathcal{R}$, $r\in g(z_{r,t,s})$ for some NP $z_{r,t,s}\in\mathcal{Z}$. Since $\mathcal{Z}$ is a NLe-compact, there is a ϵ-nbd $Q=\{z,\mu_Q(z),\sigma_Q(z),\gamma_Q(z)\}$ such that $Q(z)=1_N$ is a N-compact relative to $\mathcal{Z}$. Since g is NO function, $g(Q)$ is a ϵ-nbd of r with $(g(Q))(r)=\langle r,g(\mu_Q(r)),g(\sigma_Q(r),g(\gamma_Q(r))\rangle=\langle z,\vee\mu_Q(z),\wedge\sigma_Q(z),\wedge\gamma_Q(z)\rangle=\langle z,1_N,0_N>\rangle=1_N$. Moreover, since g is Ne-cont., hence by Corollary 3.4 $g(Q)$ is a Ne-compact relative to $\mathcal{R}$. Then $\mathcal{R}$ is a NLe-compact. □

Proposition 4.1. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NO surjection function. Now if g is N-cont. and $(\mathcal{Z},\Gamma_1)$ is a NLe-compact, hence $(\mathcal{R},\Gamma_2)$ is a NLe-compact.*

Proof. The proof is obvious from Remark 4.1. □

Proposition 4.2. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NO surjection function. If g is N-irrl. and $(\mathcal{Z},\Gamma_1)$ is a NLe-compact, hence $(\mathcal{R},\Gamma_2)$ is a NLe-compact.*

Proof. The proof is obvious from Remark 4.1. □

Theorem 4.2. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NeO bijection function. Now if g is Ne*-irrl. and $(\mathcal{Z},\Gamma_1)$ is a NLe-compact, hence $(\mathcal{R},\Gamma_2)$ is a NLe-compact.*

Proof. Let $z\in\mathcal{Z}$, $g(z)=r$. Since $\mathcal{R}$ is a NLe-compact, there is an ϵ-nbd $Q=\{z,\mu_Q(z),\sigma_Q(z),\gamma_Q(z)\}$ of $g(z)$ such that $Q(g(x))=\{z,\mu_Q(g(z)),\sigma_Q$
$(g(z)),\gamma_Q(g(z))\}=1_N$ and Q is a Ne-compact relative to $\mathcal{R}$. By Theorem 4.1, $g^{-1}(Q)$ is a NL-compact relative to $\mathcal{Z}$. Since g is Ne^*-irrl., $g^{-1}(Q)$ is a ϵ-nbd of z and $(g^{-1}(Q))(z)=\langle z,g^{-1}(\mu_Q)(z),g^{-1}(\sigma_Q)(z),g^{-1}(\gamma_Q)(z)\rangle=1_N$. Then for $z\in\mathcal{Z}$, there is an ϵ-nbd $g^{-1}(Q)$ of z such that $g^{-1}(Q)(z)=1_N$ and $g^{-1}(Q)$ is a N-compact relative to $\mathcal{Z}$. So that $\mathcal{Z}$ is a NL-compact. □

Theorem 4.3. *Let $(\mathcal{Z},\Gamma_1)$, $(\mathcal{R},\Gamma_2)$ be two neutrosophic topological spaces's (NTSs). If $g:\mathcal{Z}\longrightarrow\mathcal{R}$ is a NeO bijection function. If g is Ne-irrl. and $(\mathcal{Z},\Gamma_1)$ is a NLe-compact, hence $(\mathcal{R},\Gamma_2)$ is a NLe-compact.*

Proof. By using Proposition 4.2, the proof is similar to Theorem 4.2 □

5. Conclusion

The concepts of e-open sets, e-continuity, e-compactness and related studies in topological spaces are due to many authors. This present paper contains the next steps of intuitionistic fuzzy e-open sets, intuitionistic fuzzy e^*-open sets, intuitionistic fuzzy α-δ-open sets, intuitionistic fuzzy e-continuity and intuitionistic fuzzy e-compactness in intuitionistic fuzzy topological spaces are studied. After giving the fundamental concepts of neutrosophic sets and neutrosophic topological spaces, we present neutrosophic e-compactness sets and neutrosophic e-irresolute and other results related topological concepts. Several preservation characterizations and some properties concerning neutrosophic locally e-compactness have been studded and obtained.

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Data Availability: On request, the data used to support the findings of this study can be obtained from the corresponding author.

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