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Effect of Approximate Probability Distributions on Single and Double Acceptance Sampling Plans for Attributes

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Abstract. An acceptance sampling plan is a statement of the sample size to be used and the associated acceptance or rejection criteria for sentencing individual lots. An important measure of the performance of an acceptance sampling plan, such as the operating characteristic curve, is related to probability distributions. This research investigates the effect of binomial, Poisson and normal approximations to single and double acceptance sampling plans for attributes. For single-sampling plans, type-A OC curves show that the binomial approximation tends to overestimate the probability of acceptance P_a of the true hypergeometric distribution when the lot size is at most 10 times the sample size. The single-sampling plan with type-B OC curve displays that the P_a from Poisson is a slight overestimate of the true P_a for the binomial distribution with small n and large p, moreover, the P_a from normal approximation can be a significant underestimation, exact value, or overestimation of the binomial, even with small p. On double-sampling plans, the Poisson approximation results in a tiny overestimation, while the normal approximation appears to be a major underestimation of the binomial. In rectifying inspection, the characteristics of AOQL are very similar to the sampling plan.

1. Introduction

Statistical process control has been the centerpiece for modern statistical quality assurance, acceptance sampling remains a useful tool for a company to control the quality of raw materials or parts shipped from the suppliers, particularly when no company representatives are present at the suppliers' manufacturing facilities. There are a number of different ways to classify acceptance

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sampling plans, one major classification is by variables and attributes. Variables are quality characteristics that are measured on a numerical scale such as length, width, height, diameter, etc. A quality characteristic that cannot be measured on a numerical scale is expressed as an attribute, a set of system functional and non-functional requirements used to evaluate system performance. The attributes include performance, reliability, appearance, etc. This research deals with lot-by-lot acceptance sampling plans for attributes.

Among the categories of single-, double-, multiple-, and sequential-sampling plans for attributes, single-sampling plan is effective and the most popular. A single-sampling plan is defined by lot size N, sample size n, acceptance number c, and the number of defective items d. If $d \leq c$, the lot will be accepted. On the other hand, if $d > c$ then the lot will be rejected. For a double-sampling plan, a random sample of n_1 is selected from the lot, and the number of defectives in the sample d_1, is observed. If $d_1 \leq c_1$ (acceptance number of the first sample), the lot is accepted on the first sample. If $d_1 > c_2$ (acceptance number for both samples), the lot is rejected on the first sample. If $c_1 < d_1 \leq c_2$, a second random sample of size n_2 is drawn from the lot, and the number of defectives in this second sample d_2 is observed. If $d_1 + d_2 \leq c_2$, the lot is accepted, however, if $d_1 + d_2 > c_2$, the lot is rejected. A multiple-sampling plan is an extension of double-sampling in that more than two samples can be required to sentence a lot. Sequential-sampling plans are often applied where sample size is critical, so that a minimum sample must be taken. Under sequential sampling, samples are taken one at a time until a decision is made on the lot or process sampled. After each item is taken, a decision is made to: 1) accept; 2) reject; or 3) continue sampling. Samples are taken until an accept or reject decision is made. Thus, the procedure is open-ended, the sample size not being determined until the lot is accepted or rejected [1].

An important measure of the performance of an acceptance sampling plan is the operating characteristic (OC) curve. The OC curve depicts the discriminatory power of an acceptance sampling plan. The OC curve plots the probabilities of accepting a lot versus the fraction defective. When the OC curve is plotted, the sampling risks are obvious. Two types of OC curves are recognized: Type-A OC curve assumes that the sample is chosen from an isolated lot of finite size, and the probability of accepting the lot is calculated based on a hypergeometric distribution; Type-B assumes that the sample is chosen from a process (such as the producer's process, which produced the lot), and the binomial distribution is the exact probability distribution for calculating the probability of lot acceptance.

In acceptance sampling plan, the consumers accept the lot if the acceptance criteria are satisfied, otherwise, they reject the lot. This increases the process control producer's risk. Rectifying sampling plans are used to reduce the producer's risk. In such a plan, the entire lot is not rejected, instead, each and every unit/item of the lot is inspected. It means that 100% inspection of the rejected lot is carried out and the defective units found in the lot are replaced by non-defective units. This procedure is known as *rectifying* or *screening* the rejected lots. Average outgoing quality (AOQ) is widely used for the evaluation of a rectifying sampling plan and is the quality in the lot that results from the application of rectifying inspection. The maximum value of AOQ represents the worst possible average quality that would result from the rectifying inspection program, and this value is called the average outgoing quality limit (AOQL).

The probability distributions such as hypergeometric, binomial, Poisson, and normal have proved useful in quality control and computing the probabilities associated with the OC curve and other acceptance sampling characteristics. The binomial distribution can be used to approximate the hypergeometric distribution when the sample size n is small compared to the lot size N, and the approximation is good when $n/N \leq 0.1$. The Poisson distribution is related to the binomial distribution with parameters n and p, if n is large and p is close to 0, the Poisson distribution can be used, with $\mu = np$, to approximate binomial probabilities. The binomial distribution is also nicely approximated by the normal distribution for p of approximately $1/2$ and $n > 10$ or other values of p and larger values of n.

According to some useful approximations, this research studies the effect of approximate probability distributions on single- and double- acceptance sampling plans for attributes. The type-A and type-B OC curves are constructed to display how approximate distribution affects the probability of acceptance. Moreover, the values of AOQL are investigated to consider the yield of distribution approximations on rectifying inspection.

2. Acceptance Sampling

Generally, there are three approaches to lot sentencing: 1) accept with no inspection; 2) 100% inspection; and 3) acceptance sampling. Acceptance sampling is a middle ground between the extremes of 100% inspection and no inspection [2], and is concerned with inspection and decision-making regarding products, one of the oldest aspects of quality assurance. Acceptance sampling was popularized by Dodge and Romig [3] was originally applied by the U.S. military to the testing of bullets during World War II. In the 1930s and 1940s, acceptance sampling was one of the major components of the field of statistical quality control and was used primarily for incoming or receiving inspection. In more recent years, it has become typical to work with suppliers to improve their process performance through the use of statistical process control and designed experiments, and not to rely as much on acceptance sampling as a primary quality assurance tool. Al-Nasser and Alhroub [4] proposed new single acceptance sampling plans assuming that the lifetime distribution is the Q-Weibull distribution of a product. For a finite population size, they applied hypergeometric theory to compute the probability of acceptance, and the procedure is used to compute the minimum sample size and the operating characteristics of the sampling plans. Bilal, Mohsin and Abbas [5] proposed an acceptance sampling plan for the life length of a product which follows Weibull-Rayleigh distribution and demonstrated the OC curves to derive the efficiency of the proposed acceptance sampling plan.

2.1. **Single-Sampling Plan for Attribute.**

A single-sampling plan is a lot-sentencing procedure in which one sample of n items is selected at random from the lot, and the disposition of the lot is determined based on the information contained in that sample. For example, a single-sampling plan for attributes would consist of a sample size n and an acceptance number c. The procedure would operate as follows: 1) select n items at random from the lot; 2) if there are c or fewer defective items (d) in the sample, accept the lot, and if there are more than c defective items in the sample, reject the lot.

The probability of acceptance P_a is the probability that defective items $d \leq c$. It is a probabilistic measure, and the result can vary from 0 to 1.

2.2. **Double-Sampling Plan for Attribute.** A double-sampling plan is a procedure in which a decision about the acceptance or rejection of a lot is based on two samples that have been inspected. The double-sampling plan is defined by four parameters: n_1 is the sample size of the first sample; c_1 is the acceptance number of the first sample; n_2 is the sample size of the second sample; and c_2 is the acceptance number for both samples.

In a double-sampling plan, the number of defective items d_1 in an initial sample of size n_1 is determined. There are then three possible courses of action: 1) immediately accept the lot if $d_1 \leq c_1$; 2) immediately reject the lot if $d_1 > c_2$; or 3) take a second sample of n_2 items and reject or accept the lot depending on the total number of defective items in both samples – accept the lot if $d_1 + d_2 \leq c_2$ or reject the lot if $d_1 + d_2 > c_2$. If P_a^{I} and P_a^{II} denote the probability of acceptance on the first and second samples, respectively, and P_a denotes the probability of acceptance on the combined samples, then

$$P_a = P_a^{\mathrm{I}} + P_a^{\mathrm{II}}.$$

However, it is customary to terminate inspection of the second sample if the number of defectives is sufficient to justify rejection before all items have been examined. This is referred to as *curtailment* in the second sample. Under curtailment, it can be shown that the expected number of items inspected in double-sampling plan is smaller than the number of items examined in single-sampling plan when the OC curves of the two plans are close to being identical [6].

2.3. **Operating Characteristic Curve.** The Operating characteristic (OC) curve is an important measure of the performance of an acceptance-sampling plan. This curve plots the probability of accepting the lot versus the lot fraction defective. Thus, the OC curve displays the discriminatory power of the sampling plan. That is, it shows the probability that a lot submitted with a certain fraction defective will be either accepted or rejected.

A type-A OC curve is based on the hypergeometric distribution and is used to calculate probabilities of acceptance for an isolated lot of finite size. In the construction of the OC curve which assumes that the samples come from a large lot or sampling from a stream of lots selected at random from a process, in this situation, the binomial distribution is the exact probability distribution for calculating the probability of lot acceptance. This OC curve is referred to as a type-B OC curve.

Some literature describes the OC curves based on their distributions. Schilling [7] introduced the f-binomial distribution as a Poisson type finite analog to the hypergeometric distribution for use in constructing appropriate type-*A* OC curves for defects, moreover, the use of average run length in characterizing type-*B* sampling plans was presented as a missing measure in the evaluation of such plans. Samohyl [8] suggested the use of the hypergeometric distribution to calculate the parameters of sampling plans to reduce the error caused by approximations from binomial or Poisson distributions for finite lot size. In hypothesis testing, he evaluated the null hypothesis statement from both producer's and customer's standpoints, considering the producer's risk (the probability of rejecting a good lot when it is not) and customer's risk (the probability of accepting a bad lot when it is not). The procedure was based on individual lot acceptance sampling plan evaluation rather than the totality of lots produced by the producer.

Recent research from Alashaari and Alshammari [9] found that probability distributions such as hypergeometric, binomial and Poisson can be incorporated into sampling plan to control the quality of industrial products within production processes. Their research was conducted to investigate some industries in the Middle East that were facing strong competition and better-quality products from foreign counterparts. Also, their research was based on experiments using the defective ratio in the samples taken as an approximation to the probability of defective units in the production process. Chukhrova and Johannssen [10] presented a new binomial-type approximation for the type-*A* OC function, derived its properties, and compared this approximation via an extensive numerical study. They found that it can reduce the computational effort in relation to the type-*A* OC function and strongly recommended it for calculating sampling plans. Dewi, Gunawan and Alamsjah [11] considered two cases applicable to using the hypergeometric type-*A* OC curve; non-returned sample for destructive inspection and returned sample. They found that for a non-returned sample, a larger sample size n is required, even more than that of using the binomial distribution's sample size, which has traditionally been considered conservative. In addition, the OC curve can be used to describe the ability of the control charts to detect shifts in process quality. Nidsunkid, Budsaba and Duangsaphon [12] investigated the OC curves of the well-known Shewhart $\bar{x}$ chart when the normality assumption is violated and they found that a small size of shift is more sensitive to departures from normality than the large one.

2.4. **Rectifying Inspection.** After inspection, accepted lots go to the consumer and rejected lots may be handled as follows: destroyed; resubmitted; or screened. Acceptance sampling schemes which incorporate 100% inspection or *screening* of rejected lots are called *rectifying inspection schemes*.

In a sampling inspection plan, the items manufactured by the producer are formed in lots. The average quality level of the lots is set by the producer and the consumer through negotiation, and the producer sends the lots to the consumer for inspection. The quality of the lots before the inspection is known as *incoming quality*, and the quality of the lots which have been accepted after the inspection is known as *outgoing quality*. In an acceptance sampling plan, the lots are either accepted or rejected, so, the outgoing quality is the same as the incoming quality. However, in a rectifying inspection, the rejected lots are rectified or screened, so the outgoing quality will differ from the incoming quality. Therefore, the concept of average outgoing quality is particularly useful for the evaluation of a rectifying inspection. If the number of defective items in the sample is less than the acceptance number, the lot is accepted by replacing all defective items found in the sample with non-defective items. If the number of defective items is greater than the acceptance number, each and every unit of the lot is inspected. It means that 100% inspection is carried out for each rejected lot and all defective items found in the lot are replaced by non-defective items. Therefore, these lots are accepted after 100% inspection with zero percent defective. As a result, the accepted stores will consist of lots of varying quality level, ranging from quality levels lower than the acceptance quality level to lots with zero defective. When all lots are considered together, their average quality level may be considerably different from the incoming quality. The expected quality of the lots after the application of sampling inspection is called the average outgoing quality (AQL) and is defined as follows:

$$\text{AOQ} = \frac{\text{Number of defective items in the lot after inspection}}{\text{Lot size}}.$$

For single-sampling plan, the AOQ is calculated as follows:

$$\text{AOQ} = \frac{P_a p(N-n)}{N}. \tag{2.1}$$

When rectifying inspection is performed with double-sampling, the AOQ is given by:

$$\text{AOQ} = \frac{[P_a^{\text{I}}(N-n_1) + P_a^{\text{II}}(N-n_1-n_2)]p}{N}. \tag{2.2}$$

Note that as the lot size N becomes large relative to the sample size n, we may write (2.1) and (2.2) as:

$$\text{AOQ} = P_a p.$$

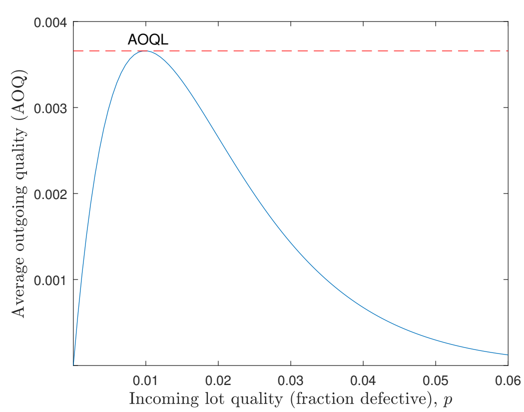


Figure 1. The AOQ curve

The curve that plots average outgoing quality against incoming lot quality is called an *AOQ curve*, which is shown in Figure 1. When the incoming quality is good, a large proportion of the lots will be accepted by the rectifying sampling plan and only a smaller fraction will be screened, hence, the outgoing quality will be good. However, when the incoming quality is not good, a large proportion of the lots will be screened. In such cases, the outgoing quality will also be good because defective items will either be replaced or rectified. Between these extremes, the AOQ increases up to a maximum and then decreases. The maximum value of AOQ represents the worst possible average for the outgoing quality, and it is known as the average outgoing quality limit (AOQL).

The AOQ and AOQL are particularly useful for the evaluation of a rectifying inspection. Salvia [13] determined the acceptance probability and AOQ that were achieved by using the standard stoplight rules, assuming that the measured characteristics were normally distributed and also determined the AOQL for any distribution of the characteristic. Yen et al. [14] proposed a quality cost model of repetitive sampling to develop a rectifying acceptance sampling plan based on the one-sided process capability index and also used the AOQ curve and AOQL for describing their proposed sampling plan.

3. Distribution Approximations

Probability distributions have an importance role in quality control and acceptance sampling. Many manufacturing processes follow a normal distribution, the mean and standard deviation of the process are used to set control limits to ensure that the process stays within acceptable bounds. The hypergeometric distribution can be used when sampling defectives from a finite lot without replacement and the binomial distribution can be applied when the defectives are sampled from an infinite lot (very large lot size) or from a finite lot with replacement. In addition, the operating characteristic curves for control chart for nonconformities (c chart) and control chart for nonconformities per unit (u chart) can be obtained from the Poisson distribution.

The hypergeometric distribution is one of the important discrete probability distributions that can be used to design acceptance sampling procedures. It is the appropriate probability model for selecting a random sample of n items without replacement from a lot of N items of which D is nonconforming or defective. In a random sample selected in such a way that all possible samples have an equal chance of being chosen, let x represents the number of nonconforming items found in the sample. Then x is a hypergeometric random variable with the probability distribution defined as follows:

$$f(x) = \frac{\binom{D}{x}\binom{N-D}{n-x}}{\binom{N}{n}}, \qquad x = 0, 1, 2, ..., \min(n, D) \ . \tag{3.1}$$

The mean and variance of the hypergeometric distribution are $\mu = \frac{nD}{N}$ and $\sigma^2 = \frac{nD}{N}\left(1 - \frac{D}{N}\right)\left(\frac{N-n}{N-1}\right)$, respectively.

The binomial distribution is used frequently in the quality inspection. It is the appropriate probability model for sampling from an infinitely large population, where p represents the fraction of defective (or damage or dismatch or nonconforming) items in the population. In these applications, x usually represents the number of defective items found in a random sample of n items. When a production unit is selected randomly from the production process, either this unit is defective (non-matching with specifications) or is intact (matching with specifications). If this process is repeated n times under the same circumstances with replacement, the results are independent. Inspection with replacement confirms that the probability of success p or failure $1-p$ remain constant from one attempt to another. The random variable x, that we are interested in is the number of defective items observed from the sample units of size n, has the binomial distribution with parameters n and p, defined as follows:

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x}, \qquad x = 0, 1, 2, ..., n\,. \tag{3.2}$$

The mean and variance of the binomial distribution are $\mu = np$ and $\sigma^2 = np(1-p)$, respectively.

The Poisson distribution can be applied in quality control as a model for the number of defects or nonconformities that occur in a unit of product. In fact, any random phenomenon that occurs on a per unit (or per unit area, per unit volume, per unit time, etc.) basis is often well approximated by the Poisson distribution. This distribution is also used in preparing quality control charts for samples and calculating probabilities for acceptance inspection plans. If the average number of defects occurring in the unit is λ ($\lambda > 0$), then the random variable x representing the actual number of defects occurring in the unit, is said to have a Poisson distribution with the probability distribution defined as follows:

$$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, \qquad x = 0, 1, 2, \ldots\,. \tag{3.3}$$

The mean and variance of the Poisson distribution are $\mu = \lambda$ and $\sigma^2 = \lambda$, respectively.

For control charts for variables, the normal distribution is an important assumption that is used to describe the behavior of variables or quality characteristics. If x is a normal random variable, then the probability distribution of x is defined as follows:

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, \qquad -\infty < x < \infty\,.$$

The mean and variance of the normal distribution are μ ($-\infty < \mu < \infty$) and $\sigma^2 > 0$, respectively.

It is sometimes useful to approximate one probability distribution with another. This is particularly helpful in situations where the original distribution is difficult to handle analytically. These approximations help to save time and effort in calculations, especially in cases which it is not feasible to use the original distributions.

3.1. **The binomial approximation to the hypergeometric.** This approximation is useful in the design of acceptance sampling plans. Recall that the hypergeometric distribution is the appropriate model for the number of nonconforming items obtained in a random sample of n items from a lot of finite size N. Thus, if the sample size n is small relative to the lot size N (often called the *sampling fraction* is small or $n/N \leq 0.1$), the binomial distribution with parameters $p = D/N$ and n is a good approximation to the hypergeometric, which usually simplifies the calculations considerably.

Since $D = Np$, $\binom{D}{x} = \frac{D!}{x!(D-x)!}$, $\binom{N-D}{n-x} = \frac{(N-D)!}{(n-x)!((N-D)-(n-x))!}$ and $\binom{N}{n} = \frac{N!}{n!(N-n)!}$.

From (3.1), we get

$$
\begin{aligned}
f(x) &= \frac{D!(N-D)!n!(N-n)!}{x!(D-x)!(n-x)!((N-D)-(n-x))!N!} \\
&= \frac{n!}{x!(n-x)!}\frac{(Np)!(N-Np)!(N-n)!}{(Np-x)!((N-Np)-(n-x))!N!} \\
&= \frac{n!}{x!(n-x)!}\frac{(Np)!(N(1-p))!(N-n)!}{(Np-x)!(N(1-p)-(n-x))!N!} \\
&= \frac{n!}{x!(n-x)!}\frac{[Np(Np-1)\cdots(Np-x+1)][N(1-p)(N(1-p)-1)\cdots(N(1-p)-(n-x)+1)]}{N(N-1)\cdots(N-n+1)} \\
&= \frac{n!}{x!(n-x)!}\frac{\left[N^x p\left(p-\frac{1}{N}\right)\cdots\left(p-\frac{x-1}{N}\right)\right]\left[N^{n-x}(1-p)\left((1-p)-\frac{1}{N}\right)\cdots\left((1-p)-\frac{n-x-1}{N}\right)\right]}{N^n\left(1-\frac{1}{N}\right)\cdots\left(1-\frac{n-1}{N}\right)} \\
&= \frac{n!}{x!(n-x)!}\frac{\left[p\left(p-\frac{1}{N}\right)\cdots\left(p-\frac{x-1}{N}\right)\right]\left[(1-p)\left((1-p)-\frac{1}{N}\right)\cdots\left((1-p)-\frac{n-x-1}{N}\right)\right]}{\left(1-\frac{1}{N}\right)\cdots\left(1-\frac{n-1}{N}\right)} \\
&\to \binom{n}{x}p^x(1-p)^{n-x}, \text{as } N\to\infty, \text{ or (3.2) which is the binomial distribution.}
\end{aligned}
$$

3.2. **The Poisson approximation to the binomial.** For binomial distribution with parameters n and p, if we let n approach infinity and p approach zero in such a way that $np = \lambda$ is constant, then the Poisson distribution results. It is possible to derive the Poisson distribution as a limiting form of the binomial distribution [15].

From equation (3.2) and $p = \frac{\lambda}{n}$, consider

$$
\begin{aligned}
\lim_{x\to\infty} f(x) &= \lim_{x\to\infty}\binom{n}{x}p^x(1-p)^{n-x} \\
&= \lim_{x\to\infty}\frac{n!}{x!(n-x)!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x} \\
&= \lim_{x\to\infty}\frac{n(n-1)(n-2)\cdots(n-x+1)}{x!}\left(\frac{\lambda}{n}\right)^x\left(1-\frac{\lambda}{n}\right)^{n-x} \\
&= \lim_{x\to\infty}\frac{n}{n}\cdot\frac{n-1}{n}\cdots\frac{n-x+1}{n}\cdot\frac{\lambda^x}{x!}\left(1-\frac{\lambda}{n}\right)^n\left(1-\frac{\lambda}{n}\right)^{-x} \\
&= \frac{\lambda^x}{x!}\lim_{x\to\infty}\left(1+\frac{-\lambda}{n}\right)^n\cdot\lim_{x\to\infty}\left(1-\frac{\lambda}{n}\right)^{-x} \\
&= \frac{\lambda^x}{x!}e^{-\lambda}(1) \qquad \left(\text{since } \lim_{x\to\infty}\left(1+\frac{-\lambda}{n}\right)^n = e^{-\lambda}\right) \\
&= \frac{e^{-\lambda}\lambda^x}{x!}, \text{or (3.3) which is the Poisson distribution.}
\end{aligned}
$$

3.3. **The normal approximation to the binomial.** With probability of success p, if the number of trials n is large, then we may use the *central limit theorem* to justify the normal distribution with mean np and variance $np(1-p)$ as an approximation to the binomial [16]. That is,

$$
f(x) = \binom{n}{x}p^x(1-p)^{n-x} \sim \frac{1}{\sqrt{2\pi np(1-p)}}e^{-\frac{1}{2}\left(\frac{x-np}{\sqrt{np(1-p)}}\right)^2}.
$$

Since the binomial distribution is discrete and the normal distribution is continuous, it is common practice to use *continuity corrections* in the approximation, so that

$$P\{x = a\} \cong \Phi\left(\frac{a + \frac{1}{2} - np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a - \frac{1}{2} - np}{\sqrt{np(1-p)}}\right)$$

where Φ denotes the standard normal cumulative distribution function. Other types of probability statements are evaluated similarly, such as

$$P\{a \le x \le b\} \cong \Phi\left(\frac{b + \frac{1}{2} - np}{\sqrt{np(1-p)}}\right) - \Phi\left(\frac{a - \frac{1}{2} - np}{\sqrt{np(1-p)}}\right).$$

The normal approximation to the binomial is known to be satisfactory for p approximately $1/2$ and $n > 10$. For other values of p, larger values of n are required. In general, the approximation is not adequate for $p < 1/(n+1)$ or $p > n/(n+1)$, or for values of the random variable outside an interval six standard deviations wide centered about the mean, i.e., the interval $np \pm 3\sqrt{np(1-p)}$ [17].

We may use the normal approximation for the sample fraction defective $\hat{p} = x/n$. The random variable $\hat{p}$ is approximately normally distributed with mean p and variance $p(1-p)/n$, so that

$$P\{a \le \hat{p} \le b\} \cong \Phi\left(\frac{b - p}{\sqrt{p(1-p)/n}}\right) - \Phi\left(\frac{a - p}{\sqrt{p(1-p)/n}}\right).$$

4. Research Scope

For a single-sampling plan, we consider the acceptance number c as 0, 1 and 2. For the type-A OC curve, the probability of acceptance P_a is calculated based on hypergeometric and binomial approximation with various lot fraction defective p. The sampling fraction n/N is defined as 0.20 ($N = 500, n = 100$ and $N = 1{,}000, n = 200$), 0.10 ($N = 500, n = 50$ and $N = 1{,}000, n = 100$) and 0.05 ($N = 500, n = 25$ and $N = 1{,}000, n = 50$). Also, for the type-B OC curve, the P_a based on binomial, Poisson approximation and normal approximation are performed with lot size $N = \infty$ and sample sizes n = 30, 50, 100, 200, 500 and 1,000.

In a double-sampling plan, the scope of acceptance number for the first sample c_1 and acceptance number for both sample c_2 are: $c_1 = 0, c_2 = 1$; $c_1 = 1, c_2 = 3$; $c_1 = 2, c_2 = 5$; $c_1 = 3, c_2 = 7$; $c_1 = 4, c_2 = 9$; and $c_1 = 5, c_2 = 11$. The sample size for the first sample n_1 and the sample size for the second sample n_2 are inspected in two cases: $n_2 = n_1$; and $n_2 = 2n_1$, with $n_1 = 50, 100$ and 200.

The AOQL values are investigated to show how sensitive the rectifying inspection procedure based on single- and double-sampling plans is to approximate distributions.

5. Results and Discussion

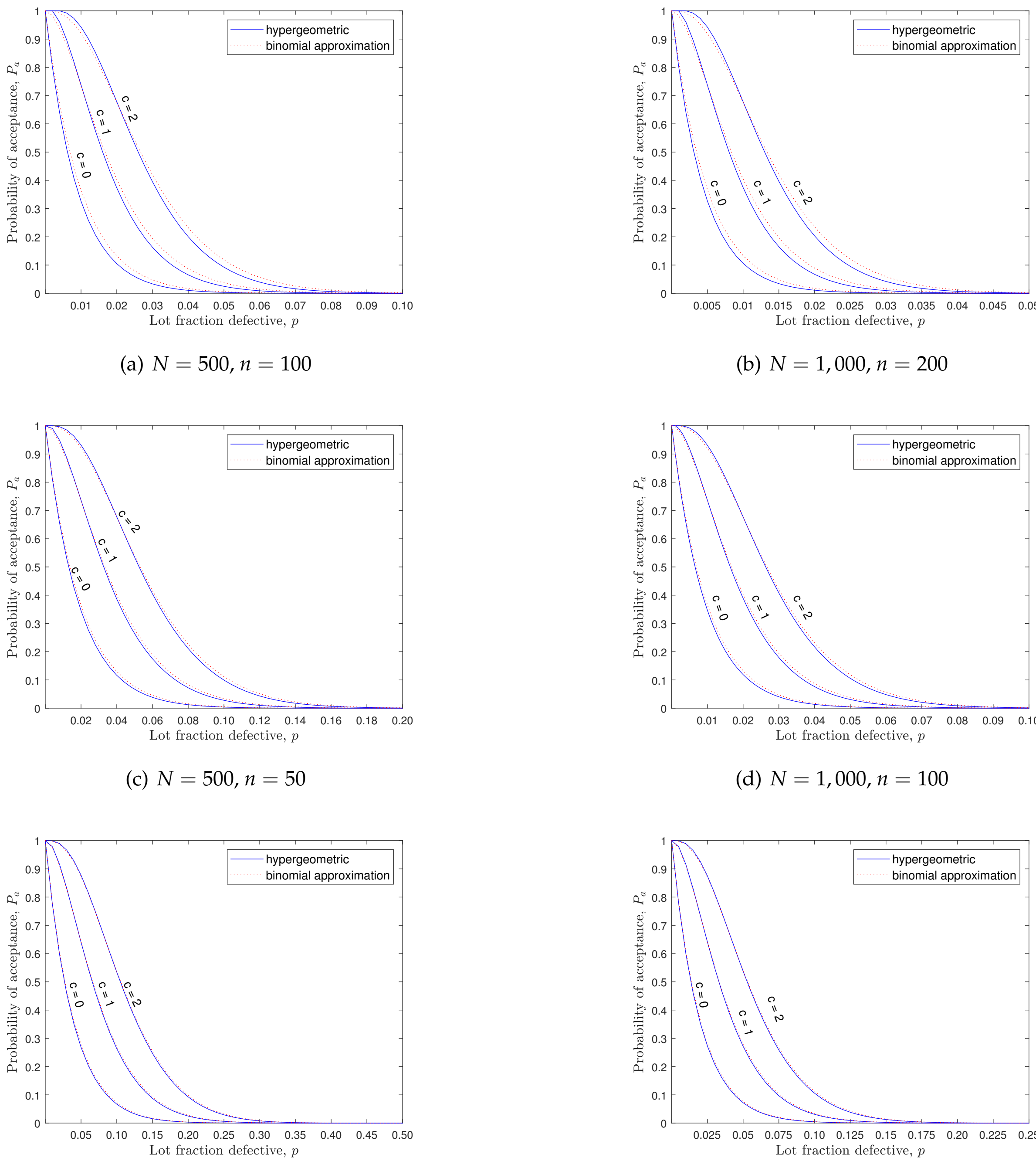
The type-A OC curves for a single-sampling plan based on hypergeometric and binomial approximation with acceptance numbers c = 0, 1 and 2 are shown in Figure 2. Generally, as the size of c increases, the probability of acceptance P_a increases for all lot fraction defective $p > 0$. At the same sampling fraction, the OC curves with larger sizes of N and n tend to reach $P_a = 0$ more quickly than smaller ones. When the lot size is 5 times the sample size or the sampling fraction $n/N = 0.20$, which are shown in Figures 2(a) and 2(b), the probabilities of acceptance from binomial approximation are always higher than the hypergeometric, exact probability distribution, with zero acceptance numbers ($c = 0$); when $c > 0$, the values of P_a from binomial approximation are smaller than hypergeometric with small p, however, P_a values from binomial approximation are larger than hypergeometric as p increases. As the $n/N = 0.20$, the P_a when $n/N = 0.10$ in Figures 2(c) and 2(d) have similar patterns and binomial approximation OC curves are quite close to hypergeometric. If $n/N = 0.05$ in Figures 2(e) and 2(f), the OC curves based on hypergeometric and binomial approximation are virtually indistinguishable.

The type-A OC curves ensure the theoretical fact that if the lot size is greater than 10 times the sample size (sampling fraction $n/N < 0.10$), then the binomial distribution will be a good approximation to the hypergeometric.

Figure 3 shows the type-B OC curve for a single-sampling plan based on binomial, Poisson approximation, and normal approximation with acceptance numbers $c = 0, 1$ and 2. The probability of acceptance P_a from Poisson distribution is very close to binomial, exact probability distribution with small lot fraction defective p. When p is larger, the increasing sample sizes n can reduce the difference of P_a between binomial and Poisson. For normal approximation, sampling plans with $c = 0$ have OC curves that are smaller and more convex than binomial throughout small values of p. As a result of this shape, the probability of acceptance begins to drop very rapidly. For $c > 0$ and very tiny p values, the OC curves from normal approximation are lower than binomial, especially with $c = 2$. However, once lot fraction defective p increases, the normal approximation curves tend to be close to binomial curve and then lift from them. The rising of normal approximation after attaching binomial is obviously seen for larger n.

These graphical results illustrate that, although the normal approximation is often justified by the Central Limit Theorem (CLT) for large n, it may yield inaccurate estimates of P_a in acceptance sampling contexts where small values of p and strict decision thresholds are involved. The CLT assumes convergence in distribution as $n \to \infty$, but in practice, the accuracy of approximations like normal or Poisson depends not only on n, but also on acceptance number c, lot fraction defective p, and the sampling fraction n/N. Therefore, using normal approximation based solely on large sample size may be misleading.

The graphical characteristics offer further insights into the behavior of different approximations. The rapid early drop and pronounced convexity of the normal-based OC curves for $c = 0$ imply that the normal approximation tends to be overly pessimistic in detecting small defect rates, potentially rejecting lots too aggressively. This could result in unnecessary rejections in high-quality manufacturing environments. For $c > 0$, the initial underestimation at low p followed by overestimation at higher p reflects a non-uniform bias that varies depending on defect level. In contrast, the smooth and gradual decline of binomial and Poisson OC curves suggests more balanced detection performance across the entire range of p. These patterns reinforce the notion that normal approximation is not uniformly conservative or liberal, it may shift from under-to overestimation depending on the region of p, which complicates risk assessment. Thus, the shape and curvature of OC curves are not merely visual, they carry decision-theoretic implications for producers and consumers alike.



(a) $N = 500, n = 100$

(b) $N = 1,000, n = 200$

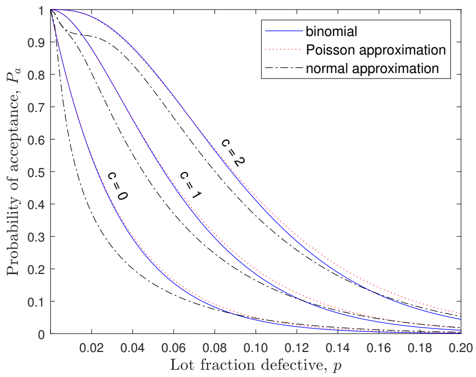
(c) $N = 500, n = 50$

(d) $N = 1,000, n = 100$

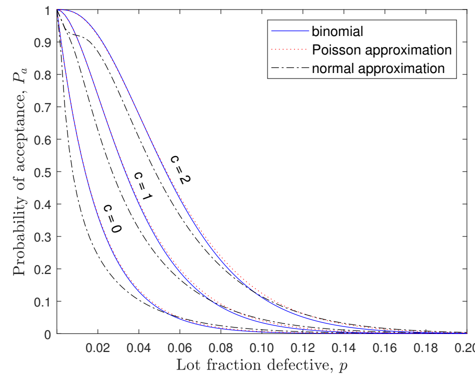
(e) $N = 500, n = 25$

(f) $N = 1,000, n = 50$

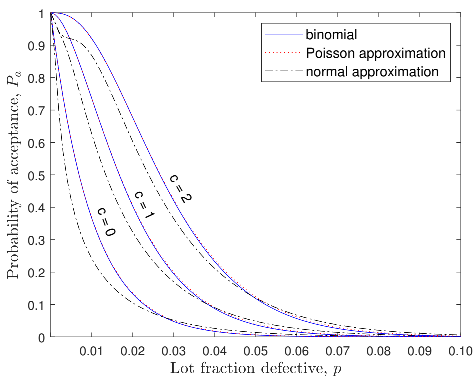
Figure 2. Type-A OC curve for single-sampling plan based on hypergeometric and binomial approximation.



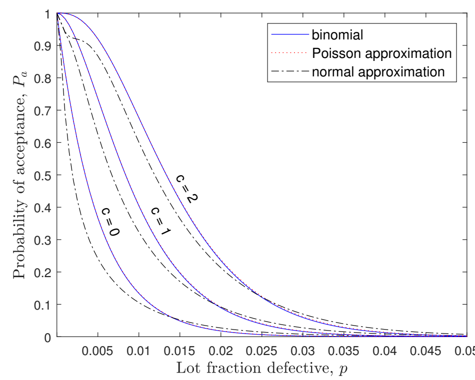
(a) $n = 30$



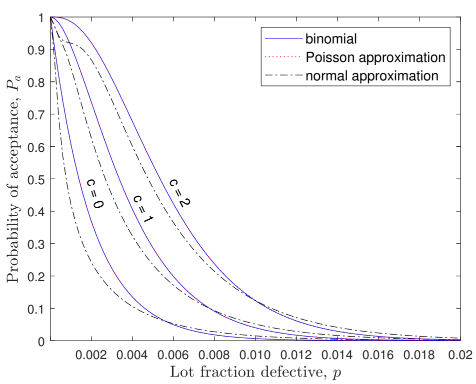
(b) $n = 50$



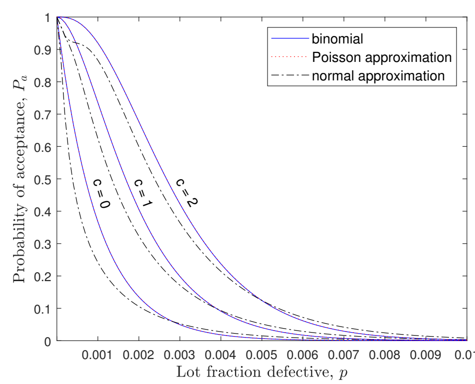
(c) $n = 100$



(d) $n = 200$

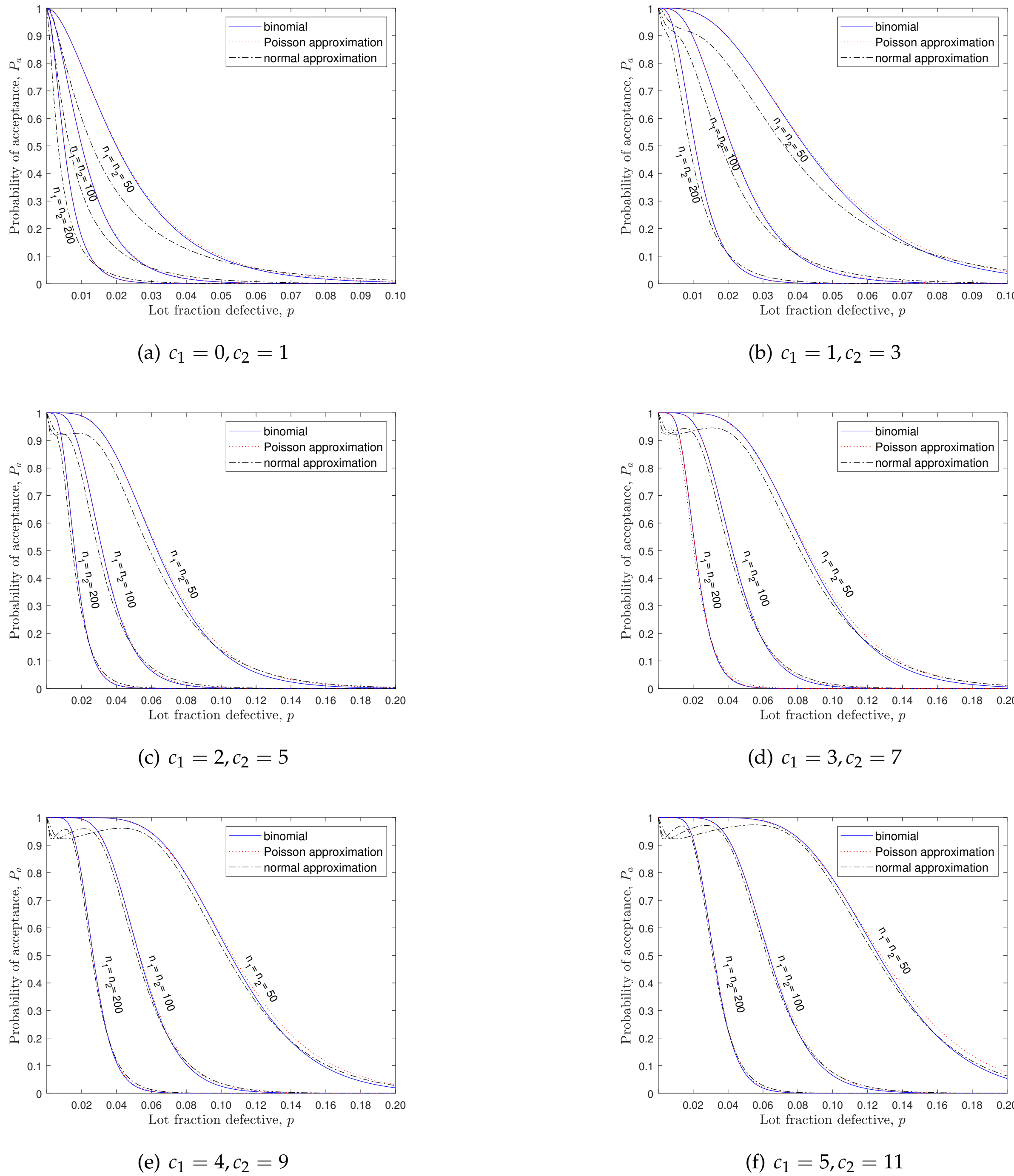


(e) $n = 500$



(f) $n = 1,000$

Figure 3. Type-*B* OC curve for single-sampling plan based on binomial, Poisson approximation and normal approximation.



(a) $c_1 = 0, c_2 = 1$

(b) $c_1 = 1, c_2 = 3$

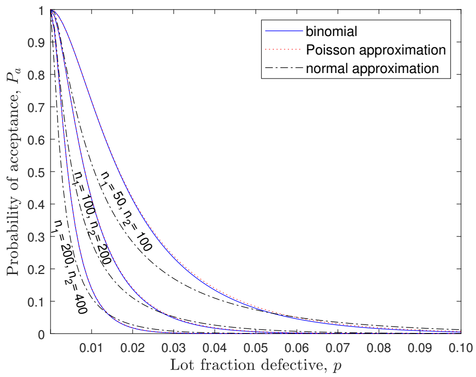
(c) $c_1 = 2, c_2 = 5$

(d) $c_1 = 3, c_2 = 7$

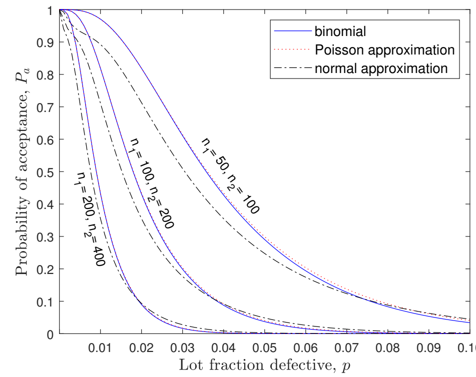
(e) $c_1 = 4, c_2 = 9$

(f) $c_1 = 5, c_2 = 11$

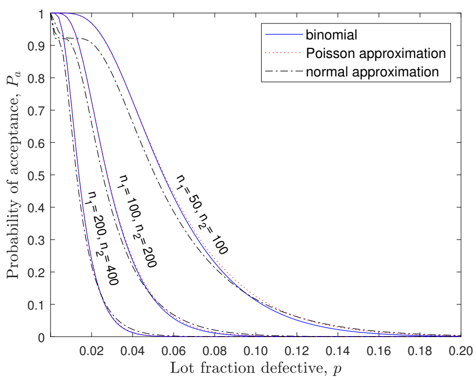
Figure 4. Type-B OC curve for double-sampling plan based on binomial, Poisson approximation and normal approximation when $n_2 = n_1$.



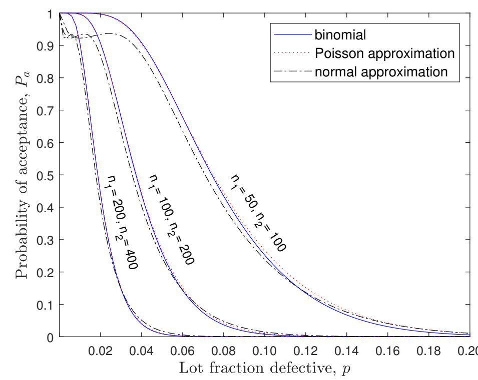
(a) $c_1 = 0, c_2 = 1$



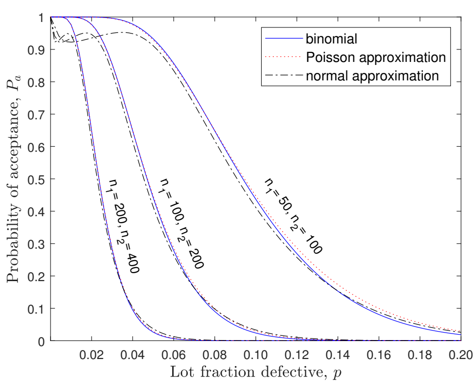
(b) $c_1 = 1, c_2 = 3$



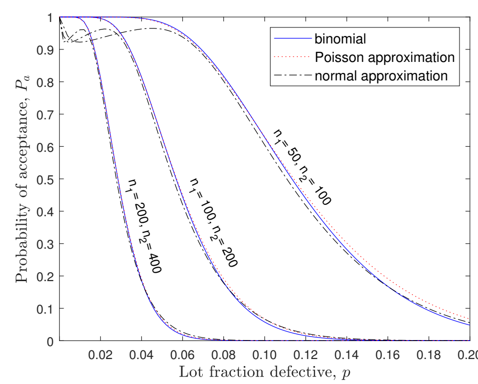
(c) $c_1 = 2, c_2 = 5$



(d) $c_1 = 3, c_2 = 7$



(e) $c_1 = 4, c_2 = 9$



(f) $c_1 = 5, c_2 = 11$

Figure 5. Type-*B* OC curve for double-sampling plan based on binomial, Poisson approximation and normal approximation when $n_2 = 2n_1$.

Table 1. The AOQL when rectifying inspection is performed with a single-sampling plan based on hypergeometric and binomial approximation.

		$c=0$		$c=1$		$c=2$	
		p	AOQL	p	AOQL	p	AOQL
$N=500, n=100$	hypergeometric	0.0100	0.0026	0.0140	0.0065	0.0220	0.0109
	binomial approximation	0.0100	0.0037	0.0160	0.0084	0.0220	0.0137
$N=1,000, n=200$	hypergeometric	0.0040	0.0013	0.0080	0.0032	0.0100	0.0054
	binomial approximation	0.0040	0.0018	0.0080	0.0042	0.0100	0.0068
$N=500, n=50$	hypergeometric	0.0200	0.0062	0.0300	0.0148	0.0420	0.0245
	binomial approximation	0.0200	0.0073	0.0300	0.0167	0.0420	0.0273
$N=1,000, n=100$	hypergeometric	0.0100	0.0031	0.0160	0.0074	0.0220	0.0123
	binomial approximation	0.0100	0.0037	0.0160	0.0084	0.0220	0.0137
$N=500, n=25$	hypergeometric	0.0400	0.0133	0.0600	0.0313	0.0900	0.0517
	binomial approximation	0.0400	0.0144	0.0600	0.0332	0.0900	0.0546
$N=1,000, n=50$	hypergeometric	0.0200	0.0067	0.0320	0.0157	0.0440	0.0259
	binomial approximation	0.0200	0.0073	0.0320	0.0167	0.0440	0.0273

Table 2. The AOQL when rectifying inspection is performed with a single-sampling plan based on binomial, Poisson approximation and normal approximation.

		$c=0$		$c=1$		$c=2$	
		p	AOQL	p	AOQL	p	AOQL
$n=30$	binomial	0.0320	0.0121	0.0540	0.0277	0.0740	0.0455
	Poisson approximation	0.0320	0.0123	0.0540	0.0280	0.0740	0.0457
	normal approximation	0.0320	0.0081	0.0500	0.0227	0.0680	0.0407
$n=50$	binomial	0.0200	0.0073	0.0320	0.0167	0.0440	0.0273
	Poisson approximation	0.0200	0.0074	0.0320	0.0168	0.0440	0.0274
	normal approximation	0.0200	0.0048	0.0320	0.0135	0.0420	0.0243
$n=100$	binomial	0.0100	0.0037	0.0160	0.0084	0.0220	0.0137
	Poisson approximation	0.0100	0.0037	0.0160	0.0084	0.0220	0.0137
	normal approximation	0.0100	0.0024	0.0150	0.0068	0.0220	0.0121
$n=200$	binomial	0.0040	0.0018	0.0075	0.0042	0.0115	0.0068
	Poisson approximation	0.0040	0.0018	0.0075	0.0042	0.0115	0.0069
	normal approximation	0.0040	0.0012	0.0075	0.0034	0.0115	0.0060
$n=500$	binomial	0.0020	0.00074	0.0028	0.0017	0.0038	0.0027
	Poisson approximation	0.0020	0.00074	0.0028	0.0017	0.0038	0.0027
	normal approximation	0.0020	0.00048	0.0028	0.0013	0.0038	0.0024
$n=1,000$	binomial	0.0010	0.00037	0.0016	0.00084	0.0020	0.0014
	Poisson approximation	0.0010	0.00037	0.0016	0.00084	0.0020	0.0014
	normal approximation	0.0011	0.00024	0.0015	0.00067	0.0020	0.0012

Table 3. The AOQL when rectifying inspection is performed with a double-sampling plan based on binomial, Poisson approximation and normal approximation for $n_2 = n_1$.

		$n_1 = n_2 = 50$		$n_1 = n_2 = 100$		$n_1 = n_2 = 200$	
		p	AOQL	p	AOQL	p	AOQL
$c_1 = 0, c_2 = 1$	binomial	0.0200	0.0100	0.0090	0.0050	0.0050	0.0025
	Poisson approximation	0.0200	0.0101	0.0090	0.0050	0.0050	0.0025
	normal approximation	0.0150	0.0067	0.0080	0.0034	0.0040	0.0017
$c_1 = 1, c_2 = 3$	binomial	0.0330	0.0220	0.0170	0.0110	0.0080	0.0055
	Poisson approximation	0.0330	0.0221	0.0170	0.0111	0.0080	0.0055
	normal approximation	0.0300	0.0183	0.0150	0.0091	0.0080	0.0046
$c_1 = 2, c_2 = 5$	binomial	0.0480	0.0351	0.0240	0.0176	0.0120	0.0088
	Poisson approximation	0.0480	0.0352	0.0240	0.0176	0.0120	0.0088
	normal approximation	0.0460	0.0318	0.0240	0.0158	0.0120	0.0079
$c_1 = 3, c_2 = 7$	binomial	0.0640	0.0490	0.0320	0.0245	0.0160	0.0122
	Poisson approximation	0.0640	0.0489	0.0320	0.0245	0.0160	0.0122
	normal approximation	0.0620	0.0462	0.0320	0.0229	0.0160	0.0114
$c_1 = 4, c_2 = 9$	binomial	0.0940	0.0782	0.0480	0.0390	0.0240	0.0195
	Poisson approximation	0.0940	0.0777	0.0480	0.0389	0.0240	0.0194
	normal approximation	0.0920	0.0764	0.0480	0.0379	0.0240	0.0189
$c_1 = 5, c_2 = 11$	binomial	0.0800	0.0634	0.0400	0.0316	0.0200	0.0158
	Poisson approximation	0.0800	0.0632	0.0400	0.0316	0.0200	0.0158
	normal approximation	0.0760	0.0611	0.0400	0.0303	0.0200	0.0151

Table 4. The AOQL when rectifying inspection is performed with a double-sampling plan based on binomial, Poisson approximation and normal approximation for $n_2 = 2n_1$.

		$n_1 = 50, n_2 = 100$		$n_1 = 100, n_2 = 200$		$n_1 = 200, n_2 = 400$	
		p	AOQL	p	AOQL	p	AOQL
$c_1 = 0, c_2 = 1$	binomial	0.0160	0.0083	0.0080	0.0042	0.0040	0.0021
	Poisson approximation	0.0160	0.0084	0.0080	0.0042	0.0040	0.0021
	normal approximation	0.0150	0.0057	0.0070	0.0028	0.0030	0.0014
$c_1 = 1, c_2 = 3$	binomial	0.0290	0.0183	0.0150	0.0092	0.0070	0.0046
	Poisson approximation	0.0290	0.0184	0.0150	0.0092	0.0070	0.0046
	normal approximation	0.0270	0.0153	0.0130	0.0076	0.0070	0.0038
$c_1 = 2, c_2 = 5$	binomial	0.0440	0.0293	0.0220	0.0146	0.0100	0.0073
	Poisson approximation	0.0440	0.0293	0.0220	0.0147	0.0100	0.0073
	normal approximation	0.0420	0.0265	0.0220	0.0132	0.0100	0.0066
$c_1 = 3, c_2 = 7$	binomial	0.0560	0.0409	0.0280	0.0204	0.0140	0.0102
	Poisson approximation	0.0560	0.0409	0.0280	0.0204	0.0140	0.0102
	normal approximation	0.0540	0.0385	0.0280	0.0192	0.0140	0.0096
$c_1 = 4, c_2 = 9$	binomial	0.0700	0.0531	0.0360	0.0265	0.0180	0.0132
	Poisson approximation	0.0700	0.0529	0.0360	0.0256	0.0180	0.0132
	normal approximation	0.0680	0.0512	0.0340	0.0255	0.0180	0.0127
$c_1 = 5, c_2 = 11$	binomial	0.0860	0.0657	0.0420	0.0328	0.0220	0.0163
	Poisson approximation	0.0860	0.0654	0.0420	0.0327	0.0220	0.0163
	normal approximation	0.0840	0.0643	0.0400	0.0319	0.0200	0.0159

To investigate the effect of approximate probability distributions on double-sampling plans, the results are revealed in Figures 4 and 5. The Poisson distribution is still a productive approximation of the binomial distribution, often yielding a slightly larger P_a than the original. Nevertheless, the P_a of Poisson obviously differs from binomial when the sample sizes n_1, n_2 are not many and the acceptance numbers c_1, c_2 are large, which are presented in Figures 4(e), 4(f), 5(e) and 5(f). At small values of p, the normal approximation OC curves depart from binomial curves, however, with the same sizes of n_1, n_2, an expansion of acceptance numbers c_1, c_2 can reduce these departures.

When the rectifying inspection is operated with a sampling plan, the average outgoing quality limit, AOQL, is given by the maximum value of the average outgoing quality (AOQ). The AOQL refers to the fact that no matter how poor the incoming quality is, on an average, the outgoing quality will never be poorer than AOQL. For rectifying inspection, which is dealing with a single-sampling plan, the AOQL values increase as acceptance number c increases. Table 1 shows that the binomial approximation offers the larger AOQL than hypergeometric distribution at the same or similar lot fraction defective p. Table 2 shows that AOQL values of Poisson approximation are very close to binomial especially when sample size n tends to infinity, in contrast, AOQL values from normal approximation are always smaller than binomial.

The graphical AOQL curves presented alongside the tabulated values reveal important behavioral differences between the approximations. The AOQL curve based on normal approximation typically lies below that of the binomial across most of the defect proportion range, indicating a persistent underestimation of outgoing quality risk. This is particularly visible at low-to-moderate defect levels, where normal approximation implies an overly optimistic outlook. In contrast, Poisson-based AOQL curves tend to align more closely with the binomial, not only in magnitude but also in shape, offering a consistent and stable representation of outgoing quality expectations.

From a strategic decision-making perspective, this has practical implications. A sampling plan based on normal approximation may lead to overly strict quality control limits, increasing inspection costs or unnecessary rejections, especially when defect levels are already low. Conversely, if Poisson approximation is selected, the resulting AOQL estimation is more in line with binomial-based plans, helping ensure that the quality targets are realistic without being excessively lenient. For producers operating under tight quality thresholds or regulatory limits, the choice of approximation directly affects the balance between producer's risk and consumer's risk. Hence, careful attention should be paid not just to analytical convenience, but to the decision-theoretic consequences shown in AOQL behavior.

The AOQL values when rectifying inspection is performed with a double-sampling plan, which are indicated in Tables 3 and 4 show similar patterns to the AOQL when rectifying inspection is done with a single-sampling plan based on binomial, Poisson approximation and normal approximation in Table 2. These mean that Poisson approximation produces more suitable values of the worst possible average for the outgoing quality than approximation by normal.

6. Conclusions

The effect of approximate probability distributions on single- and double- acceptance sampling plans for attributes are considered by the probability of acceptance from type-A and type-B OC curves. The values of AOQL are also investigated to consider how distribution approximations affect the rectifying inspection.

For single-sampling plan based on type-A OC curve, the binomial approximation tends to provide larger probability of acceptance P_a than hypergeometric when the lot size is at most 10 times the sample size ($n/N \geq 0.10$). For type-B OC curve of single-sampling plan, the Poisson performs as a good approximate probability distribution to the binomial especially when n is large, however, the P_a from Poisson is slightly overestimate of the true P_a for the binomial distribution with small n and large p. The normal approximation is not appropriate to estimate type-B OC. The P_a from normal approximation can be a significant underestimation, exact value, or overestimation of the binomial, even with small values p.

For double-sampling plan, the Poisson approximation results in a tiny overestimation of binomial, while the normal appears to be an inappropriate approximation because of the major lower values of P_a than binomial. The size of acceptance numbers c_1, c_2 also have an influence on the difference from exact distribution.

In rectifying inspection based on single-sampling plan, the AOQL from binomial approximation is overestimate to hypergeometric. Also, when the rectifying inspection is performed with single- and double-sampling plan, the AOQL from Poisson can be a nice approximation for binomial, however, the AOQL from normal shows the results as being the underestimation.

Although the normal approximation is considered a standard approximation in statistical theory and is widely used for binomial distributions in practice, particularly as justified by the Central Limit Theorem (CLT), our study shows that P_a tends to be underestimated in acceptance sampling plans, even when the sample size is large. In contrast, the Poisson approximation performs well, particularly with small acceptance numbers and low defect rates.

These findings highlight the importance of considering not only the sample size n, but also the proportion defective p, lot size N, and the structure of the sampling plan when selecting an appropriate probability model. Using an inappropriate approximation may result in misleading acceptance decisions and increased quality risk.

Author Contributions:
Peang-or Yeesa: Data curation, Formal analysis, Investigation, Methodology, Resources, Software, Supervision, Validation, Writing – original draft, Writing – review & editing.
Onuma Thonglor: Resources, Visualization, Writing – review & editing.
Sudarat Nidsunkid: Conceptualization, Formal analysis, Methodology, Software, Validation, Project administration, Visualization, Writing – review & editing.
All authors have read and approved the final version of the manuscript for publication.

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