Int. J. Anal. Appl. (2025), 23:227

International Journal of Analysis and Applications

Numerical Analysis of a SEIHRD Model for a Hypothetical Ebola Outbreak During the 2025 Hajj Season in Saudi Arabia

Abeer Al-Nana[1,*], Iqbal M. Batiha[2,3], Ahmed Bouchenak[4], Shaimaa A. A. Ahmed[5]

[1]*Department of Mathematics, Prince Sattam Bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia*

[2]*Department of Mathematics, Al Zaytoonah University of Jordan, Amman 11733, Jordan*

[3]*Nonlinear Dynamics Research Center (NDRC), Ajman University, Ajman 346, UAE*

[4]*Department of Mathematics, Faculty of Exact Sciences, University Mustapha Stambouli of Mascara, Mascara 29000, Algeria*

[5]*Department of Mathematics, College of Science and Humanities in Al-Kharj, Prince Sattam Bin Abdulaziz University, Al-Kharj 11942, Saudi Arabia*

[*]*Corresponding author:* a.alnana@psau.edu.sa

Abstract. Saudi Arabia, one of the world's most frequented destinations for foreign visitors and pilgrims—particularly during the Hajj season—faces an elevated risk of infectious disease outbreaks due to the high influx of international travelers. One such potential threat is the Ebola virus, which may be introduced by individuals arriving from affected regions, especially in parts of Africa. In this study, we explore the SEIHRD epidemiological model, specifically adapted to a hypothetical Ebola outbreak scenario in Saudi Arabia. The model is represented as a system of nonlinear ordinary differential equations and is numerically solved using the classical fourth-order Runge-Kutta method. The results yield critical insights into disease progression and offer strategic guidance for preparedness, control measures, and isolation protocols.

1. Introduction

Recent advances in numerical analysis and fractional modeling have produced efficient solvers for Volterra integro–differential and fractional differential equations, including variable–order and noise–perturbed settings [1–3]. In mathematical biology, these tools have been used to analyze transmission dynamics and design controls across emerging infections—Zika, COVID-19, and Mpox—while establishing stability and accuracy properties for continuous and discrete formulations [4–11]. Related efforts also link fractional modeling to clinical decision and control design

Received: Jul. 5, 2025.

2020 *Mathematics Subject Classification.* 92D30, 65L06, 65L05.

Key words and phrases. SEIHRD model; Runge-Kutta method; Ebola virus; ordinary equation; Hajj season.

https://doi.org/10.28924/2291-8639-23-2025-227

ISSN: 2291-8639

and to chronic disease dynamics [12,13]. Building on these developments, we investigate a mass-gathering scenario for Ebola in Saudi Arabia using an SEIHRD framework and classical numerical integration, aiming to quantify outbreak progression and to inform preparedness and control strategies.

Over the past two decades, the world has witnessed the emergence and re-emergence of several infectious disease outbreaks, underscoring the persistent threat posed by zoonotic and viral epidemics. Between 2010 and 2020, numerous significant epidemics emerged, including novel strains of influenza, the Middle East Respiratory Syndrome Coronavirus (MERS-CoV), and the H1N1 swine flu pandemic. These followed earlier outbreaks such as Severe Acute Respiratory Syndrome (SARS) in 2002 and H5N1 avian influenza in 2003, both of which highlighted the vulnerability of global public health systems to respiratory pathogens [14–17].

In the past decade, additional high-fatality viruses have emerged, with Ebola Virus Disease (EVD) and the Zika virus drawing considerable international attention. The 2015–2016 Zika virus outbreak in South and Central America, which was associated with congenital malformations, added a new dimension to arboviral threats [18,19]. Among these, however, the Ebola virus remains one of the deadliest. First identified in 1976, Ebola re-emerged with unprecedented severity during the 2013–2016 West African epidemic, which became the largest and deadliest Ebola outbreak on record [20,21].

The West African Ebola outbreak began in December 2013 in the village of Meliandou, Guinea, where a two-year-old boy, Emile Ouamouno, was identified as the index case [22]. The virus rapidly spread across national borders into Liberia and Sierra Leone, leading to widespread mortality, socio-economic disruption, and the near-collapse of healthcare infrastructure in the region [23–25]. By the end of 2015, the outbreak had caused over 11,000 deaths from nearly 29,000 reported cases, making it the most severe Ebola outbreak in recorded history [26].

In 2018, Ebola re-emerged in the Democratic Republic of the Congo (DRC), resulting in more than 2,200 deaths from approximately 3,300 confirmed cases, despite advances in diagnostics, vaccines, and outbreak response capabilities [27,28]. These recurrent outbreaks highlight the ongoing threat posed by filoviruses and underscore the critical need for continued research into epidemic modeling, transmission dynamics, and effective intervention strategies.

Hajj, one of the Five Pillars of Islam, is a pilgrimage to Mecca that is obligatory for Muslims who are physically and financially capable of undertaking it. Saudi Arabia receives one of the highest numbers of international visitors annually, especially during the Hajj season. This influx of travelers significantly elevates the risk of epidemic outbreaks due to large-scale human gatherings and cross-border interactions.

In this study, we investigate the dynamics of a hypothetical Ebola virus outbreak using a compartmental SEIHRD model (Susceptible; Exposed; Infectious; Hospitalized; Recovered; Deceased). The model is formulated as a system of ordinary differential equations (ODEs) and numerically solved using the classical fourth-order Runge-Kutta method. This method is widely recognized for its accuracy, efficiency, and stability in solving nonlinear ODE systems, particularly in epidemiological modeling [29–32]. We examine the effects of key intervention strategies—namely quarantine measures and vaccination programs—on the transmission, control, and mitigation of the disease within the simulated population.

2. Model Development

Although Ebola is not endemic in Saudi Arabia, the Hajj season attracts millions of visitors, including individuals from Ebola-affected regions such as parts of Africa. Global travel facilitates the possibility of imported cases. Mathematical modeling in this context supports outbreak preparedness, timely intervention strategies, and effective isolation protocols.

2.1. **Mathematical Analysis.** The SEIHRD model for a hypothetical Ebola outbreak in Saudi Arabia is expressed as a system of nonlinear ordinary differential equations, given by:

$$\begin{aligned}
\frac{dS}{dt} &= -\beta\frac{S(I+\theta H+\omega D)}{N},\\
\frac{dE}{dt} &= \beta\frac{S(I+\theta H+\omega D)}{N}-\sigma E,\\
\frac{dI}{dt} &= \sigma E-(\gamma+\delta)I,\\
\frac{dH}{dt} &= \delta I-(\eta+\mu)H,\\
\frac{dR}{dt} &= \gamma I+\eta H,\\
\frac{dD}{dt} &= \mu H.
\end{aligned}$$

The model is solved subject to the following initial conditions:

$$\begin{aligned}
S(0) &= N-1,\\
E(0) &= 1,\\
I(0) &= 0,\\
H(0) &= 0,\\
R(0) &= 0,\\
D(0) &= 0.
\end{aligned}$$

The variables in the system are defined as follows:

(1) $S(t)$: Number of susceptible individuals at time t.
(2) $E(t)$: Number of exposed individuals (infected but not yet infectious).
(3) $I(t)$: Number of infectious (symptomatic) individuals.
(4) $H(t)$: Number of hospitalized individuals.
(5) $R(t)$: Number of recovered individuals.
(6) $D(t)$: Number of deceased individuals (still infectious in the case of Ebola).

The model parameters are defined as follows:

(1) β: Transmission rate of the disease.
(2) θ: Infectiousness modifier for hospitalized individuals.
(3) ω: Infectiousness modifier for deceased individuals.
(4) σ: Rate at which exposed individuals become infectious (incubation rate).
(5) γ: Recovery rate of infectious individuals.
(6) δ: Rate of hospitalization of infectious individuals.
(7) η: Recovery rate of hospitalized individuals.
(8) μ: Mortality rate of hospitalized individuals.

2.2. **Regarding the Number of People Exposed to the Epidemic Infection and Treatment.** During the 2025 Hajj season, over 1.67 million pilgrims participated, including both domestic and international attendees, with the overwhelming majority being international pilgrims. According to the Saudi General Authority for Statistics (GASTAT), international pilgrims accounted for approximately 90% of the total. To accommodate the 1,673,230 pilgrims, around 420,070 personnel from both public and private sectors—including security forces—were mobilized to provide services and ensure safety and organization throughout the pilgrimage. Based on data from the Saudi General Authority for Statistics, the details of the aforementioned information are summarized in Table 1.

Table 1. Pilgrim and workforce statistics during the 2025 Hajj season

1,673,230	Total pilgrims
1,506,576	External (foreign) pilgrims
166,654	Internal (Saudi citizens and residents) pilgrims
420,070	Total number of employees
Participating sectors	Public sector, private sector, and security sector
Approximate ratio	One employee served for every four pilgrims

Based on the information in Table 1 and the potential for infection spread among individuals in Saudi Arabia, we classify the study into two distinct cases:

- **Case 1:** The infection is assumed to originate at one of the Saudi airports receiving international pilgrims. In this case, the number of individuals potentially exposed to the virus is estimated at 1,506,576, representing the total number of external (foreign) pilgrims.
- **Case 2:** The infection is assumed to originate at one of the Hajj ritual sites (e.g., Masjid al-Haram, Arafat, Mina, Muzdalifah), where pilgrims and service personnel interact. This scenario increases the number of potentially exposed individuals to 2,093,300, accounting for both pilgrims and employees.

2.3. **Regarding the Time Period Allocated to Study the Spread of the Epidemic and Its Treatment.** The duration of the 2025 Hajj season spans from the arrival of the first group of pilgrims to the departure of the last group. This period includes two main phases: arrival and departure.

- **Arrival of Pilgrims:** The Kingdom of Saudi Arabia begins receiving pilgrims from the start of the month of Dhul-Qi'dah until the end of the 4th day of Dhul-Hijjah.
- **Departure of Pilgrims:** The departure phase begins on the 13th of Dhul-Hijjah and extends until the 15th of Muharram of the following Hijri year.

In the same regard, the detailed timeline is described as follows

- **Pilgrims' Arrival:** This phase starts at the beginning of Dhul-Qi'dah (approximately early May) and continues until the end of the 4th day of Dhul-Hijjah (around the end of May).
- **Pilgrims' Departure:** This phase begins on the 13th of Dhul-Hijjah (around mid-June) and lasts until the end of the 15th of Muharram (approximately the end of July).

In general, the Hajj season spans approximately two months, considering the staggered arrival and departure schedules. Accordingly, it is very important to take the following points into account:

- All dates are approximate and based on the Hijri calendar, which may vary slightly depending on the official moon sighting.
- Dedicated flights are arranged for transporting pilgrims during both arrival and departure phases.
- Arrival and departure schedules may differ by country.
- The Hajj season typically involves enhanced services and extensive security measures to ensure pilgrim safety and facilitate ritual performance.

Therefore, we conclude that the duration of the 2025 Hajj season is approximately 60 days. However, in light of a potential epidemic, we extend the study period by at least 30 additional days to allow for investigation, containment, and treatment efforts. In conclusion, the total time period allocated for studying the spread and treatment of the epidemic is 90 days.

3. Runge-Kutta 4th Order Method

The methods used to solve the SEIHRD model—or any epidemiological model based on ordinary differential equations (ODEs)—depend on the type of analysis required, whether analytical or numerical. For complex systems like the SEIHRD model, obtaining a closed-form analytical solution is often infeasible or overly complicated. Therefore, numerical methods become essential.

In this study, we apply the classical fourth-order Runge-Kutta (RK4) method, which is widely regarded for its accuracy, reliability, and ease of implementation in solving nonlinear ODE systems in epidemiological modeling. The SEIHRD model can be used to simulate the following scenarios:

- The impact of varying the number of people exposed to infection.
- The effect of changing model parameters, such as the transmission rate β or the hospitalization rate δ.

- The effectiveness of intervention strategies such as quarantine, vaccination, or improvements in healthcare infrastructure.
- The evaluation of different outbreak scenarios during the Hajj season.

We can summarize the comparative advantages and complexity of different solution approaches as shown in Table 2.

Table 2. Comparison of solution approaches in terms of benefit and complexity

Solution Type	**Benefit**	**Complexity**
Mathematical Analysis	Theoretical understanding of disease dynamics	High
Numerical Solution	Simulation of real-life epidemic scenarios	Medium
Simulation	Evaluation of policy and intervention strategies	Medium

We propose different parameter scenarios for the SEIHRD model to investigate their impact on the spread of the Ebola virus. Selected parameters will be varied intentionally, with justifications provided for each modification. These parameter sets will then be used for numerical and graphical comparisons. The primary goal of this approach is to examine how changes in key parameters—such as infection rates, hospitalization rates, and recovery rates—affect critical outcomes, including the total number of infections, hospitalizations, recoveries, and deaths. Table 3 presents three main parameter scenarios for the SEIHRD model.

Table 3. Parameter scenarios used in the SEIHRD model simulation, where EC represents the Essential Case, EHI represents the Effective Health Intervention, and HCE represents the Highly Contagious Epidemic

Parameter	**EC**	**EHI**	**HCE**
β (Transmission rate)	0.4	0.2	0.6
θ (Hospital infection modifier)	0.5	0.3	0.8
ω (Cadaver infection modifier)	1.2	0.5	1.5
σ (Incubation rate)	$\frac{1}{7}$	$\frac{1}{6}$	$\frac{1}{9}$
γ (Recovery rate)	0.1	$\frac{1}{7}$	$\frac{1}{12}$
δ (Hospitalization rate)	$\frac{1}{6}$	0.25	0.125
η (Recovery rate from hospital)	0.1	$\frac{1}{7}$	$\frac{1}{12}$
μ (Death rate)	0.125	$\frac{1}{12}$	$\frac{1}{6}$

In light of Table 3, it is worth mention that the *Essential Case (EC)* scenario assumes no intervention and represents a hypothetical base case. The *Effective Health Intervention (EHI)* scenario includes early isolation, hospital preparedness, and safe burial practices. The *Highly Contagious Epidemic (HCE)* scenario reflects delayed response, collapse of the healthcare system, and lack of containment measures. However, the rationale for varying the coefficients in the three scenarios presented in Table 3 is as follows:

- β reflects the impact of population density—specifically, the degree of spacing or crowding among individuals.
- θ corresponds to the effectiveness of hospital isolation measures.
- ω is influenced by the method of corpse disposal, whether safe and regulated or otherwise.
- σ represents the speed of early diagnosis and identification of infected individuals.
- γ is associated with the quality and accessibility of medical care provided to infected individuals.
- δ depends on the efficiency of transferring infectious individuals to appropriate healthcare facilities.
- η reflects the availability and effectiveness of treatment options for hospitalized patients.
- μ indicates the intensity and quality of hospital care, including whether patients receive intensive care or deteriorate due to inadequate treatment.

3.1. **Numerical Approach under the First Initial Conditions.** The numerical simulation is carried out for the two previously defined cases using the three parameter scenarios presented in Table 3. These simulations are based on the initial conditions described earlier and aim to evaluate the impact of each scenario on the progression of the Ebola outbreak.

Case 1: In this case, the number of people initially exposed to infection is 1,506,576, representing international pilgrims. The total time period allocated for studying the spread and treatment of the epidemic is 90 days.

TABLE 4. Numerical results under the essential scenario using the Runge-Kutta 4th order method (Case 1)

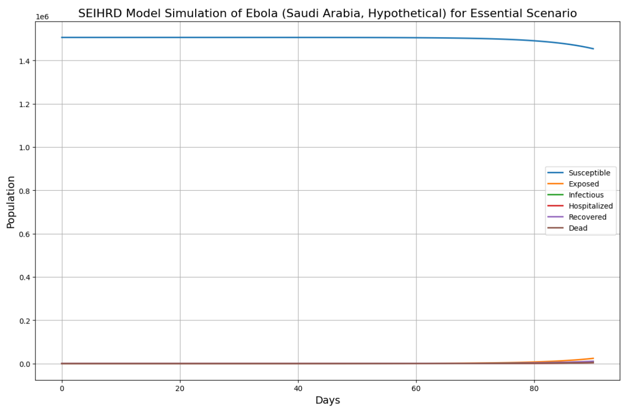
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506575e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	1.506575e+06	0.891521	0.117579	0.009714	0.006613	0.000417
2	1.506575e+06	0.837362	0.197804	0.031896	0.024637	0.002925
3	1.506575e+06	0.822145	0.255144	0.059659	0.051982	0.008610
4	1.506575e+06	0.836289	0.299160	0.089254	0.087219	0.017908

TABLE 5. Numerical results under the effective health intervention scenario using the Runge-Kutta 4th order method (Case 1)

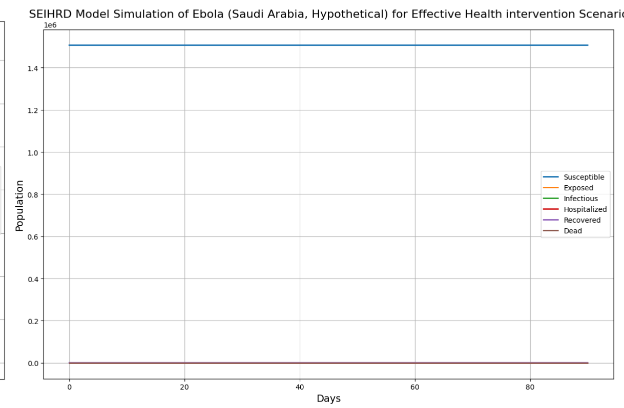
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506575e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	1.506575e+06	0.860005	0.126894	0.016173	0.010760	0.000465
2	1.506575e+06	0.760610	0.196475	0.050234	0.039012	0.003156
3	1.506575e+06	0.688211	0.231877	0.088647	0.079838	0.008937
4	1.506575e+06	0.634135	0.247236	0.124663	0.129529	0.017852

TABLE 6. Numerical results under the highly contagious epidemic scenario using the Runge-Kutta 4th order method (Case 1)

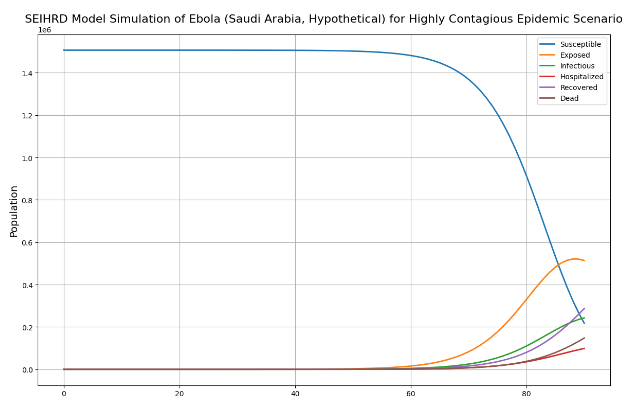
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506575e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	1.506575e+06	0.924935	0.095811	0.005793	0.004358	0.000331
2	1.506575e+06	0.910752	0.169363	0.019509	0.016540	0.002356
3	1.506575e+06	0.945952	0.230299	0.037522	0.035609	0.007063
4	1.506575e+06	1.024290	0.285581	0.057907	0.061087	0.014988



(A) The essential scenario



(B) The effective health intervention scenario



(C) The highly contagious epidemic scenario

FIGURE 1. SEIHRD model simulation results for a hypothetical Ebola outbreak in Saudi Arabia under three different scenarios (Case 1) using the Runge-Kutta 4th order method.

Case 2: In this case, the number of individuals exposed to the infection is 2,093,300, reflecting both international pilgrims and service personnel at key Hajj sites. The total time period allocated to study the spread of the epidemic and implement control and treatment measures is 90 days.

Table 7. Numerical results under the essential scenario using the Runge-Kutta 4th order method (Case 2)

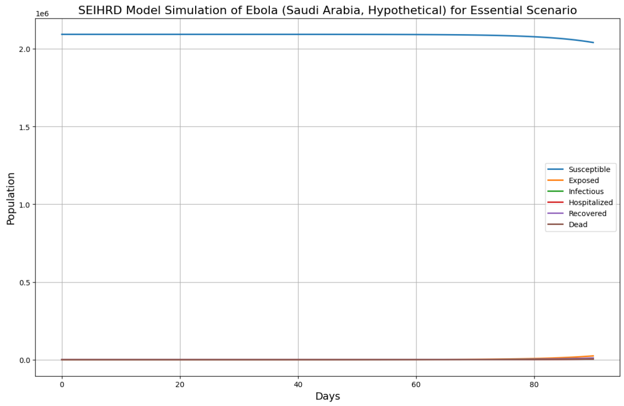
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093299e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	2.093299e+06	0.891521	0.117579	0.009714	0.006613	0.000417
2	2.093299e+06	0.837362	0.197804	0.031896	0.024637	0.002925
3	2.093299e+06	0.822145	0.255144	0.059659	0.051982	0.008610
4	2.093299e+06	0.836289	0.299160	0.089254	0.087219	0.017908

Table 8. Numerical results under the effective health intervention scenario using the Runge-Kutta 4th order method (Case 2)

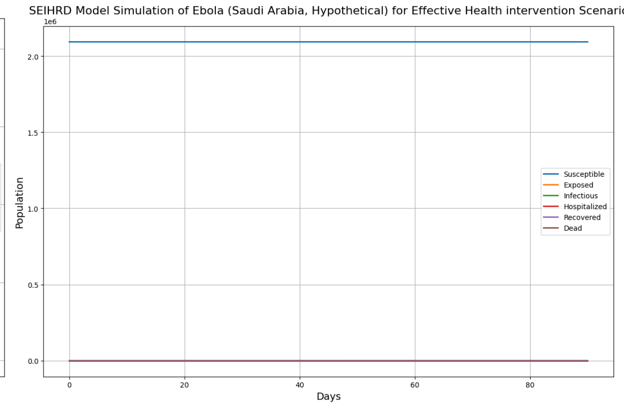
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093299e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	2.093299e+06	0.860005	0.126894	0.016173	0.010760	0.000465
2	2.093299e+06	0.760610	0.196475	0.050234	0.039012	0.003156
3	2.093299e+06	0.688211	0.231877	0.088647	0.079838	0.008937
4	2.093299e+06	0.634135	0.247236	0.124663	0.129529	0.017852

Table 9. Numerical results under the highly contagious epidemic scenario using the Runge-Kutta 4th order method (Case 2)

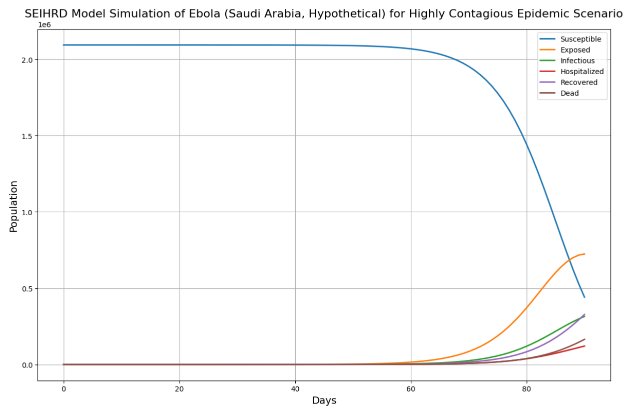
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093299e+06	1.000000	0.000000	0.000000	0.000000	0.000000
1	2.093299e+06	0.924935	0.095811	0.005793	0.004358	0.000331
2	2.093299e+06	0.910752	0.169363	0.019509	0.016540	0.002356
3	2.093299e+06	0.945952	0.230299	0.037522	0.035609	0.007063
4	2.093299e+06	1.024290	0.285581	0.057907	0.061087	0.014988



(A) The essential scenario



(B) The effective health intervention scenario



(C) The highly contagious epidemic scenario

Figure 2. SEIHRD model simulation results for a hypothetical Ebola outbreak in Saudi Arabia under three different scenarios (Case 2) using the Runge-Kutta 4th order method.

3.2. **Numerical Approach under the Second Initial Conditions.** In this scenario, it is assumed that one individual is in the exposed class (infected but not yet infectious), and another is in the infectious class (actively symptomatic) among the pilgrims. Consequently, the number of susceptible individuals is reduced by two from the total population. The second set of initial conditions for the SEIHRD model, applied to simulate a hypothetical Ebola outbreak, is given by:

$$
\begin{aligned}
S(0) &= N-2, \\
E(0) &= 1, \\
I(0) &= 1, \\
H(0) &= 0, \\
R(0) &= 0, \\
D(0) &= 0.
\end{aligned}
$$

The numerical approach under the second initial conditions yields the following results:
Case 1: In this case, the number of individuals exposed to the infection is 1,506,576, and the total time period allocated to study the spread of the epidemic and implement control and treatment measures is 90 days.

Table 10. Numerical results under the essential scenario using the Runge-Kutta 4th order method (Case 1, Second Initial Conditions)

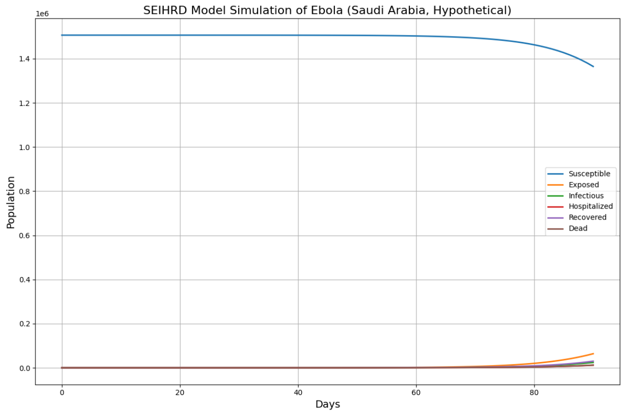
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506574e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	1.506574e+06	1.235742	0.907198	0.141303	0.102330	0.009334
2	1.506573e+06	1.445695	0.863736	0.244327	0.210065	0.033760
3	1.506573e+06	1.648998	0.856164	0.323023	0.324326	0.069421
4	1.506572e+06	1.859059	0.876176	0.386953	0.446327	0.113913

Table 11. Numerical results under the effective health intervention scenario using the Runge-Kutta 4th order method (Case 1, Second Initial Conditions)

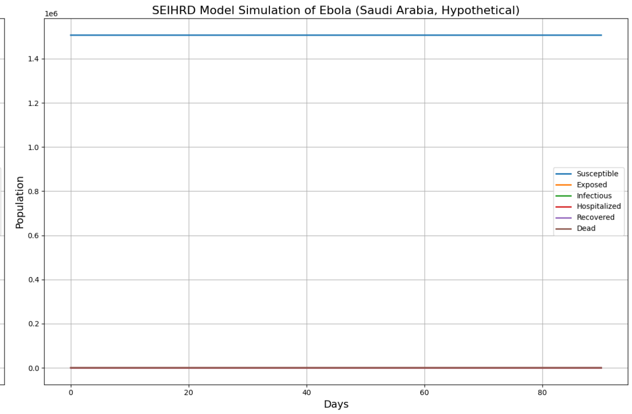
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506574e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	1.506574e+06	1.018379	0.814686	0.200738	0.144150	0.009017
2	1.506574e+06	1.016357	0.690440	0.327006	0.289489	0.031430
3	1.506573e+06	1.003738	0.605398	0.404803	0.434412	0.062198
4	1.506573e+06	0.986408	0.545837	0.450994	0.577829	0.098036

Table 12. Numerical results under the highly contagious epidemic scenario using the Runge-Kutta 4th order method (Case 1, Second Initial Conditions)

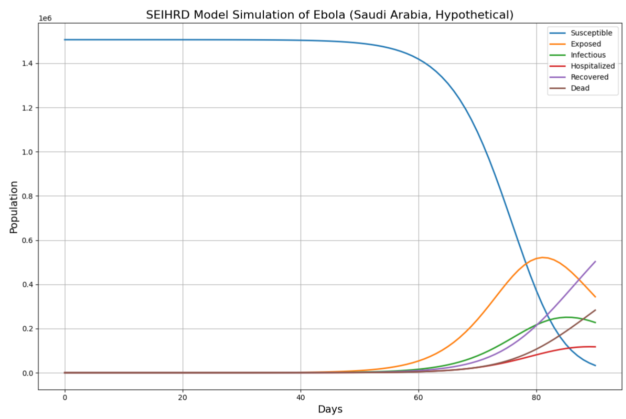
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	1.506574e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	1.506573e+06	1.470019	0.936911	0.106339	0.084919	0.009351
2	1.506573e+06	1.928673	0.931923	0.185656	0.174734	0.033975
3	1.506572e+06	2.408865	0.974739	0.249706	0.272083	0.070408
4	1.506571e+06	2.937985	1.059987	0.306802	0.379790	0.116837



(A) The essential scenario



(B) The effective health intervention scenario



(C) The highly contagious epidemic scenario

FIGURE 3. SEIHRD model simulation of Ebola (Saudi Arabia, Hypothetical) for the three scenarios under case 1 with second initial conditions

Case 2: The number of people exposed to the infection is 2,093,300, and the time period allocated to study the spread of the epidemic and its treatment is 90 days.

Table 13. Numerical results under the essential scenario using the Runge-Kutta 4th order method (Case 2)

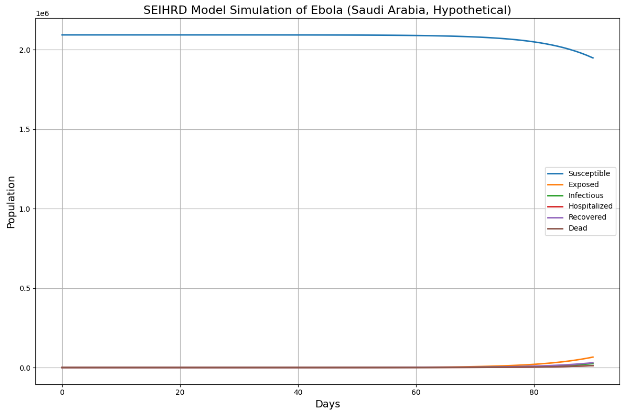
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093298e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	2.093298e+06	1.235742	0.907198	0.141303	0.102330	0.009334
2	2.093297e+06	1.445695	0.863736	0.244327	0.210065	0.033760
3	2.093297e+06	1.648998	0.856164	0.323023	0.324326	0.069421
4	2.093296e+06	1.859060	0.876176	0.386953	0.446328	0.113913

Table 14. Numerical results under the effective health intervention scenario using the Runge-Kutta 4th order method (Case 2)

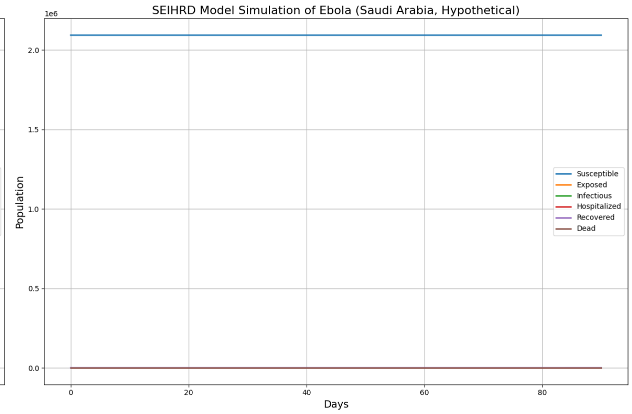
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093298e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	2.093298e+06	1.018379	0.814686	0.200738	0.144150	0.009017
2	2.093298e+06	1.016358	0.690440	0.327006	0.289489	0.031430
3	2.093297e+06	1.003739	0.605398	0.404803	0.434412	0.062198
4	2.093297e+06	0.986408	0.545837	0.450994	0.577829	0.098036

Table 15. Numerical results under the highly contagious epidemic scenario using the Runge-Kutta 4th order method (Case 2)

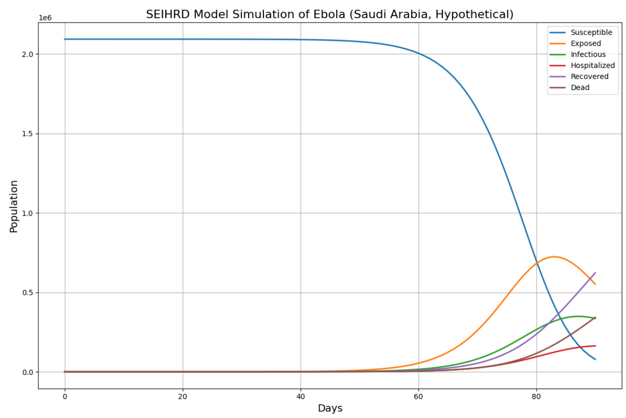
Day	Susceptible	Exposed	Infectious	Hospitalized	Recovered	Dead
0	2.093298e+06	1.000000	1.000000	0.000000	0.000000	0.000000
1	2.093297e+06	1.470019	0.936911	0.106339	0.084919	0.009351
2	2.093297e+06	1.928674	0.931923	0.185656	0.174734	0.033975
3	2.093296e+06	2.408866	0.974739	0.249706	0.272083	0.070408
4	2.093295e+06	2.937987	1.059987	0.306802	0.379790	0.116837



(A) The essential scenario



(B) The effective health intervention scenario



(C) The highly contagious epidemic scenario

FIGURE 4. SEIHRD model simulation of Ebola (Saudi Arabia, Hypothetical) for the three scenarios under Case 2

3.3. **Comparative Study.**

3.3.1. *Comparative study between the three scenarios under the first initial conditions.* **Case 1:** According to Tables 4, 5, and 6, which present the solution trajectories of the SEIHRD model for a hypothetical Ebola outbreak, we observe the following trends:

- The proportions of *Infectious*, *Hospitalized*, *Recovered*, and *Dead* individuals increase daily across all three scenarios.
- The *Exposed* compartment behaves differently:
  - In the *essential* and *highly contagious epidemic* scenarios, the percentage of exposed individuals fluctuates and is not monotonic.
  - In contrast, in the *effective health intervention* scenario, the percentage of exposed individuals shows a clear decreasing trend, indicating successful containment and early detection.

- The rate of increase for key compartments (e.g., infectious and dead) is highest in the highly contagious epidemic scenario, followed by the essential scenario, and is slowest in the effective intervention case.

According to Figure 1, which visualizes the SEIHRD model simulation of the Ebola virus under the second initial condition, we observe the following:

- **Essential Scenario:** For the first 70 days, the Ebola virus shows no significant spread. In the last 20 days, there is a noticeable decrease in the number of susceptible individuals and an increase in the numbers of exposed, infectious, hospitalized, recovered, and deceased individuals. However, none of these compartments exceed 35,000 individuals during the entire 90-day period.
- **Effective Health Intervention Scenario:** All compartments—susceptible, exposed, infectious, hospitalized, recovered, and deceased—remain constant throughout the 90 days, consistent with the initial conditions. This reflects the containment of the outbreak under optimal intervention strategies.
- **Highly Contagious Epidemic Scenario:**
  - The number of susceptible individuals remains unchanged for the first 50 days. Afterward, it begins to decline—dropping to 1,400,000 on day 68, around 1,000,000 on day 70, and down to approximately 220,000 by day 90.
  - The number of exposed individuals remains stable through day 50. Starting from day 51, it increases gradually—reaching 100,000 by day 70, 400,000 by day 83, peaking at 450,000 on day 87, and then declining slightly over the final three days.
  - The number of infectious individuals remains constant until day 60. Thereafter, it increases gradually—reaching 100,000 by day 80 and 230,000 by day 90.
  - The hospitalized population remains steady through day 60, then increases—reaching 50,000 by day 80 and peaking at 100,000 by day 90.
  - The number of recovered individuals remains unchanged for the first 60 days, then increases; reaching 80,000 by day 80 and a peak of 300,000 by day 90.
  - The number of deaths remains unchanged for the first 60 days, followed by a rise; reaching 50,000 by day 80 and up to 150,000 by day 90.

Case 2: According to Tables 7, 8, and 9, which presents the numerical solution values of the SEIHRD model for a hypothetical Ebola outbreak, we observe that the proportions of infectious, hospitalized, recovered, and deceased individuals consistently increase with time across all scenarios. However, the proportion of exposed individuals exhibits fluctuations in the essential (baseline) scenario and in the highly contagious epidemic scenario. In contrast, under the effective health intervention scenario, the proportion of exposed individuals decreases steadily throughout the simulation period. Furthermore, it is noteworthy that the growth rates of the affected compartments are highest in the highly contagious scenario, followed by the essential scenario, and lowest in the effective intervention scenario.

According to Figure 2, which illustrates the SEIHRD model simulation for the Ebola virus:

- **Baseline (essential) scenario:** The virus does not spread during the first 70 days. Over the final 20 days, there is a decline in the number of susceptible individuals and a rise in the numbers of exposed, infected, hospitalized, recovered, and deceased individuals. However, none of these groups exceeds 750,000 people within the 90-day period.
- **Effective health intervention scenario:** The numbers for all categories (susceptible, exposed, infectious, hospitalized, recovered, and deceased) remain constant throughout the 90 days, staying equal to the initial conditions.
- **Highly contagious epidemic scenario:**
  - The number of susceptible individuals stays constant for the first 50 days, then drops to approximately 2,000,000 by day 68, around 1,900,000 by day 70, and eventually reaches about 400,000 by day 90.
  - The number of exposed individuals remains unchanged for the initial 50 days, then begins to rise, reaching around 100,000 by day 70 and 400,000 by day 83. It peaks at about 450,000 on day 87 before gradually declining over the final three days.
  - The number of infectious individuals stays steady for the first 60 days, then gradually increases to about 150,000 by day 80 and reaches approximately 300,000 by day 90.
  - The number of hospitalized individuals remains at the baseline for 60 days, then gradually grows to 90,000 by day 80 and peaks at around 150,000 on day 90.
  - The number of recovered individuals remains stable during the first 60 days, then begins to increase to reach 125,000 on day 80, and reaches a maximum of 300,000 on day 90.
  - The number of deaths remains similar to the initial conditions during the first 60 days, then begins to increase to reach 100,000 by day 80 and 200,000 by day 90.

As closing remarks, one might observe that the second scenario (effective health intervention scenario) is better than the first scenario (essential scenario), and the first scenario is better than the third scenario (highly contagious epidemic scenario). This conclusion is based on several key observations:

- The number of deaths in the second scenario is lower than in the first, and in the first scenario it is lower than in the third. This is the primary reason the second scenario is considered the most favorable.
- The number of healthy (susceptible) individuals remains high throughout the second scenario, while it declines in the first scenario more gradually than in the third.
- The first and second cases are similar in terms of the Ebola spread pattern during the Hajj season, with only a slight difference due to population size. The second case includes a larger population (2,093,300) compared to the first case (1,506,576).
- The numbers of exposed, infectious, hospitalized, recovered, and deceased individuals are directly influenced by the initial number of susceptible people.

3.3.2. *Comparative Study of the Results Between the First and Second Initial Conditions.* To conduct a comparative analysis between the results obtained under the first and second initial conditions, it is sufficient to focus on the outcomes of **Case 1**, as both sets share the same scenarios.

According to Tables 10, 11, and 12, which illustrates the SEIHRD model solutions for the Ebola virus under the second initial conditions, we observe the following:

- The percentages of exposed, hospitalized, recovered, and deceased individuals increase progressively each day.
- The percentage of infectious individuals exhibits instability in both the essential and highly contagious epidemic scenarios.
- In the effective health intervention scenario, the percentage of exposed individuals decreases over time, indicating successful containment.
- It is important to note that the rate of increase is highest in the highly contagious epidemic scenario, followed by the essential scenario, and is lowest in the effective health intervention scenario.

According to Figure 3, which presents the visualization of the SEIHRD model simulation of the Ebola virus, we observe the following:

- **Essential scenario:** For the first 70 days, the Ebola virus does not spread. In the last 20 days, there is a noticeable decrease in the number of susceptible individuals and an increase in the number of exposed, infected, hospitalized, recovered, and deceased individuals. However, none of these populations exceed 80,000 individuals over the 90-day period.
- **Effective health intervention scenario:** All compartments (susceptible, exposed, infectious, hospitalized, recovered, deceased) remain constant and equal to the initial conditions throughout the 90 days.
- **Highly contagious epidemic scenario:**
  - The number of susceptible individuals remains stable for the first 50 days, then decreases to approximately 1,200,000 on day 68, about 1,100,000 on day 70, and reaches 50,000 by day 90.
  - The number of exposed individuals remains constant for the initial 50 days, then begins to rise—reaching 250,000 on day 70 and peaking at 510,000 on day 83—before gradually decreasing during the final 7 days.
  - The number of infectious individuals remains steady during the first 60 days, then increases gradually to 210,000 on both day 80 and day 90.
  - The number of hospitalized individuals stays at baseline for the first 60 days, then increases to 95,000 by day 80 and peaks at 110,000 on day 90.
  - The number of recovered individuals is stable for the first 60 days, then grows to 220,000 on day 80 and ultimately reaches 500,000 by day 90.
  - The number of deaths remains unchanged for the first 60 days, then rises to 110,000 on day 80 and 250,000 by day 90.

After a careful comparison of the results of the hypothetical Ebola outbreak model in Saudi Arabia under both the first and second initial conditions, we conclude the following:

- The second scenario (effective health intervention) is superior to the first (baseline), and the first is better than the third (highly contagious epidemic), primarily based on the number of deaths recorded in each case.
- The outcomes under the second initial condition are significantly worse than those under the first initial condition, due to the faster spread of the virus and the higher number of deaths.
- The number of individuals in each compartment—susceptible, exposed, infectious, hospitalized, recovered, and deceased—plays a critical role in the epidemic's dynamics, especially given that the Ebola virus can continue to spread even after an infected individual has died.
- The study highlights how a simple public health intervention, such as early and effective isolation, can drastically alter the trajectory of the outbreak.

4. Conclusion

The Runge-Kutta 4th order method was successfully applied to the SEIHRD model to simulate a hypothetical Ebola outbreak during the 2025 Hajj season in Saudi Arabia. This study combines a numerical approach with real-world data based on official reports from the 2025 pilgrimage, enhancing its relevance and practical application. The key contributions of this study include:

- Simulating disease dynamics to support preparedness and response planning.
- Evaluating the effectiveness of intervention strategies, such as isolation, hospitalization, and safe burial practices.
- Highlighting the critical role of deceased individuals in postmortem transmission—a distinct characteristic of Ebola.

Based on the numerical results and scenario analysis, we recommend the following measures:

- Implement early isolation protocols, enhance hospital readiness, and ensure safe burial of corpses.
- Develop contingency plans for temporary quarantine zones during the Hajj season.
- Issue temporary religious directives to limit traditional practices such as corpse washing in the event of infection.
- Distribute personal protective equipment (PPE) to workers handling infected individuals or corpses.
- Enforce temporary travel restrictions if necessary.
- Train personnel in outbreak response and infection control.
- Raise public and community awareness regarding the transmission and prevention of Ebola.

Acknowledgment: The authors extend their appreciation to Prince Sattam bin Abdulaziz University for funding this research work through the project number (PSAU/2025/01/32960).

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] N.R. Anakira, A. Almalki, D. Katatbeh, G.B. Hani, A.F. Jameel, K.S. Al Kalbani, M. Abu-Dawas, An Algorithm for Solving Linear and Non-Linear Volterra Integro-Differential Equations, Int. J. Adv. Soft Comput. Appl. 15 (2023), 70–83.

[2] G. Farraj, B. Maayah, R. Khalil, W. Beghami, An Algorithm for Solving Fractional Differential Equations Using Conformable Optimized Decomposition Method, Int. J. Adv. Soft Comput. Appl. 15 (2023), 187–196.

[3] M. Berir, Analysis of the Effect of White Noise on the Halvorsen System of Variable-Order Fractional Derivatives Using a Novel Numerical Method, Int. J. Adv. Soft Comput. Appl. 16 (2024), 294–306.

[4] K.M. Dharmalingam, N. Jeeva, N. Ali, R.K. Al-Hamido, S.E. Fadugba, K. Malesela, F.T. Tolasa, H.S. El-Bahkiry, M. Qousini, Mathematical Analysis of Zika Virus Transmission: Exploring Semi-Analytical Solutions and Effective Controls, Commun. Math. Biol. Neurosci. 2024 (2024), 112. https://doi.org/10.28919/cmbn/8719.

[5] I. Iqbal, I.M. Batiha, M.S. Hijazi, I. Bendib, A. Ouannas, N. Anakira, Fractional-Order SEIR Model for COVID-19: Finite-Time Stability Analysis and Numerical Validation, Int. J. Neutrosophic Sci. 26 (2025), 266–282. https://doi.org/10.54216/ijns.260123.

[6] S. Momani, I.M. Batiha, I. Bendib, A. Al-Nana, A. Ouannas, M. Dalah, On Finite-Time Stability of Some COVID-19 Models Using Fractional Discrete Calculus, Comput. Methods Programs Biomed. Update 7 (2025), 100188. https://doi.org/10.1016/j.cmpbup.2025.100188.

[7] I.M. Batiha, A.A. Abubaker, I.H. Jebril, S.B. Al-Shaikh, K. Matarneh, M. Almuzini, A Mathematical Study on a Fractional-Order Seir Mpox Model: Analysis and Vaccination Influence, Algorithms 16 (2023), 418. https://doi.org/10.3390/a16090418.

[8] I.M. Batiha, A. Obeidat, S. Alshorm, A. Alotaibi, H. Alsubaie, S. Momani, M. Albdareen, F. Zouidi, S.M. Eldin, H. Jahanshahi, A Numerical Confirmation of a Fractional-Order COVID-19 Model's Efficiency, Symmetry 14 (2022), 2583. https://doi.org/10.3390/sym14122583.

[9] A. Dababneh, N. Djenina, A. Ouannas, G. Grassi, I.M. Batiha, I.H. Jebril, A New Incommensurate Fractional-Order Discrete COVID-19 Model with Vaccinated Individuals Compartment, Fractal Fract. 6 (2022), 456. https://doi.org/10.3390/fractalfract6080456.

[10] I.M. Batiha, S. Momani, A. Ouannas, Z. Momani, S.B. Hadid, Fractional-Order COVID-19 Pandemic Outbreak: Modeling and Stability Analysis, Int. J. Biomath. 15 (2021), 2150090. https://doi.org/10.1142/s179352452150090x.

[11] I.M. Batiha, A.A. Al-Nana, R.B. Albadarneh, A. Ouannas, A. Al-Khasawneh, S. Momani, Fractional-Order Coronavirus Models with Vaccination Strategies Impacted on Saudi Arabia's Infections, AIMS Math. 7 (2022), 12842–12858. https://doi.org/10.3934/math.2022711.

[12] I.M. Batiha, R. El-Khazali, O.Y. Ababneh, A. Ouannas, R.M. Batyha, S. Momani, Optimal Design of $PI^{\rho}D^{\mu}$-Controller for Artificial Ventilation Systems for Covid-19 Patients, AIMS Math. 8 (2023), 657–675. https://doi.org/10.3934/math.2023031.

[13] I.M. Batiha, H.O. Al-Khawaldeh, M. Almuzini, W.G. Alshanti, N. Anakira, A. Amourah, A Fractional Mathematical Examination on Breast Cancer Progression for the Healthcare System of Jordan, Commun. Math. Biol. Neurosci. 2025 (2025), 29. https://doi.org/10.28919/cmbn/9138.

[14] World Health Organization, Pandemic (H1N1) 2009 – Update 112, World Health Organization, Geneva, 2010.

[15] A.M. Zaki, S. van Boheemen, T.M. Bestebroer, A.D. Osterhaus, R.A. Fouchier, Isolation of a Novel Coronavirus From a Man with Pneumonia in Saudi Arabia, N. Engl. J. Med. 367 (2012), 1814–1820. https://doi.org/10.1056/nejmoa1211721.

[16] Y. Chen, W. Liang, S. Yang, N. Wu, H. Gao, et al. Human Infections with the Emerging Avian Influenza a H7n9 Virus From Wet Market Poultry: Clinical Analysis and Characterisation of Viral Genome, Lancet 381 (2013), 1916–1925. https://doi.org/10.1016/s0140-6736(13)60903-4.

[17] L. Poon, Y. Guan, J. Nicholls, K. Yuen, J. Peiris, The Aetiology, Origins, and Diagnosis of Severe Acute Respiratory Syndrome, Lancet Infect. Dis. 4 (2004), 663–671. https://doi.org/10.1016/s1473-3099(04)01172-7.

[18] L.R. Petersen, D.J. Jamieson, A.M. Powers, M.A. Honein, Zika Virus, N. Engl. J. Med. 374 (2016), 1552–1563. https://doi.org/10.1056/nejmra1602113.

[19] D. Musso, D.J. Gubler, Zika Virus, Clin. Microbiol. Rev. 29 (2016), 487–524. https://doi.org/10.1128/cmr.00072-15.

[20] World Health Organization, Ebola Situation Report – Final, World Health Organization, Geneva, 2016.

[21] J.H. Kuhn, T. Adachi, N.K.J. Adhikari, J.R. Arribas, I.E. Bah, et al. New Filovirus Disease Classification and Nomenclature, Nat. Rev. Microbiol. 17 (2019), 261–263. https://doi.org/10.1038/s41579-019-0187-4.

[22] S. Baize, D. Pannetier, L. Oestereich, T. Rieger, L. Koivogui, et al. Emergence of Zaire Ebola Virus Disease in Guinea, N. Engl. J. Med. 371 (2014), 1418–1425. https://doi.org/10.1056/nejmoa1404505.

[23] J.M. Shultz, J.L. Cooper, F. Baingana, M.A. Oquendo, Z. Espinel, et al. The Role of Fear-Related Behaviors in the 2013–2016 West Africa Ebola Virus Disease Outbreak, Curr. Psychiatry Rep. 18 (2016), 104. https://doi.org/10.1007/s11920-016-0741-y.

[24] C. Huber, L. Finelli, W. Stevens, The Economic and Social Burden of the 2014 Ebola Outbreak in West Africa, J. Infect. Dis. 218 (2018), S698–S704. https://doi.org/10.1093/infdis/jiy213.

[25] L.O. Gostin, H.A. Waxman, W. Foege, The President's National Security Agenda: Curtailing Ebola, Safeguarding the Future, JAMA 313 (2015), 27–28. https://doi.org/10.1001/jama.2014.16572.

[26] Centers for Disease Control and Prevention, 2014–2016 Ebola Outbreak in West Africa, CDC, Atlanta, 2016.

[27] World Health Organization, Ebola Virus Disease – Democratic Republic of the Congo, World Health Organization, Geneva, 2019.

[28] D. Malvy, A.K. McElroy, H. de Clerck, S. Günther, J. van Griensven, Ebola Virus Disease, Lancet 393 (2019), 936–948. https://doi.org/10.1016/s0140-6736(18)33132-5.

[29] J.C. Butcher, Numerical Methods for Ordinary Differential Equations, Wiley, 2008.

[30] E. Hairer, S.P. Nørsett, G. Wanner, Solving Ordinary Differential Equations I: Nonstiff Problems, Springer, Berlin, 2008. https://doi.org/10.1007/978-3-540-78862-1.

[31] A. Iserles, A First Course in the Numerical Analysis of Differential Equations, Cambridge University Press, Cambridge, 2009.

[32] H.W. Hethcote, The Mathematics of Infectious Diseases, SIAM Rev. 42 (2000), 599–653. https://doi.org/10.1137/s0036144500371907.