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Statistical Analysis for the Unit Exponential Distribution Under Ranked Set Sampling with Applications to Engineering Data

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Abstract. In numerous research endeavors, cost-effective sampling is paramount, particularly when measuring the feature of interest is costly, intrusive, or requires a lot of time. Ranked set sampling (RSS) offers a valuable approach to optimize observational efficiency and enhance data collection. The two-parameter unit exponential distribution (UED) has emerged as a valuable tool for analyzing asymmetrical complex datasets. Its density function can exhibit various right-skewed and left-skewed shapes, making it well-suited for modeling a wide range of data. In this study, RSS is used to investigate the performance of ten classical estimation techniques for the UED parameters. Using a variety of accuracy criteria, the suggested RSS-based estimators' performance was compared to that of simple random sampling (SRS) through a simulation study. Partial and overall rankings of the estimators were computed to identify the best estimate approach. As evidenced by simulation studies, the maximum likelihood and maximum product spacing methods demonstrate significant promise in accurately assessing the estimated quality of RSS and SRS, respectively. Due to its higher efficiency compared to SRS, RSS demonstrates superior performance in terms of mean squared error and other relevant metrics. Two practical implementations support our findings. The first set of data examines the performance and dependability of 20 components by focusing on their failure times. The second data explores the proportion of crude oil converted to gasoline, assessing its efficiency in the refining process. By effectively analyzing both failure time data and the proportion of crude oil data, industries can make informed decisions, improve efficiency, and optimize their operations.

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1. Introduction

In numerous studies involving sampling, such as those in environmental management, ecology, sociology, and agriculture, precise measurement of individual units can be challenging, costly, or time-consuming. Nevertheless, visual inspection or the use of auxiliary variables often make ranking a small group of selected units easy. For efficient estimation of mean pasture and forage yields, especially in situations where measurement is costly, McIntyre [1] proposed ranked set sampling (RSS) as a more effective sampling design than traditional simple random sampling (SRS). Takahasi and Wakimoto [2] created the mathematical framework that RSS requires. Two scenarios can be taken into consideration in RSS. The first is a perfect ranking, which happens when sampling items are ranked without any errors. On the other hand, an imperfect ranking arises when errors are expected during the ranking process. Dell and Clutter [3] further confirmed the superiority of RSS over SRS, even in the presence of ranking errors. Although RSS was provided by McIntyre [1] in the field of agriculture, it has subsequently been successfully used in a number of other fields, including forestry [4], environmental [5–7], auditing [8], ecological [9], entomology [10], and medicine [11,12].

To design an RSS study, we typically select a small set size, generally between 2 and 5 (see [13]), to minimize potential ranking errors that may occur when using visual methods. The RSS procedure is outlined as follows:

(1) Randomly select s^2 sample units, denoted by $W_{d_1 d_2}$, where $d_1, d_2 = 1, 2, \ldots, s$, from the population. Then, randomly assign the s^2 selected units to s sets, each containing s units.
(2) Next, we rank each set's units from smallest to largest. Any technique that doesn't need the direct measurement of the variable of interest can be used to determine this ranking. This is followed by choosing the unit with rank one from the first set, the unit with rank two from the second set, and so on, to create a sample for measurement. Consequently, $(W_{(1)1}, W_{(2)2}, ..., W_{(s)s})$ represents the RSS related to this cycle. Keep in mind that, $W_{(s)s}$ represents the s-th order statistics of order s.
(3) Repeat steps 1 and 2 (c) times until the desired sample size, $n = s \times c$ is achieved. This will result in an RSS $(W_{(d_1)d_1d_2}, d_1 = 1, 2, ...s,\ d_2 = 1, 2, ..., c)$ composed of c cycles, each of size s, such that $n = s \times c$. For simplified form, $W_{(d_1)d_1d_2}$ will be written as $W_{d_1d_2}$.

RSS-based estimation has been thoroughly studied recently for a variety of parametric models, and studies have repeatedly shown that it is more efficient than other SRS. Notable examples include Chuiv et al. [14] for Cauchy distribution; Adatia [15] for half-logistic distribution; He et al. [16] for log-logistic distribution; Shaibu and Muttlak [17] for normal, exponential, and gamma distributions; Qian et al. [18] and Omar and Ibrahim [19] for Pareto distribution; Bantan et al. [20] for Zubair Lomax distribution; Hassan et al. [21] for inverse power Cauchy distribution; Al-Omari et al. [22] for xgamma distribution; Nagy et al. [23] and Hassan et al. [24] for inverted Kumaraswamy distribution; Al-Omari et al. [25] for exponentiated Pareto distribution; Esemen and Gurler [26] for generalized Rayleigh distribution; Pedroso et al. [27] for the Birnbaum-Saunders distribution;

Samuh et al. [28] for the new Weibull-Pareto distribution; Hassan et al. [29] for exponentiated exponential distribution; Yousef et al. [30] for inverted Topp-Leone distribution. For more recent studies the reader can refer to [31–36].

The proliferation of new probability distributions in the past two decades has gained significant attention due to their relevance in modern data analysis. Selecting an appropriate parametric model necessitates prior knowledge of the underlying data distribution. The skill of choosing the most suitable distribution for a given analysis minimizes information loss and enables accurate inferences. Unit distributions specified on [0, 1] are used to model random variables constrained to this range. These distributions have found applications in various fields, including health, biology, meteorology, hydrology, financial modeling, and other sciences. The beta distribution is a widely used statistical distribution for modeling data within the interval (0, 1). Despite being a flexible tool in many statistical domains, its drawbacks could force investigation into other distributions. In recent years, unit distributions that exhibit specific characteristics, such as increasing, decreasing, or bathtub-shaped hazard rate functions (HRFs), have become particularly attractive. Some notable examples of these newly introduced distributions include, unit-Gompertz distribution [37] unit-Birnbaum-Saunders distribution [38], unit exponentiated half logistic distribution [39], unit Burr-XII distribution [40], unit power Burr X distribution [41], unit-Weibull distribution [42], unit exponential distribution [43], unit logistic exponential distribution [44], among others.

In this context, a relatively recent bounded probability distribution is the unit exponential distribution (UED), which was first presented by Bakouch et al. [43]. The UED is quite flexible and can show both positive and negative skewness [43]. Furthermore, HRF of the UED is exclusively increasing. As far as we are aware, no prior research has comprehensively examined the parameter estimators of the UED using the RSS methodology. Given the UED's suitability for modeling skewed and increasing failure rate data, we propose a comparative analysis of different frequentist estimation techniques for this distribution. To achieve this, we investigate various sample sizes and parameter combinations. Thus, the effort can be summed up as follows:

- Several different estimation techniques are considered, including ordinary least squares (OLS), Cramér-von Mises (CVM), maximum likelihood (ML), Anderson Darling (AD), maximum product of spacings (MPS). In addition to minimum spacing (MS) methods include MS square log-distance (MSSLD), MS absolute distance (MSAD), MS absolute-log distance (MSALD), MS Linex distance (MSLND), MS square distance (MSSD).
- A simulation study is conducted to compare the proposed RSS-based estimators, assuming perfect ranking with their SRS counterparts using the same sample size. Various precision metrics are employed to assess estimator performance, and to identify the optimal estimation method Additionally, graphical representations based on these measures are provided for further clarification.
- Two real-world applications; one utilizing failure time data and the other concentrating on the percentage of crude oil data, validate our study.

The format of the paper is as follows: The ML estimators (MLEs) for the UED parameters within the RSS framework is presented in Section 2. Minimum distance estimators, such as CVM and AD, are covered in Section 3. The UED's maximum and minimum spacing distance estimators are given in Sections 4 and 5. We explore the OLS estimators in Section 6. In Section 7, a Monte Carlo simulation is run to evaluate the effectiveness of RSS-based estimators compared with SRS estimators. Section 8 presents two real data analysis from real-world sources. Concluding remarks are provided in Section 9.

2. The ML Estimator of the UED

This section presents MLEs for the UED parameters based on the RSS framework. We begin by providing a brief overview of the probability density function (PDF), cumulative distribution function (CDF), and HRF of the UED, accompanied by graphical representations of its PDF and HRF. Next, we discuss parameter estimation for the UED under the RSS framework.

Bakouch et al. [43] introduced the UED as a new flexible lifetime distribution designed to effectively analyze real-world datasets. The PDF and CDF of the UED are, respectively, given by

$$g(w) = \frac{2\phi\chi}{1-w^2}\left[\frac{w+1}{1-w}\right]^{\chi} e^{\phi-\phi\left[\frac{w+1}{1-w}\right]^{\chi}}; \qquad w \in (0,1), \tag{2.1}$$

and

$$G(w) = 1 - e^{\phi\left(1-\left[\frac{w+1}{1-w}\right]^{\chi}\right)}; \qquad w \in (0,1), \tag{2.2}$$

where ϕ is the scale parameter and $\chi > 0$ is the shape parameter. The HRF of the UED is as follows:

$$h(w) = \frac{2\phi\chi}{1-w^2}\left[\frac{w+1}{1-w}\right]^{\chi}.$$

The flexibility of the PDF of the UED is illustrated in Figure 1, which showcases various PDF shapes depending on the parameter values $\chi > 0$ and $\phi > 0$. From this figure we conclude that the PDF of the UED exhibits both positive and negative skewed for all values of $\chi > 0$ and $\phi > 0$. Additionally, we might infer from this figure that the HRF is increasing.

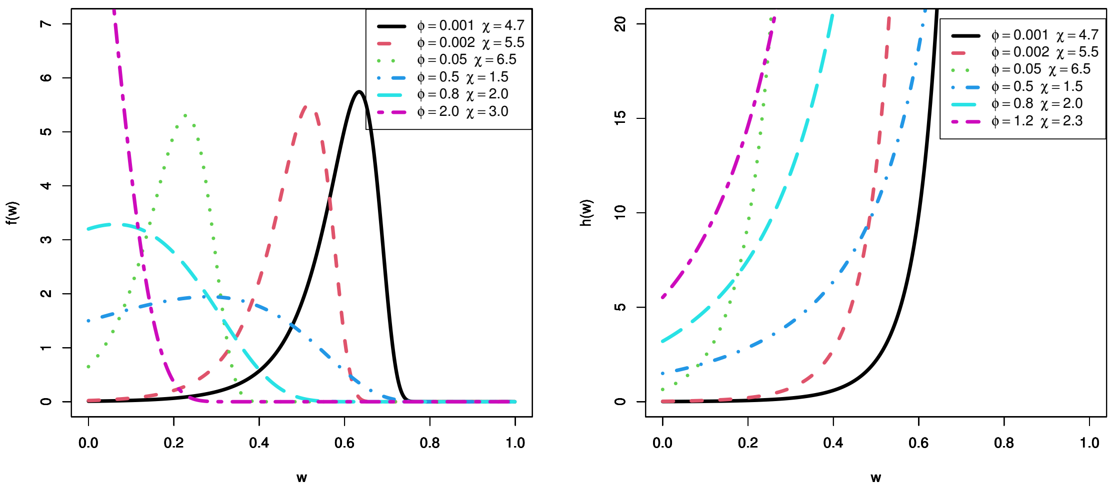

Figure 1. Plots of the PDF and HRF for the UED

Now to obtain the MLEs of the parameters χ and ϕ, assuming that the $W_{d_1d_2}$, where $d_1 = 1, 2, \ldots, s$ and $d_2 = 1, 2, \ldots, c$ be a RSS taken from the UED with sample size $n = s \times c$, where s is the set size and c is the cycle count. According to Arnold et al. [45], the PDF of $W_{d_1d_2}$, for perfect ranking, is given by:

$$g(w_{d_1d_2}) = \frac{s!}{(d_1-1)(s-d_1)}\Big[G(w_{d_1d_2})\Big]^{d_1-1}\Big[1-G(w_{d_1d_2})\Big]^{s-d_1} g(w_{d_1d_2}). \tag{2.3}$$

Substituting Equations (2.1) and (2.2) in Equation (2.3) the likelihood function (LF) of the UED is given by:

$$L(\chi,\phi) = \prod_{d_2=1}^{c}\prod_{d_1=1}^{s} \frac{2\phi\chi(s!)}{(d_1-1)(s-d_1)}\Big[1-e^{\phi\left(1-\Delta(w_{d_1d_2},\chi)\right)}\Big]^{d_1-1} e^{(s-d_1+1)\phi\left(1-\Delta(w_{d_1d_2},\chi)\right)}\frac{\Delta(w_{d_1d_2},\chi)}{1-w_{d_1d_2}^2}, \tag{2.4}$$

where, $\Delta(w_{d_1d_2},\chi) = \left[\frac{w_{d_1d_2}+1}{1-w_{d_1d_2}}\right]^{\chi}$.

The log-likelihood function of Equation (2.4) is expressed as follows:

$$\begin{aligned} \log l \propto\; & n\log(\phi\chi) + \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}(d_1-1)\log\Big[1-e^{\phi\left(1-\Delta(w_{d_1d_2},\chi)\right)}\Big] + \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}\log\Big[\Delta(w_{d_1d_2},\chi)\Big] \\ & - \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}\log(1-w_{d_1d_2}^2) + \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}(s-d_1+1)\,\phi\Big(1-\Delta(w_{d_1d_2},\chi)\Big). \end{aligned} \tag{2.5}$$

Equation (2.5) is maximized to determine the MLEs $\hat{\phi}_1$ of ϕ and $\hat{\chi}_1$ of χ for the UED. This optimization issue has a numerical solution that can be achieved by solving the following nonlinear equations

$$\frac{\partial \log l}{\partial \phi} = \frac{n}{\phi} - \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}\frac{(d_1-1)\Big(1-\Delta(w_{d_1d_2},\chi)\Big)}{\Big[e^{-\phi\left(1-\Delta(w_{d_1d_2},\chi)\right)}-1\Big]} + \sum_{d_2=1}^{c}\sum_{d_1=1}^{s}(s-d_1+1)\,\Big(1-\Delta(w_{d_1d_2},\chi)\Big), \tag{2.6}$$

and

$$\frac{\partial \log l}{\partial \chi}=\frac{n}{\chi}+\sum_{d_2=1}^{c}\sum_{d_1=1}^{s}\frac{(d_1-1)\phi\Delta'_{\chi}(w_{d_1d_2},\chi)}{e^{-\phi\left(1-\Delta(w_{d_1d_2},\chi)\right)}-1}+\sum_{d_2=1}^{c}\sum_{d_1=1}^{s}\frac{\Delta'_{\chi}(w_{d_1d_2},\chi)}{\Delta(w_{d_1d_2},\chi)}-\sum_{d_2=1}^{c}\sum_{d_1=1}^{s}(s-d_1+1)\,\phi\Delta'_{\chi}(w_{d_1d_2},\chi), \tag{2.7}$$

where, $\Delta'_{\chi}(w_{d_1d_2},\chi)=\left[\frac{w_{d_1d_2}+1}{1-w_{d_1d_2}}\right]^{\chi}\log\left[\frac{w_{d_1d_2}+1}{1-w_{d_1d_2}}\right]$.

Equations (2.6) and (2.7) can be solved numerically to determine the MLEs $\hat{\phi}_1$ of ϕ and $\hat{\chi}_1$ of χ for the UED using the Newton-Raphson method or other suitable numerical approaches.

3. Minimum Distance Estimators

This section presents the AD and CVM techniques for estimating UED parameters based on the RSS framework. MacDonald [46] empirically demonstrated that CVM-type minimum distance estimators are less biased than other minimum distance estimators. This finding provides strong evidence of the effectiveness of CVM-type estimators in real-world applications.

Suppose that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ represent an ordered sample of size $n=s\times c$, drawn from the UED, where s is the set size and c is the cycle count. To estimate the parameters ϕ and χ of the UED, we minimize the following function concerning ϕ and χ

$$C^{\bullet}=\frac{1}{12n}+\sum_{l=1}^{n}\left\{G\left(w_{(l:n)}\left|\phi,\chi\right.\right)-\frac{2l-1}{2n}\right\}^2. \tag{3.1}$$

Rather than minimizing Equation (3.1), the CVM estimate (CVME) $\hat{\phi}_2$ of ϕ and $\hat{\chi}_2$ of χ one could obtain by solving the subsequent nonlinear equations:

$$\frac{\partial C^{\bullet}}{\partial \phi}=\sum_{l=1}^{n}\left\{G\left(w_{(l:n)}\left|\phi,\chi\right.\right)-\frac{2l-1}{2n}\right\}M_{\phi}(\,w_{(l:n)}\left|\phi,\chi\right.)\;=0, \tag{3.2}$$

and

$$\frac{\partial C^{\bullet}}{\partial \chi}=\sum_{l=1}^{n}\left\{G\left(w_{(l:n)}\left|\phi,\chi\right.\right)-\frac{2l-1}{2n}\right\}M_{\chi}(\,w_{(l:n)}\left|\phi,\chi\right.)\;=0, \tag{3.3}$$

where,

$$M_{\phi}(\,w_{(l:n)}\left|\phi,\chi\right.)=\frac{\partial G(\,w_{(l:n)}\left|\phi,\chi\right.)}{\partial\phi}=-\left(1-\left[\frac{w_{(l:n)}+1}{1-w_{(l:n)}}\right]^{\chi}\right)e^{\phi\left(1-\left[\frac{w_{(l:n)}+1}{1-w_{(l:n)}}\right]^{\chi}\right)}, \tag{3.4}$$

and

$$M_{\chi}(\,w_{(l:n)}\left|\phi,\chi\right.)=\frac{\partial G(\,w_{(l:n)}\left|\phi,\chi\right.)}{\partial\chi}=e^{\phi\left(1-\left[\frac{w_{(l:n)}+1}{1-w_{(l:n)}}\right]^{\chi}\right)}\left[\frac{w_{(l:n)}+1}{1-w_{(l:n)}}\right]^{\chi}\log\left[\frac{w_{(l:n)}+1}{1-w_{(l:n)}}\right]. \tag{3.5}$$

By solving numerically Equations (3.2) and (3.3) using Newton-Raphson method or other suitable numerical approaches, the CVMEs $\hat{\phi}_2$ and $\hat{\chi}_2$ are generated.

After that the AD estimate (ADE) of the parameters ϕ and χ are determined for the UED under RSS by assuming that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ be an ordered sample of size $n=s\times c$, where c is the cycle count and s is the set size. To obtain the ADE $\hat{\phi}_3$ of ϕ and $\hat{\chi}_3$ of χ minimizing the following function with respect to ϕ and χ

$$\ddot{A} = -n - \frac{1}{n}\sum_{l=1}^{n}(2l-1)\left\{\log G\left(w_{(l:n)}\left|\phi,\chi\right.\right) + \log\left[1 - G\left(w_{(1-l+n:n)}\left|\phi,\chi\right.\right)\right]\right\}, \tag{3.6}$$

Alternative to minimize Equation (3.6), one can obtain the ADEs $\hat{\phi}_3$ and $\hat{\chi}_3$ by solving, numerically, following nonlinear equations:

$$\frac{\partial \ddot{A}}{\partial \phi} = \sum_{l=1}^{n}(2l-1)\left\{\frac{M_{\phi}(w_{(l:n)}\left|\phi,\chi\right.)}{G\left(w_{(l:n)}\left|\phi,\chi\right.\right)} - \frac{M_{\phi}(w_{(1-l+n:n)}\left|\phi,\chi\right.)}{1 - G\left(w_{(1-l+n:n)}\left|\phi,\chi\right.\right)}\right\} = 0, \tag{3.7}$$

and

$$\frac{\partial \ddot{A}}{\partial \chi} = \sum_{l=1}^{n}(2l-1)\left\{\frac{M_{\chi}(w_{(l:n)}\left|\phi,\chi\right.)}{G\left(w_{(l:n)}\left|\phi,\chi\right.\right)} - \frac{M_{\chi}(w_{(1-l+n:n)}\left|\phi,\chi\right.)}{1 - G\left(w_{(l-l+n:n)}\left|\phi,\chi\right.\right)}\right\} = 0, \tag{3.8}$$

where, $M_{\phi}(\,.\left|\phi,\chi\right.)$ and $M_{\chi}(\,.\left|\phi,\chi\right.)$ are provided in Equations (3.4) and (3.5). Equations (3.7) and (3.8) can be solved numerically, by using an appropriate approximation.

4. Maximum Product Spacing Distance Estimator

The MPS method provides a powerful approach for estimating parameters of probability distributions, particularly when dealing with censored or truncated data. Its robustness, efficiency, and applicability in various fields make it a valuable tool in statistical analysis (see Cheng and Amin [47]). In this section, the MPS estimators (MPSEs) of the parameters ϕ and χ of the UED are determined based on the optimization criterion.

Let $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ represent an ordered sample of size $n = s \times c$, drawn from the UED, where s is the set size and c is the cycle count. For a random sample with uniform spacings from the UED are given by

$$\gamma_l(\phi,\chi) = G\left(w_{(l:n)}\left|\phi,\chi\right.\right) - G\left(w_{(l-1:n)}\left|\phi,\chi\right.\right), \quad l = 1, 2, ..., n+1.$$

Note that $G\left(w_{(0:n)}\left|\phi,\chi\right.\right) = 0, \quad G\left(w_{(n+1:n)}\left|\phi,\chi\right.\right) = 1$, and $\sum_{l=1}^{n+1}\gamma_l = 1$.

To ger the MPSEs of the parameters ϕ and χ of the UED, so, we maximize the following function with respect to ϕ and χ to give $\hat{\phi}_4$ and $\hat{\chi}_4$

$$\zeta(\phi,\chi) = \frac{1}{n+1}\sum_{l=1}^{n+1}\log\left[\gamma_l(\phi,\chi)\right]. \tag{4.1}$$

Or the MPSEs $\hat{\phi}_4$ of ϕ and MPSE $\hat{\chi}_4$ of χ are produced by solving the nonlinear equations

$$\frac{\partial}{\partial \phi}\zeta(\phi,\chi) = \frac{1}{n+1}\sum_{l=1}^{n+1}\frac{M_{\phi}(w_{(l:n)}\left|\phi,\chi\right.) - M_{\phi}(w_{(l-1:n)}\left|\phi,\chi\right.)}{\left[\gamma_l(\phi,\chi)\right]} = 0,$$

and

$$\frac{\partial}{\partial \chi}\zeta(\phi,\chi) = \frac{1}{n+1}\sum_{l=1}^{n+1}\frac{M_{\chi}(w_{(l:n)}\left|\phi,\chi\right.) - M_{\chi}(w_{(l-1:n)}\left|\phi,\chi\right.)}{\left[\gamma_l(\phi,\chi)\right]} = 0,$$

where, $M_{\phi}(\,.\,|\phi,\chi)$ and $M_{\chi}(\,.\,|\phi,\chi)$ are provided in Equations (3.4) and (3.5).

5. Minimum Product Spacing Distance Estimator

This section gives some MS estimators, this foundational method was prepared by Torabi [48]. Several estimators of the parameters ϕ and χ are generated based on minimization criteria using RSS design. The aforementioned estimators are the MSAD estimates (MSADEs; $\hat{\phi}_5$ of ϕ and $\hat{\chi}_5$ of χ), the MSALD estimates (MSALDEs; $\hat{\phi}_6$ of ϕ and $\hat{\chi}_6$ of χ), MSLND estimates (MSLNDEs; $\hat{\phi}_7$ of ϕ and $\hat{\chi}_7$ of χ), the MSSLD estimates (MSSLDEs; $\hat{\phi}_8$ of ϕ and $\hat{\chi}_8$ of χ), and the MSSD estimates (MSSDEs; $\hat{\phi}_9$ of ϕ and $\hat{\chi}_9$ of χ).

Firstly, MSADEs $\hat{\phi}_5$ and $\hat{\chi}_5$ for the UED are obtained, so suppose that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ represent an ordered sample of size $n = s \times c$, drawn from the UED, where s is the set size and c is the cycle count. Minimizing the following $\vartheta_1^*(\phi,\chi)$, with respect ϕ and χ

$$\vartheta_1^*(\phi,\chi) = \sum_{l=1}^{n+1}\left|\gamma_l(\phi,\chi) - \frac{1}{n+1}\right|. \tag{5.1}$$

The non-linear equations listed below can be used instead of Equation (5.1) to find $\hat{\phi}_5$ and $\hat{\chi}_5$

$$\frac{\partial\vartheta_1^*(\phi,\chi)}{\partial\phi} = \sum_{l=1}^{n+1}\frac{\left|\gamma_l(\phi,\chi) - \frac{1}{n+1}\right|}{\gamma_l(\phi,\chi) - \frac{1}{n+1}}\left[M_{\phi}(\,w_{(l:n)}\,|\phi,\chi) - M_{\phi}(\,w_{(l-1:n)}\,|\phi,\chi)\right],$$

and

$$\frac{\partial\vartheta_1^*(\phi,\chi)}{\partial\chi} = \sum_{l=1}^{n+1}\frac{\left|\gamma_l(\phi,\chi) - \frac{1}{n+1}\right|}{\gamma_l(\phi,\chi) - \frac{1}{n+1}}\left[M_{\chi}(\,w_{(l:n)}\,|\phi,\chi) - M_{\chi}(\,w_{(l-1:n)}\,|\phi,\chi)\right],$$

where, $M_{\phi}(\,.\,|\phi,\chi)$ and $M_{\chi}(\,.\,|\phi,\chi)$ are provided in Equations (3.4) and (3.5).

Secondly, given that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ be an ordered sample of size $n = s \times c$, drawn from the UED, where s is the set size and c is the cycle count. The MSALDEs; $\hat{\phi}_6$ and $\hat{\chi}_6$ for the UED are produced by minimizing the following $\vartheta_2^*(\phi,\chi)$, with respect ϕ and χ

$$\vartheta_2^*(\phi,\chi) = \sum_{l=1}^{n+1}\left|\log(\gamma_l(\phi,\chi)) - \log\frac{1}{n+1}\right|. \tag{5.2}$$

The non-linear equations listed below can be used instead of Equation (5.2) to find $\hat{\phi}_6$ and $\hat{\chi}_6$

$$\frac{\partial\vartheta_2^*(\phi,\chi)}{\partial\phi} = \sum_{l=1}^{n+1}\frac{\left|\log\gamma_l(\phi,\chi) - \log\frac{1}{n+1}\right|}{\log\gamma_l(\phi,\chi) - \frac{1}{n+1}}\frac{1}{\gamma_l(\phi,\chi)}\left[M_{\phi}(\,w_{(l:n)}\,|\phi,\chi) - M_{\phi}(\,w_{(l-1:n)}\,|\phi,\chi)\right],$$

and

$$\frac{\partial\vartheta_2^*(\phi,\chi)}{\partial\chi} = \sum_{l=1}^{n+1}\frac{\left|\log\gamma_l(\phi,\chi) - \log\frac{1}{n+1}\right|}{\log\gamma_l(\phi,\chi) - \frac{1}{n+1}}\frac{1}{\gamma_l(\phi,\chi)}\left[M_{\chi}(\,w_{(l:n)}\,|\phi,\chi) - M_{\chi}(\,w_{(l-1:n)}\,|\phi,\chi)\right],$$

where, $M_{\phi}(\,.\,|\phi,\chi)$ and $M_{\chi}(\,.\,|\phi,\chi)$ are provided in Equations (3.4) and (3.5).

Thirdly, given that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ be an ordered sample of size $n = s \times c$, drawn from the

UED, where s is the set size and c is the cycle count. The MSLNDEs $\hat{\phi}_7$ of ϕ and $\hat{\chi}_7$ of χ for the UED are created by minimizing the following $\vartheta_3^*(\phi,\chi)$, with respect ϕ and χ

$$\vartheta_3^*(\phi,\chi)=\sum_{l=1}^{n+1}\left|e^{\gamma_l(\phi,\chi)-\frac{1}{n+1}}\left(\gamma_l(\phi,\chi)-\frac{1}{n+1}\right)-1\right|. \tag{5.3}$$

The non-linear equations listed below can be used instead of Equation (5.3) to find $\hat{\phi}_7$ and $\hat{\chi}_7$

$$\frac{\partial\vartheta_3^*(\phi,\chi)}{\partial\phi}=\sum_{j_1=1}^{n+1}\left(e^{\gamma_l(\phi,\chi)-\frac{1}{n+1}}-1\right)\left[M_\phi(\,w_{(l:n)}\left|\phi,\chi\right.)-M_\phi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right]=0,$$

and

$$\frac{\partial\vartheta_3^*(\phi,\chi)}{\partial\chi}=\sum_{j_1=1}^{n+1}\left(e^{\gamma_l(\phi,\chi)-\frac{1}{n+1}}-1\right)\left[M_\chi(\,w_{(l:n)}\left|\phi,\chi\right.)-M_\chi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right]=0,$$

where, $M_\phi(\,.\left|\phi,\chi\right.)$ and $M_\chi(\,.\left|\phi,\chi\right.)$ are provided in Equations (3.4) and (3.5).
Fourthly, let $W_{(1:n)},W_{(2:n)},...,W_{(n:n)}$ be an ordered sample of size $n=s\times c$, drawn from the UED, where s is the set size and c is the cycle count. Minimizing the following $\vartheta_4^*(\phi,\chi)$, with respect ϕ and χ, the MSSLDEs $\hat{\phi}_8$ and $\hat{\chi}_8$ for the UED are generated

$$\vartheta_4^*(\phi,\chi)=\sum_{l=1}^{n+1}\left(\log\gamma_l(\phi,\chi)-\log\frac{1}{n+1}\right)^2. \tag{5.4}$$

The following non-linear equations can be employed rather than Equation (5.4) to produce $\hat{\phi}_8$ and $\hat{\chi}_8$

$$\frac{\partial\vartheta_4^*(\phi,\chi)}{\partial\phi}=\sum_{l=1}^{n+1}\left(\log\gamma_l(\phi,\chi)-\log\frac{1}{n+1}\right)\frac{1}{\gamma_l(\phi,\chi)}\left[M_\phi(\,w_{(l:n)}\left|\phi,\chi\right.)-M_\phi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right]=0,$$

and

$$\frac{\partial\vartheta_4^*(\phi,\chi)}{\partial\chi}=\sum_{l=1}^{n+1}\left(\log\gamma_l(\phi,\chi)-\log\frac{1}{n+1}\right)\frac{1}{\gamma_l(\phi,\chi)}\left[M_\chi(\,w_{(l:n)}\left|\phi,\chi\right.)-M_\chi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right]=0,$$

where, $M_\phi(\,.\left|\phi,\chi\right.)$ and $M_\chi(\,.\left|\phi,\chi\right.)$ are provided in Equations (3.4) and (3.5).
Finally, let $W_{(1:n)},W_{(2:n)},...,W_{(n:n)}$ be an ordered sample of size $n=s\times c$, drawn from the UED, where s is the set size and c is the cycle count. Minimizing the following $\vartheta_5^*(\phi,\chi)$, with respect ϕ and χ, the MSSLDEs $\hat{\phi}_9$ and $\hat{\chi}_9$ for the UED are generated

$$\vartheta_5^*(\phi,\chi)=\sum_{l=1}^{n+1}\left(\gamma_l(\phi,\chi)-\frac{1}{n+1}\right)^2. \tag{5.5}$$

An alternative to (5.5), solving the following non-linear equations can be generated to provide $\hat{\phi}_9$ and $\hat{\chi}_9$

$$\frac{\partial\vartheta_5^*(\phi,\chi)}{\partial\phi}=\sum_{l=1}^{n+1}\left(\gamma_l(\phi,\chi)-\frac{1}{n+1}\right)\left[M_\phi(\,w_{(l:n)}\left|\phi,\chi\right.)-M_\phi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right]=0,$$

and

$$\frac{\partial \vartheta_5^*(\phi,\chi)}{\partial \chi} = \sum_{l=1}^{n+1}\left(\gamma_l(\phi,\chi) - \frac{1}{n+1}\right)\left[M_\chi(\,w_{(l:n)}\left|\phi,\chi\right.) - M_\chi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right] = 0,$$

where, $M_\phi(\,.\left|\phi,\chi\right.)$ and $M_\chi(\,.\left|\phi,\chi\right.)$ are provided in Equations (3.4) and (3.5).

6. Ordinary Least Squares Estimators

The least squares method is a widely used statistical technique employed to fit a line or curve to a set of data points. Its primary goal is to minimize the sum of the squared differences between the observed data points and the values predicted by the fitted model (Swain et al. [49]). This minimization process ensures that the resulting line or curve is as close as possible to the data points.

Given that $W_{(1:n)}, W_{(2:n)}, ..., W_{(n:n)}$ be an ordered sample of size $n = s \times c$, drawn from the UED, where s is the set size and c is the cycle count. Minimizing the function $S^\bullet(\phi,\chi)$, with respect to ϕ and χ, the OLS estimates (OLSEs) $\hat{\phi}_{10}$ and $\hat{\chi}_{10}$ for the UED are generated

$$S^\bullet(\phi,\chi) = \sum_{l=1}^{n}\left(G\left(w_{(l:n)}\left|\phi,\chi\right.\right) - \frac{l}{n+1}\right)^2. \tag{6.1}$$

Equivalently to (6.1), OLSEs of ϕ and χ, can be obtained by solving the following non-linear equations:

$$\frac{\partial S^\bullet(\phi,\chi)}{\partial \phi} = \sum_{l=1}^{n}\left(G\left(w_{(l:n)}\left|\phi,\chi\right.\right) - \frac{l}{n+1}\right)\left[M_\phi(\,w_{(l:n)}\left|\phi,\chi\right.) - M_\phi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right] = 0,$$

and

$$\frac{\partial S^\bullet(\phi,\chi)}{\partial \chi} = \sum_{l=1}^{n}\left(G\left(w_{(l:n)}\left|\phi,\chi\right.\right) - \frac{l}{n+1}\right)\left[M_\chi(\,w_{(l:n)}\left|\phi,\chi\right.) - M_\chi(\,w_{(l-1:n)}\left|\phi,\chi\right.)\right] = 0,$$

where, $M_\phi(\,.\left|\phi,\chi\right.)$ and $M_\chi(\,.\left|\phi,\chi\right.)$ are provided in Equations (3.4) and (3.5).

7. Numerical Simulation

This section evaluates the effectiveness of several estimating approaches for the UED distribution. We will construct and rank randomly generated synthetic datasets and then use several ways to choose the most appropriate one. Additionally, the datasets will be rated, and estimate techniques will be used to determine the best option. The simulation is done by using R software and it will be carried out utilizing the following steps:

- Create SRSs of varying sizes with the proposed model $n = 20, 60, 100, 150, 200,$ and 300.
- The suggested distribution is used to generate an RSS with a fixed set size ($d = 5$) and varying cycle numbers (c = 4, 15, 25, 30, 50, and 60). The findings are: $n = 20, 60, 100, 150, 200,$ and 300.

- Obtain the recommended distribution estimators ($\hat{\Phi}$, $\hat{\chi}$) for each of the two generating processes. Six distinct indicators are used to assess estimation procedures. These are characterized as follows:
  (1) The absolute bias average (BIAS), calculated using the following formula: $|Bias(\widehat{\boldsymbol{\Omega}})| = \frac{1}{M}\sum_{i=1}^{M}|\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega}|$, M=1000 is the number of iterations.
  (2) The following formula yielded the mean squared error (MSE): $MSE = \frac{1}{M}\sum_{i=1}^{M}(\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega})^2$.
  (3) The following formula is used to get the mean absolute relative error (MRE):: $MRE = \frac{1}{M}\sum_{i=1}^{M}|\widehat{\boldsymbol{\Omega}} - \boldsymbol{\Omega}|/\boldsymbol{\Omega}$.
  (4) The average absolute difference, D_{abs}, is computed as follows: $D_{abs} = \frac{1}{nM}\sum_{i=1}^{M}\sum_{j=1}^{n}|G(w_{ij};\boldsymbol{\Omega}) - G(w_{ij};\widehat{\boldsymbol{\Omega}})|$, where $G(w;\boldsymbol{\Omega}) = G(w)$ and w_{ij} represents values obtained at the i-th iteration sample and j-th component of this sample.
  (5) The maximum absolute difference, D_{max}, is computed as follows: $D_{max} = \frac{1}{M}\sum_{i=1}^{M}\max_{j=1,\ldots,n}|G(w_{ij};\boldsymbol{\Omega}) - G(w_{ij};\widehat{\boldsymbol{\Omega}})|$.
  (6) The average squared absolute error (ASAE), determined by: $ASAE = \frac{1}{M}\sum_{i=1}^{M}\frac{|w_{(i)}-\hat{w}(i)|}{w(n)-w_{(1)}}$, where $w_{(i)}$ denotes the ascending ordered observations, $\boldsymbol{\Omega} = (\Phi,\ \chi)$.
- Tables 1 through 10 show the evaluation metrics' results. They include an overview of the findings and allow users to compare various estimating approaches.
- Figures 2 to 6 visually illustrate the numerical data from Tables 3 and 4.
- Table 13 shows the ratio of SRS to RSS MSE, allowing for comparison of sampling methodologies' performance.
- Tables 11 and 12 give a comprehensive performance analysis by displaying the partial and total ranks of estimations for SRS and RSS, respectively.

Following a detailed review of the simulation's ranks and outcomes, the following conclusions have been reached:

- Beginning model estimates show the simulation results fit with the theory that the estimators are consistent across both the SRS and RSS datasets, indicating that they will converge to true parameter values as sample sizes increase.
- The MLE and MPSE techniques appear to be fairly beneficial for assessing the estimated quality of RSS and SRS, respectively.
- Given RSS's higher efficiency than SRS, it is a more effective sampling method with lower MSE and other metrics.

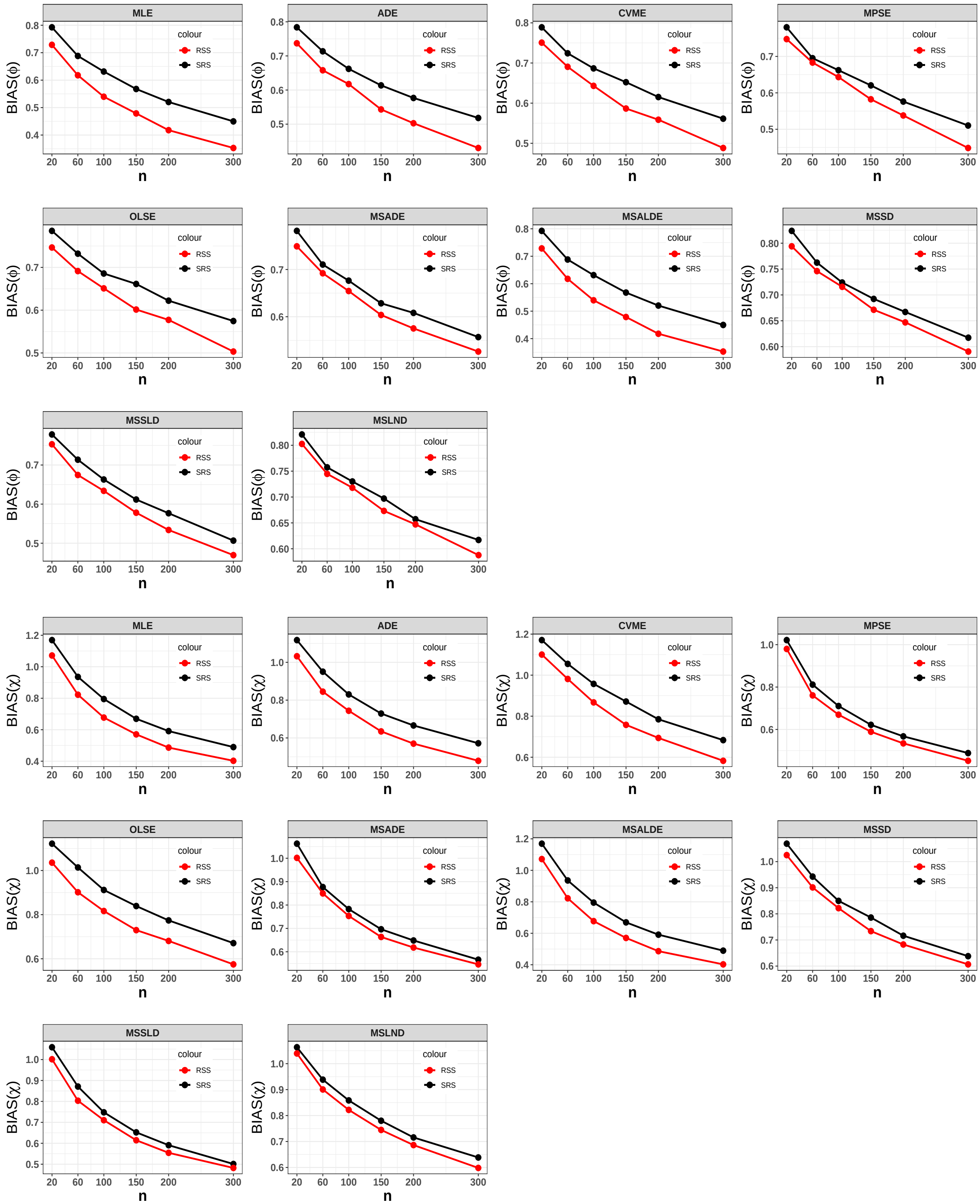

FIGURE 2. A Graphical representation for the numerical values of BIAS presented in Tables 3 and 4.

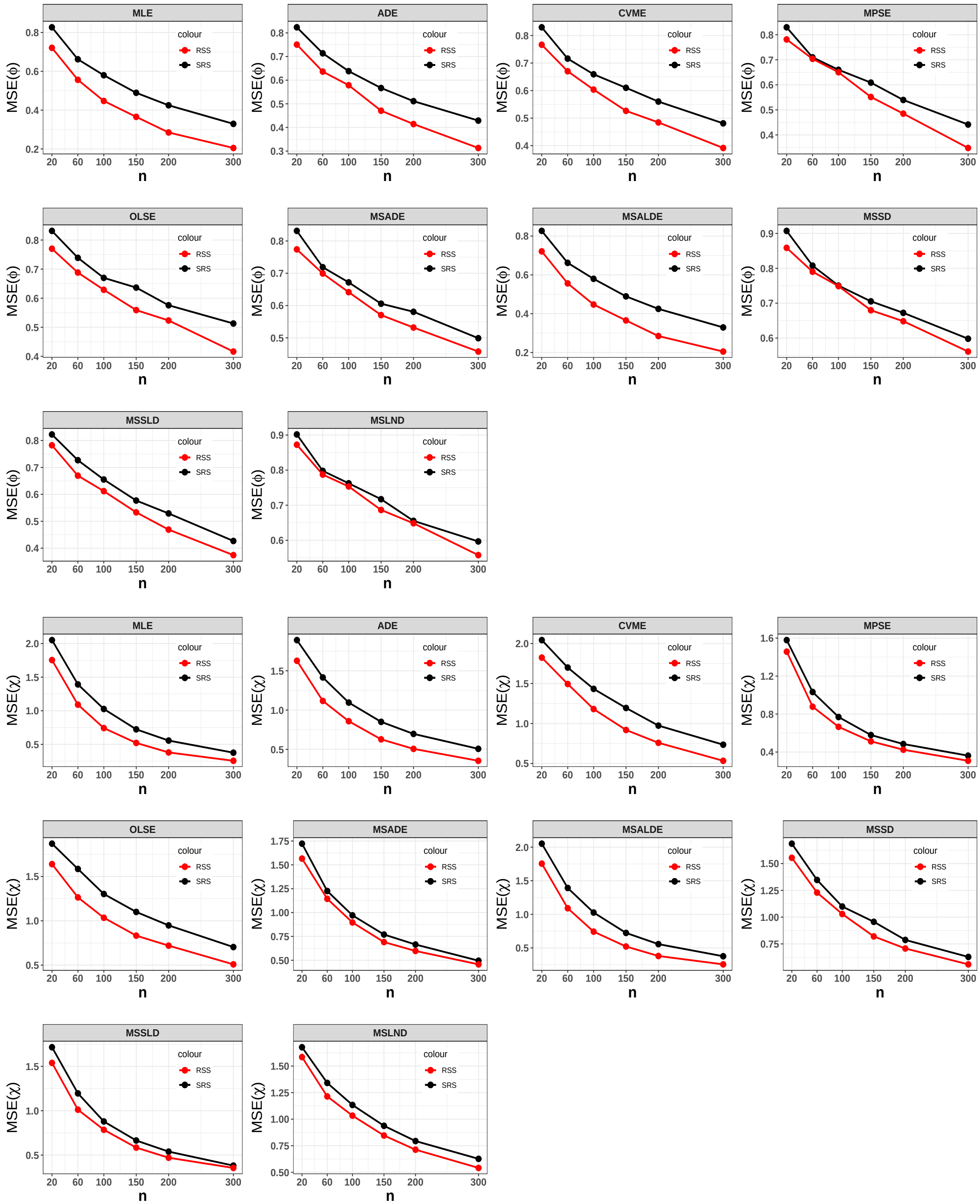

FIGURE 3. A Graphical representation for the numerical values of MSE presented in Tables 3 and 4.

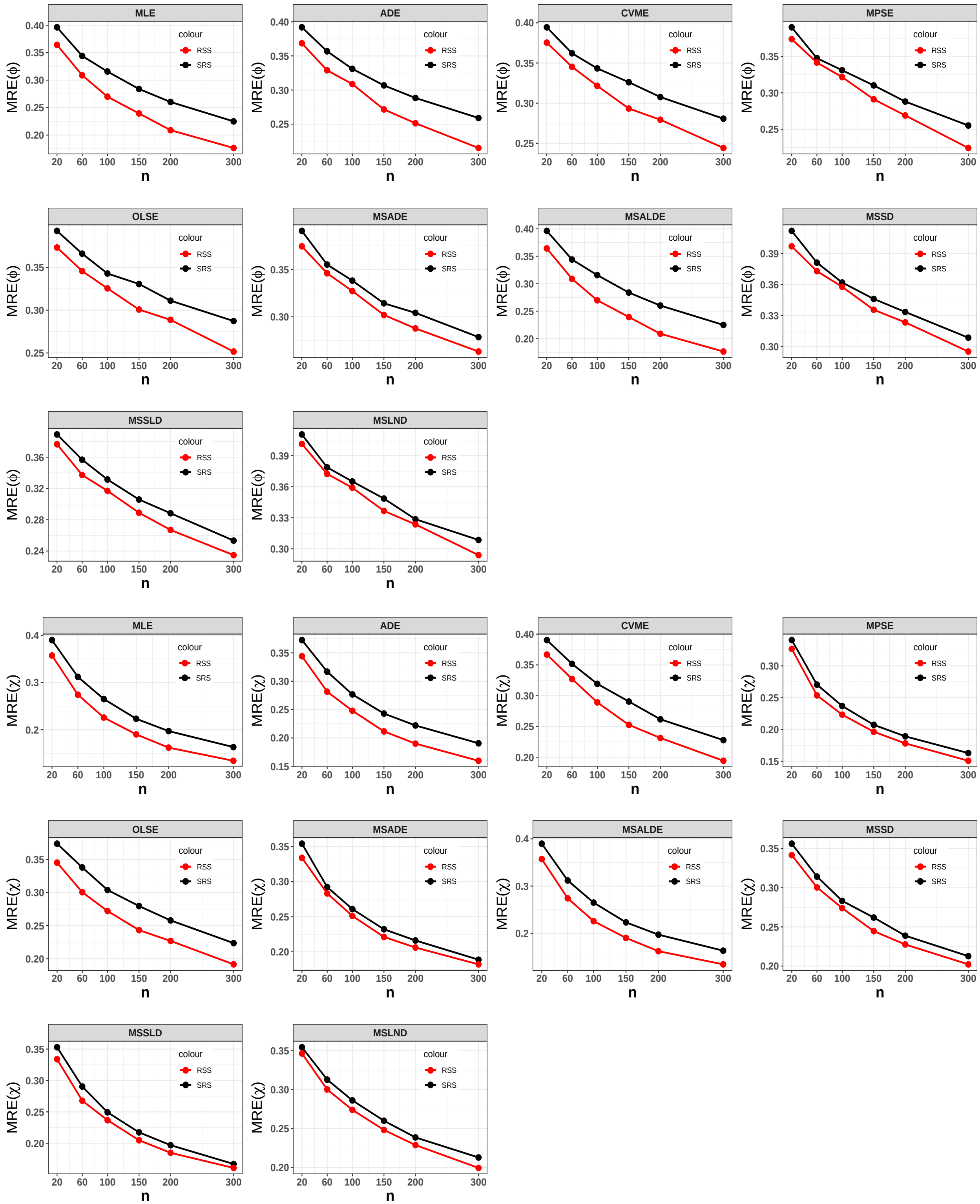

Figure 4. A Graphical representation for the numerical values of MRE presented in Tables 3 and 4.

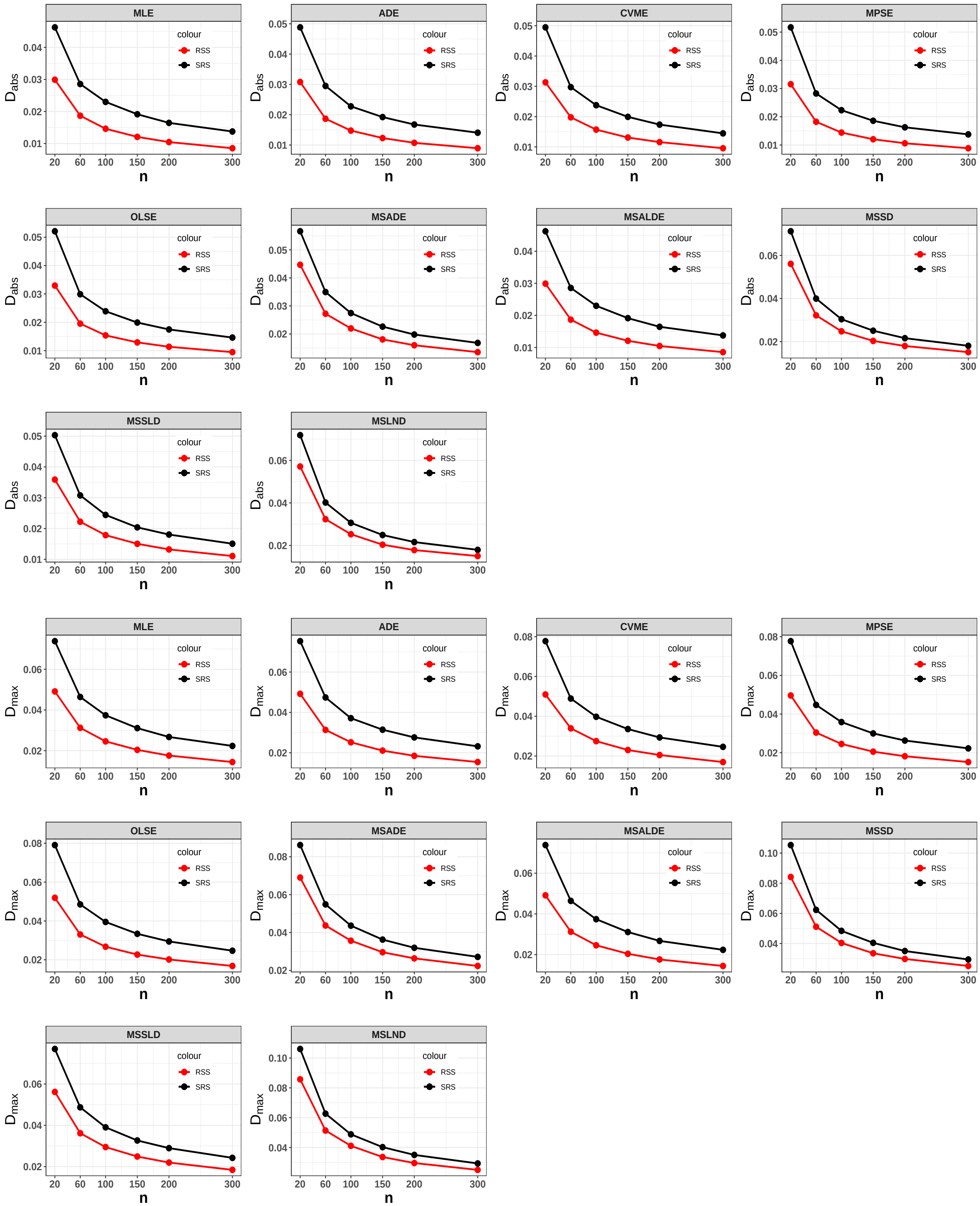

Figure 5. A Graphical representation for the numerical values of D_{abs} and D_{max} presented in Tables 3 and 4.

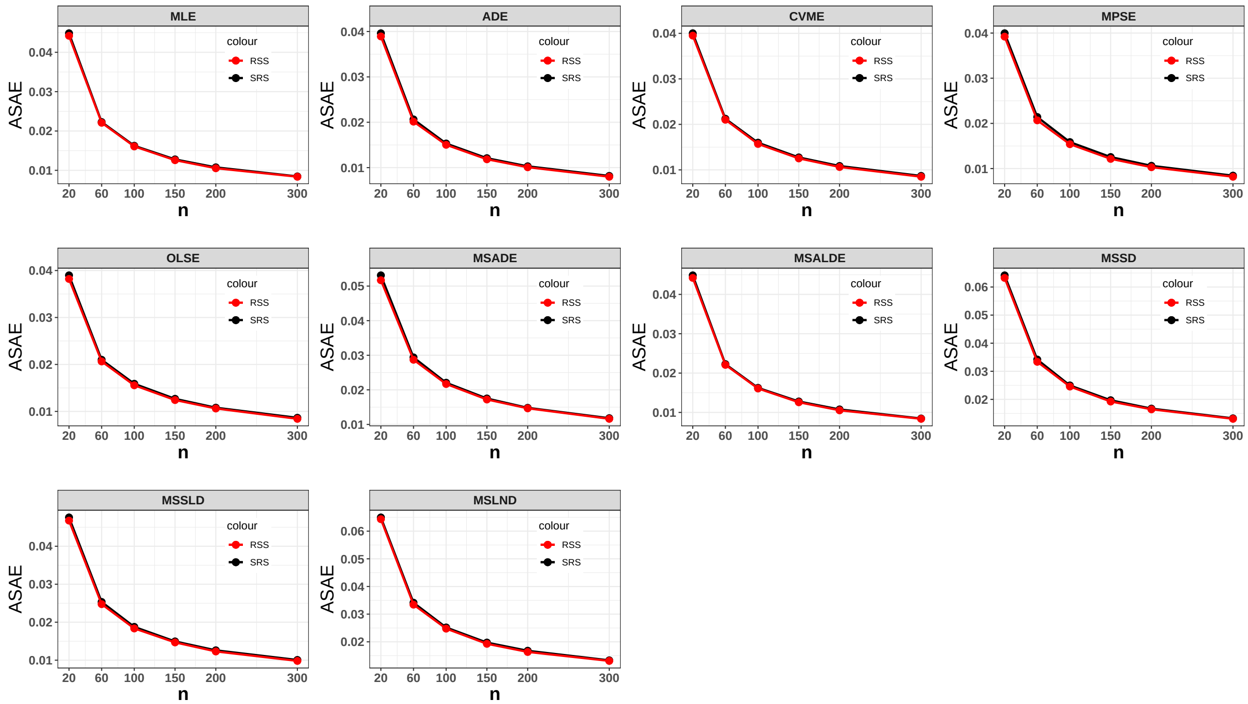

Figure 6. A Graphical representation for the numerical values of ASAE presented in Tables 3 and 4.

Table 1. Numerical values of simulation measures for $\Phi = 0.8$ and $\chi = 0.2$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.32732^{\{6\}}$	$0.32201^{\{3\}}$	$0.33205^{\{8\}}$	$0.31829^{\{2\}}$	$0.32837^{\{7\}}$	$0.31819^{\{1\}}$	$0.32372^{\{4\}}$	$0.33383^{\{9\}}$	$0.32602^{\{5\}}$	$0.33907^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.06695^{\{9\}}$	$0.06331^{\{7\}}$	$0.07088^{\{10\}}$	$0.05638^{\{1\}}$	$0.06558^{\{8\}}$	$0.05939^{\{2\}}$	$0.05999^{\{4\}}$	$0.06124^{\{5\}}$	$0.05979^{\{3\}}$	$0.06281^{\{6\}}$
	MSE($\hat{\Phi}$)	$0.1426^{\{4\}}$	$0.14004^{\{2\}}$	$0.14552^{\{8\}}$	$0.1397^{\{1\}}$	$0.14482^{\{7\}}$	$0.14027^{\{3\}}$	$0.14334^{\{5\}}$	$0.15153^{\{9\}}$	$0.14423^{\{6\}}$	$0.15468^{\{10\}}$
	MSE($\hat{\chi}$)	$0.00712^{\{9\}}$	$0.00644^{\{7\}}$	$0.0078^{\{10\}}$	$0.00509^{\{1\}}$	$0.00675^{\{8\}}$	$0.00571^{\{2\}}$	$0.00575^{\{3\}}$	$0.00578^{\{4.5\}}$	$0.00578^{\{4.5\}}$	$0.00605^{\{6\}}$
	MRE($\hat{\Phi}$)	$0.40916^{\{6\}}$	$0.40251^{\{3\}}$	$0.41507^{\{8\}}$	$0.39786^{\{2\}}$	$0.41047^{\{7\}}$	$0.39774^{\{1\}}$	$0.40465^{\{4\}}$	$0.41728^{\{9\}}$	$0.40752^{\{5\}}$	$0.42384^{\{10\}}$
	MRE($\hat{\chi}$)	$0.33473^{\{9\}}$	$0.31654^{\{7\}}$	$0.35442^{\{10\}}$	$0.2819^{\{1\}}$	$0.3279^{\{8\}}$	$0.29694^{\{2\}}$	$0.29993^{\{4\}}$	$0.30621^{\{5\}}$	$0.29894^{\{3\}}$	$0.31405^{\{6\}}$
	D_{abs}	$0.04943^{\{1\}}$	$0.05057^{\{2\}}$	$0.05186^{\{4\}}$	$0.05092^{\{3\}}$	$0.05379^{\{6\}}$	$0.05916^{\{8\}}$	$0.05576^{\{7\}}$	$0.06942^{\{10\}}$	$0.05218^{\{5\}}$	$0.06861^{\{9\}}$
	D_{max}	$0.08119^{\{3\}}$	$0.08092^{\{2\}}$	$0.08533^{\{5\}}$	$0.07964^{\{1\}}$	$0.0855^{\{6\}}$	$0.09271^{\{8\}}$	$0.0872^{\{7\}}$	$0.10693^{\{10\}}$	$0.08241^{\{4\}}$	$0.10595^{\{9\}}$
	ASAE	$0.05143^{\{3\}}$	$0.04407^{\{2\}}$	$0.04412^{\{5\}}$	$0.04864^{\{1\}}$	$0.04357^{\{6\}}$	$0.06063^{\{8\}}$	$0.05562^{\{7\}}$	$0.0627^{\{10\}}$	$0.05425^{\{4\}}$	$0.06335^{\{9\}}$
	$\sum Ranks$	$52^{\{6\}}$	$35^{\{2.5\}}$	$66^{\{8\}}$	$16^{\{1\}}$	$58^{\{7\}}$	$35^{\{2.5\}}$	$45^{\{5\}}$	$70.5^{\{9\}}$	$41.5^{\{4\}}$	$76^{\{10\}}$
60	BIAS($\hat{\Phi}$)	$0.25824^{\{1\}}$	$0.26984^{\{2\}}$	$0.28202^{\{7\}}$	$0.27192^{\{3\}}$	$0.2846^{\{8\}}$	$0.27379^{\{5\}}$	$0.28015^{\{6\}}$	$0.2988^{\{10\}}$	$0.27273^{\{4\}}$	$0.2949^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.0436^{\{5\}}$	$0.04564^{\{6\}}$	$0.05383^{\{10\}}$	$0.03892^{\{1\}}$	$0.05051^{\{9\}}$	$0.04335^{\{4\}}$	$0.04221^{\{3\}}$	$0.04935^{\{8\}}$	$0.04141^{\{2\}}$	$0.04861^{\{7\}}$
	MSE($\hat{\Phi}$)	$0.09706^{\{1\}}$	$0.10474^{\{2\}}$	$0.1121^{\{6\}}$	$0.11124^{\{5\}}$	$0.11615^{\{8\}}$	$0.11106^{\{4\}}$	$0.11575^{\{7\}}$	$0.12739^{\{10\}}$	$0.11015^{\{3\}}$	$0.12506^{\{9\}}$
	MSE($\hat{\chi}$)	$0.00318^{\{5\}}$	$0.00341^{\{6\}}$	$0.00475^{\{10\}}$	$0.00234^{\{1\}}$	$0.00412^{\{9\}}$	$0.0031^{\{4\}}$	$0.00286^{\{3\}}$	$0.00387^{\{8\}}$	$0.00276^{\{2\}}$	$0.00376^{\{7\}}$
	MRE($\hat{\Phi}$)	$0.3228^{\{1\}}$	$0.3373^{\{2\}}$	$0.35253^{\{7\}}$	$0.3399^{\{3\}}$	$0.35575^{\{8\}}$	$0.34223^{\{5\}}$	$0.35019^{\{6\}}$	$0.3735^{\{10\}}$	$0.34091^{\{4\}}$	$0.36862^{\{9\}}$
	MRE($\hat{\chi}$)	$0.218^{\{5\}}$	$0.2282^{\{6\}}$	$0.26917^{\{10\}}$	$0.19462^{\{1\}}$	$0.25254^{\{9\}}$	$0.21674^{\{4\}}$	$0.21107^{\{3\}}$	$0.24673^{\{8\}}$	$0.20707^{\{2\}}$	$0.24304^{\{7\}}$
	D_{abs}	$0.03019^{\{2\}}$	$0.03064^{\{3\}}$	$0.03165^{\{5\}}$	$0.02933^{\{1\}}$	$0.03142^{\{4\}}$	$0.03603^{\{8\}}$	$0.03329^{\{7\}}$	$0.03929^{\{10\}}$	$0.03207^{\{6\}}$	$0.03873^{\{9\}}$
	D_{max}	$0.04959^{\{2\}}$	$0.05037^{\{3\}}$	$0.05376^{\{7\}}$	$0.04737^{\{1\}}$	$0.05271^{\{5\}}$	$0.05798^{\{8\}}$	$0.05353^{\{6\}}$	$0.06388^{\{10\}}$	$0.05169^{\{4\}}$	$0.06299^{\{9\}}$
	ASAE	$0.02636^{\{2\}}$	$0.02307^{\{3\}}$	$0.02291^{\{7\}}$	$0.02603^{\{1\}}$	$0.02296^{\{5\}}$	$0.03293^{\{8\}}$	$0.03073^{\{6\}}$	$0.03398^{\{10\}}$	$0.02942^{\{4\}}$	$0.0336^{\{9\}}$
	$\sum Ranks$	$27^{\{2\}}$	$33^{\{3.5\}}$	$63^{\{8\}}$	$20^{\{1\}}$	$62^{\{7\}}$	$50^{\{6\}}$	$48^{\{5\}}$	$84^{\{10\}}$	$33^{\{3.5\}}$	$75^{\{9\}}$
100	BIAS($\hat{\Phi}$)	$0.21952^{\{1\}}$	$0.23916^{\{3\}}$	$0.25384^{\{8\}}$	$0.23845^{\{2\}}$	$0.25168^{\{6\}}$	$0.24721^{\{5\}}$	$0.25311^{\{7\}}$	$0.2754^{\{9\}}$	$0.2426^{\{4\}}$	$0.27596^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.03413^{\{3\}}$	$0.03697^{\{6\}}$	$0.04511^{\{10\}}$	$0.03232^{\{1\}}$	$0.04159^{\{7\}}$	$0.03573^{\{5\}}$	$0.03561^{\{4\}}$	$0.04192^{\{8\}}$	$0.03362^{\{2\}}$	$0.04241^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.07352^{\{1\}}$	$0.08776^{\{2\}}$	$0.09523^{\{6\}}$	$0.09058^{\{3\}}$	$0.09525^{\{7\}}$	$0.09513^{\{5\}}$	$0.0995^{\{8\}}$	$0.11241^{\{9\}}$	$0.09125^{\{4\}}$	$0.11258^{\{10\}}$
	MSE($\hat{\chi}$)	$0.0019^{\{3\}}$	$0.00219^{\{6\}}$	$0.00332^{\{10\}}$	$0.00158^{\{1\}}$	$0.00276^{\{7\}}$	$0.00205^{\{5\}}$	$0.00196^{\{4\}}$	$0.00279^{\{8\}}$	$0.00176^{\{2\}}$	$0.00287^{\{9\}}$
	MRE($\hat{\Phi}$)	$0.2744^{\{1\}}$	$0.29895^{\{3\}}$	$0.3173^{\{8\}}$	$0.29806^{\{2\}}$	$0.3146^{\{6\}}$	$0.30901^{\{5\}}$	$0.31638^{\{7\}}$	$0.34426^{\{9\}}$	$0.30325^{\{4\}}$	$0.34495^{\{10\}}$
	MRE($\hat{\chi}$)	$0.17067^{\{3\}}$	$0.18486^{\{6\}}$	$0.22555^{\{10\}}$	$0.16162^{\{1\}}$	$0.20793^{\{7\}}$	$0.17866^{\{5\}}$	$0.17805^{\{4\}}$	$0.2096^{\{8\}}$	$0.16809^{\{2\}}$	$0.21204^{\{9\}}$
	D_{abs}	$0.02407^{\{2\}}$	$0.02423^{\{3\}}$	$0.02503^{\{5\}}$	$0.02311^{\{1\}}$	$0.02466^{\{4\}}$	$0.02834^{\{8\}}$	$0.02639^{\{7\}}$	$0.0306^{\{9\}}$	$0.02552^{\{6\}}$	$0.03085^{\{10\}}$
	D_{max}	$0.03935^{\{2\}}$	$0.03987^{\{3\}}$	$0.04271^{\{6\}}$	$0.03748^{\{1\}}$	$0.04157^{\{5\}}$	$0.04572^{\{8\}}$	$0.04277^{\{7\}}$	$0.05024^{\{9\}}$	$0.04115^{\{4\}}$	$0.05059^{\{10\}}$
	ASAE	$0.02011^{\{2\}}$	$0.01757^{\{3\}}$	$0.0175^{\{6\}}$	$0.02^{\{1\}}$	$0.01734^{\{5\}}$	$0.02513^{\{8\}}$	$0.02392^{\{7\}}$	$0.02613^{\{9\}}$	$0.02252^{\{4\}}$	$0.02623^{\{10\}}$
	$\sum Ranks$	$21^{\{2\}}$	$35^{\{4\}}$	$65^{\{8\}}$	$16^{\{1\}}$	$50^{\{5\}}$	$54^{\{6\}}$	$55^{\{7\}}$	$78^{\{9\}}$	$34^{\{3\}}$	$87^{\{10\}}$
150	BIAS($\hat{\Phi}$)	$0.18894^{\{1\}}$	$0.21107^{\{3\}}$	$0.22751^{\{5\}}$	$0.20951^{\{2\}}$	$0.22821^{\{6\}}$	$0.22951^{\{7\}}$	$0.23027^{\{8\}}$	$0.25625^{\{9\}}$	$0.21842^{\{4\}}$	$0.25651^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.02789^{\{2\}}$	$0.03142^{\{5\}}$	$0.0375^{\{10\}}$	$0.02757^{\{1\}}$	$0.03577^{\{7\}}$	$0.03199^{\{6\}}$	$0.0306^{\{4\}}$	$0.03686^{\{8\}}$	$0.02923^{\{3\}}$	$0.03711^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.05667^{\{1\}}$	$0.07046^{\{2\}}$	$0.07954^{\{5\}}$	$0.07251^{\{3\}}$	$0.08137^{\{6\}}$	$0.0846^{\{8\}}$	$0.08424^{\{7\}}$	$0.10123^{\{10\}}$	$0.07777^{\{4\}}$	$0.10033^{\{9\}}$
	MSE($\hat{\chi}$)	$0.00124^{\{2\}}$	$0.00153^{\{5\}}$	$0.00224^{\{10\}}$	$0.00114^{\{1\}}$	$0.00201^{\{7\}}$	$0.0016^{\{6\}}$	$0.00142^{\{4\}}$	$0.00213^{\{9\}}$	$0.0013^{\{3\}}$	$0.00212^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.23618^{\{1\}}$	$0.26384^{\{3\}}$	$0.28438^{\{5\}}$	$0.26189^{\{2\}}$	$0.28527^{\{6\}}$	$0.28688^{\{7\}}$	$0.28784^{\{8\}}$	$0.32031^{\{9\}}$	$0.27302^{\{4\}}$	$0.32064^{\{10\}}$
	MRE($\hat{\chi}$)	$0.13944^{\{2\}}$	$0.15708^{\{5\}}$	$0.1875^{\{10\}}$	$0.13784^{\{1\}}$	$0.17885^{\{7\}}$	$0.15993^{\{6\}}$	$0.15302^{\{4\}}$	$0.18429^{\{8\}}$	$0.14617^{\{3\}}$	$0.18554^{\{9\}}$
	D_{abs}	$0.01982^{\{2\}}$	$0.02017^{\{3\}}$	$0.02099^{\{5\}}$	$0.01935^{\{1\}}$	$0.02079^{\{4\}}$	$0.0239^{\{8\}}$	$0.02274^{\{7\}}$	$0.02542^{\{10\}}$	$0.02158^{\{6\}}$	$0.02523^{\{9\}}$
	D_{max}	$0.03232^{\{2\}}$	$0.03329^{\{3\}}$	$0.03567^{\{6\}}$	$0.03143^{\{1\}}$	$0.03509^{\{5\}}$	$0.03879^{\{8\}}$	$0.03671^{\{7\}}$	$0.04177^{\{10\}}$	$0.03493^{\{4\}}$	$0.04167^{\{9\}}$
	ASAE	$0.01621^{\{2\}}$	$0.01427^{\{3\}}$	$0.01417^{\{6\}}$	$0.01628^{\{1\}}$	$0.01414^{\{5\}}$	$0.02072^{\{8\}}$	$0.01956^{\{7\}}$	$0.02165^{\{10\}}$	$0.01847^{\{4\}}$	$0.02159^{\{9\}}$
	$\sum Ranks$	$17^{\{1.5\}}$	$32^{\{3\}}$	$58^{\{7\}}$	$17^{\{1.5\}}$	$49^{\{5\}}$	$64^{\{8\}}$	$56^{\{6\}}$	$83^{\{10\}}$	$37^{\{4\}}$	$82^{\{9\}}$
200	BIAS($\hat{\Phi}$)	$0.16602^{\{1\}}$	$0.19126^{\{4\}}$	$0.20535^{\{5\}}$	$0.18899^{\{2\}}$	$0.2119^{\{8\}}$	$0.211^{\{7\}}$	$0.20844^{\{6\}}$	$0.23732^{\{10\}}$	$0.19016^{\{3\}}$	$0.23568^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.02438^{\{1\}}$	$0.02797^{\{5\}}$	$0.03283^{\{8\}}$	$0.02475^{\{2\}}$	$0.03243^{\{7\}}$	$0.02884^{\{6\}}$	$0.02742^{\{4\}}$	$0.03329^{\{9.5\}}$	$0.02539^{\{3\}}$	$0.03329^{\{9.5\}}$
	MSE($\hat{\Phi}$)	$0.04473^{\{1\}}$	$0.05895^{\{2\}}$	$0.06659^{\{5\}}$	$0.06056^{\{4\}}$	$0.07203^{\{7\}}$	$0.07368^{\{8\}}$	$0.07145^{\{6\}}$	$0.08944^{\{10\}}$	$0.06009^{\{3\}}$	$0.08822^{\{9\}}$
	MSE($\hat{\chi}$)	$0.00095^{\{2\}}$	$0.00121^{\{5\}}$	$0.00172^{\{9.5\}}$	$0.00093^{\{1\}}$	$0.00164^{\{7\}}$	$0.0013^{\{6\}}$	$0.00115^{\{4\}}$	$0.00171^{\{8\}}$	$0.001^{\{3\}}$	$0.00172^{\{9.5\}}$
	MRE($\hat{\Phi}$)	$0.20753^{\{1\}}$	$0.23908^{\{4\}}$	$0.25669^{\{5\}}$	$0.23624^{\{2\}}$	$0.26488^{\{8\}}$	$0.26375^{\{7\}}$	$0.26055^{\{6\}}$	$0.29665^{\{10\}}$	$0.2377^{\{3\}}$	$0.2946^{\{9\}}$
	MRE($\hat{\chi}$)	$0.12192^{\{1\}}$	$0.13987^{\{5\}}$	$0.16414^{\{8\}}$	$0.12375^{\{2\}}$	$0.16214^{\{7\}}$	$0.14422^{\{6\}}$	$0.1371^{\{4\}}$	$0.16643^{\{9\}}$	$0.12693^{\{3\}}$	$0.16645^{\{10\}}$
	D_{abs}	$0.01718^{\{2\}}$	$0.01761^{\{3\}}$	$0.01795^{\{4\}}$	$0.017^{\{1\}}$	$0.01822^{\{5\}}$	$0.02095^{\{8\}}$	$0.01979^{\{7\}}$	$0.02247^{\{9\}}$	$0.01876^{\{6\}}$	$0.02253^{\{10\}}$
	D_{max}	$0.02804^{\{2\}}$	$0.02911^{\{3\}}$	$0.0305^{\{5\}}$	$0.02767^{\{1\}}$	$0.03083^{\{6\}}$	$0.03404^{\{8\}}$	$0.03205^{\{7\}}$	$0.03699^{\{9\}}$	$0.03037^{\{4\}}$	$0.03704^{\{10\}}$
	ASAE	$0.014^{\{2\}}$	$0.01228^{\{3\}}$	$0.01223^{\{5\}}$	$0.01403^{\{1\}}$	$0.01215^{\{6\}}$	$0.01763^{\{8\}}$	$0.01685^{\{7\}}$	$0.01885^{\{9\}}$	$0.01593^{\{4\}}$	$0.0187^{\{10\}}$
	$\sum Ranks$	$15^{\{1\}}$	$34^{\{3.5\}}$	$51.5^{\{6\}}$	$20^{\{2\}}$	$56^{\{7\}}$	$64^{\{8\}}$	$51^{\{5\}}$	$84.5^{\{9\}}$	$34^{\{3.5\}}$	$85^{\{10\}}$
300	BIAS($\hat{\Phi}$)	$0.13824^{\{1\}}$	$0.16146^{\{3\}}$	$0.17964^{\{6\}}$	$0.15292^{\{2\}}$	$0.18117^{\{7\}}$	$0.18523^{\{8\}}$	$0.17675^{\{5\}}$	$0.20562^{\{9\}}$	$0.1635^{\{4\}}$	$0.20767^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.01992^{\{1\}}$	$0.02323^{\{5\}}$	$0.0276^{\{8\}}$	$0.02022^{\{2\}}$	$0.02733^{\{7\}}$	$0.025^{\{6\}}$	$0.02319^{\{4\}}$	$0.028^{\{9\}}$	$0.02185^{\{3\}}$	$0.02835^{\{10\}}$
	MSE($\hat{\Phi}$)	$0.03094^{\{1\}}$	$0.04411^{\{3\}}$	$0.05313^{\{6\}}$	$0.04006^{\{2\}}$	$0.05382^{\{7\}}$	$0.05711^{\{8\}}$	$0.0527^{\{5\}}$	$0.06933^{\{9\}}$	$0.04507^{\{4\}}$	$0.07078^{\{10\}}$
	MSE($\hat{\chi}$)	$0.00063^{\{1.5\}}$	$0.00085^{\{5\}}$	$0.00119^{\{8\}}$	$0.00063^{\{1.5\}}$	$0.00116^{\{7\}}$	$0.00097^{\{6\}}$	$0.00082^{\{4\}}$	$0.00121^{\{9\}}$	$0.00074^{\{3\}}$	$0.00124^{\{10\}}$
	MRE($\hat{\Phi}$)	$0.17281^{\{1\}}$	$0.20182^{\{3\}}$	$0.22455^{\{6\}}$	$0.19115^{\{2\}}$	$0.22647^{\{7\}}$	$0.23154^{\{8\}}$	$0.22094^{\{5\}}$	$0.25703^{\{9\}}$	$0.20437^{\{4\}}$	$0.25959^{\{10\}}$
	MRE($\hat{\chi}$)	$0.09958^{\{1\}}$	$0.11617^{\{5\}}$	$0.138^{\{8\}}$	$0.10111^{\{2\}}$	$0.13667^{\{7\}}$	$0.12499^{\{6\}}$	$0.11595^{\{4\}}$	$0.14^{\{9\}}$	$0.10925^{\{3\}}$	$0.14175^{\{10\}}$
	D_{abs}	$0.01413^{\{1\}}$	$0.0145^{\{3\}}$	$0.01508^{\{5\}}$	$0.01421^{\{2\}}$	$0.01505^{\{4\}}$	$0.01765^{\{8\}}$	$0.01657^{\{7\}}$	$0.01896^{\{10\}}$	$0.01572^{\{6\}}$	$0.01881^{\{9\}}$
	D_{max}	$0.02301^{\{1\}}$	$0.02397^{\{3\}}$	$0.02559^{\{6\}}$	$0.02305^{\{2\}}$	$0.02547^{\{5\}}$	$0.02868^{\{8\}}$	$0.02687^{\{7\}}$	$0.03108^{\{10\}}$	$0.02545^{\{4\}}$	$0.0309^{\{9\}}$
	ASAE	$0.0114^{\{1\}}$	$0.01008^{\{3\}}$	$0.01001^{\{6\}}$	$0.01147^{\{2\}}$	$0.00991^{\{5\}}$	$0.01468^{\{8\}}$	$0.01373^{\{7\}}$	$0.0156^{\{10\}}$	$0.01305^{\{4\}}$	$0.01562^{\{9\}}$
	$\sum Ranks$	$12.5^{\{1\}}$	$33^{\{3\}}$	$55^{\{7\}}$	$20.5^{\{2\}}$	$52^{\{6\}}$	$66^{\{8\}}$	$48^{\{5\}}$	$83^{\{9\}}$	$37^{\{4\}}$	$88^{\{10\}}$

TABLE 2. Numerical values of simulation measures for $\Phi = 0.8$ and $\chi = 0.2$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS(Φ)	$0.28725^{(1)}$	$0.29602^{(2)}$	$0.30698^{(7)}$	$0.29749^{(3)}$	$0.30244^{(4)}$	$0.30498^{(6)}$	$0.30428^{(5)}$	$0.32216^{(10)}$	$0.30812^{(8)}$	$0.32213^{(9)}$
	BIAS(χ)	$0.05787^{(7)}$	$0.05517^{(4)}$	$0.06389^{(10)}$	$0.04983^{(1)}$	$0.05727^{(6)}$	$0.05622^{(5)}$	$0.05504^{(3)}$	$0.05905^{(9)}$	$0.05486^{(2)}$	$0.0579^{(8)}$
	MSE(Φ)	$0.11494^{(1)}$	$0.1228^{(2)}$	$0.12837^{(3.5)}$	$0.12837^{(3.5)}$	$0.12888^{(5)}$	$0.13164^{(7)}$	$0.13018^{(6)}$	$0.1438^{(10)}$	$0.13224^{(8)}$	$0.14348^{(9)}$
	MSE(χ)	$0.0055^{(9)}$	$0.00487^{(3.5)}$	$0.00645^{(10)}$	$0.0039^{(1)}$	$0.00521^{(7)}$	$0.00515^{(6)}$	$0.00487^{(3.5)}$	$0.00533^{(8)}$	$0.00483^{(2)}$	$0.0051^{(5)}$
	MRE(Φ)	$0.35907^{(1)}$	$0.37002^{(2)}$	$0.38372^{(7)}$	$0.37186^{(3)}$	$0.37806^{(4)}$	$0.38123^{(6)}$	$0.38035^{(5)}$	$0.40269^{(10)}$	$0.38515^{(8)}$	$0.40266^{(9)}$
	MRE(χ)	$0.28937^{(7)}$	$0.27583^{(4)}$	$0.31943^{(10)}$	$0.24916^{(1)}$	$0.28633^{(6)}$	$0.2811^{(5)}$	$0.27519^{(3)}$	$0.29527^{(9)}$	$0.27429^{(2)}$	$0.28949^{(8)}$
	D_{abs}	$0.03266^{(3)}$	$0.03216^{(2)}$	$0.03419^{(5)}$	$0.03193^{(1)}$	$0.03401^{(4)}$	$0.04595^{(8)}$	$0.04044^{(7)}$	$0.05425^{(9)}$	$0.03637^{(6)}$	$0.05455^{(10)}$
	D_{max}	$0.05479^{(3)}$	$0.05349^{(2)}$	$0.058^{(5)}$	$0.05258^{(1)}$	$0.05663^{(4)}$	$0.07356^{(8)}$	$0.06518^{(7)}$	$0.08596^{(9)}$	$0.05929^{(6)}$	$0.08611^{(10)}$
	ASAE	$0.04695^{(3)}$	$0.04228^{(2)}$	$0.04277^{(5)}$	$0.04634^{(1)}$	$0.04191^{(4)}$	$0.0584^{(8)}$	$0.05299^{(7)}$	$0.06074^{(9)}$	$0.05213^{(6)}$	$0.06096^{(10)}$
	$\sum Ranks$	$37^{(3)}$	$23.5^{(2)}$	$60.5^{(8)}$	$18.5^{(1)}$	$41^{(4)}$	$59^{(7)}$	$46.5^{(5)}$	$83^{(10)}$	$48^{(6)}$	$78^{(9)}$
60	BIAS(Φ)	$0.2123^{(1)}$	$0.23602^{(2)}$	$0.25253^{(3)}$	$0.25643^{(5)}$	$0.25897^{(6)}$	$0.26166^{(7)}$	$0.26761^{(8)}$	$0.29144^{(9)}$	$0.25566^{(4)}$	$0.29173^{(10)}$
	BIAS(χ)	$0.0355^{(1)}$	$0.03792^{(4)}$	$0.04615^{(8)}$	$0.03663^{(2)}$	$0.04361^{(7)}$	$0.04036^{(6)}$	$0.03996^{(5)}$	$0.04622^{(9)}$	$0.03785^{(3)}$	$0.04634^{(10)}$
	MSE(Φ)	$0.06921^{(1)}$	$0.0866^{(2)}$	$0.0949^{(3)}$	$0.1038^{(6)}$	$0.1012^{(5)}$	$0.10477^{(7)}$	$0.10899^{(8)}$	$0.12459^{(10)}$	$0.10077^{(4)}$	$0.12458^{(9)}$
	MSE(χ)	$0.00205^{(2)}$	$0.00227^{(4)}$	$0.00349^{(10)}$	$0.002^{(1)}$	$0.00299^{(7)}$	$0.00262^{(6)}$	$0.00247^{(5)}$	$0.00331^{(8)}$	$0.00225^{(3)}$	$0.00334^{(9)}$
	MRE(Φ)	$0.26538^{(1)}$	$0.29502^{(2)}$	$0.31567^{(3)}$	$0.32054^{(5)}$	$0.32371^{(6)}$	$0.32707^{(7)}$	$0.33451^{(8)}$	$0.3643^{(9)}$	$0.31958^{(4)}$	$0.36466^{(10)}$
	MRE(χ)	$0.17748^{(1)}$	$0.18961^{(4)}$	$0.23073^{(8)}$	$0.18313^{(2)}$	$0.21804^{(7)}$	$0.20179^{(6)}$	$0.19979^{(5)}$	$0.23109^{(9)}$	$0.18926^{(3)}$	$0.23169^{(10)}$
	D_{abs}	$0.01941^{(2)}$	$0.01963^{(3)}$	$0.02115^{(5)}$	$0.01911^{(1)}$	$0.0204^{(4)}$	$0.02896^{(8)}$	$0.02547^{(7)}$	$0.03136^{(9)}$	$0.02347^{(6)}$	$0.03207^{(10)}$
	D_{max}	$0.03277^{(2)}$	$0.03351^{(3)}$	$0.03707^{(5)}$	$0.03276^{(1)}$	$0.03586^{(4)}$	$0.04745^{(8)}$	$0.04241^{(7)}$	$0.05249^{(9)}$	$0.03897^{(6)}$	$0.05343^{(10)}$
	ASAE	$0.02466^{(2)}$	$0.02261^{(3)}$	$0.02258^{(5)}$	$0.02497^{(1)}$	$0.0223^{(4)}$	$0.03192^{(8)}$	$0.0297^{(7)}$	$0.03282^{(9)}$	$0.02837^{(6)}$	$0.03317^{(10)}$
	$\sum Ranks$	$15^{(1)}$	$27^{(2)}$	$47^{(5.5)}$	$28^{(3)}$	$47^{(5.5)}$	$63^{(8)}$	$60^{(7)}$	$81^{(9)}$	$39^{(4)}$	$88^{(10)}$
100	BIAS(Φ)	$0.17363^{(1)}$	$0.20101^{(2)}$	$0.22165^{(5)}$	$0.21859^{(3)}$	$0.22955^{(6)}$	$0.23927^{(7)}$	$0.24043^{(8)}$	$0.26936^{(10)}$	$0.22152^{(4)}$	$0.2672^{(9)}$
	BIAS(χ)	$0.02796^{(1)}$	$0.03135^{(3)}$	$0.03767^{(8)}$	$0.03044^{(2)}$	$0.03661^{(7)}$	$0.03469^{(6)}$	$0.03342^{(5)}$	$0.03999^{(10)}$	$0.03136^{(4)}$	$0.03985^{(9)}$
	MSE(Φ)	$0.04808^{(1)}$	$0.06539^{(2)}$	$0.07724^{(3)}$	$0.07871^{(4)}$	$0.08337^{(6)}$	$0.09117^{(7)}$	$0.09296^{(8)}$	$0.11033^{(10)}$	$0.07951^{(5)}$	$0.1088^{(9)}$
	MSE(χ)	$0.00125^{(1)}$	$0.00153^{(4)}$	$0.00228^{(8)}$	$0.00138^{(2)}$	$0.00208^{(7)}$	$0.00188^{(6)}$	$0.00168^{(5)}$	$0.00244^{(9)}$	$0.00151^{(3)}$	$0.00245^{(10)}$
	MRE(Φ)	$0.21704^{(1)}$	$0.25127^{(2)}$	$0.27706^{(5)}$	$0.27324^{(3)}$	$0.28693^{(6)}$	$0.29909^{(7)}$	$0.30054^{(8)}$	$0.3367^{(10)}$	$0.27691^{(4)}$	$0.334^{(9)}$
	MRE(χ)	$0.13981^{(1)}$	$0.15676^{(3)}$	$0.18836^{(8)}$	$0.15219^{(2)}$	$0.18307^{(7)}$	$0.17343^{(6)}$	$0.1671^{(5)}$	$0.19993^{(10)}$	$0.15682^{(4)}$	$0.19924^{(9)}$
	D_{abs}	$0.0151^{(1)}$	$0.01547^{(3)}$	$0.01668^{(5)}$	$0.01531^{(2)}$	$0.01631^{(4)}$	$0.02284^{(8)}$	$0.02058^{(7)}$	$0.02513^{(9)}$	$0.01888^{(6)}$	$0.02524^{(10)}$
	D_{max}	$0.02559^{(1)}$	$0.02663^{(3)}$	$0.02931^{(5)}$	$0.02638^{(2)}$	$0.02886^{(4)}$	$0.03779^{(8)}$	$0.03442^{(7)}$	$0.04235^{(9)}$	$0.03145^{(6)}$	$0.04249^{(10)}$
	ASAE	$0.01858^{(1)}$	$0.01729^{(3)}$	$0.01721^{(5)}$	$0.01915^{(2)}$	$0.01703^{(4)}$	$0.02433^{(8)}$	$0.02291^{(7)}$	$0.02533^{(9)}$	$0.02172^{(6)}$	$0.02557^{(10)}$
	$\sum Ranks$	$12^{(1)}$	$25^{(2.5)}$	$49^{(6)}$	$25^{(2.5)}$	$48^{(5)}$	$63^{(8)}$	$60^{(7)}$	$86^{(10)}$	$42^{(4)}$	$85^{(9)}$
150	BIAS(Φ)	$0.14333^{(1)}$	$0.17188^{(2)}$	$0.19083^{(4)}$	$0.1853^{(3)}$	$0.19813^{(6)}$	$0.21514^{(8)}$	$0.21374^{(7)}$	$0.24537^{(9)}$	$0.19343^{(5)}$	$0.24638^{(10)}$
	BIAS(χ)	$0.02263^{(1)}$	$0.02648^{(3)}$	$0.03132^{(8)}$	$0.0259^{(2)}$	$0.03092^{(7)}$	$0.03041^{(6)}$	$0.02921^{(5)}$	$0.03474^{(9)}$	$0.02704^{(4)}$	$0.03506^{(10)}$
	MSE(Φ)	$0.03317^{(1)}$	$0.04935^{(2)}$	$0.05965^{(4)}$	$0.05822^{(3)}$	$0.06496^{(6)}$	$0.07622^{(8)}$	$0.07524^{(7)}$	$0.09481^{(10)}$	$0.06186^{(5)}$	$0.0947^{(9)}$
	MSE(χ)	$0.00081^{(1)}$	$0.00111^{(3.5)}$	$0.00156^{(8)}$	$0.001^{(2)}$	$0.00149^{(7)}$	$0.00142^{(6)}$	$0.00128^{(5)}$	$0.00182^{(9)}$	$0.00111^{(3.5)}$	$0.00187^{(10)}$
	MRE(Φ)	$0.17916^{(1)}$	$0.21485^{(2)}$	$0.23854^{(4)}$	$0.23163^{(3)}$	$0.24766^{(6)}$	$0.26892^{(8)}$	$0.26717^{(7)}$	$0.30671^{(9)}$	$0.24179^{(5)}$	$0.30798^{(10)}$
	MRE(χ)	$0.11313^{(1)}$	$0.13242^{(3)}$	$0.15658^{(8)}$	$0.12951^{(2)}$	$0.1546^{(7)}$	$0.15205^{(6)}$	$0.14603^{(5)}$	$0.17368^{(9)}$	$0.13522^{(4)}$	$0.1753^{(10)}$
	D_{abs}	$0.01235^{(1)}$	$0.01301^{(3)}$	$0.01378^{(5)}$	$0.01288^{(2)}$	$0.01375^{(4)}$	$0.0191^{(8)}$	$0.01743^{(7)}$	$0.0208^{(9)}$	$0.01578^{(6)}$	$0.02121^{(10)}$
	D_{max}	$0.02091^{(1)}$	$0.0224^{(3)}$	$0.02427^{(5)}$	$0.02218^{(2)}$	$0.02424^{(4)}$	$0.03179^{(8)}$	$0.02919^{(7)}$	$0.03509^{(9)}$	$0.0264^{(6)}$	$0.03578^{(10)}$
	ASAE	$0.01493^{(1)}$	$0.01407^{(3)}$	$0.01385^{(5)}$	$0.0156^{(2)}$	$0.01378^{(4)}$	$0.01997^{(8)}$	$0.01896^{(7)}$	$0.02079^{(9)}$	$0.01786^{(6)}$	$0.02104^{(10)}$
	$\sum Ranks$	$12^{(1)}$	$24.5^{(3)}$	$48^{(5.5)}$	$24^{(2)}$	$48^{(5.5)}$	$66^{(8)}$	$57^{(7)}$	$82^{(9)}$	$44.5^{(4)}$	$89^{(10)}$
200	BIAS(Φ)	$0.12475^{(1)}$	$0.15049^{(2)}$	$0.16953^{(4)}$	$0.15826^{(3)}$	$0.17689^{(6)}$	$0.19663^{(8)}$	$0.18668^{(7)}$	$0.22574^{(9)}$	$0.16978^{(5)}$	$0.2263^{(10)}$
	BIAS(χ)	$0.01951^{(1)}$	$0.02308^{(3)}$	$0.0275^{(7)}$	$0.02238^{(2)}$	$0.02769^{(8)}$	$0.02745^{(6)}$	$0.02565^{(5)}$	$0.03183^{(10)}$	$0.02363^{(4)}$	$0.03168^{(9)}$
	MSE(Φ)	$0.02529^{(1)}$	$0.038^{(2)}$	$0.04744^{(4)}$	$0.04354^{(3)}$	$0.05271^{(6)}$	$0.06489^{(8)}$	$0.05898^{(7)}$	$0.08227^{(9)}$	$0.04951^{(5)}$	$0.08317^{(10)}$
	MSE(χ)	$0.00061^{(1)}$	$0.00083^{(3)}$	$0.00119^{(7.5)}$	$0.00076^{(2)}$	$0.00119^{(7.5)}$	$0.00117^{(6)}$	$0.001^{(5)}$	$0.00154^{(9.5)}$	$0.00086^{(4)}$	$0.00154^{(9.5)}$
	MRE(Φ)	$0.15594^{(1)}$	$0.18812^{(2)}$	$0.21191^{(4)}$	$0.19782^{(3)}$	$0.22111^{(6)}$	$0.24579^{(8)}$	$0.23334^{(7)}$	$0.28218^{(9)}$	$0.21222^{(5)}$	$0.28288^{(10)}$
	MRE(χ)	$0.09753^{(1)}$	$0.11542^{(3)}$	$0.13751^{(7)}$	$0.11191^{(2)}$	$0.13844^{(8)}$	$0.13727^{(6)}$	$0.12827^{(5)}$	$0.15917^{(10)}$	$0.11817^{(4)}$	$0.15842^{(9)}$
	D_{abs}	$0.01073^{(1)}$	$0.01121^{(3)}$	$0.01202^{(5)}$	$0.01114^{(2)}$	$0.01196^{(4)}$	$0.01701^{(8)}$	$0.01518^{(7)}$	$0.01868^{(9)}$	$0.01384^{(6)}$	$0.01873^{(10)}$
	D_{max}	$0.01813^{(1)}$	$0.01931^{(3)}$	$0.02121^{(5)}$	$0.01914^{(2)}$	$0.02118^{(4)}$	$0.02826^{(8)}$	$0.02544^{(7)}$	$0.03149^{(9)}$	$0.02314^{(6)}$	$0.03152^{(10)}$
	ASAE	$0.01299^{(1)}$	$0.01207^{(3)}$	$0.01195^{(5)}$	$0.01345^{(2)}$	$0.01194^{(4)}$	$0.01741^{(8)}$	$0.01625^{(7)}$	$0.01849^{(9)}$	$0.0155^{(6)}$	$0.01843^{(10)}$
	$\sum Ranks$	$12^{(1)}$	$24^{(2.5)}$	$45.5^{(5)}$	$24^{(2.5)}$	$50.5^{(6)}$	$66^{(8)}$	$57^{(7)}$	$84.5^{(9)}$	$45^{(4)}$	$86.5^{(10)}$
300	BIAS(Φ)	$0.10329^{(1)}$	$0.12257^{(2)}$	$0.13966^{(5)}$	$0.12487^{(3)}$	$0.14696^{(6)}$	$0.1657^{(8)}$	$0.15363^{(7)}$	$0.19138^{(10)}$	$0.13853^{(4)}$	$0.19115^{(9)}$
	BIAS(χ)	$0.01609^{(1)}$	$0.01903^{(3)}$	$0.02252^{(6)}$	$0.0181^{(2)}$	$0.02314^{(7)}$	$0.02339^{(8)}$	$0.02133^{(5)}$	$0.02673^{(10)}$	$0.01969^{(4)}$	$0.02633^{(9)}$
	MSE(Φ)	$0.01727^{(1)}$	$0.02529^{(2)}$	$0.03262^{(4)}$	$0.02647^{(3)}$	$0.03673^{(6)}$	$0.04728^{(8)}$	$0.0404^{(7)}$	$0.06077^{(9)}$	$0.03279^{(5)}$	$0.06152^{(10)}$
	MSE(χ)	$0.00041^{(1)}$	$0.00057^{(3)}$	$0.00081^{(6)}$	$5e-04^{(2)}$	$0.00084^{(7)}$	$0.00086^{(8)}$	$7e-04^{(5)}$	$0.00109^{(10)}$	$6e-04^{(4)}$	$0.00107^{(9)}$
	MRE(Φ)	$0.12912^{(1)}$	$0.15322^{(2)}$	$0.17458^{(5)}$	$0.15609^{(3)}$	$0.1837^{(6)}$	$0.20712^{(8)}$	$0.19203^{(7)}$	$0.23923^{(10)}$	$0.17317^{(4)}$	$0.23894^{(9)}$
	MRE(χ)	$0.08047^{(1)}$	$0.09513^{(3)}$	$0.1126^{(6)}$	$0.09048^{(2)}$	$0.11569^{(7)}$	$0.11693^{(8)}$	$0.10663^{(5)}$	$0.13367^{(10)}$	$0.09843^{(4)}$	$0.13165^{(9)}$
	D_{abs}	$0.00885^{(1)}$	$0.00924^{(3)}$	$0.00981^{(4.5)}$	$0.00909^{(2)}$	$0.00981^{(4.5)}$	$0.01397^{(8)}$	$0.01259^{(7)}$	$0.01581^{(9)}$	$0.01142^{(6)}$	$0.01583^{(10)}$
	D_{max}	$0.01497^{(1)}$	$0.01597^{(3)}$	$0.01732^{(4)}$	$0.01558^{(2)}$	$0.01738^{(5)}$	$0.02335^{(8)}$	$0.02102^{(7)}$	$0.02652^{(10)}$	$0.01911^{(6)}$	$0.0265^{(9)}$
	ASAE	$0.01051^{(1)}$	$0.00975^{(3)}$	$0.00974^{(4)}$	$0.01089^{(2)}$	$0.00969^{(5)}$	$0.01413^{(8)}$	$0.0134^{(7)}$	$0.01524^{(10)}$	$0.01268^{(6)}$	$0.01524^{(9)}$
	$\sum Ranks$	$12^{(1)}$	$23^{(2)}$	$39.5^{(4)}$	$30^{(3)}$	$46.5^{(5)}$	$69^{(8)}$	$62^{(7)}$	$84^{(10)}$	$48^{(6)}$	$81^{(9)}$

TABLE 3. Numerical values of simulation measures for $\Phi = 2.0$ and $\chi = 3.0$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.79248^{(8)}$	$0.78388^{(5)}$	$0.78893^{(7)}$	$0.78005^{(2)}$	$0.7854^{(6)}$	$0.78179^{(3)}$	$0.78383^{(4)}$	$0.82381^{(10)}$	$0.7782^{(1)}$	$0.82095^{(9)}$
	BIAS($\hat{\chi}$)	$1.16935^{(9)}$	$1.11761^{(7)}$	$1.17085^{(10)}$	$1.02152^{(1)}$	$1.12171^{(8)}$	$1.06244^{(4)}$	$1.05136^{(2)}$	$1.06875^{(6)}$	$1.05877^{(3)}$	$1.0633^{(5)}$
	MSE($\hat{\Phi}$)	$0.82682^{(3)}$	$0.82367^{(2)}$	$0.8299^{(5)}$	$0.82973^{(4)}$	$0.83155^{(8)}$	$0.83126^{(7)}$	$0.83082^{(6)}$	$0.90729^{(10)}$	$0.82262^{(1)}$	$0.90169^{(9)}$
	MSE($\hat{\chi}$)	$2.05162^{(10)}$	$1.89105^{(8)}$	$2.04283^{(9)}$	$1.57916^{(1)}$	$1.87069^{(7)}$	$1.72213^{(6)}$	$1.69985^{(4)}$	$1.6856^{(3)}$	$1.71526^{(5)}$	$1.6771^{(2)}$
	MRE($\hat{\Phi}$)	$0.39624^{(8)}$	$0.39194^{(5)}$	$0.39447^{(7)}$	$0.39002^{(2)}$	$0.3927^{(6)}$	$0.3909^{(3)}$	$0.39191^{(4)}$	$0.4119^{(10)}$	$0.3891^{(1)}$	$0.41047^{(9)}$
	MRE($\hat{\chi}$)	$0.38978^{(9)}$	$0.37254^{(7)}$	$0.39028^{(10)}$	$0.34051^{(1)}$	$0.3739^{(8)}$	$0.35415^{(4)}$	$0.35045^{(2)}$	$0.35625^{(6)}$	$0.35292^{(3)}$	$0.35443^{(5)}$
	D_{abs}	$0.04625^{(1)}$	$0.04882^{(2)}$	$0.04947^{(3)}$	$0.05168^{(5)}$	$0.0521^{(6)}$	$0.05664^{(8)}$	$0.05462^{(7)}$	$0.07119^{(9)}$	$0.05032^{(4)}$	$0.07187^{(10)}$
	D_{max}	$0.07383^{(1)}$	$0.0753^{(2)}$	$0.07775^{(5)}$	$0.07772^{(4)}$	$0.07906^{(6)}$	$0.08616^{(8)}$	$0.08243^{(7)}$	$0.10528^{(9)}$	$0.07695^{(3)}$	$0.10608^{(10)}$
	ASAE	$0.04485^{(1)}$	$0.03962^{(2)}$	$0.04003^{(5)}$	$0.03996^{(4)}$	$0.03898^{(6)}$	$0.05311^{(8)}$	$0.05012^{(7)}$	$0.06416^{(9)}$	$0.0476^{(3)}$	$0.06497^{(10)}$
	$\sum Ranks$	$54^{(6)}$	$40^{(3)}$	$60^{(8)}$	$23^{(1)}$	$56^{(7)}$	$51^{(5)}$	$43^{(4)}$	$72^{(10)}$	$27^{(2)}$	$69^{(9)}$
60	BIAS($\hat{\Phi}$)	$0.68812^{(1)}$	$0.71353^{(4)}$	$0.72421^{(7)}$	$0.69519^{(2)}$	$0.7321^{(8)}$	$0.71046^{(3)}$	$0.71754^{(6)}$	$0.76246^{(10)}$	$0.7136^{(5)}$	$0.75739^{(9)}$
	BIAS($\hat{\chi}$)	$0.93566^{(5)}$	$0.95056^{(8)}$	$1.05492^{(10)}$	$0.81143^{(1)}$	$1.0139^{(9)}$	$0.87688^{(4)}$	$0.86391^{(2)}$	$0.94264^{(7)}$	$0.87103^{(3)}$	$0.93844^{(6)}$
	MSE($\hat{\Phi}$)	$0.66205^{(1)}$	$0.71374^{(3)}$	$0.71613^{(4)}$	$0.70984^{(2)}$	$0.7389^{(8)}$	$0.71875^{(5)}$	$0.73531^{(7)}$	$0.80759^{(10)}$	$0.72694^{(6)}$	$0.79791^{(9)}$
	MSE($\hat{\chi}$)	$1.39229^{(7)}$	$1.41569^{(8)}$	$1.69957^{(10)}$	$1.0327^{(1)}$	$1.58547^{(9)}$	$1.22603^{(4)}$	$1.17779^{(2)}$	$1.34724^{(6)}$	$1.19531^{(3)}$	$1.34133^{(5)}$
	MRE($\hat{\Phi}$)	$0.34406^{(1)}$	$0.35676^{(4)}$	$0.3621^{(7)}$	$0.34759^{(2)}$	$0.36605^{(8)}$	$0.35523^{(3)}$	$0.35877^{(6)}$	$0.38123^{(10)}$	$0.3568^{(5)}$	$0.37869^{(9)}$
	MRE($\hat{\chi}$)	$0.31189^{(5)}$	$0.31685^{(8)}$	$0.35164^{(10)}$	$0.27048^{(1)}$	$0.33797^{(9)}$	$0.29229^{(4)}$	$0.28797^{(2)}$	$0.31421^{(7)}$	$0.29034^{(3)}$	$0.31281^{(6)}$
	D_{abs}	$0.02857^{(2)}$	$0.02949^{(3)}$	$0.02973^{(4)}$	$0.02826^{(1)}$	$0.02989^{(5)}$	$0.03497^{(8)}$	$0.03298^{(7)}$	$0.03996^{(9)}$	$0.03076^{(6)}$	$0.0402^{(10)}$
	D_{max}	$0.0464^{(2)}$	$0.0474^{(3)}$	$0.0489^{(6)}$	$0.04473^{(1)}$	$0.04852^{(4)}$	$0.05487^{(8)}$	$0.05186^{(7)}$	$0.06234^{(9)}$	$0.0487^{(5)}$	$0.06271^{(10)}$
	ASAE	$0.02228^{(2)}$	$0.02063^{(3)}$	$0.02124^{(6)}$	$0.02142^{(1)}$	$0.021^{(4)}$	$0.02939^{(8)}$	$0.02745^{(7)}$	$0.03418^{(9)}$	$0.02534^{(5)}$	$0.03414^{(10)}$
	$\sum Ranks$	$29^{(2)}$	$42^{(3.5)}$	$61^{(7)}$	$15^{(1)}$	$62^{(8)}$	$47^{(6)}$	$46^{(5)}$	$78^{(10)}$	$42^{(3.5)}$	$73^{(9)}$
100	BIAS($\hat{\Phi}$)	$0.63133^{(1)}$	$0.66208^{(2)}$	$0.6866^{(8)}$	$0.66216^{(3)}$	$0.68573^{(7)}$	$0.67644^{(6)}$	$0.67207^{(5)}$	$0.724^{(9)}$	$0.66298^{(4)}$	$0.73006^{(10)}$
	BIAS($\hat{\chi}$)	$0.79501^{(5)}$	$0.83022^{(6)}$	$0.95744^{(10)}$	$0.71057^{(1)}$	$0.91183^{(9)}$	$0.78258^{(4)}$	$0.75679^{(3)}$	$0.84976^{(7)}$	$0.74802^{(2)}$	$0.85846^{(8)}$
	MSE($\hat{\Phi}$)	$0.57984^{(1)}$	$0.63806^{(2)}$	$0.65924^{(4)}$	$0.66008^{(5)}$	$0.66995^{(6)}$	$0.67178^{(8)}$	$0.67168^{(7)}$	$0.75039^{(9)}$	$0.65517^{(3)}$	$0.76243^{(10)}$
	MSE($\hat{\chi}$)	$1.02674^{(5)}$	$1.09566^{(6)}$	$1.43276^{(10)}$	$0.76906^{(1)}$	$1.30274^{(9)}$	$0.97105^{(4)}$	$0.90432^{(3)}$	$1.09892^{(7)}$	$0.87877^{(2)}$	$1.13382^{(8)}$
	MRE($\hat{\Phi}$)	$0.31567^{(1)}$	$0.33104^{(2)}$	$0.3433^{(8)}$	$0.33108^{(3)}$	$0.34286^{(7)}$	$0.33822^{(6)}$	$0.33604^{(5)}$	$0.362^{(9)}$	$0.33149^{(4)}$	$0.36503^{(10)}$
	MRE($\hat{\chi}$)	$0.265^{(5)}$	$0.27674^{(6)}$	$0.31915^{(10)}$	$0.23686^{(1)}$	$0.30394^{(9)}$	$0.26086^{(4)}$	$0.25226^{(3)}$	$0.28325^{(7)}$	$0.24934^{(2)}$	$0.28615^{(8)}$
	D_{abs}	$0.023^{(3)}$	$0.02275^{(2)}$	$0.02377^{(4)}$	$0.02233^{(1)}$	$0.02388^{(5)}$	$0.02746^{(8)}$	$0.02529^{(7)}$	$0.03041^{(9)}$	$0.02442^{(6)}$	$0.03065^{(10)}$
	D_{max}	$0.03739^{(3)}$	$0.03713^{(2)}$	$0.03974^{(6)}$	$0.03586^{(1)}$	$0.03949^{(5)}$	$0.04364^{(8)}$	$0.04039^{(7)}$	$0.04841^{(9)}$	$0.03903^{(4)}$	$0.04882^{(10)}$
	ASAE	$0.01625^{(3)}$	$0.01532^{(2)}$	$0.01598^{(6)}$	$0.01587^{(1)}$	$0.01589^{(5)}$	$0.02206^{(8)}$	$0.02045^{(7)}$	$0.02497^{(9)}$	$0.01878^{(4)}$	$0.02519^{(10)}$
	$\sum Ranks$	$29^{(2.5)}$	$29^{(2.5)}$	$64^{(8)}$	$18^{(1)}$	$60^{(7)}$	$56^{(6)}$	$47^{(5)}$	$75^{(9)}$	$33^{(4)}$	$84^{(10)}$
150	BIAS($\hat{\Phi}$)	$0.56777^{(1)}$	$0.61366^{(3)}$	$0.65217^{(7)}$	$0.62048^{(4)}$	$0.66129^{(8)}$	$0.62845^{(5)}$	$0.63238^{(6)}$	$0.6923^{(9)}$	$0.61175^{(2)}$	$0.69706^{(10)}$
	BIAS($\hat{\chi}$)	$0.66944^{(4)}$	$0.72904^{(6)}$	$0.87143^{(10)}$	$0.62187^{(1)}$	$0.83898^{(9)}$	$0.69631^{(5)}$	$0.66943^{(3)}$	$0.78585^{(8)}$	$0.65228^{(2)}$	$0.7801^{(7)}$
	MSE($\hat{\Phi}$)	$0.48932^{(1)}$	$0.56681^{(2)}$	$0.61016^{(6)}$	$0.60888^{(5)}$	$0.63647^{(8)}$	$0.60577^{(4)}$	$0.61453^{(7)}$	$0.70528^{(9)}$	$0.5771^{(3)}$	$0.71713^{(10)}$
	MSE($\hat{\chi}$)	$0.72349^{(4)}$	$0.85025^{(6)}$	$1.19376^{(10)}$	$0.57913^{(1)}$	$1.10013^{(9)}$	$0.76994^{(5)}$	$0.69475^{(3)}$	$0.95638^{(8)}$	$0.66427^{(2)}$	$0.9376^{(7)}$
	MRE($\hat{\Phi}$)	$0.28389^{(1)}$	$0.30683^{(3)}$	$0.32609^{(7)}$	$0.31024^{(4)}$	$0.33064^{(8)}$	$0.31423^{(5)}$	$0.31619^{(6)}$	$0.34615^{(9)}$	$0.30588^{(2)}$	$0.34853^{(10)}$
	MRE($\hat{\chi}$)	$0.22315^{(4)}$	$0.24301^{(6)}$	$0.29048^{(10)}$	$0.20729^{(1)}$	$0.27966^{(9)}$	$0.2321^{(5)}$	$0.22314^{(3)}$	$0.26195^{(8)}$	$0.21743^{(2)}$	$0.26003^{(7)}$
	D_{abs}	$0.01914^{(2)}$	$0.01921^{(3)}$	$0.01992^{(5)}$	$0.01859^{(1)}$	$0.01991^{(4)}$	$0.02263^{(8)}$	$0.02107^{(7)}$	$0.02508^{(10)}$	$0.02038^{(6)}$	$0.02492^{(9)}$
	D_{max}	$0.03109^{(2)}$	$0.03143^{(3)}$	$0.03356^{(6)}$	$0.02998^{(1)}$	$0.03338^{(5)}$	$0.03627^{(8)}$	$0.03382^{(7)}$	$0.04047^{(10)}$	$0.03266^{(4)}$	$0.04024^{(9)}$
	ASAE	$0.01279^{(2)}$	$0.01209^{(3)}$	$0.01275^{(6)}$	$0.01256^{(1)}$	$0.01269^{(5)}$	$0.01751^{(8)}$	$0.01622^{(7)}$	$0.0197^{(10)}$	$0.01495^{(4)}$	$0.01973^{(9)}$
	$\sum Ranks$	$24^{(2)}$	$33^{(4)}$	$65^{(8)}$	$20^{(1)}$	$63^{(7)}$	$53^{(6)}$	$49^{(5)}$	$80^{(10)}$	$29^{(3)}$	$79^{(9)}$
200	BIAS($\hat{\Phi}$)	$0.52042^{(1)}$	$0.57674^{(4)}$	$0.6152^{(7)}$	$0.5761^{(2)}$	$0.62223^{(8)}$	$0.60817^{(6)}$	$0.59478^{(5)}$	$0.66707^{(10)}$	$0.57665^{(3)}$	$0.65711^{(9)}$
	BIAS($\hat{\chi}$)	$0.59129^{(3)}$	$0.666^{(6)}$	$0.78484^{(10)}$	$0.5675^{(1)}$	$0.77394^{(9)}$	$0.64839^{(5)}$	$0.60548^{(4)}$	$0.71633^{(8)}$	$0.59089^{(2)}$	$0.71573^{(7)}$
	MSE($\hat{\Phi}$)	$0.4249^{(1)}$	$0.51118^{(2)}$	$0.56007^{(5)}$	$0.53962^{(4)}$	$0.5757^{(7)}$	$0.58072^{(8)}$	$0.56232^{(6)}$	$0.6723^{(10)}$	$0.52913^{(3)}$	$0.65546^{(9)}$
	MSE($\hat{\chi}$)	$0.55659^{(3)}$	$0.69684^{(6)}$	$0.97336^{(10)}$	$0.48447^{(1)}$	$0.94832^{(9)}$	$0.66521^{(5)}$	$0.55799^{(4)}$	$0.78703^{(7)}$	$0.53912^{(2)}$	$0.79408^{(8)}$
	MRE($\hat{\Phi}$)	$0.26021^{(1)}$	$0.28837^{(4)}$	$0.3076^{(7)}$	$0.28805^{(2)}$	$0.31111^{(8)}$	$0.30409^{(6)}$	$0.29739^{(5)}$	$0.33354^{(10)}$	$0.28832^{(3)}$	$0.32855^{(9)}$
	MRE($\hat{\chi}$)	$0.1971^{(3)}$	$0.222^{(6)}$	$0.26161^{(10)}$	$0.18917^{(1)}$	$0.25798^{(9)}$	$0.21613^{(5)}$	$0.20183^{(4)}$	$0.23878^{(8)}$	$0.19696^{(2)}$	$0.23858^{(7)}$
	D_{abs}	$0.01645^{(2)}$	$0.01675^{(3)}$	$0.01736^{(4)}$	$0.01627^{(1)}$	$0.01747^{(5)}$	$0.01981^{(8)}$	$0.01839^{(7)}$	$0.0216^{(9.5)}$	$0.01804^{(6)}$	$0.0216^{(9.5)}$
	D_{max}	$0.02673^{(2)}$	$0.0276^{(3)}$	$0.02934^{(5)}$	$0.02629^{(1)}$	$0.02944^{(6)}$	$0.03195^{(8)}$	$0.02959^{(7)}$	$0.03503^{(9.5)}$	$0.02895^{(4)}$	$0.03503^{(9.5)}$
	ASAE	$0.01077^{(2)}$	$0.0103^{(3)}$	$0.01086^{(5)}$	$0.01062^{(1)}$	$0.0108^{(6)}$	$0.01481^{(8)}$	$0.01372^{(7)}$	$0.01672^{(9.5)}$	$0.01261^{(4)}$	$0.01678^{(9.5)}$
	$\sum Ranks$	$19^{(2)}$	$35^{(4)}$	$63^{(7)}$	$15^{(1)}$	$65^{(8)}$	$59^{(6)}$	$49^{(5)}$	$81^{(10)}$	$31^{(3)}$	$78^{(9)}$
300	BIAS($\hat{\Phi}$)	$0.44981^{(1)}$	$0.51811^{(4)}$	$0.56135^{(7)}$	$0.51026^{(3)}$	$0.57462^{(8)}$	$0.55693^{(6)}$	$0.53706^{(5)}$	$0.61736^{(10)}$	$0.50666^{(2)}$	$0.61701^{(9)}$
	BIAS($\hat{\chi}$)	$0.48966^{(2)}$	$0.57178^{(6)}$	$0.6833^{(10)}$	$0.48859^{(1)}$	$0.67098^{(9)}$	$0.56623^{(5)}$	$0.52472^{(4)}$	$0.6379^{(7)}$	$0.50106^{(3)}$	$0.63852^{(8)}$
	MSE($\hat{\Phi}$)	$0.32939^{(1)}$	$0.42902^{(3)}$	$0.48101^{(6)}$	$0.44153^{(4)}$	$0.51295^{(8)}$	$0.49887^{(7)}$	$0.4747^{(5)}$	$0.59806^{(10)}$	$0.42681^{(2)}$	$0.59674^{(9)}$
	MSE($\hat{\chi}$)	$0.37635^{(2)}$	$0.5067^{(6)}$	$0.73461^{(10)}$	$0.36172^{(1)}$	$0.70385^{(9)}$	$0.49573^{(5)}$	$0.41308^{(4)}$	$0.62839^{(8)}$	$0.38149^{(3)}$	$0.62662^{(7)}$
	MRE($\hat{\Phi}$)	$0.2249^{(1)}$	$0.25905^{(4)}$	$0.28067^{(7)}$	$0.25513^{(3)}$	$0.28731^{(8)}$	$0.27846^{(6)}$	$0.26853^{(5)}$	$0.30868^{(10)}$	$0.25333^{(2)}$	$0.30851^{(9)}$
	MRE($\hat{\chi}$)	$0.16322^{(2)}$	$0.19059^{(6)}$	$0.22777^{(10)}$	$0.16286^{(1)}$	$0.22366^{(9)}$	$0.18874^{(5)}$	$0.17491^{(4)}$	$0.21263^{(7)}$	$0.16702^{(3)}$	$0.21284^{(8)}$
	D_{abs}	$0.01376^{(1)}$	$0.01405^{(3)}$	$0.01447^{(4)}$	$0.01377^{(2)}$	$0.01461^{(5)}$	$0.01682^{(8)}$	$0.0157^{(7)}$	$0.01807^{(10)}$	$0.01507^{(6)}$	$0.01794^{(9)}$
	D_{max}	$0.02234^{(2)}$	$0.02318^{(3)}$	$0.02461^{(5)}$	$0.02224^{(1)}$	$0.02468^{(6)}$	$0.02715^{(8)}$	$0.02529^{(7)}$	$0.02944^{(10)}$	$0.02425^{(4)}$	$0.0293^{(9)}$
	ASAE	$0.00846^{(2)}$	$0.00818^{(3)}$	$0.00868^{(5)}$	$0.00844^{(1)}$	$0.00865^{(6)}$	$0.01182^{(8)}$	$0.01093^{(7)}$	$0.01326^{(10)}$	$0.01007^{(4)}$	$0.01331^{(9)}$
	$\sum Ranks$	$15^{(1)}$	$36^{(4)}$	$64^{(7)}$	$18^{(2)}$	$66^{(8)}$	$58^{(6)}$	$48^{(5)}$	$81^{(10)}$	$31^{(3)}$	$78^{(9)}$

TABLE 4. Numerical values of simulation measures for $\Phi = 2.0$ and $\chi = 3.0$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.72865^{(1)}$	$0.73705^{(2)}$	$0.75074^{(7)}$	$0.74765^{(5)}$	$0.74652^{(4)}$	$0.74908^{(6)}$	$0.73979^{(3)}$	$0.79397^{(9)}$	$0.75309^{(8)}$	$0.80263^{(10)}$
	BIAS($\hat{\chi}$)	$1.0716^{(9)}$	$1.03227^{(6)}$	$1.10034^{(10)}$	$0.97978^{(1)}$	$1.03573^{(7)}$	$1.00173^{(3)}$	$1.01095^{(4)}$	$1.02496^{(5)}$	$1.00143^{(2)}$	$1.03932^{(8)}$
	MSE($\hat{\Phi}$)	$0.72121^{(1)}$	$0.75051^{(2)}$	$0.76656^{(4)}$	$0.78121^{(7)}$	$0.77016^{(5)}$	$0.77396^{(6)}$	$0.75842^{(3)}$	$0.8587^{(9)}$	$0.78247^{(8)}$	$0.8724^{(10)}$
	MSE($\hat{\chi}$)	$1.75446^{(9)}$	$1.62791^{(7)}$	$1.82366^{(10)}$	$1.45732^{(1)}$	$1.63994^{(8)}$	$1.56583^{(4)}$	$1.57031^{(5)}$	$1.55387^{(3)}$	$1.53896^{(2)}$	$1.58455^{(6)}$
	MRE($\hat{\Phi}$)	$0.36432^{(1)}$	$0.36853^{(2)}$	$0.37537^{(7)}$	$0.37383^{(5)}$	$0.37326^{(4)}$	$0.37454^{(6)}$	$0.3699^{(3)}$	$0.39699^{(9)}$	$0.37654^{(8)}$	$0.40131^{(10)}$
	MRE($\hat{\chi}$)	$0.3572^{(9)}$	$0.34409^{(6)}$	$0.36678^{(10)}$	$0.32659^{(1)}$	$0.34524^{(7)}$	$0.33391^{(3)}$	$0.33698^{(4)}$	$0.34165^{(5)}$	$0.33381^{(2)}$	$0.34644^{(8)}$
	D_{abs}	$0.02991^{(1)}$	$0.03078^{(2)}$	$0.03131^{(3)}$	$0.03154^{(4)}$	$0.03294^{(5)}$	$0.04468^{(8)}$	$0.03879^{(7)}$	$0.05608^{(9)}$	$0.03589^{(6)}$	$0.05714^{(10)}$
	D_{max}	$0.04912^{(1)}$	$0.04921^{(2)}$	$0.05094^{(4)}$	$0.04961^{(3)}$	$0.05192^{(5)}$	$0.06901^{(8)}$	$0.06045^{(7)}$	$0.08413^{(9)}$	$0.05617^{(6)}$	$0.08573^{(10)}$
	ASAE	$0.0442^{(1)}$	$0.03892^{(2)}$	$0.03951^{(4)}$	$0.03921^{(3)}$	$0.03821^{(5)}$	$0.05169^{(8)}$	$0.04863^{(7)}$	$0.06324^{(9)}$	$0.04676^{(6)}$	$0.06433^{(10)}$
	$\sum Ranks$	$37^{(3)}$	$31^{(2)}$	$59^{(8)}$	$30^{(1)}$	$46^{(5)}$	$52^{(7)}$	$43^{(4)}$	$67^{(9)}$	$48^{(6)}$	$82^{(10)}$
60	BIAS($\hat{\Phi}$)	$0.61776^{(1)}$	$0.65788^{(2)}$	$0.69055^{(5)}$	$0.68316^{(4)}$	$0.69141^{(6)}$	$0.69202^{(7)}$	$0.70241^{(8)}$	$0.74605^{(10)}$	$0.67449^{(3)}$	$0.74477^{(9)}$
	BIAS($\hat{\chi}$)	$0.82238^{(3)}$	$0.84483^{(5)}$	$0.98137^{(10)}$	$0.76071^{(1)}$	$0.90158^{(9)}$	$0.84962^{(6)}$	$0.83267^{(4)}$	$0.90125^{(8)}$	$0.80341^{(2)}$	$0.90055^{(7)}$
	MSE($\hat{\Phi}$)	$0.55626^{(1)}$	$0.63629^{(2)}$	$0.67033^{(4)}$	$0.70383^{(7)}$	$0.68814^{(5)}$	$0.69925^{(6)}$	$0.72272^{(8)}$	$0.7903^{(10)}$	$0.66951^{(3)}$	$0.7873^{(9)}$
	MSE($\hat{\chi}$)	$1.09073^{(4)}$	$1.11654^{(5)}$	$1.49371^{(10)}$	$0.87761^{(1)}$	$1.26437^{(9)}$	$1.14418^{(6)}$	$1.07947^{(3)}$	$1.22906^{(8)}$	$1.01157^{(2)}$	$1.21413^{(7)}$
	MRE($\hat{\Phi}$)	$0.30888^{(1)}$	$0.32894^{(2)}$	$0.34527^{(5)}$	$0.34158^{(4)}$	$0.34571^{(6)}$	$0.34601^{(7)}$	$0.3512^{(8)}$	$0.37302^{(10)}$	$0.33724^{(3)}$	$0.37238^{(9)}$
	MRE($\hat{\chi}$)	$0.27413^{(3)}$	$0.28161^{(5)}$	$0.32712^{(10)}$	$0.25357^{(1)}$	$0.30053^{(9)}$	$0.28321^{(6)}$	$0.27756^{(4)}$	$0.30042^{(8)}$	$0.2678^{(2)}$	$0.30018^{(7)}$
	D_{abs}	$0.01868^{(3)}$	$0.01866^{(2)}$	$0.01977^{(5)}$	$0.01824^{(1)}$	$0.01953^{(4)}$	$0.02723^{(8)}$	$0.02497^{(7)}$	$0.0322^{(9)}$	$0.0222^{(6)}$	$0.03236^{(10)}$
	D_{max}	$0.03123^{(2)}$	$0.03132^{(3)}$	$0.03392^{(5)}$	$0.03042^{(1)}$	$0.03304^{(4)}$	$0.04371^{(8)}$	$0.04031^{(7)}$	$0.05115^{(9)}$	$0.03617^{(6)}$	$0.05138^{(10)}$
	ASAE	$0.02209^{(2)}$	$0.02014^{(3)}$	$0.02102^{(5)}$	$0.02068^{(1)}$	$0.02064^{(4)}$	$0.0287^{(8)}$	$0.02688^{(7)}$	$0.03337^{(9)}$	$0.02476^{(6)}$	$0.03343^{(10)}$
	$\sum Ranks$	$23^{(1.5)}$	$27^{(3)}$	$58^{(7)}$	$23^{(1.5)}$	$54^{(5)}$	$62^{(8)}$	$56^{(6)}$	$81^{(10)}$	$33^{(4)}$	$78^{(9)}$
100	BIAS($\hat{\Phi}$)	$0.53968^{(1)}$	$0.61745^{(2)}$	$0.64311^{(4)}$	$0.64335^{(5)}$	$0.65103^{(6)}$	$0.65445^{(7)}$	$0.65894^{(8)}$	$0.71571^{(9)}$	$0.63393^{(3)}$	$0.71795^{(10)}$
	BIAS($\hat{\chi}$)	$0.67717^{(2)}$	$0.74381^{(5)}$	$0.86727^{(10)}$	$0.66974^{(1)}$	$0.81647^{(7)}$	$0.75329^{(6)}$	$0.72011^{(4)}$	$0.8218^{(8)}$	$0.71035^{(3)}$	$0.82189^{(9)}$
	MSE($\hat{\Phi}$)	$0.4473^{(1)}$	$0.57858^{(2)}$	$0.60363^{(3)}$	$0.65038^{(7)}$	$0.62893^{(5)}$	$0.64147^{(6)}$	$0.6567^{(8)}$	$0.74895^{(9)}$	$0.61206^{(4)}$	$0.75341^{(10)}$
	MSE($\hat{\chi}$)	$0.74254^{(2)}$	$0.8583^{(5)}$	$1.18026^{(10)}$	$0.66573^{(1)}$	$1.0347^{(9)}$	$0.89657^{(6)}$	$0.79602^{(4)}$	$1.02909^{(7)}$	$0.78523^{(3)}$	$1.03339^{(8)}$
	MRE($\hat{\Phi}$)	$0.26984^{(1)}$	$0.30872^{(2)}$	$0.32155^{(4)}$	$0.32167^{(5)}$	$0.32551^{(6)}$	$0.32723^{(7)}$	$0.32947^{(8)}$	$0.35786^{(9)}$	$0.31697^{(3)}$	$0.35897^{(10)}$
	MRE($\hat{\chi}$)	$0.22572^{(2)}$	$0.24794^{(5)}$	$0.28909^{(10)}$	$0.22325^{(1)}$	$0.27216^{(7)}$	$0.2511^{(6)}$	$0.24004^{(4)}$	$0.27393^{(8)}$	$0.23678^{(3)}$	$0.27396^{(9)}$
	D_{abs}	$0.01462^{(2)}$	$0.01475^{(3)}$	$0.0157^{(5)}$	$0.01441^{(1)}$	$0.01538^{(4)}$	$0.02198^{(8)}$	$0.01949^{(7)}$	$0.02479^{(9)}$	$0.01785^{(6)}$	$0.0253^{(10)}$
	D_{max}	$0.02457^{(2)}$	$0.02519^{(3)}$	$0.02749^{(5)}$	$0.02452^{(1)}$	$0.02674^{(4)}$	$0.03569^{(8)}$	$0.03195^{(7)}$	$0.0404^{(9)}$	$0.02947^{(6)}$	$0.04111^{(10)}$
	ASAE	$0.01608^{(2)}$	$0.01503^{(3)}$	$0.0157^{(5)}$	$0.0154^{(1)}$	$0.01556^{(4)}$	$0.02169^{(8)}$	$0.01992^{(7)}$	$0.02456^{(9)}$	$0.01836^{(6)}$	$0.02476^{(10)}$
	$\sum Ranks$	$18^{(1)}$	$28^{(3)}$	$55^{(6)}$	$24^{(2)}$	$51^{(5)}$	$62^{(8)}$	$57^{(7)}$	$77^{(9)}$	$37^{(4)}$	$86^{(10)}$
150	BIAS($\hat{\Phi}$)	$0.47879^{(1)}$	$0.54317^{(2)}$	$0.58659^{(5)}$	$0.58233^{(4)}$	$0.60149^{(6)}$	$0.60373^{(7)}$	$0.61968^{(8)}$	$0.67133^{(9)}$	$0.57801^{(3)}$	$0.6732^{(10)}$
	BIAS($\hat{\chi}$)	$0.57045^{(1)}$	$0.63461^{(4)}$	$0.75775^{(10)}$	$0.58924^{(2)}$	$0.72985^{(7)}$	$0.66344^{(6)}$	$0.64849^{(5)}$	$0.73385^{(8)}$	$0.61458^{(3)}$	$0.74464^{(9)}$
	MSE($\hat{\Phi}$)	$0.36536^{(1)}$	$0.4707^{(2)}$	$0.52632^{(3)}$	$0.55163^{(5)}$	$0.55904^{(6)}$	$0.57045^{(7)}$	$0.6062^{(8)}$	$0.67971^{(9)}$	$0.53322^{(4)}$	$0.68646^{(10)}$
	MSE($\hat{\chi}$)	$0.52138^{(2)}$	$0.62765^{(4)}$	$0.91856^{(10)}$	$0.51266^{(1)}$	$0.83255^{(8)}$	$0.69096^{(6)}$	$0.63181^{(5)}$	$0.81996^{(7)}$	$0.58436^{(3)}$	$0.84626^{(9)}$
	MRE($\hat{\Phi}$)	$0.2394^{(1)}$	$0.27159^{(2)}$	$0.29329^{(5)}$	$0.29117^{(4)}$	$0.30075^{(6)}$	$0.30187^{(7)}$	$0.30984^{(8)}$	$0.33567^{(9)}$	$0.289^{(3)}$	$0.3366^{(10)}$
	MRE($\hat{\chi}$)	$0.19015^{(1)}$	$0.21154^{(4)}$	$0.25258^{(10)}$	$0.19641^{(2)}$	$0.24328^{(7)}$	$0.22115^{(6)}$	$0.21616^{(5)}$	$0.24462^{(8)}$	$0.20486^{(3)}$	$0.24821^{(9)}$
	D_{abs}	$0.01208^{(2)}$	$0.01229^{(3)}$	$0.01309^{(5)}$	$0.01204^{(1)}$	$0.01291^{(4)}$	$0.01807^{(8)}$	$0.01617^{(7)}$	$0.02034^{(9)}$	$0.01504^{(6)}$	$0.02037^{(10)}$
	D_{max}	$0.0204^{(1)}$	$0.02106^{(3)}$	$0.02302^{(5)}$	$0.02054^{(2)}$	$0.02269^{(4)}$	$0.0296^{(8)}$	$0.02682^{(7)}$	$0.03356^{(9)}$	$0.02489^{(6)}$	$0.03361^{(10)}$
	ASAE	$0.01258^{(1)}$	$0.01184^{(3)}$	$0.01252^{(5)}$	$0.01215^{(2)}$	$0.01242^{(4)}$	$0.01721^{(8)}$	$0.01577^{(7)}$	$0.01923^{(9)}$	$0.01468^{(6)}$	$0.01927^{(10)}$
	$\sum Ranks$	$15^{(1)}$	$25^{(3)}$	$57^{(6)}$	$23^{(2)}$	$51^{(5)}$	$63^{(8)}$	$60^{(7)}$	$77^{(9)}$	$37^{(4)}$	$87^{(10)}$
200	BIAS($\hat{\Phi}$)	$0.41779^{(1)}$	$0.50262^{(2)}$	$0.55873^{(5)}$	$0.53788^{(4)}$	$0.57729^{(8)}$	$0.57525^{(7)}$	$0.56788^{(6)}$	$0.64695^{(9)}$	$0.53384^{(3)}$	$0.64711^{(10)}$
	BIAS($\hat{\chi}$)	$0.48643^{(1)}$	$0.5699^{(4)}$	$0.69387^{(10)}$	$0.53437^{(2)}$	$0.68091^{(7)}$	$0.61815^{(6)}$	$0.58^{(5)}$	$0.68267^{(8)}$	$0.55451^{(3)}$	$0.68631^{(9)}$
	MSE($\hat{\Phi}$)	$0.28496^{(1)}$	$0.41433^{(2)}$	$0.48438^{(4)}$	$0.48498^{(5)}$	$0.52338^{(6)}$	$0.53193^{(8)}$	$0.52666^{(7)}$	$0.64828^{(9)}$	$0.46898^{(3)}$	$0.64853^{(10)}$
	MSE($\hat{\chi}$)	$0.38007^{(1)}$	$0.50636^{(4)}$	$0.7582^{(10)}$	$0.42454^{(2)}$	$0.71977^{(9)}$	$0.59776^{(6)}$	$0.5092^{(5)}$	$0.70723^{(7)}$	$0.47009^{(3)}$	$0.71303^{(8)}$
	MRE($\hat{\Phi}$)	$0.2089^{(1)}$	$0.25131^{(2)}$	$0.27936^{(5)}$	$0.26894^{(4)}$	$0.28865^{(8)}$	$0.28763^{(7)}$	$0.28394^{(6)}$	$0.32347^{(9)}$	$0.26692^{(3)}$	$0.32356^{(10)}$
	MRE($\hat{\chi}$)	$0.16214^{(1)}$	$0.18997^{(4)}$	$0.23129^{(10)}$	$0.17812^{(2)}$	$0.22697^{(7)}$	$0.20605^{(6)}$	$0.19333^{(5)}$	$0.22756^{(8)}$	$0.18484^{(3)}$	$0.22877^{(9)}$
	D_{abs}	$0.01046^{(1)}$	$0.01071^{(3)}$	$0.01158^{(5)}$	$0.01062^{(2)}$	$0.01136^{(4)}$	$0.01599^{(8)}$	$0.0143^{(7)}$	$0.01798^{(10)}$	$0.01321^{(6)}$	$0.01784^{(9)}$
	D_{max}	$0.0176^{(1)}$	$0.01847^{(3)}$	$0.02051^{(5)}$	$0.01817^{(2)}$	$0.02017^{(4)}$	$0.02636^{(8)}$	$0.02373^{(7)}$	$0.02979^{(10)}$	$0.02197^{(6)}$	$0.02957^{(9)}$
	ASAE	$0.01053^{(1)}$	$0.01008^{(3)}$	$0.01063^{(5)}$	$0.0103^{(2)}$	$0.01062^{(4)}$	$0.01465^{(8)}$	$0.01339^{(7)}$	$0.01645^{(10)}$	$0.0123^{(6)}$	$0.01633^{(9)}$
	$\sum Ranks$	$11^{(1)}$	$25^{(2.5)}$	$59^{(7)}$	$25^{(2.5)}$	$57^{(6)}$	$64^{(8)}$	$55^{(5)}$	$80^{(9)}$	$36^{(4)}$	$83^{(10)}$
300	BIAS($\hat{\Phi}$)	$0.3529^{(1)}$	$0.43011^{(2)}$	$0.48832^{(5)}$	$0.44856^{(3)}$	$0.50327^{(6)}$	$0.52628^{(8)}$	$0.50835^{(7)}$	$0.59055^{(10)}$	$0.46951^{(4)}$	$0.58769^{(9)}$
	BIAS($\hat{\chi}$)	$0.40264^{(1)}$	$0.47915^{(3)}$	$0.5826^{(8)}$	$0.45224^{(2)}$	$0.57469^{(7)}$	$0.5462^{(6)}$	$0.50824^{(5)}$	$0.60658^{(10)}$	$0.48229^{(4)}$	$0.59822^{(9)}$
	MSE($\hat{\Phi}$)	$0.20498^{(1)}$	$0.31308^{(2)}$	$0.39147^{(5)}$	$0.34793^{(3)}$	$0.41647^{(6)}$	$0.45752^{(8)}$	$0.43529^{(7)}$	$0.56132^{(10)}$	$0.37421^{(4)}$	$0.55775^{(9)}$
	MSE($\hat{\chi}$)	$0.25568^{(1)}$	$0.35443^{(3)}$	$0.53331^{(8)}$	$0.30789^{(2)}$	$0.50856^{(7)}$	$0.45732^{(6)}$	$0.38574^{(5)}$	$0.55907^{(10)}$	$0.35526^{(4)}$	$0.54108^{(9)}$
	MRE($\hat{\Phi}$)	$0.17645^{(1)}$	$0.21506^{(2)}$	$0.24416^{(5)}$	$0.22428^{(3)}$	$0.25164^{(6)}$	$0.26314^{(8)}$	$0.25417^{(7)}$	$0.29527^{(10)}$	$0.23475^{(4)}$	$0.29385^{(9)}$
	MRE($\hat{\chi}$)	$0.13421^{(1)}$	$0.15972^{(3)}$	$0.1942^{(8)}$	$0.15075^{(2)}$	$0.19156^{(7)}$	$0.18207^{(6)}$	$0.16941^{(5)}$	$0.20219^{(10)}$	$0.16076^{(4)}$	$0.19941^{(9)}$
	D_{abs}	$0.00854^{(1)}$	$0.00893^{(3)}$	$0.00956^{(5)}$	$0.00886^{(2)}$	$0.00947^{(4)}$	$0.01353^{(8)}$	$0.01205^{(7)}$	$0.01513^{(10)}$	$0.01106^{(6)}$	$0.01503^{(9)}$
	D_{max}	$0.01441^{(1)}$	$0.01539^{(3)}$	$0.01697^{(5)}$	$0.01516^{(2)}$	$0.01683^{(4)}$	$0.02236^{(8)}$	$0.02002^{(7)}$	$0.02514^{(10)}$	$0.01843^{(6)}$	$0.02499^{(9)}$
	ASAE	$0.00837^{(1)}$	$0.00799^{(3)}$	$0.00849^{(5)}$	$0.00819^{(2)}$	$0.00842^{(4)}$	$0.01162^{(8)}$	$0.01069^{(7)}$	$0.01307^{(10)}$	$0.00981^{(6)}$	$0.01309^{(9)}$
	$\sum Ranks$	$11^{(1)}$	$22^{(3)}$	$54^{(6)}$	$21^{(2)}$	$51^{(5)}$	$66^{(8)}$	$57^{(7)}$	$89^{(10)}$	$42^{(4)}$	$82^{(9)}$

TABLE 5. Numerical values of simulation measures for $\Phi = 1.5$ and $\chi = 0.7$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.58761^{\{4\}}$	$0.59714^{\{5\}}$	$0.59813^{\{8\}}$	$0.58476^{\{1\}}$	$0.59775^{\{7\}}$	$0.58605^{\{2\}}$	$0.58628^{\{3\}}$	$0.6207^{\{10\}}$	$0.59759^{\{6\}}$	$0.61777^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.25434^{\{9\}}$	$0.24919^{\{7\}}$	$0.26603^{\{10\}}$	$0.22536^{\{1\}}$	$0.24976^{\{8\}}$	$0.23796^{\{3\}}$	$0.2358^{\{2\}}$	$0.2422^{\{6\}}$	$0.23875^{\{5\}}$	$0.2381^{\{4\}}$
	MSE($\hat{\Phi}$)	$0.46083^{\{1\}}$	$0.47631^{\{6\}}$	$0.47331^{\{4\}}$	$0.47367^{\{5\}}$	$0.47962^{\{7\}}$	$0.47118^{\{3\}}$	$0.47015^{\{2\}}$	$0.51739^{\{10\}}$	$0.48384^{\{8\}}$	$0.51246^{\{9\}}$
	MSE($\hat{\chi}$)	$0.10027^{\{9\}}$	$0.09534^{\{8\}}$	$0.10663^{\{10\}}$	$0.07912^{\{1\}}$	$0.09474^{\{7\}}$	$0.08801^{\{5\}}$	$0.08621^{\{3\}}$	$0.08723^{\{4\}}$	$0.08865^{\{6\}}$	$0.08467^{\{2\}}$
	MRE($\hat{\Phi}$)	$0.39174^{\{4\}}$	$0.3981^{\{5\}}$	$0.39875^{\{8\}}$	$0.38984^{\{1\}}$	$0.3985^{\{7\}}$	$0.3907^{\{2\}}$	$0.39086^{\{3\}}$	$0.4138^{\{10\}}$	$0.39839^{\{6\}}$	$0.41185^{\{9\}}$
	MRE($\hat{\chi}$)	$0.36334^{\{9\}}$	$0.35598^{\{7\}}$	$0.38005^{\{10\}}$	$0.32194^{\{1\}}$	$0.3568^{\{8\}}$	$0.33994^{\{3\}}$	$0.33686^{\{2\}}$	$0.34599^{\{6\}}$	$0.34107^{\{5\}}$	$0.34014^{\{4\}}$
	D_{abs}	$0.04714^{\{1\}}$	$0.04891^{\{2\}}$	$0.05035^{\{3\}}$	$0.05054^{\{4\}}$	$0.05271^{\{6\}}$	$0.05812^{\{8\}}$	$0.0557^{\{7\}}$	$0.06944^{\{9\}}$	$0.05062^{\{5\}}$	$0.07064^{\{10\}}$
	D_{max}	$0.07589^{\{1\}}$	$0.07634^{\{2\}}$	$0.08026^{\{5\}}$	$0.07719^{\{3\}}$	$0.08111^{\{6\}}$	$0.08913^{\{8\}}$	$0.08496^{\{7\}}$	$0.10419^{\{9\}}$	$0.07839^{\{4\}}$	$0.10571^{\{10\}}$
	ASAE	$0.04542^{\{1\}}$	$0.03979^{\{2\}}$	$0.03963^{\{5\}}$	$0.0414^{\{3\}}$	$0.03857^{\{6\}}$	$0.05646^{\{8\}}$	$0.05265^{\{7\}}$	$0.06326^{\{9\}}$	$0.04906^{\{4\}}$	$0.06416^{\{10\}}$
	$\sum Ranks$	$43^{\{4\}}$	$45^{\{5\}}$	$60^{\{8\}}$	$21^{\{1\}}$	$57^{\{7\}}$	$42^{\{3\}}$	$36^{\{2\}}$	$73^{\{10\}}$	$51^{\{6\}}$	$67^{\{9\}}$
60	BIAS($\hat{\Phi}$)	$0.50418^{\{1\}}$	$0.51991^{\{2\}}$	$0.54823^{\{8\}}$	$0.5253^{\{4\}}$	$0.53912^{\{7\}}$	$0.5238^{\{3\}}$	$0.52625^{\{5\}}$	$0.56947^{\{10\}}$	$0.53287^{\{6\}}$	$0.56927^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.19532^{\{5\}}$	$0.20013^{\{6\}}$	$0.23483^{\{10\}}$	$0.17359^{\{1\}}$	$0.21777^{\{9\}}$	$0.18975^{\{4\}}$	$0.1835^{\{2\}}$	$0.20382^{\{7\}}$	$0.18562^{\{3\}}$	$0.20449^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.36013^{\{1\}}$	$0.38254^{\{2\}}$	$0.41124^{\{8\}}$	$0.40434^{\{5\}}$	$0.40655^{\{7\}}$	$0.39841^{\{3\}}$	$0.40373^{\{4\}}$	$0.45399^{\{9\}}$	$0.40595^{\{6\}}$	$0.45574^{\{10\}}$
	MSE($\hat{\chi}$)	$0.06224^{\{5\}}$	$0.06365^{\{6\}}$	$0.08543^{\{10\}}$	$0.0469^{\{1\}}$	$0.07482^{\{9\}}$	$0.05739^{\{4\}}$	$0.05339^{\{2\}}$	$0.06405^{\{7\}}$	$0.05488^{\{3\}}$	$0.06412^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.33612^{\{1\}}$	$0.34661^{\{2\}}$	$0.36549^{\{8\}}$	$0.3502^{\{4\}}$	$0.35941^{\{7\}}$	$0.3492^{\{3\}}$	$0.35084^{\{5\}}$	$0.37965^{\{10\}}$	$0.35524^{\{6\}}$	$0.37951^{\{9\}}$
	MRE($\hat{\chi}$)	$0.27903^{\{5\}}$	$0.2859^{\{6\}}$	$0.33548^{\{10\}}$	$0.24799^{\{1\}}$	$0.3111^{\{9\}}$	$0.27107^{\{4\}}$	$0.26214^{\{2\}}$	$0.29118^{\{7\}}$	$0.26517^{\{3\}}$	$0.29212^{\{8\}}$
	D_{abs}	$0.02945^{\{2\}}$	$0.02947^{\{3\}}$	$0.03044^{\{4\}}$	$0.0291^{\{1\}}$	$0.03085^{\{5\}}$	$0.0354^{\{8\}}$	$0.0327^{\{7\}}$	$0.03986^{\{9\}}$	$0.03143^{\{6\}}$	$0.0399^{\{10\}}$
	D_{max}	$0.04797^{\{3\}}$	$0.04788^{\{2\}}$	$0.0509^{\{6\}}$	$0.04648^{\{1\}}$	$0.05072^{\{5\}}$	$0.05616^{\{8\}}$	$0.05204^{\{7\}}$	$0.06304^{\{9\}}$	$0.05014^{\{4\}}$	$0.0632^{\{10\}}$
	ASAE	$0.02358^{\{3\}}$	$0.02137^{\{2\}}$	$0.02173^{\{6\}}$	$0.02282^{\{1\}}$	$0.02123^{\{5\}}$	$0.03192^{\{8\}}$	$0.0296^{\{7\}}$	$0.03499^{\{9\}}$	$0.02719^{\{4\}}$	$0.0351^{\{10\}}$
	$\sum Ranks$	$28^{\{2\}}$	$31^{\{3\}}$	$67^{\{8\}}$	$22^{\{1\}}$	$59^{\{7\}}$	$45^{\{6\}}$	$41^{\{4\}}$	$77^{\{9\}}$	$43^{\{5\}}$	$82^{\{10\}}$
100	BIAS($\hat{\Phi}$)	$0.44733^{\{1\}}$	$0.47966^{\{2\}}$	$0.51303^{\{8\}}$	$0.48514^{\{4\}}$	$0.5041^{\{7\}}$	$0.48559^{\{5\}}$	$0.50112^{\{6\}}$	$0.54031^{\{10\}}$	$0.482^{\{3\}}$	$0.53595^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.16171^{\{5\}}$	$0.1716^{\{6\}}$	$0.2062^{\{10\}}$	$0.14724^{\{1\}}$	$0.19241^{\{9\}}$	$0.16094^{\{4\}}$	$0.15989^{\{3\}}$	$0.18283^{\{8\}}$	$0.15449^{\{2\}}$	$0.18201^{\{7\}}$
	MSE($\hat{\Phi}$)	$0.29653^{\{1\}}$	$0.33871^{\{2\}}$	$0.37466^{\{7\}}$	$0.36587^{\{5\}}$	$0.36767^{\{6\}}$	$0.35631^{\{4\}}$	$0.37864^{\{8\}}$	$0.42225^{\{10\}}$	$0.35315^{\{3\}}$	$0.4181^{\{9\}}$
	MSE($\hat{\chi}$)	$0.04299^{\{5\}}$	$0.04679^{\{6\}}$	$0.06726^{\{10\}}$	$0.03308^{\{1\}}$	$0.05879^{\{9\}}$	$0.04151^{\{4\}}$	$0.03971^{\{3\}}$	$0.05193^{\{8\}}$	$0.03784^{\{2\}}$	$0.05178^{\{7\}}$
	MRE($\hat{\Phi}$)	$0.29822^{\{1\}}$	$0.31977^{\{2\}}$	$0.34202^{\{8\}}$	$0.32342^{\{4\}}$	$0.33607^{\{7\}}$	$0.32373^{\{5\}}$	$0.33408^{\{6\}}$	$0.36021^{\{10\}}$	$0.32133^{\{3\}}$	$0.3573^{\{9\}}$
	MRE($\hat{\chi}$)	$0.23101^{\{5\}}$	$0.24514^{\{6\}}$	$0.29457^{\{10\}}$	$0.21034^{\{1\}}$	$0.27487^{\{9\}}$	$0.22991^{\{4\}}$	$0.22841^{\{3\}}$	$0.26118^{\{8\}}$	$0.2207^{\{2\}}$	$0.26002^{\{7\}}$
	D_{abs}	$0.02328^{\{2\}}$	$0.02335^{\{3\}}$	$0.02434^{\{5\}}$	$0.02259^{\{1\}}$	$0.0241^{\{4\}}$	$0.02813^{\{8\}}$	$0.02582^{\{7\}}$	$0.03045^{\{9\}}$	$0.02468^{\{6\}}$	$0.03082^{\{10\}}$
	D_{max}	$0.038^{\{2\}}$	$0.03831^{\{3\}}$	$0.04119^{\{6\}}$	$0.03647^{\{1\}}$	$0.04035^{\{5\}}$	$0.04495^{\{8\}}$	$0.04148^{\{7\}}$	$0.04917^{\{9\}}$	$0.03966^{\{4\}}$	$0.04974^{\{10\}}$
	ASAE	$0.01755^{\{2\}}$	$0.01615^{\{3\}}$	$0.01642^{\{6\}}$	$0.01731^{\{1\}}$	$0.0163^{\{5\}}$	$0.02431^{\{8\}}$	$0.02243^{\{7\}}$	$0.02684^{\{9\}}$	$0.02073^{\{4\}}$	$0.02673^{\{10\}}$
	$\sum Ranks$	$27^{\{2\}}$	$31^{\{3.5\}}$	$67^{\{8\}}$	$22^{\{1\}}$	$58^{\{7\}}$	$50^{\{5.5\}}$	$50^{\{5.5\}}$	$82^{\{10\}}$	$31^{\{3.5\}}$	$77^{\{9\}}$
150	BIAS($\hat{\Phi}$)	$0.40135^{\{1\}}$	$0.43881^{\{2\}}$	$0.46948^{\{8\}}$	$0.43892^{\{3\}}$	$0.46935^{\{7\}}$	$0.45457^{\{5\}}$	$0.46027^{\{6\}}$	$0.50962^{\{9\}}$	$0.44117^{\{4\}}$	$0.51049^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.13437^{\{2\}}$	$0.14834^{\{6\}}$	$0.17961^{\{10\}}$	$0.12692^{\{1\}}$	$0.16805^{\{9\}}$	$0.14268^{\{5\}}$	$0.13882^{\{4\}}$	$0.16334^{\{7\}}$	$0.13465^{\{3\}}$	$0.16389^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.24805^{\{1\}}$	$0.29444^{\{2\}}$	$0.32441^{\{6\}}$	$0.30999^{\{4\}}$	$0.33159^{\{7\}}$	$0.32248^{\{5\}}$	$0.33185^{\{8\}}$	$0.38854^{\{9\}}$	$0.30623^{\{3\}}$	$0.3893^{\{10\}}$
	MSE($\hat{\chi}$)	$0.02886^{\{3\}}$	$0.03451^{\{6\}}$	$0.05162^{\{10\}}$	$0.02412^{\{1\}}$	$0.04472^{\{9\}}$	$0.03214^{\{5\}}$	$0.02939^{\{4\}}$	$0.04109^{\{7\}}$	$0.02848^{\{2\}}$	$0.04166^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.26757^{\{1\}}$	$0.29254^{\{2\}}$	$0.31299^{\{8\}}$	$0.29262^{\{3\}}$	$0.3129^{\{7\}}$	$0.30305^{\{5\}}$	$0.30685^{\{6\}}$	$0.33975^{\{9\}}$	$0.29412^{\{4\}}$	$0.34033^{\{10\}}$
	MRE($\hat{\chi}$)	$0.19195^{\{2\}}$	$0.21191^{\{6\}}$	$0.25659^{\{10\}}$	$0.18132^{\{1\}}$	$0.24007^{\{9\}}$	$0.20383^{\{5\}}$	$0.19832^{\{4\}}$	$0.23334^{\{7\}}$	$0.19235^{\{3\}}$	$0.23413^{\{8\}}$
	D_{abs}	$0.0192^{\{2\}}$	$0.01958^{\{3\}}$	$0.0204^{\{5\}}$	$0.01882^{\{1\}}$	$0.01992^{\{4\}}$	$0.02315^{\{8\}}$	$0.02181^{\{7\}}$	$0.02495^{\{10\}}$	$0.02095^{\{6\}}$	$0.02492^{\{9\}}$
	D_{max}	$0.03132^{\{2\}}$	$0.0322^{\{3\}}$	$0.03458^{\{6\}}$	$0.03047^{\{1\}}$	$0.03354^{\{4\}}$	$0.03724^{\{8\}}$	$0.03507^{\{7\}}$	$0.04061^{\{10\}}$	$0.03367^{\{5\}}$	$0.0406^{\{9\}}$
	ASAE	$0.01402^{\{2\}}$	$0.01305^{\{3\}}$	$0.01322^{\{6\}}$	$0.01401^{\{1\}}$	$0.01312^{\{4\}}$	$0.01965^{\{8\}}$	$0.01815^{\{7\}}$	$0.02145^{\{10\}}$	$0.0166^{\{5\}}$	$0.02152^{\{9\}}$
	$\sum Ranks$	$19^{\{1.5\}}$	$31^{\{3\}}$	$66^{\{8\}}$	$19^{\{1.5\}}$	$58^{\{7\}}$	$54^{\{6\}}$	$53^{\{5\}}$	$77^{\{9\}}$	$36^{\{4\}}$	$82^{\{10\}}$
200	BIAS($\hat{\Phi}$)	$0.35979^{\{1\}}$	$0.40326^{\{2\}}$	$0.43909^{\{7\}}$	$0.40587^{\{4\}}$	$0.44817^{\{8\}}$	$0.42983^{\{5\}}$	$0.43018^{\{6\}}$	$0.47714^{\{9\}}$	$0.40566^{\{3\}}$	$0.48753^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.11793^{\{2\}}$	$0.13226^{\{6\}}$	$0.16101^{\{10\}}$	$0.11599^{\{1\}}$	$0.15725^{\{9\}}$	$0.13164^{\{5\}}$	$0.12498^{\{4\}}$	$0.14909^{\{7\}}$	$0.11971^{\{3\}}$	$0.15031^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.20518^{\{1\}}$	$0.25855^{\{2\}}$	$0.29141^{\{5\}}$	$0.27322^{\{4\}}$	$0.30456^{\{8\}}$	$0.29861^{\{7\}}$	$0.29467^{\{6\}}$	$0.35334^{\{9\}}$	$0.26555^{\{3\}}$	$0.3645^{\{10\}}$
	MSE($\hat{\chi}$)	$0.02235^{\{3\}}$	$0.02736^{\{5\}}$	$0.04131^{\{10\}}$	$0.0202^{\{1\}}$	$0.03871^{\{9\}}$	$0.02741^{\{6\}}$	$0.02341^{\{4\}}$	$0.0346^{\{8\}}$	$0.02209^{\{2\}}$	$0.03456^{\{7\}}$
	MRE($\hat{\Phi}$)	$0.23986^{\{1\}}$	$0.26884^{\{2\}}$	$0.29272^{\{7\}}$	$0.27058^{\{4\}}$	$0.29878^{\{8\}}$	$0.28656^{\{5\}}$	$0.28679^{\{6\}}$	$0.31809^{\{9\}}$	$0.27044^{\{3\}}$	$0.32502^{\{10\}}$
	MRE($\hat{\chi}$)	$0.16847^{\{2\}}$	$0.18894^{\{6\}}$	$0.23001^{\{10\}}$	$0.1657^{\{1\}}$	$0.22464^{\{9\}}$	$0.18806^{\{5\}}$	$0.17854^{\{4\}}$	$0.21299^{\{7\}}$	$0.17101^{\{3\}}$	$0.21472^{\{8\}}$
	D_{abs}	$0.01684^{\{2\}}$	$0.0171^{\{3\}}$	$0.01785^{\{5\}}$	$0.01643^{\{1\}}$	$0.01753^{\{4\}}$	$0.0202^{\{8\}}$	$0.01909^{\{7\}}$	$0.02169^{\{9\}}$	$0.0185^{\{6\}}$	$0.02192^{\{10\}}$
	D_{max}	$0.02741^{\{2\}}$	$0.02818^{\{3\}}$	$0.03028^{\{6\}}$	$0.02673^{\{1\}}$	$0.02972^{\{4\}}$	$0.03261^{\{8\}}$	$0.03081^{\{7\}}$	$0.03543^{\{9\}}$	$0.02978^{\{5\}}$	$0.03578^{\{10\}}$
	ASAE	$0.01205^{\{2\}}$	$0.01118^{\{3\}}$	$0.01138^{\{6\}}$	$0.012^{\{1\}}$	$0.0113^{\{4\}}$	$0.01689^{\{8\}}$	$0.01553^{\{7\}}$	$0.01849^{\{9\}}$	$0.01429^{\{5\}}$	$0.01858^{\{10\}}$
	$\sum Ranks$	$19^{\{1\}}$	$30^{\{3\}}$	$63^{\{8\}}$	$21^{\{2\}}$	$61^{\{7\}}$	$57^{\{6\}}$	$51^{\{5\}}$	$76^{\{9\}}$	$34^{\{4\}}$	$83^{\{10\}}$
300	BIAS($\hat{\Phi}$)	$0.30257^{\{1\}}$	$0.3539^{\{4\}}$	$0.39254^{\{7\}}$	$0.34121^{\{2\}}$	$0.40207^{\{8\}}$	$0.38874^{\{6\}}$	$0.38207^{\{5\}}$	$0.43034^{\{9\}}$	$0.35278^{\{3\}}$	$0.4348^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.0959^{\{1\}}$	$0.11241^{\{5\}}$	$0.13494^{\{10\}}$	$0.09684^{\{2\}}$	$0.13396^{\{9\}}$	$0.11495^{\{6\}}$	$0.10865^{\{4\}}$	$0.1283^{\{7\}}$	$0.10244^{\{3\}}$	$0.12946^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.15068^{\{1\}}$	$0.2058^{\{3\}}$	$0.2446^{\{6\}}$	$0.19756^{\{2\}}$	$0.25711^{\{8\}}$	$0.25143^{\{7\}}$	$0.24425^{\{5\}}$	$0.29838^{\{9\}}$	$0.20998^{\{4\}}$	$0.30273^{\{10\}}$
	MSE($\hat{\chi}$)	$0.01467^{\{2\}}$	$0.01949^{\{5\}}$	$0.02872^{\{10\}}$	$0.01417^{\{1\}}$	$0.02768^{\{9\}}$	$0.02067^{\{6\}}$	$0.01783^{\{4\}}$	$0.0253^{\{7\}}$	$0.01603^{\{3\}}$	$0.02572^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.20171^{\{1\}}$	$0.23594^{\{4\}}$	$0.2617^{\{7\}}$	$0.22747^{\{2\}}$	$0.26805^{\{8\}}$	$0.25916^{\{6\}}$	$0.25471^{\{5\}}$	$0.28689^{\{9\}}$	$0.23519^{\{3\}}$	$0.28986^{\{10\}}$
	MRE($\hat{\chi}$)	$0.13701^{\{1\}}$	$0.16058^{\{5\}}$	$0.19277^{\{10\}}$	$0.13835^{\{2\}}$	$0.19137^{\{9\}}$	$0.16421^{\{6\}}$	$0.15521^{\{4\}}$	$0.18329^{\{7\}}$	$0.14634^{\{3\}}$	$0.18494^{\{8\}}$
	D_{abs}	$0.01392^{\{1\}}$	$0.01425^{\{3\}}$	$0.01469^{\{5\}}$	$0.01399^{\{2\}}$	$0.01464^{\{4\}}$	$0.01716^{\{8\}}$	$0.0161^{\{7\}}$	$0.01828^{\{9\}}$	$0.01517^{\{6\}}$	$0.0183^{\{10\}}$
	D_{max}	$0.02261^{\{1\}}$	$0.0235^{\{3\}}$	$0.02489^{\{6\}}$	$0.02267^{\{2\}}$	$0.0248^{\{5\}}$	$0.02773^{\{8\}}$	$0.02605^{\{7\}}$	$0.02985^{\{9\}}$	$0.02453^{\{4\}}$	$0.02991^{\{10\}}$
	ASAE	$0.00963^{\{1\}}$	$0.00902^{\{3\}}$	$0.00914^{\{6\}}$	$0.00971^{\{2\}}$	$0.00917^{\{5\}}$	$0.01363^{\{8\}}$	$0.0126^{\{7\}}$	$0.01501^{\{9\}}$	$0.01156^{\{4\}}$	$0.01505^{\{10\}}$
	$\sum Ranks$	$13^{\{1\}}$	$33^{\{3\}}$	$63^{\{7.5\}}$	$20^{\{2\}}$	$63^{\{7.5\}}$	$61^{\{6\}}$	$48^{\{5\}}$	$75^{\{9\}}$	$35^{\{4\}}$	$84^{\{10\}}$

TABLE 6. Numerical values of simulation measures for $\Phi = 1.5$ and $\chi = 0.7$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	0.54235$^{(1)}$	0.55137$^{(2)}$	0.56233$^{(4)}$	0.56618$^{(6)}$	0.57189$^{(8)}$	0.56284$^{(5)}$	0.5616$^{(3)}$	0.59766$^{(9)}$	0.5672$^{(7)}$	0.59856$^{(10)}$
	BIAS($\hat{\chi}$)	0.23667$^{(9)}$	0.22635$^{(4)}$	0.25062$^{(10)}$	0.21197$^{(1)}$	0.23172$^{(8)}$	0.22511$^{(3)}$	0.22661$^{(5)}$	0.23009$^{(6)}$	0.21989$^{(2)}$	0.23118$^{(7)}$
	MSE($\hat{\Phi}$)	0.40141$^{(1)}$	0.42353$^{(2)}$	0.42964$^{(3)}$	0.45283$^{(8)}$	0.4523$^{(7)}$	0.44534$^{(5)}$	0.441$^{(4)}$	0.48925$^{(9)}$	0.44737$^{(6)}$	0.49231$^{(10)}$
	MSE($\hat{\chi}$)	0.08734$^{(9)}$	0.08009$^{(7)}$	0.09632$^{(10)}$	0.06844$^{(1)}$	0.08178$^{(8)}$	0.07906$^{(3)}$	0.07988$^{(6)}$	0.07924$^{(4)}$	0.07557$^{(2)}$	0.07971$^{(5)}$
	MRE($\hat{\Phi}$)	0.36157$^{(1)}$	0.36758$^{(2)}$	0.37488$^{(4)}$	0.37746$^{(6)}$	0.38126$^{(8)}$	0.37523$^{(5)}$	0.3744$^{(3)}$	0.39844$^{(9)}$	0.37813$^{(7)}$	0.39904$^{(10)}$
	MRE($\hat{\chi}$)	0.33811$^{(9)}$	0.32335$^{(4)}$	0.35803$^{(10)}$	0.30281$^{(1)}$	0.33103$^{(8)}$	0.32159$^{(3)}$	0.32373$^{(5)}$	0.32869$^{(6)}$	0.31412$^{(2)}$	0.33026$^{(7)}$
	D_{abs}	0.03102$^{(1)}$	0.03154$^{(2)}$	0.03198$^{(4)}$	0.03183$^{(3)}$	0.03298$^{(5)}$	0.04474$^{(8)}$	0.0393$^{(7)}$	0.05576$^{(9.5)}$	0.03542$^{(6)}$	0.05576$^{(9.5)}$
	D_{max}	0.05146$^{(3)}$	0.051$^{(2)}$	0.05301$^{(4)}$	0.05093$^{(1)}$	0.05307$^{(5)}$	0.07008$^{(8)}$	0.06214$^{(7)}$	0.08502$^{(10)}$	0.05633$^{(6)}$	0.08492$^{(9)}$
	ASAE	0.04422$^{(3)}$	0.03918$^{(2)}$	0.03898$^{(4)}$	0.0402$^{(1)}$	0.03796$^{(5)}$	0.05455$^{(8)}$	0.05075$^{(7)}$	0.06272$^{(10)}$	0.04781$^{(6)}$	0.06282$^{(9)}$
	$\sum Ranks$	39$^{(3)}$	28$^{(1)}$	51$^{(7)}$	31$^{(2)}$	58$^{(8)}$	48$^{(6)}$	47$^{(5)}$	71.5$^{(9)}$	44$^{(4)}$	77.5$^{(10)}$
60	BIAS($\hat{\Phi}$)	0.44198$^{(1)}$	0.4833$^{(2)}$	0.50219$^{(4)}$	0.50483$^{(5)}$	0.51457$^{(7)}$	0.51027$^{(6)}$	0.515$^{(8)}$	0.55888$^{(10)}$	0.50037$^{(3)}$	0.55085$^{(9)}$
	BIAS($\hat{\chi}$)	0.16883$^{(2)}$	0.17766$^{(5)}$	0.2066$^{(10)}$	0.15869$^{(1)}$	0.19648$^{(8)}$	0.17953$^{(6)}$	0.17609$^{(4)}$	0.19698$^{(9)}$	0.17156$^{(3)}$	0.19318$^{(7)}$
	MSE($\hat{\Phi}$)	0.29106$^{(1)}$	0.34858$^{(2)}$	0.36285$^{(3)}$	0.39292$^{(7)}$	0.38601$^{(5)}$	0.38776$^{(6)}$	0.39315$^{(8)}$	0.44838$^{(10)}$	0.37506$^{(4)}$	0.43982$^{(9)}$
	MSE($\hat{\chi}$)	0.04645$^{(2)}$	0.04984$^{(5)}$	0.06765$^{(10)}$	0.03796$^{(1)}$	0.06071$^{(9)}$	0.05117$^{(6)}$	0.04823$^{(4)}$	0.05928$^{(8)}$	0.04685$^{(3)}$	0.057$^{(7)}$
	MRE($\hat{\Phi}$)	0.29465$^{(1)}$	0.3222$^{(2)}$	0.33479$^{(4)}$	0.33655$^{(5)}$	0.34305$^{(7)}$	0.34018$^{(6)}$	0.34333$^{(8)}$	0.37258$^{(10)}$	0.33358$^{(3)}$	0.36723$^{(9)}$
	MRE($\hat{\chi}$)	0.24119$^{(2)}$	0.2538$^{(5)}$	0.29514$^{(10)}$	0.2267$^{(1)}$	0.28069$^{(8)}$	0.25647$^{(6)}$	0.25156$^{(4)}$	0.28139$^{(9)}$	0.24509$^{(3)}$	0.27598$^{(7)}$
	D_{abs}	0.01895$^{(2)}$	0.01919$^{(3)}$	0.02019$^{(5)}$	0.01835$^{(1)}$	0.02012$^{(4)}$	0.02807$^{(8)}$	0.02453$^{(7)}$	0.03231$^{(10)}$	0.02265$^{(6)}$	0.03224$^{(9)}$
	D_{max}	0.03188$^{(2)}$	0.0325$^{(3)}$	0.035$^{(5)}$	0.03102$^{(1)}$	0.03466$^{(4)}$	0.04555$^{(8)}$	0.04025$^{(7)}$	0.05229$^{(10)}$	0.03725$^{(6)}$	0.0522$^{(9)}$
	ASAE	0.02278$^{(2)}$	0.0209$^{(3)}$	0.02128$^{(5)}$	0.022$^{(1)}$	0.02094$^{(4)}$	0.03115$^{(8)}$	0.02846$^{(7)}$	0.0347$^{(10)}$	0.02641$^{(6)}$	0.03474$^{(9)}$
	$\sum Ranks$	18$^{(1)}$	28$^{(3)}$	54$^{(5.5)}$	26$^{(2)}$	54$^{(5.5)}$	60$^{(8)}$	57$^{(7)}$	85$^{(10)}$	37$^{(4)}$	76$^{(9)}$
100	BIAS($\hat{\Phi}$)	0.37785$^{(1)}$	0.42483$^{(2)}$	0.46334$^{(5)}$	0.45803$^{(4)}$	0.47159$^{(7)}$	0.46926$^{(6)}$	0.48232$^{(8)}$	0.5253$^{(10)}$	0.45255$^{(3)}$	0.51878$^{(9)}$
	BIAS($\hat{\chi}$)	0.13418$^{(1)}$	0.14842$^{(4)}$	0.17817$^{(10)}$	0.13782$^{(2)}$	0.17073$^{(7)}$	0.15464$^{(6)}$	0.15002$^{(5)}$	0.17256$^{(8)}$	0.14306$^{(3)}$	0.17298$^{(9)}$
	MSE($\hat{\Phi}$)	0.22318$^{(1)}$	0.28354$^{(2)}$	0.3213$^{(3)}$	0.33645$^{(5)}$	0.3377$^{(6)}$	0.34087$^{(7)}$	0.36089$^{(8)}$	0.41009$^{(10)}$	0.322$^{(4)}$	0.40157$^{(9)}$
	MSE($\hat{\chi}$)	0.02885$^{(2)}$	0.03507$^{(5)}$	0.05068$^{(10)}$	0.02814$^{(1)}$	0.0458$^{(8)}$	0.03785$^{(6)}$	0.03422$^{(4)}$	0.04518$^{(7)}$	0.03187$^{(3)}$	0.04608$^{(9)}$
	MRE($\hat{\Phi}$)	0.2519$^{(1)}$	0.28322$^{(2)}$	0.30889$^{(5)}$	0.30536$^{(4)}$	0.31439$^{(7)}$	0.31284$^{(6)}$	0.32155$^{(8)}$	0.3502$^{(10)}$	0.3017$^{(3)}$	0.34585$^{(9)}$
	MRE($\hat{\chi}$)	0.19169$^{(1)}$	0.21203$^{(4)}$	0.25453$^{(10)}$	0.19689$^{(2)}$	0.2439$^{(7)}$	0.22092$^{(6)}$	0.21432$^{(5)}$	0.24651$^{(8)}$	0.20437$^{(3)}$	0.24712$^{(9)}$
	D_{abs}	0.01484$^{(2)}$	0.01502$^{(3)}$	0.01614$^{(5)}$	0.01457$^{(1)}$	0.01591$^{(4)}$	0.02213$^{(8)}$	0.01988$^{(7)}$	0.02476$^{(9)}$	0.01819$^{(6)}$	0.02498$^{(10)}$
	D_{max}	0.02503$^{(2)}$	0.02568$^{(3)}$	0.02834$^{(5)}$	0.02493$^{(1)}$	0.02792$^{(4)}$	0.03629$^{(8)}$	0.03284$^{(7)}$	0.04092$^{(9)}$	0.03008$^{(6)}$	0.04126$^{(10)}$
	ASAE	0.01699$^{(2)}$	0.01583$^{(3)}$	0.01609$^{(5)}$	0.01672$^{(1)}$	0.01589$^{(4)}$	0.02375$^{(8)}$	0.0218$^{(7)}$	0.02605$^{(9)}$	0.0202$^{(6)}$	0.02636$^{(10)}$
	$\sum Ranks$	16$^{(1)}$	26$^{(3)}$	56$^{(6)}$	24$^{(2)}$	52$^{(5)}$	61$^{(8)}$	59$^{(7)}$	80$^{(9)}$	37$^{(4)}$	84$^{(10)}$
150	BIAS($\hat{\Phi}$)	0.32128$^{(1)}$	0.3827$^{(2)}$	0.42332$^{(5)}$	0.40893$^{(4)}$	0.43416$^{(7)}$	0.42898$^{(6)}$	0.43965$^{(8)}$	0.50171$^{(10)}$	0.40882$^{(3)}$	0.49415$^{(9)}$
	BIAS($\hat{\chi}$)	0.11081$^{(1)}$	0.12792$^{(4)}$	0.15405$^{(8)}$	0.121$^{(2)}$	0.15102$^{(7)}$	0.13584$^{(6)}$	0.13236$^{(5)}$	0.15842$^{(10)}$	0.12499$^{(3)}$	0.1563$^{(9)}$
	MSE($\hat{\Phi}$)	0.16572$^{(1)}$	0.23967$^{(2)}$	0.27935$^{(4)}$	0.27941$^{(5)}$	0.2959$^{(7)}$	0.2956$^{(6)}$	0.30977$^{(8)}$	0.38294$^{(10)}$	0.27121$^{(3)}$	0.37371$^{(9)}$
	MSE($\hat{\chi}$)	0.01944$^{(1)}$	0.02551$^{(4)}$	0.03718$^{(9)}$	0.02177$^{(2)}$	0.03538$^{(7)}$	0.02898$^{(6)}$	0.02652$^{(5)}$	0.03816$^{(10)}$	0.02382$^{(3)}$	0.03713$^{(8)}$
	MRE($\hat{\Phi}$)	0.21419$^{(1)}$	0.25513$^{(2)}$	0.28221$^{(5)}$	0.27262$^{(4)}$	0.28944$^{(7)}$	0.28598$^{(6)}$	0.2931$^{(8)}$	0.33447$^{(10)}$	0.27255$^{(3)}$	0.32943$^{(9)}$
	MRE($\hat{\chi}$)	0.1583$^{(1)}$	0.18274$^{(4)}$	0.22008$^{(8)}$	0.17285$^{(2)}$	0.21574$^{(7)}$	0.19406$^{(6)}$	0.18909$^{(5)}$	0.22631$^{(10)}$	0.17856$^{(3)}$	0.22329$^{(9)}$
	D_{abs}	0.01221$^{(1)}$	0.01268$^{(3)}$	0.01349$^{(5)}$	0.01228$^{(2)}$	0.01318$^{(4)}$	0.0184$^{(8)}$	0.01662$^{(7)}$	0.02051$^{(9)}$	0.01525$^{(6)}$	0.02062$^{(10)}$
	D_{max}	0.02061$^{(1)}$	0.02179$^{(3)}$	0.02389$^{(5)}$	0.02113$^{(2)}$	0.0234$^{(4)}$	0.03031$^{(8)}$	0.02763$^{(7)}$	0.03429$^{(9)}$	0.02538$^{(6)}$	0.03444$^{(10)}$
	ASAE	0.01359$^{(1)}$	0.01268$^{(3)}$	0.01294$^{(5)}$	0.01348$^{(2)}$	0.01286$^{(4)}$	0.01918$^{(8)}$	0.01766$^{(7)}$	0.02099$^{(9)}$	0.01622$^{(6)}$	0.02113$^{(10)}$
	$\sum Ranks$	13$^{(1)}$	25$^{(2)}$	52$^{(5.5)}$	27$^{(3)}$	52$^{(5.5)}$	60$^{(7.5)}$	60$^{(7.5)}$	87$^{(10)}$	36$^{(4)}$	83$^{(9)}$
200	BIAS($\hat{\Phi}$)	0.28335$^{(1)}$	0.34517$^{(2)}$	0.38478$^{(5)}$	0.36489$^{(3)}$	0.39252$^{(6)}$	0.4091$^{(8)}$	0.4049$^{(7)}$	0.47095$^{(9)}$	0.3715$^{(4)}$	0.47254$^{(10)}$
	BIAS($\hat{\chi}$)	0.09652$^{(1)}$	0.11405$^{(4)}$	0.13694$^{(8)}$	0.10764$^{(2)}$	0.13275$^{(7)}$	0.12427$^{(6)}$	0.11943$^{(5)}$	0.14402$^{(9)}$	0.11253$^{(3)}$	0.14491$^{(10)}$
	MSE($\hat{\Phi}$)	0.132$^{(1)}$	0.19851$^{(2)}$	0.23703$^{(5)}$	0.22544$^{(3)}$	0.25074$^{(6)}$	0.27393$^{(8)}$	0.27355$^{(7)}$	0.34811$^{(9)}$	0.23113$^{(4)}$	0.3506$^{(10)}$
	MSE($\hat{\chi}$)	0.01483$^{(1)}$	0.02008$^{(4)}$	0.02959$^{(8)}$	0.01724$^{(2)}$	0.02734$^{(7)}$	0.02393$^{(6)}$	0.02138$^{(5)}$	0.03137$^{(9)}$	0.01935$^{(3)}$	0.03206$^{(10)}$
	MRE($\hat{\Phi}$)	0.1889$^{(1)}$	0.23011$^{(2)}$	0.25652$^{(5)}$	0.24326$^{(3)}$	0.26168$^{(6)}$	0.27273$^{(8)}$	0.26994$^{(7)}$	0.31397$^{(9)}$	0.24767$^{(4)}$	0.31503$^{(10)}$
	MRE($\hat{\chi}$)	0.13788$^{(1)}$	0.16294$^{(4)}$	0.19563$^{(8)}$	0.15377$^{(2)}$	0.18965$^{(7)}$	0.17753$^{(6)}$	0.17061$^{(5)}$	0.20575$^{(9)}$	0.16075$^{(3)}$	0.20702$^{(10)}$
	D_{abs}	0.0107$^{(1)}$	0.01096$^{(3)}$	0.01174$^{(5)}$	0.01073$^{(2)}$	0.0115$^{(4)}$	0.01624$^{(8)}$	0.0146$^{(7)}$	0.01815$^{(10)}$	0.01355$^{(6)}$	0.01813$^{(9)}$
	D_{max}	0.018$^{(1)}$	0.01894$^{(3)}$	0.02079$^{(5)}$	0.01843$^{(2)}$	0.02038$^{(4)}$	0.02685$^{(8)}$	0.02438$^{(7)}$	0.03035$^{(9)}$	0.02256$^{(6)}$	0.03039$^{(10)}$
	ASAE	0.01159$^{(1)}$	0.01092$^{(3)}$	0.01108$^{(5)}$	0.01156$^{(2)}$	0.01104$^{(4)}$	0.0165$^{(8)}$	0.01509$^{(7)}$	0.01814$^{(9)}$	0.01394$^{(6)}$	0.01819$^{(10)}$
	$\sum Ranks$	13$^{(1)}$	25$^{(3)}$	52$^{(6)}$	23$^{(2)}$	49$^{(5)}$	66$^{(8)}$	57$^{(7)}$	82$^{(9)}$	39$^{(4)}$	89$^{(10)}$
300	BIAS($\hat{\Phi}$)	0.23476$^{(1)}$	0.28767$^{(2)}$	0.32851$^{(5)}$	0.29349$^{(3)}$	0.33664$^{(6)}$	0.36422$^{(8)}$	0.34741$^{(7)}$	0.4196$^{(10)}$	0.31289$^{(4)}$	0.41719$^{(9)}$
	BIAS($\hat{\chi}$)	0.07885$^{(1)}$	0.0954$^{(4)}$	0.11448$^{(8)}$	0.08853$^{(2)}$	0.11268$^{(7)}$	0.11017$^{(6)}$	0.10216$^{(5)}$	0.12487$^{(10)}$	0.09477$^{(3)}$	0.12421$^{(9)}$
	MSE($\hat{\Phi}$)	0.09091$^{(1)}$	0.14031$^{(2)}$	0.1785$^{(5)}$	0.14943$^{(3)}$	0.19189$^{(6)}$	0.22333$^{(8)}$	0.20533$^{(7)}$	0.2888$^{(10)}$	0.16849$^{(4)}$	0.28598$^{(9)}$
	MSE($\hat{\chi}$)	0.00985$^{(1)}$	0.01416$^{(4)}$	0.02061$^{(8)}$	0.01191$^{(2)}$	0.01962$^{(7)}$	0.01859$^{(6)}$	0.01584$^{(5)}$	0.02378$^{(10)}$	0.01385$^{(3)}$	0.02351$^{(9)}$
	MRE($\hat{\Phi}$)	0.15651$^{(1)}$	0.19178$^{(2)}$	0.21901$^{(5)}$	0.19566$^{(3)}$	0.22443$^{(6)}$	0.24282$^{(8)}$	0.23161$^{(7)}$	0.27973$^{(10)}$	0.20859$^{(4)}$	0.27813$^{(9)}$
	MRE($\hat{\chi}$)	0.11264$^{(1)}$	0.13629$^{(4)}$	0.16355$^{(8)}$	0.12647$^{(2)}$	0.16097$^{(7)}$	0.15739$^{(6)}$	0.14595$^{(5)}$	0.17839$^{(10)}$	0.13539$^{(3)}$	0.17744$^{(9)}$
	D_{abs}	0.00868$^{(1)}$	0.00908$^{(3)}$	0.00965$^{(5)}$	0.0089$^{(2)}$	0.00961$^{(4)}$	0.01361$^{(8)}$	0.01227$^{(7)}$	0.01528$^{(9)}$	0.01117$^{(6)}$	0.01532$^{(10)}$
	D_{max}	0.0146$^{(1)}$	0.0157$^{(3)}$	0.01714$^{(5)}$	0.01526$^{(2)}$	0.01707$^{(4)}$	0.02264$^{(8)}$	0.02044$^{(7)}$	0.02558$^{(9.5)}$	0.01866$^{(6)}$	0.02558$^{(9.5)}$
	ASAE	0.00932$^{(1)}$	0.00884$^{(3)}$	0.00898$^{(5)}$	0.00929$^{(2)}$	0.00893$^{(4)}$	0.01341$^{(8)}$	0.01226$^{(7)}$	0.01486$^{(9.5)}$	0.01127$^{(6)}$	0.01483$^{(9.5)}$
	$\sum Ranks$	13$^{(1)}$	25$^{(3)}$	52$^{(6)}$	23$^{(2)}$	49$^{(5)}$	66$^{(8)}$	57$^{(7)}$	88.5$^{(10)}$	39$^{(4)}$	82.5$^{(9)}$

Table 7. Numerical values of simulation measures for $\Phi = 0.5$ and $\chi = 2.0$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.20582^{\{6\}}$	$0.20436^{\{3\}}$	$0.21205^{\{10\}}$	$0.19838^{\{1\}}$	$0.2062^{\{7\}}$	$0.20374^{\{2\}}$	$0.20479^{\{4\}}$	$0.21204^{\{9\}}$	$0.20487^{\{5\}}$	$0.21192^{\{8\}}$
	BIAS($\hat{\chi}$)	$0.59824^{\{9\}}$	$0.57701^{\{7\}}$	$0.64866^{\{10\}}$	$0.49182^{\{1\}}$	$0.58066^{\{8\}}$	$0.52704^{\{3\}}$	$0.52315^{\{2\}}$	$0.56646^{\{6\}}$	$0.53871^{\{4\}}$	$0.56537^{\{5\}}$
	MSE($\hat{\Phi}$)	$0.05688^{\{3\}}$	$0.05696^{\{4\}}$	$0.05964^{\{8\}}$	$0.05506^{\{1\}}$	$0.05781^{\{7\}}$	$0.05683^{\{2\}}$	$0.05761^{\{6\}}$	$0.06113^{\{10\}}$	$0.05747^{\{5\}}$	$0.0611^{\{9\}}$
	MSE($\hat{\chi}$)	$0.59135^{\{9\}}$	$0.55089^{\{8\}}$	$0.67733^{\{10\}}$	$0.3946^{\{1\}}$	$0.55021^{\{7\}}$	$0.46696^{\{3\}}$	$0.45485^{\{2\}}$	$0.50778^{\{6\}}$	$0.48338^{\{4\}}$	$0.50774^{\{5\}}$
	MRE($\hat{\Phi}$)	$0.41163^{\{6\}}$	$0.40872^{\{3\}}$	$0.4241^{\{10\}}$	$0.39677^{\{1\}}$	$0.41239^{\{7\}}$	$0.40748^{\{2\}}$	$0.40957^{\{4\}}$	$0.42407^{\{9\}}$	$0.40975^{\{5\}}$	$0.42383^{\{8\}}$
	MRE($\hat{\chi}$)	$0.29912^{\{9\}}$	$0.2885^{\{7\}}$	$0.32433^{\{10\}}$	$0.24591^{\{1\}}$	$0.29033^{\{8\}}$	$0.26352^{\{3\}}$	$0.26158^{\{2\}}$	$0.28323^{\{6\}}$	$0.26935^{\{4\}}$	$0.28268^{\{5\}}$
	D_{abs}	$0.0509^{\{2\}}$	$0.05204^{\{3\}}$	$0.05276^{\{5\}}$	$0.05082^{\{1\}}$	$0.05366^{\{6\}}$	$0.05967^{\{8\}}$	$0.05603^{\{7\}}$	$0.06744^{\{9\}}$	$0.05273^{\{4\}}$	$0.06821^{\{10\}}$
	D_{max}	$0.08431^{\{4\}}$	$0.08418^{\{3\}}$	$0.08828^{\{7\}}$	$0.08025^{\{1\}}$	$0.08677^{\{5\}}$	$0.09388^{\{8\}}$	$0.08807^{\{6\}}$	$0.10607^{\{9\}}$	$0.08406^{\{2\}}$	$0.107^{\{10\}}$
	ASAE	$0.04548^{\{4\}}$	$0.04043^{\{3\}}$	$0.04091^{\{7\}}$	$0.04186^{\{1\}}$	$0.03954^{\{5\}}$	$0.05635^{\{8\}}$	$0.05274^{\{6\}}$	$0.0618^{\{9\}}$	$0.04928^{\{2\}}$	$0.06237^{\{10\}}$
	$\sum Ranks$	$53^{\{6\}}$	$40^{\{4.5\}}$	$73^{\{9.5\}}$	$12^{\{1\}}$	$56^{\{7\}}$	$39^{\{2.5\}}$	$40^{\{4.5\}}$	$73^{\{9.5\}}$	$39^{\{2.5\}}$	$70^{\{8\}}$
60	BIAS($\hat{\Phi}$)	$0.15608^{\{1\}}$	$0.16294^{\{2\}}$	$0.1722^{\{5\}}$	$0.16415^{\{3\}}$	$0.17458^{\{8\}}$	$0.17351^{\{7\}}$	$0.17256^{\{6\}}$	$0.18615^{\{10\}}$	$0.16659^{\{4\}}$	$0.18528^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.36501^{\{4\}}$	$0.37341^{\{6\}}$	$0.45323^{\{10\}}$	$0.32793^{\{1\}}$	$0.42196^{\{7\}}$	$0.37328^{\{5\}}$	$0.35723^{\{3\}}$	$0.42858^{\{9\}}$	$0.34483^{\{2\}}$	$0.42315^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.03577^{\{1\}}$	$0.03925^{\{2\}}$	$0.04238^{\{5\}}$	$0.04105^{\{3\}}$	$0.04417^{\{6\}}$	$0.04452^{\{8\}}$	$0.0443^{\{7\}}$	$0.04991^{\{10\}}$	$0.04173^{\{4\}}$	$0.04985^{\{9\}}$
	MSE($\hat{\chi}$)	$0.22288^{\{4\}}$	$0.22643^{\{5\}}$	$0.34239^{\{10\}}$	$0.16462^{\{1\}}$	$0.28945^{\{7\}}$	$0.23074^{\{6\}}$	$0.19974^{\{3\}}$	$0.29516^{\{9\}}$	$0.19191^{\{2\}}$	$0.29239^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.31215^{\{1\}}$	$0.32589^{\{2\}}$	$0.3444^{\{5\}}$	$0.3283^{\{3\}}$	$0.34915^{\{8\}}$	$0.34701^{\{7\}}$	$0.34511^{\{6\}}$	$0.3723^{\{10\}}$	$0.33317^{\{4\}}$	$0.37057^{\{9\}}$
	MRE($\hat{\chi}$)	$0.18251^{\{4\}}$	$0.1867^{\{6\}}$	$0.22662^{\{10\}}$	$0.16396^{\{1\}}$	$0.21098^{\{7\}}$	$0.18664^{\{5\}}$	$0.17861^{\{3\}}$	$0.21429^{\{9\}}$	$0.17241^{\{2\}}$	$0.21158^{\{8\}}$
	D_{abs}	$0.03076^{\{3\}}$	$0.03046^{\{2\}}$	$0.03188^{\{4\}}$	$0.02986^{\{1\}}$	$0.03203^{\{6\}}$	$0.03681^{\{8\}}$	$0.03373^{\{7\}}$	$0.03907^{\{9\}}$	$0.03192^{\{5\}}$	$0.0394^{\{10\}}$
	D_{max}	$0.0507^{\{3\}}$	$0.05021^{\{2\}}$	$0.05444^{\{6\}}$	$0.04832^{\{1\}}$	$0.05373^{\{5\}}$	$0.05933^{\{8\}}$	$0.05457^{\{7\}}$	$0.06421^{\{9\}}$	$0.05155^{\{4\}}$	$0.06458^{\{10\}}$
	ASAE	$0.02326^{\{3\}}$	$0.02145^{\{2\}}$	$0.022^{\{6\}}$	$0.02275^{\{1\}}$	$0.02153^{\{5\}}$	$0.03157^{\{8\}}$	$0.02932^{\{7\}}$	$0.03403^{\{9\}}$	$0.02693^{\{4\}}$	$0.03421^{\{10\}}$
	$\sum Ranks$	$26^{\{2\}}$	$28^{\{3\}}$	$58^{\{7\}}$	$18^{\{1\}}$	$56^{\{6\}}$	$62^{\{8\}}$	$49^{\{5\}}$	$84^{\{10\}}$	$33^{\{4\}}$	$81^{\{9\}}$
100	BIAS($\hat{\Phi}$)	$0.13107^{\{1\}}$	$0.14169^{\{2\}}$	$0.15188^{\{6\}}$	$0.14417^{\{3\}}$	$0.15131^{\{5\}}$	$0.1558^{\{8\}}$	$0.15281^{\{7\}}$	$0.17003^{\{9\}}$	$0.1485^{\{4\}}$	$0.17149^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.28098^{\{2\}}$	$0.30187^{\{5\}}$	$0.36304^{\{9\}}$	$0.27138^{\{1\}}$	$0.33899^{\{7\}}$	$0.31485^{\{6\}}$	$0.29413^{\{4\}}$	$0.35978^{\{8\}}$	$0.28615^{\{3\}}$	$0.36418^{\{10\}}$
	MSE($\hat{\Phi}$)	$0.02651^{\{1\}}$	$0.03136^{\{2\}}$	$0.03448^{\{5\}}$	$0.03348^{\{3\}}$	$0.03504^{\{6\}}$	$0.03751^{\{8\}}$	$0.03638^{\{7\}}$	$0.04353^{\{9\}}$	$0.03446^{\{4\}}$	$0.04391^{\{10\}}$
	MSE($\hat{\chi}$)	$0.12682^{\{3\}}$	$0.14337^{\{5\}}$	$0.21314^{\{10\}}$	$0.11109^{\{1\}}$	$0.1816^{\{7\}}$	$0.15769^{\{6\}}$	$0.13232^{\{4\}}$	$0.205^{\{8\}}$	$0.12671^{\{2\}}$	$0.20806^{\{9\}}$
	MRE($\hat{\Phi}$)	$0.26214^{\{1\}}$	$0.28339^{\{2\}}$	$0.30376^{\{6\}}$	$0.28834^{\{3\}}$	$0.30262^{\{5\}}$	$0.31161^{\{8\}}$	$0.30561^{\{7\}}$	$0.34006^{\{9\}}$	$0.29701^{\{4\}}$	$0.34298^{\{10\}}$
	MRE($\hat{\chi}$)	$0.14049^{\{2\}}$	$0.15094^{\{5\}}$	$0.18152^{\{9\}}$	$0.13569^{\{1\}}$	$0.16949^{\{7\}}$	$0.15742^{\{6\}}$	$0.14707^{\{4\}}$	$0.17989^{\{8\}}$	$0.14307^{\{3\}}$	$0.18209^{\{10\}}$
	D_{abs}	$0.02453^{\{3\}}$	$0.02431^{\{2\}}$	$0.02562^{\{5\}}$	$0.02379^{\{1\}}$	$0.0252^{\{4\}}$	$0.02921^{\{8\}}$	$0.02693^{\{7\}}$	$0.03094^{\{9\}}$	$0.02638^{\{6\}}$	$0.03096^{\{10\}}$
	D_{max}	$0.04014^{\{2.5\}}$	$0.04014^{\{2.5\}}$	$0.04354^{\{6\}}$	$0.03862^{\{1\}}$	$0.04235^{\{4\}}$	$0.04743^{\{8\}}$	$0.04368^{\{7\}}$	$0.05107^{\{9\}}$	$0.04263^{\{5\}}$	$0.05118^{\{10\}}$
	ASAE	$0.01742^{\{2.5\}}$	$0.01628^{\{2.5\}}$	$0.01655^{\{6\}}$	$0.0172^{\{1\}}$	$0.0164^{\{4\}}$	$0.0241^{\{8\}}$	$0.0221^{\{7\}}$	$0.02623^{\{9\}}$	$0.02046^{\{5\}}$	$0.02628^{\{10\}}$
	$\sum Ranks$	$20.5^{\{2\}}$	$26.5^{\{3\}}$	$59^{\{7\}}$	$18^{\{1\}}$	$47^{\{5\}}$	$66^{\{8\}}$	$54^{\{6\}}$	$78^{\{9\}}$	$37^{\{4\}}$	$89^{\{10\}}$
150	BIAS($\hat{\Phi}$)	$0.11225^{\{1\}}$	$0.12415^{\{3\}}$	$0.13284^{\{5\}}$	$0.12389^{\{2\}}$	$0.13691^{\{7\}}$	$0.13875^{\{8\}}$	$0.13667^{\{6\}}$	$0.15451^{\{10\}}$	$0.12719^{\{4\}}$	$0.15425^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.23131^{\{2\}}$	$0.25646^{\{5\}}$	$0.2968^{\{8\}}$	$0.22882^{\{1\}}$	$0.29305^{\{7\}}$	$0.27308^{\{6\}}$	$0.25462^{\{4\}}$	$0.30954^{\{9\}}$	$0.23882^{\{3\}}$	$0.31031^{\{10\}}$
	MSE($\hat{\Phi}$)	$0.01989^{\{1\}}$	$0.02476^{\{2\}}$	$0.02778^{\{5\}}$	$0.02552^{\{3\}}$	$0.02958^{\{6\}}$	$0.03058^{\{8\}}$	$0.0301^{\{7\}}$	$0.03705^{\{9\}}$	$0.02648^{\{4\}}$	$0.03706^{\{10\}}$
	MSE($\hat{\chi}$)	$0.08564^{\{2\}}$	$0.10145^{\{5\}}$	$0.13949^{\{8\}}$	$0.07895^{\{1\}}$	$0.1333^{\{7\}}$	$0.11629^{\{6\}}$	$0.09762^{\{4\}}$	$0.14873^{\{9\}}$	$0.08771^{\{3\}}$	$0.15077^{\{10\}}$
	MRE($\hat{\Phi}$)	$0.22451^{\{1\}}$	$0.2483^{\{3\}}$	$0.26568^{\{5\}}$	$0.24777^{\{2\}}$	$0.27382^{\{7\}}$	$0.27751^{\{8\}}$	$0.27335^{\{6\}}$	$0.30902^{\{10\}}$	$0.25438^{\{4\}}$	$0.30849^{\{9\}}$
	MRE($\hat{\chi}$)	$0.11566^{\{2\}}$	$0.12823^{\{5\}}$	$0.1484^{\{8\}}$	$0.11441^{\{1\}}$	$0.14653^{\{7\}}$	$0.13654^{\{6\}}$	$0.12731^{\{4\}}$	$0.15477^{\{9\}}$	$0.11941^{\{3\}}$	$0.15515^{\{10\}}$
	D_{abs}	$0.02022^{\{3\}}$	$0.02021^{\{2\}}$	$0.02099^{\{4\}}$	$0.01992^{\{1\}}$	$0.02138^{\{5\}}$	$0.02434^{\{8\}}$	$0.02286^{\{7\}}$	$0.02588^{\{9\}}$	$0.0219^{\{6\}}$	$0.02589^{\{10\}}$
	D_{max}	$0.03299^{\{2\}}$	$0.03346^{\{3\}}$	$0.03555^{\{5\}}$	$0.03232^{\{1\}}$	$0.03591^{\{6\}}$	$0.03961^{\{8\}}$	$0.03712^{\{7\}}$	$0.04269^{\{9\}}$	$0.0354^{\{4\}}$	$0.04274^{\{10\}}$
	ASAE	$0.01382^{\{2\}}$	$0.01301^{\{3\}}$	$0.01327^{\{5\}}$	$0.01377^{\{1\}}$	$0.01318^{\{6\}}$	$0.01932^{\{8\}}$	$0.01784^{\{7\}}$	$0.02121^{\{9\}}$	$0.01652^{\{4\}}$	$0.02114^{\{10\}}$
	$\sum Ranks$	$19^{\{2\}}$	$29^{\{3\}}$	$51^{\{5\}}$	$16^{\{1\}}$	$54^{\{7\}}$	$66^{\{8\}}$	$52^{\{6\}}$	$84^{\{9\}}$	$37^{\{4\}}$	$87^{\{10\}}$
200	BIAS($\hat{\Phi}$)	$0.09787^{\{1\}}$	$0.11154^{\{3\}}$	$0.1179^{\{5\}}$	$0.11056^{\{2\}}$	$0.12216^{\{6\}}$	$0.12777^{\{8\}}$	$0.1225^{\{7\}}$	$0.14131^{\{10\}}$	$0.11614^{\{4\}}$	$0.14053^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.19841^{\{1\}}$	$0.22736^{\{5\}}$	$0.25862^{\{7\}}$	$0.20605^{\{2\}}$	$0.26153^{\{8\}}$	$0.24341^{\{6\}}$	$0.22626^{\{4\}}$	$0.27693^{\{10\}}$	$0.21698^{\{3\}}$	$0.27479^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.0155^{\{1\}}$	$0.02037^{\{2\}}$	$0.02255^{\{5\}}$	$0.02063^{\{3\}}$	$0.02417^{\{6\}}$	$0.02691^{\{8\}}$	$0.02496^{\{7\}}$	$0.03197^{\{10\}}$	$0.0225^{\{4\}}$	$0.03156^{\{9\}}$
	MSE($\hat{\chi}$)	$0.06231^{\{1\}}$	$0.08021^{\{5\}}$	$0.10633^{\{8\}}$	$0.06457^{\{2\}}$	$0.10557^{\{7\}}$	$0.09139^{\{6\}}$	$0.07823^{\{4\}}$	$0.11869^{\{10\}}$	$0.07241^{\{3\}}$	$0.11672^{\{9\}}$
	MRE($\hat{\Phi}$)	$0.19574^{\{1\}}$	$0.22308^{\{3\}}$	$0.23581^{\{5\}}$	$0.22112^{\{2\}}$	$0.24433^{\{6\}}$	$0.25555^{\{8\}}$	$0.24499^{\{7\}}$	$0.28263^{\{10\}}$	$0.23227^{\{4\}}$	$0.28106^{\{9\}}$
	MRE($\hat{\chi}$)	$0.0992^{\{1\}}$	$0.11368^{\{5\}}$	$0.12931^{\{7\}}$	$0.10302^{\{2\}}$	$0.13077^{\{8\}}$	$0.1217^{\{6\}}$	$0.11313^{\{4\}}$	$0.13847^{\{10\}}$	$0.10849^{\{3\}}$	$0.1374^{\{9\}}$
	D_{abs}	$0.01744^{\{2\}}$	$0.01787^{\{3\}}$	$0.01828^{\{4\}}$	$0.01741^{\{1\}}$	$0.01835^{\{5\}}$	$0.02143^{\{8\}}$	$0.02023^{\{7\}}$	$0.0229^{\{10\}}$	$0.01919^{\{6\}}$	$0.0227^{\{9\}}$
	D_{max}	$0.02847^{\{2\}}$	$0.02955^{\{3\}}$	$0.03089^{\{4\}}$	$0.02837^{\{1\}}$	$0.03099^{\{5\}}$	$0.03493^{\{8\}}$	$0.03281^{\{7\}}$	$0.03771^{\{10\}}$	$0.03116^{\{6\}}$	$0.03742^{\{9\}}$
	ASAE	$0.01182^{\{2\}}$	$0.01115^{\{3\}}$	$0.01131^{\{4\}}$	$0.0118^{\{1\}}$	$0.01133^{\{5\}}$	$0.01666^{\{8\}}$	$0.01528^{\{7\}}$	$0.01822^{\{10\}}$	$0.01408^{\{6\}}$	$0.01826^{\{9\}}$
	$\sum Ranks$	$15^{\{1\}}$	$30^{\{3\}}$	$47^{\{5\}}$	$19^{\{2\}}$	$54^{\{6.5\}}$	$66^{\{8\}}$	$54^{\{6.5\}}$	$89^{\{10\}}$	$39^{\{4\}}$	$82^{\{9\}}$
300	BIAS($\hat{\Phi}$)	$0.08063^{\{1\}}$	$0.09206^{\{3\}}$	$0.10153^{\{6\}}$	$0.08721^{\{2\}}$	$0.10272^{\{7\}}$	$0.10745^{\{8\}}$	$0.10147^{\{5\}}$	$0.12202^{\{9\}}$	$0.09544^{\{4\}}$	$0.1223^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.16251^{\{1\}}$	$0.1875^{\{4\}}$	$0.21733^{\{8\}}$	$0.16482^{\{2\}}$	$0.21688^{\{7\}}$	$0.20627^{\{6\}}$	$0.18863^{\{5\}}$	$0.2334^{\{10\}}$	$0.17905^{\{3\}}$	$0.23333^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.01043^{\{1\}}$	$0.01407^{\{3\}}$	$0.01714^{\{5\}}$	$0.01298^{\{2\}}$	$0.01772^{\{7\}}$	$0.01935^{\{8\}}$	$0.01756^{\{6\}}$	$0.02475^{\{9\}}$	$0.01522^{\{4\}}$	$0.02501^{\{10\}}$
	MSE($\hat{\chi}$)	$0.04171^{\{1\}}$	$0.05509^{\{4\}}$	$0.07468^{\{8\}}$	$0.04247^{\{2\}}$	$0.07392^{\{7\}}$	$0.06626^{\{6\}}$	$0.05528^{\{5\}}$	$0.08409^{\{9\}}$	$0.0498^{\{3\}}$	$0.08562^{\{10\}}$
	MRE($\hat{\Phi}$)	$0.16127^{\{1\}}$	$0.18412^{\{3\}}$	$0.20306^{\{6\}}$	$0.17441^{\{2\}}$	$0.20543^{\{7\}}$	$0.21489^{\{8\}}$	$0.20293^{\{5\}}$	$0.24404^{\{9\}}$	$0.19088^{\{4\}}$	$0.24461^{\{10\}}$
	MRE($\hat{\chi}$)	$0.08125^{\{1\}}$	$0.09375^{\{4\}}$	$0.10867^{\{8\}}$	$0.08241^{\{2\}}$	$0.10844^{\{7\}}$	$0.10314^{\{6\}}$	$0.09432^{\{5\}}$	$0.1167^{\{10\}}$	$0.08953^{\{3\}}$	$0.11667^{\{9\}}$
	D_{abs}	$0.01423^{\{2\}}$	$0.01463^{\{3\}}$	$0.01535^{\{5\}}$	$0.01419^{\{1\}}$	$0.01527^{\{4\}}$	$0.0178^{\{8\}}$	$0.01673^{\{7\}}$	$0.01941^{\{10\}}$	$0.01591^{\{6\}}$	$0.01927^{\{9\}}$
	D_{max}	$0.0232^{\{2\}}$	$0.02421^{\{3\}}$	$0.02586^{\{6\}}$	$0.02311^{\{1\}}$	$0.02579^{\{4\}}$	$0.02906^{\{8\}}$	$0.02713^{\{7\}}$	$0.03181^{\{10\}}$	$0.02584^{\{5\}}$	$0.0316^{\{9\}}$
	ASAE	$0.00949^{\{2\}}$	$0.00894^{\{3\}}$	$0.00913^{\{6\}}$	$0.00947^{\{1\}}$	$0.0091^{\{4\}}$	$0.01351^{\{8\}}$	$0.01227^{\{7\}}$	$0.01485^{\{10\}}$	$0.01143^{\{5\}}$	$0.01481^{\{9\}}$
	$\sum Ranks$	$15^{\{1\}}$	$28^{\{3\}}$	$55^{\{7\}}$	$18^{\{2\}}$	$52^{\{5.5\}}$	$66^{\{8\}}$	$52^{\{5.5\}}$	$86^{\{10\}}$	$38^{\{4\}}$	$85^{\{9\}}$

TABLE 8. Numerical values of simulation measures for $\Phi = 0.5$ and $\chi = 2.0$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	0.17811$^{(1)}$	0.18179$^{(2)}$	0.1926$^{(4)}$	0.18947$^{(3)}$	0.19261$^{(5)}$	0.19379$^{(6)}$	0.19409$^{(8)}$	0.20498$^{(10)}$	0.1938$^{(7)}$	0.20295$^{(9)}$
	BIAS($\hat{\chi}$)	0.49713$^{(6)}$	0.47193$^{(2)}$	0.57359$^{(10)}$	0.43898$^{(1)}$	0.50414$^{(7)}$	0.48857$^{(5)}$	0.48611$^{(4)}$	0.53079$^{(9)}$	0.47448$^{(3)}$	0.53022$^{(8)}$
	MSE($\hat{\Phi}$)	0.04473$^{(1)}$	0.04748$^{(2)}$	0.05124$^{(3)}$	0.05218$^{(4)}$	0.05258$^{(5)}$	0.05289$^{(6)}$	0.05322$^{(8)}$	0.05829$^{(10)}$	0.05315$^{(7)}$	0.05746$^{(9)}$
	MSE($\hat{\chi}$)	0.41694$^{(7)}$	0.36948$^{(2)}$	0.54052$^{(10)}$	0.29984$^{(1)}$	0.40978$^{(6)}$	0.40302$^{(5)}$	0.38742$^{(4)}$	0.44087$^{(9)}$	0.37124$^{(3)}$	0.44028$^{(8)}$
	MRE($\hat{\Phi}$)	0.35623$^{(1)}$	0.36358$^{(2)}$	0.3852$^{(4)}$	0.37894$^{(3)}$	0.38523$^{(5)}$	0.38757$^{(6)}$	0.38819$^{(8)}$	0.40997$^{(10)}$	0.38761$^{(7)}$	0.4059$^{(9)}$
	MRE($\hat{\chi}$)	0.24857$^{(6)}$	0.23596$^{(2)}$	0.28679$^{(10)}$	0.21949$^{(1)}$	0.25207$^{(7)}$	0.24429$^{(5)}$	0.24306$^{(4)}$	0.26539$^{(9)}$	0.23724$^{(3)}$	0.26511$^{(8)}$
	D_{abs}	0.03374$^{(3)}$	0.03312$^{(2)}$	0.03632$^{(5)}$	0.0325$^{(1)}$	0.03501$^{(4)}$	0.04645$^{(8)}$	0.04106$^{(7)}$	0.05337$^{(9)}$	0.03753$^{(6)}$	0.05421$^{(10)}$
	D_{max}	0.05667$^{(3)}$	0.05513$^{(2)}$	0.06184$^{(6)}$	0.05408$^{(1)}$	0.05906$^{(4)}$	0.07483$^{(8)}$	0.06703$^{(7)}$	0.08628$^{(9)}$	0.0614$^{(5)}$	0.08742$^{(10)}$
	ASAE	0.04422$^{(3)}$	0.03947$^{(2)}$	0.04031$^{(6)}$	0.04054$^{(1)}$	0.03868$^{(4)}$	0.05566$^{(8)}$	0.0513$^{(7)}$	0.06064$^{(9)}$	0.04801$^{(5)}$	0.062$^{(10)}$
	$\sum Ranks$	33$^{(3)}$	18$^{(1)}$	55$^{(6)}$	19$^{(2)}$	44$^{(4)}$	57$^{(7.5)}$	57$^{(7.5)}$	84$^{(10)}$	47$^{(5)}$	81$^{(9)}$
60	BIAS($\hat{\Phi}$)	0.12602$^{(1)}$	0.13994$^{(2)}$	0.14989$^{(3)}$	0.15111$^{(4)}$	0.15502$^{(5)}$	0.1639$^{(7)}$	0.1656$^{(8)}$	0.17922$^{(9)}$	0.15594$^{(6)}$	0.1815$^{(10)}$
	BIAS($\hat{\chi}$)	0.29098$^{(1)}$	0.3158$^{(3)}$	0.37731$^{(8)}$	0.30331$^{(2)}$	0.36049$^{(7)}$	0.35079$^{(6)}$	0.34065$^{(5)}$	0.40349$^{(9)}$	0.32086$^{(4)}$	0.4051$^{(10)}$
	MSE($\hat{\Phi}$)	0.02468$^{(1)}$	0.03061$^{(2)}$	0.03424$^{(3)}$	0.03683$^{(4)}$	0.03692$^{(5)}$	0.04086$^{(7)}$	0.04191$^{(8)}$	0.04761$^{(9)}$	0.03752$^{(6)}$	0.04857$^{(10)}$
	MSE($\hat{\chi}$)	0.13757$^{(2)}$	0.15676$^{(3)}$	0.23434$^{(8)}$	0.13568$^{(1)}$	0.20526$^{(7)}$	0.19583$^{(6)}$	0.17635$^{(5)}$	0.25599$^{(10)}$	0.15825$^{(4)}$	0.25547$^{(9)}$
	MRE($\hat{\Phi}$)	0.25205$^{(1)}$	0.27987$^{(2)}$	0.29978$^{(3)}$	0.30222$^{(4)}$	0.31005$^{(5)}$	0.32779$^{(7)}$	0.3312$^{(8)}$	0.35845$^{(9)}$	0.31188$^{(6)}$	0.363$^{(10)}$
	MRE($\hat{\chi}$)	0.14549$^{(1)}$	0.1579$^{(3)}$	0.18865$^{(8)}$	0.15165$^{(2)}$	0.18024$^{(7)}$	0.17539$^{(6)}$	0.17032$^{(5)}$	0.20175$^{(9)}$	0.16043$^{(4)}$	0.20255$^{(10)}$
	D_{abs}	0.01979$^{(2)}$	0.01989$^{(3)}$	0.02146$^{(5)}$	0.01952$^{(1)}$	0.02109$^{(4)}$	0.02894$^{(8)}$	0.02583$^{(7)}$	0.03159$^{(9)}$	0.02379$^{(6)}$	0.03185$^{(10)}$
	D_{max}	0.03341$^{(1)}$	0.03403$^{(3)}$	0.03758$^{(5)}$	0.03358$^{(2)}$	0.03697$^{(4)}$	0.04788$^{(8)}$	0.04324$^{(7)}$	0.05346$^{(9)}$	0.0397$^{(6)}$	0.05386$^{(10)}$
	ASAE	0.02276$^{(1)}$	0.02106$^{(3)}$	0.02159$^{(5)}$	0.02192$^{(2)}$	0.02121$^{(4)}$	0.0308$^{(8)}$	0.02851$^{(7)}$	0.03345$^{(9)}$	0.02636$^{(6)}$	0.03378$^{(10)}$
	$\sum Ranks$	15$^{(1)}$	22$^{(2)}$	46$^{(4.5)}$	24$^{(3)}$	46$^{(4.5)}$	63$^{(8)}$	60$^{(7)}$	82$^{(9)}$	48$^{(6)}$	89$^{(10)}$
100	BIAS($\hat{\Phi}$)	0.10061$^{(1)}$	0.11825$^{(2)}$	0.12893$^{(4)}$	0.12753$^{(3)}$	0.13268$^{(6)}$	0.14449$^{(8)}$	0.14414$^{(7)}$	0.16196$^{(9)}$	0.13097$^{(5)}$	0.16356$^{(10)}$
	BIAS($\hat{\chi}$)	0.2254$^{(1)}$	0.25717$^{(3)}$	0.30645$^{(8)}$	0.25285$^{(2)}$	0.29714$^{(7)}$	0.29627$^{(6)}$	0.28308$^{(5)}$	0.33944$^{(9)}$	0.25947$^{(4)}$	0.34143$^{(10)}$
	MSE($\hat{\Phi}$)	0.01614$^{(1)}$	0.02302$^{(2)}$	0.02615$^{(3)}$	0.02726$^{(4)}$	0.02837$^{(6)}$	0.03331$^{(7)}$	0.03346$^{(8)}$	0.04058$^{(9)}$	0.02807$^{(5)}$	0.04138$^{(10)}$
	MSE($\hat{\chi}$)	0.08101$^{(1)}$	0.10327$^{(4)}$	0.14892$^{(8)}$	0.09586$^{(2)}$	0.13652$^{(6)}$	0.13727$^{(7)}$	0.12044$^{(5)}$	0.17848$^{(9)}$	0.10298$^{(3)}$	0.1799$^{(10)}$
	MRE($\hat{\Phi}$)	0.20122$^{(1)}$	0.23649$^{(2)}$	0.25786$^{(4)}$	0.25505$^{(3)}$	0.26536$^{(6)}$	0.28898$^{(8)}$	0.28827$^{(7)}$	0.32393$^{(9)}$	0.26193$^{(5)}$	0.32711$^{(10)}$
	MRE($\hat{\chi}$)	0.1127$^{(1)}$	0.12858$^{(3)}$	0.15323$^{(8)}$	0.12643$^{(2)}$	0.14857$^{(7)}$	0.14814$^{(6)}$	0.14154$^{(5)}$	0.16972$^{(9)}$	0.12974$^{(4)}$	0.17072$^{(10)}$
	D_{abs}	0.01533$^{(1)}$	0.01591$^{(3)}$	0.01685$^{(4)}$	0.01563$^{(2)}$	0.01686$^{(5)}$	0.02345$^{(8)}$	0.02106$^{(7)}$	0.02493$^{(9)}$	0.01915$^{(6)}$	0.0255$^{(10)}$
	D_{max}	0.02595$^{(1)}$	0.02733$^{(3)}$	0.02967$^{(4)}$	0.02694$^{(2)}$	0.02969$^{(5)}$	0.03896$^{(8)}$	0.03531$^{(7)}$	0.04237$^{(9)}$	0.03201$^{(6)}$	0.04329$^{(10)}$
	ASAE	0.01694$^{(1)}$	0.01596$^{(3)}$	0.01619$^{(4)}$	0.01676$^{(2)}$	0.01606$^{(5)}$	0.02379$^{(8)}$	0.02177$^{(7)}$	0.0255$^{(9)}$	0.02011$^{(6)}$	0.02572$^{(10)}$
	$\sum Ranks$	13$^{(1)}$	23$^{(2)}$	46$^{(5)}$	24$^{(3)}$	50$^{(6)}$	66$^{(8)}$	58$^{(7)}$	81$^{(9)}$	44$^{(4)}$	90$^{(10)}$
150	BIAS($\hat{\Phi}$)	0.08307$^{(1)}$	0.09911$^{(2)}$	0.11011$^{(4)}$	0.10538$^{(3)}$	0.11316$^{(6)}$	0.13035$^{(8)}$	0.12353$^{(7)}$	0.14875$^{(10)}$	0.11115$^{(5)}$	0.14539$^{(9)}$
	BIAS($\hat{\chi}$)	0.18341$^{(1)}$	0.21806$^{(3)}$	0.25381$^{(7)}$	0.21099$^{(2)}$	0.25011$^{(6)}$	0.25867$^{(8)}$	0.24047$^{(5)}$	0.29672$^{(10)}$	0.22105$^{(4)}$	0.29398$^{(9)}$
	MSE($\hat{\Phi}$)	0.01126$^{(1)}$	0.01649$^{(2)}$	0.01972$^{(4)}$	0.01905$^{(3)}$	0.02138$^{(6)}$	0.02796$^{(8)}$	0.02556$^{(7)}$	0.03522$^{(10)}$	0.0207$^{(5)}$	0.034$^{(9)}$
	MSE($\hat{\chi}$)	0.05384$^{(1)}$	0.07446$^{(3)}$	0.10178$^{(7)}$	0.06809$^{(2)}$	0.09708$^{(6)}$	0.10339$^{(8)}$	0.08755$^{(5)}$	0.13411$^{(9)}$	0.07532$^{(4)}$	0.13514$^{(10)}$
	MRE($\hat{\Phi}$)	0.16614$^{(1)}$	0.19821$^{(2)}$	0.22021$^{(4)}$	0.21076$^{(3)}$	0.22633$^{(6)}$	0.2607$^{(8)}$	0.24706$^{(7)}$	0.2975$^{(10)}$	0.2223$^{(5)}$	0.29077$^{(9)}$
	MRE($\hat{\chi}$)	0.09171$^{(1)}$	0.10903$^{(3)}$	0.12691$^{(7)}$	0.10549$^{(2)}$	0.12506$^{(6)}$	0.12934$^{(8)}$	0.12023$^{(5)}$	0.14836$^{(10)}$	0.11053$^{(4)}$	0.14699$^{(9)}$
	D_{abs}	0.01266$^{(1)}$	0.01329$^{(3)}$	0.01399$^{(5)}$	0.01296$^{(2)}$	0.01392$^{(4)}$	0.01963$^{(8)}$	0.01756$^{(7)}$	0.02159$^{(10)}$	0.016$^{(6)}$	0.02147$^{(9)}$
	D_{max}	0.02138$^{(1)}$	0.02292$^{(3)}$	0.02462$^{(5)}$	0.02233$^{(2)}$	0.02454$^{(4)}$	0.03273$^{(8)}$	0.02946$^{(7)}$	0.03648$^{(10)}$	0.02683$^{(6)}$	0.03632$^{(9)}$
	ASAE	0.01347$^{(1)}$	0.01274$^{(3)}$	0.013$^{(5)}$	0.01337$^{(2)}$	0.01287$^{(4)}$	0.01907$^{(8)}$	0.01748$^{(7)}$	0.02091$^{(10)}$	0.01608$^{(6)}$	0.0209$^{(9)}$
	$\sum Ranks$	13$^{(1)}$	22$^{(2)}$	46$^{(5.5)}$	23$^{(3)}$	46$^{(5.5)}$	72$^{(8)}$	57$^{(7)}$	89$^{(10)}$	45$^{(4)}$	82$^{(9)}$
200	BIAS($\hat{\Phi}$)	0.07051$^{(1)}$	0.08304$^{(2)}$	0.09606$^{(4)}$	0.08962$^{(3)}$	0.10006$^{(6)}$	0.11542$^{(8)}$	0.10956$^{(7)}$	0.13363$^{(10)}$	0.09919$^{(5)}$	0.13119$^{(9)}$
	BIAS($\hat{\chi}$)	0.1557$^{(1)}$	0.18276$^{(3)}$	0.2211$^{(6)}$	0.18268$^{(2)}$	0.22256$^{(7)}$	0.23125$^{(8)}$	0.21327$^{(5)}$	0.26572$^{(10)}$	0.19889$^{(4)}$	0.26356$^{(9)}$
	MSE($\hat{\Phi}$)	0.00802$^{(1)}$	0.01151$^{(2)}$	0.01508$^{(4)}$	0.01359$^{(3)}$	0.01674$^{(6)}$	0.02229$^{(8)}$	0.02059$^{(7)}$	0.02913$^{(10)}$	0.01649$^{(5)}$	0.02807$^{(9)}$
	MSE($\hat{\chi}$)	0.0387$^{(1)}$	0.05269$^{(3)}$	0.07673$^{(6)}$	0.05124$^{(2)}$	0.07726$^{(7)}$	0.08287$^{(8)}$	0.06992$^{(5)}$	0.10758$^{(10)}$	0.06075$^{(4)}$	0.10656$^{(9)}$
	MRE($\hat{\Phi}$)	0.14101$^{(1)}$	0.16607$^{(2)}$	0.19213$^{(4)}$	0.17925$^{(3)}$	0.20013$^{(6)}$	0.23084$^{(8)}$	0.21911$^{(7)}$	0.26726$^{(10)}$	0.19838$^{(5)}$	0.26239$^{(9)}$
	MRE($\hat{\chi}$)	0.07785$^{(1)}$	0.09138$^{(3)}$	0.11055$^{(6)}$	0.09134$^{(2)}$	0.11128$^{(7)}$	0.11563$^{(8)}$	0.10663$^{(5)}$	0.13286$^{(10)}$	0.09945$^{(4)}$	0.13178$^{(9)}$
	D_{abs}	0.01075$^{(1)}$	0.01128$^{(3)}$	0.01213$^{(4)}$	0.01125$^{(2)}$	0.01222$^{(5)}$	0.01716$^{(8)}$	0.0154$^{(7)}$	0.01913$^{(10)}$	0.01425$^{(6)}$	0.01891$^{(9)}$
	D_{max}	0.01818$^{(1)}$	0.01942$^{(3)}$	0.02134$^{(4)}$	0.01933$^{(2)}$	0.02154$^{(5)}$	0.02877$^{(8)}$	0.02584$^{(7)}$	0.03229$^{(10)}$	0.02396$^{(6)}$	0.03196$^{(9)}$
	ASAE	0.01153$^{(1)}$	0.01086$^{(3)}$	0.0111$^{(4)}$	0.01147$^{(2)}$	0.01105$^{(5)}$	0.01641$^{(8)}$	0.01501$^{(7)}$	0.0179$^{(10)}$	0.01388$^{(6)}$	0.01802$^{(9)}$
	$\sum Ranks$	13$^{(1)}$	22$^{(2)}$	41$^{(4)}$	23$^{(3)}$	51$^{(6)}$	72$^{(8)}$	57$^{(7)}$	89$^{(10)}$	45$^{(5)}$	82$^{(9)}$
300	BIAS($\hat{\Phi}$)	0.05839$^{(1)}$	0.06755$^{(2)}$	0.0777$^{(4)}$	0.07025$^{(3)}$	0.08038$^{(6)}$	0.09547$^{(8)}$	0.08768$^{(7)}$	0.11168$^{(10)}$	0.07876$^{(5)}$	0.11013$^{(9)}$
	BIAS($\hat{\chi}$)	0.12839$^{(1)}$	0.14954$^{(3)}$	0.17966$^{(6)}$	0.14684$^{(2)}$	0.17988$^{(7)}$	0.1912$^{(8)}$	0.17445$^{(5)}$	0.21912$^{(10)}$	0.15938$^{(4)}$	0.21822$^{(9)}$
	MSE($\hat{\Phi}$)	0.00544$^{(1)}$	0.00744$^{(2)}$	0.00984$^{(4)}$	0.00826$^{(3)}$	0.01088$^{(6)}$	0.01555$^{(8)}$	0.01307$^{(7)}$	0.02084$^{(10)}$	0.01047$^{(5)}$	0.02031$^{(9)}$
	MSE($\hat{\chi}$)	0.02607$^{(1)}$	0.03535$^{(3)}$	0.05078$^{(7)}$	0.03337$^{(2)}$	0.05071$^{(6)}$	0.05751$^{(8)}$	0.04708$^{(5)}$	0.07387$^{(10)}$	0.03978$^{(4)}$	0.07274$^{(9)}$
	MRE($\hat{\Phi}$)	0.11678$^{(1)}$	0.1351$^{(2)}$	0.15541$^{(4)}$	0.14051$^{(3)}$	0.16076$^{(6)}$	0.19094$^{(8)}$	0.17537$^{(7)}$	0.22336$^{(10)}$	0.15751$^{(5)}$	0.22026$^{(9)}$
	MRE($\hat{\chi}$)	0.06419$^{(1)}$	0.07477$^{(3)}$	0.08983$^{(6)}$	0.07342$^{(2)}$	0.08994$^{(7)}$	0.0956$^{(8)}$	0.08723$^{(5)}$	0.10956$^{(10)}$	0.07969$^{(4)}$	0.10911$^{(9)}$
	D_{abs}	0.00884$^{(1)}$	0.00926$^{(3)}$	0.00986$^{(4)}$	0.00921$^{(2)}$	0.00992$^{(5)}$	0.01431$^{(8)}$	0.0127$^{(7)}$	0.01612$^{(10)}$	0.01154$^{(6)}$	0.01611$^{(9)}$
	D_{max}	0.01496$^{(1)}$	0.0159$^{(3)}$	0.01737$^{(4)}$	0.01583$^{(2)}$	0.01745$^{(5)}$	0.02389$^{(8)}$	0.02128$^{(7)}$	0.02699$^{(9)}$	0.01935$^{(6)}$	0.02708$^{(10)}$
	ASAE	0.0092$^{(1)}$	0.00873$^{(3)}$	0.00889$^{(4)}$	0.00918$^{(2)}$	0.00886$^{(5)}$	0.01332$^{(8)}$	0.01211$^{(7)}$	0.01471$^{(9)}$	0.01114$^{(6)}$	0.01471$^{(10)}$
	$\sum Ranks$	13$^{(1)}$	22$^{(2)}$	42$^{(4)}$	23$^{(3)}$	50$^{(6)}$	72$^{(8)}$	57$^{(7)}$	89$^{(10)}$	45$^{(5)}$	82$^{(9)}$

TABLE 9. Numerical values of simulation measures for $\Phi = 2.5$ and $\chi = 0.5$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	0.96388$^{(3)}$	0.96572$^{(4)}$	0.97522$^{(7)}$	0.97722$^{(8)}$	0.97304$^{(5)}$	0.93939$^{(1)}$	0.94942$^{(2)}$	1.02038$^{(10)}$	0.97316$^{(6)}$	1.01392$^{(9)}$
	BIAS($\hat{\chi}$)	0.19535$^{(9)}$	0.18938$^{(7)}$	0.19988$^{(10)}$	0.17839$^{(1)}$	0.18983$^{(8)}$	0.17887$^{(2)}$	0.18227$^{(4)}$	0.18142$^{(3)}$	0.18373$^{(6)}$	0.18301$^{(5)}$
	MSE($\hat{\Phi}$)	1.23536$^{(2)}$	1.25163$^{(3)}$	1.26959$^{(6)}$	1.29108$^{(8)}$	1.26895$^{(5)}$	1.2296$^{(1)}$	1.25444$^{(4)}$	1.39638$^{(10)}$	1.27714$^{(7)}$	1.37205$^{(9)}$
	MSE($\hat{\chi}$)	0.05686$^{(9)}$	0.05372$^{(8)}$	0.05906$^{(10)}$	0.04767$^{(1)}$	0.05355$^{(7)}$	0.04882$^{(3)}$	0.05027$^{(5)}$	0.04852$^{(2)}$	0.05123$^{(6)}$	0.04885$^{(4)}$
	MRE($\hat{\Phi}$)	0.38555$^{(3)}$	0.38629$^{(4)}$	0.39009$^{(7)}$	0.39089$^{(8)}$	0.38922$^{(5)}$	0.37576$^{(1)}$	0.37977$^{(2)}$	0.40815$^{(10)}$	0.38927$^{(6)}$	0.40557$^{(9)}$
	MRE($\hat{\chi}$)	0.3907$^{(9)}$	0.37876$^{(7)}$	0.39976$^{(10)}$	0.35678$^{(1)}$	0.37967$^{(8)}$	0.35775$^{(2)}$	0.36453$^{(4)}$	0.36285$^{(3)}$	0.36746$^{(6)}$	0.36602$^{(5)}$
	D_{abs}	0.0456$^{(1)}$	0.04846$^{(2)}$	0.04941$^{(3)}$	0.05128$^{(5)}$	0.05278$^{(6)}$	0.05748$^{(8)}$	0.05469$^{(7)}$	0.07155$^{(9)}$	0.04964$^{(4)}$	0.07272$^{(10)}$
	D_{max}	0.07202$^{(1)}$	0.07383$^{(2)}$	0.07677$^{(5)}$	0.07648$^{(4)}$	0.07907$^{(6)}$	0.08668$^{(8)}$	0.08183$^{(7)}$	0.10486$^{(9)}$	0.07532$^{(3)}$	0.10652$^{(10)}$
	ASAE	0.04565$^{(1)}$	0.03966$^{(2)}$	0.0394$^{(5)}$	0.04092$^{(4)}$	0.03837$^{(6)}$	0.05554$^{(8)}$	0.05202$^{(7)}$	0.06348$^{(9)}$	0.04862$^{(3)}$	0.06484$^{(10)}$
	$\sum Ranks$	42$^{(4.5)}$	40$^{(2.5)}$	60$^{(8)}$	40$^{(2.5)}$	51$^{(7)}$	34$^{(1)}$	42$^{(4.5)}$	65$^{(9)}$	50$^{(6)}$	71$^{(10)}$
60	BIAS($\hat{\Phi}$)	0.87681$^{(1)}$	0.89629$^{(4)}$	0.91432$^{(8)}$	0.88441$^{(2)}$	0.90793$^{(7)}$	0.89046$^{(3)}$	0.90342$^{(6)}$	0.94681$^{(9)}$	0.90042$^{(5)}$	0.95655$^{(10)}$
	BIAS($\hat{\chi}$)	0.16731$^{(7)}$	0.17007$^{(8)}$	0.18327$^{(10)}$	0.14664$^{(1)}$	0.17547$^{(9)}$	0.15655$^{(4)}$	0.15514$^{(3)}$	0.16372$^{(5)}$	0.15406$^{(2)}$	0.16397$^{(6)}$
	MSE($\hat{\Phi}$)	1.05794$^{(1)}$	1.11113$^{(2)}$	1.13251$^{(4)}$	1.12934$^{(3)}$	1.13491$^{(6)}$	1.13462$^{(5)}$	1.15883$^{(8)}$	1.23774$^{(9)}$	1.148$^{(7)}$	1.26926$^{(10)}$
	MSE($\hat{\chi}$)	0.04359$^{(7)}$	0.04459$^{(8)}$	0.05048$^{(10)}$	0.0333$^{(1)}$	0.04684$^{(9)}$	0.03862$^{(4)}$	0.03778$^{(3)}$	0.04043$^{(6)}$	0.03716$^{(2)}$	0.04032$^{(5)}$
	MRE($\hat{\Phi}$)	0.35073$^{(1)}$	0.35852$^{(4)}$	0.36573$^{(8)}$	0.35377$^{(2)}$	0.36317$^{(7)}$	0.35618$^{(3)}$	0.36137$^{(6)}$	0.37872$^{(9)}$	0.36017$^{(5)}$	0.38262$^{(10)}$
	MRE($\hat{\chi}$)	0.33463$^{(7)}$	0.34013$^{(8)}$	0.36654$^{(10)}$	0.29328$^{(1)}$	0.35094$^{(9)}$	0.31311$^{(4)}$	0.31028$^{(3)}$	0.32744$^{(5)}$	0.30813$^{(2)}$	0.32794$^{(6)}$
	D_{abs}	0.02853$^{(1)}$	0.02889$^{(3)}$	0.02957$^{(4)}$	0.02868$^{(2)}$	0.03011$^{(5)}$	0.03509$^{(8)}$	0.03262$^{(7)}$	0.04022$^{(9)}$	0.03052$^{(6)}$	0.04085$^{(10)}$
	D_{max}	0.04594$^{(2)}$	0.04613$^{(3)}$	0.04798$^{(4)}$	0.04492$^{(1)}$	0.04812$^{(6)}$	0.05462$^{(8)}$	0.05095$^{(7)}$	0.062$^{(9)}$	0.04799$^{(5)}$	0.06296$^{(10)}$
	ASAE	0.02331$^{(2)}$	0.0211$^{(3)}$	0.0213$^{(4)}$	0.0226$^{(1)}$	0.02089$^{(6)}$	0.03154$^{(8)}$	0.02934$^{(7)}$	0.03494$^{(9)}$	0.02687$^{(5)}$	0.03556$^{(10)}$
	$\sum Ranks$	32$^{(2)}$	42$^{(4)}$	61$^{(8)}$	17$^{(1)}$	59$^{(7)}$	47$^{(5)}$	50$^{(6)}$	70$^{(9)}$	40$^{(3)}$	77$^{(10)}$
100	BIAS($\hat{\Phi}$)	0.80242$^{(1)}$	0.83847$^{(3)}$	0.87676$^{(8)}$	0.83946$^{(4)}$	0.86748$^{(7)}$	0.83434$^{(2)}$	0.85467$^{(6)}$	0.91297$^{(10)}$	0.8407$^{(5)}$	0.90693$^{(9)}$
	BIAS($\hat{\chi}$)	0.14405$^{(5)}$	0.15012$^{(6)}$	0.17188$^{(10)}$	0.12738$^{(1)}$	0.16359$^{(9)}$	0.13757$^{(4)}$	0.13599$^{(3)}$	0.15156$^{(8)}$	0.13521$^{(2)}$	0.15014$^{(7)}$
	MSE($\hat{\Phi}$)	0.92081$^{(1)}$	1.00759$^{(2)}$	1.06902$^{(7)}$	1.05362$^{(5)}$	1.05867$^{(6)}$	1.02588$^{(3)}$	1.07897$^{(8)}$	1.18179$^{(10)}$	1.03276$^{(4)}$	1.17511$^{(9)}$
	MSE($\hat{\chi}$)	0.03298$^{(5)}$	0.03537$^{(8)}$	0.0453$^{(10)}$	0.02489$^{(1)}$	0.04123$^{(9)}$	0.02988$^{(4)}$	0.02879$^{(2)}$	0.03499$^{(7)}$	0.02882$^{(3)}$	0.03466$^{(6)}$
	MRE($\hat{\Phi}$)	0.32097$^{(1)}$	0.33539$^{(3)}$	0.3507$^{(8)}$	0.33578$^{(4)}$	0.34699$^{(7)}$	0.33374$^{(2)}$	0.34187$^{(6)}$	0.36519$^{(10)}$	0.33628$^{(5)}$	0.36277$^{(9)}$
	MRE($\hat{\chi}$)	0.28811$^{(5)}$	0.30024$^{(6)}$	0.34375$^{(10)}$	0.25475$^{(1)}$	0.32717$^{(9)}$	0.27514$^{(4)}$	0.27197$^{(3)}$	0.30313$^{(8)}$	0.27042$^{(2)}$	0.30027$^{(7)}$
	D_{abs}	0.02244$^{(2)}$	0.02286$^{(3)}$	0.02339$^{(5)}$	0.02194$^{(1)}$	0.02328$^{(4)}$	0.02734$^{(8)}$	0.0257$^{(7)}$	0.03068$^{(9)}$	0.02442$^{(6)}$	0.03076$^{(10)}$
	D_{max}	0.03633$^{(2)}$	0.03695$^{(3)}$	0.03871$^{(5)}$	0.03493$^{(1)}$	0.03818$^{(4)}$	0.04306$^{(8)}$	0.0406$^{(7)}$	0.04828$^{(9)}$	0.03873$^{(6)}$	0.0484$^{(10)}$
	ASAE	0.0174$^{(2)}$	0.01592$^{(3)}$	0.01616$^{(5)}$	0.01709$^{(1)}$	0.01591$^{(4)}$	0.02398$^{(8)}$	0.02207$^{(7)}$	0.02655$^{(9)}$	0.02037$^{(6)}$	0.02674$^{(10)}$
	$\sum Ranks$	27$^{(2)}$	36$^{(3)}$	66$^{(8)}$	22$^{(1)}$	56$^{(7)}$	43$^{(5)}$	49$^{(6)}$	80$^{(10)}$	39$^{(4)}$	77$^{(9)}$
150	BIAS($\hat{\Phi}$)	0.74174$^{(1)}$	0.78865$^{(2)}$	0.83177$^{(7)}$	0.7905$^{(3)}$	0.83656$^{(8)}$	0.79962$^{(5)}$	0.80448$^{(6)}$	0.87148$^{(9)}$	0.79234$^{(4)}$	0.87519$^{(10)}$
	BIAS($\hat{\chi}$)	0.1242$^{(4)}$	0.13562$^{(6)}$	0.15881$^{(10)}$	0.11371$^{(1)}$	0.15175$^{(9)}$	0.12613$^{(5)}$	0.12217$^{(3)}$	0.13826$^{(7)}$	0.11948$^{(2)}$	0.13955$^{(8)}$
	MSE($\hat{\Phi}$)	0.82614$^{(1)}$	0.91467$^{(2)}$	0.98253$^{(7)}$	0.97175$^{(4)}$	1.00273$^{(8)}$	0.97253$^{(5)}$	0.9818$^{(6)}$	1.11065$^{(9)}$	0.95919$^{(3)}$	1.12131$^{(10)}$
	MSE($\hat{\chi}$)	0.0248$^{(4)}$	0.02882$^{(6)}$	0.03947$^{(10)}$	0.01963$^{(1)}$	0.0357$^{(9)}$	0.02522$^{(5)}$	0.02312$^{(3)}$	0.02956$^{(7)}$	0.02224$^{(2)}$	0.03$^{(8)}$
	MRE($\hat{\Phi}$)	0.29669$^{(1)}$	0.31546$^{(2)}$	0.33271$^{(7)}$	0.3162$^{(3)}$	0.33462$^{(8)}$	0.31985$^{(5)}$	0.32179$^{(6)}$	0.34859$^{(9)}$	0.31694$^{(4)}$	0.35007$^{(10)}$
	MRE($\hat{\chi}$)	0.2484$^{(4)}$	0.27125$^{(6)}$	0.31762$^{(10)}$	0.22742$^{(1)}$	0.3035$^{(9)}$	0.25226$^{(5)}$	0.24434$^{(3)}$	0.27652$^{(7)}$	0.23897$^{(2)}$	0.27911$^{(8)}$
	D_{abs}	0.01858$^{(2)}$	0.01888$^{(3)}$	0.01957$^{(5)}$	0.01829$^{(1)}$	0.01941$^{(4)}$	0.02243$^{(8)}$	0.02121$^{(7)}$	0.02475$^{(9)}$	0.02032$^{(6)}$	0.02508$^{(10)}$
	D_{max}	0.03012$^{(2)}$	0.03082$^{(3)}$	0.03276$^{(6)}$	0.02935$^{(1)}$	0.03232$^{(4)}$	0.03572$^{(8)}$	0.03378$^{(7)}$	0.03953$^{(9)}$	0.03242$^{(5)}$	0.04001$^{(10)}$
	ASAE	0.01388$^{(2)}$	0.01275$^{(3)}$	0.01307$^{(6)}$	0.01384$^{(1)}$	0.0129$^{(4)}$	0.01919$^{(8)}$	0.01784$^{(7)}$	0.02109$^{(9)}$	0.01631$^{(5)}$	0.02149$^{(10)}$
	$\sum Ranks$	24$^{(2)}$	31$^{(3)}$	65$^{(8)}$	19$^{(1)}$	61$^{(7)}$	54$^{(6)}$	48$^{(5)}$	75$^{(9)}$	34$^{(4)}$	84$^{(10)}$
200	BIAS($\hat{\Phi}$)	0.68468$^{(1)}$	0.7487$^{(3)}$	0.80009$^{(8)}$	0.7435$^{(2)}$	0.79672$^{(7)}$	0.76101$^{(5)}$	0.76612$^{(6)}$	0.85512$^{(10)}$	0.75429$^{(4)}$	0.8439$^{(9)}$
	BIAS($\hat{\chi}$)	0.112$^{(3)}$	0.1241$^{(6)}$	0.14619$^{(10)}$	0.10348$^{(1)}$	0.14007$^{(9)}$	0.11554$^{(5)}$	0.11204$^{(4)}$	0.12979$^{(8)}$	0.11003$^{(2)}$	0.12896$^{(7)}$
	MSE($\hat{\Phi}$)	0.71178$^{(1)}$	0.84678$^{(2)}$	0.93114$^{(7)}$	0.88686$^{(3)}$	0.93165$^{(8)}$	0.89713$^{(5)}$	0.913$^{(6)}$	1.08394$^{(10)}$	0.88835$^{(4)}$	1.0688$^{(9)}$
	MSE($\hat{\chi}$)	0.02002$^{(4)}$	0.02417$^{(6)}$	0.03341$^{(10)}$	0.01627$^{(1)}$	0.03069$^{(9)}$	0.02107$^{(5)}$	0.01945$^{(3)}$	0.0256$^{(7)}$	0.01855$^{(2)}$	0.02566$^{(8)}$
	MRE($\hat{\Phi}$)	0.27387$^{(1)}$	0.29948$^{(3)}$	0.32004$^{(8)}$	0.2974$^{(2)}$	0.31869$^{(7)}$	0.3044$^{(5)}$	0.30645$^{(6)}$	0.34205$^{(10)}$	0.30172$^{(4)}$	0.33756$^{(9)}$
	MRE($\hat{\chi}$)	0.22401$^{(3)}$	0.2482$^{(6)}$	0.29239$^{(10)}$	0.20696$^{(1)}$	0.28014$^{(9)}$	0.23107$^{(5)}$	0.22408$^{(4)}$	0.25958$^{(8)}$	0.22007$^{(2)}$	0.25793$^{(7)}$
	D_{abs}	0.01645$^{(2)}$	0.01671$^{(3)}$	0.01723$^{(5)}$	0.01618$^{(1)}$	0.01711$^{(4)}$	0.01978$^{(8)}$	0.01845$^{(7)}$	0.02193$^{(10)}$	0.01771$^{(6)}$	0.02147$^{(9)}$
	D_{max}	0.02674$^{(2)}$	0.02739$^{(3)}$	0.02901$^{(6)}$	0.02599$^{(1)}$	0.02862$^{(5)}$	0.03162$^{(8)}$	0.02953$^{(7)}$	0.03524$^{(10)}$	0.02838$^{(4)}$	0.03451$^{(9)}$
	ASAE	0.01181$^{(2)}$	0.01094$^{(3)}$	0.01119$^{(6)}$	0.0118$^{(1)}$	0.01113$^{(5)}$	0.01658$^{(8)}$	0.01519$^{(7)}$	0.01832$^{(10)}$	0.01402$^{(4)}$	0.01818$^{(9)}$
	$\sum Ranks$	22$^{(2)}$	33$^{(3)}$	67$^{(8)}$	16$^{(1)}$	60$^{(7)}$	54$^{(6)}$	50$^{(5)}$	83$^{(10)}$	34$^{(4)}$	76$^{(9)}$
300	BIAS($\hat{\Phi}$)	0.59299$^{(1)}$	0.68395$^{(4)}$	0.73124$^{(7)}$	0.67291$^{(3)}$	0.75697$^{(8)}$	0.70971$^{(6)}$	0.70311$^{(5)}$	0.78631$^{(9)}$	0.66975$^{(2)}$	0.79917$^{(10)}$
	BIAS($\hat{\chi}$)	0.09176$^{(2)}$	0.10725$^{(6)}$	0.12897$^{(10)}$	0.09012$^{(1)}$	0.12699$^{(9)}$	0.10342$^{(5)}$	0.09808$^{(4)}$	0.11635$^{(7)}$	0.09436$^{(3)}$	0.11659$^{(8)}$
	MSE($\hat{\Phi}$)	0.5636$^{(1)}$	0.73887$^{(3)}$	0.80965$^{(6)}$	0.75532$^{(4)}$	0.86764$^{(8)}$	0.81084$^{(7)}$	0.79799$^{(5)}$	0.95254$^{(9)}$	0.72496$^{(2)}$	0.98921$^{(10)}$
	MSE($\hat{\chi}$)	0.01349$^{(2)}$	0.01779$^{(6)}$	0.02639$^{(10)}$	0.01215$^{(1)}$	0.02515$^{(9)}$	0.01665$^{(5)}$	0.01463$^{(4)}$	0.02074$^{(7)}$	0.01361$^{(3)}$	0.02095$^{(8)}$
	MRE($\hat{\Phi}$)	0.2372$^{(1)}$	0.27358$^{(4)}$	0.2925$^{(7)}$	0.26916$^{(3)}$	0.30279$^{(8)}$	0.28389$^{(6)}$	0.28125$^{(5)}$	0.31452$^{(9)}$	0.2679$^{(2)}$	0.31967$^{(10)}$
	MRE($\hat{\chi}$)	0.18352$^{(2)}$	0.21449$^{(6)}$	0.25793$^{(10)}$	0.18024$^{(1)}$	0.25398$^{(9)}$	0.20685$^{(5)}$	0.19616$^{(4)}$	0.23271$^{(7)}$	0.18872$^{(3)}$	0.23317$^{(8)}$
	D_{abs}	0.0137$^{(2)}$	0.01378$^{(3)}$	0.0143$^{(5)}$	0.01344$^{(1)}$	0.01427$^{(4)}$	0.01646$^{(8)}$	0.0156$^{(7)}$	0.01779$^{(9)}$	0.01493$^{(6)}$	0.01816$^{(10)}$
	D_{max}	0.02214$^{(2)}$	0.02272$^{(3)}$	0.02421$^{(6)}$	0.02167$^{(1)}$	0.02413$^{(5)}$	0.02648$^{(8)}$	0.02503$^{(7)}$	0.02882$^{(9)}$	0.02396$^{(4)}$	0.02937$^{(10)}$
	ASAE	0.00952$^{(2)}$	0.00888$^{(3)}$	0.00911$^{(6)}$	0.00947$^{(1)}$	0.00903$^{(5)}$	0.0134$^{(8)}$	0.01227$^{(7)}$	0.01485$^{(9)}$	0.01135$^{(4)}$	0.01489$^{(10)}$
	$\sum Ranks$	18$^{(1)}$	36$^{(4)}$	64$^{(8)}$	19$^{(2)}$	62$^{(7)}$	58$^{(6)}$	48$^{(5)}$	75$^{(9)}$	31$^{(3)}$	84$^{(10)}$

TABLE 10. Numerical values of simulation measures for $\Phi = 2.5$ and $\chi = 0.5$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
20	BIAS($\hat{\Phi}$)	$0.90251^{\{2\}}$	$0.90564^{\{3\}}$	$0.92334^{\{5\}}$	$0.93324^{\{7\}}$	$0.92935^{\{6\}}$	$0.91193^{\{4\}}$	$0.90187^{\{1\}}$	$0.99319^{\{10\}}$	$0.93741^{\{8\}}$	$0.98728^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.18463^{\{9\}}$	$0.17666^{\{4\}}$	$0.18986^{\{10\}}$	$0.16823^{\{1\}}$	$0.17938^{\{8\}}$	$0.17332^{\{2\}}$	$0.17344^{\{3\}}$	$0.17882^{\{7\}}$	$0.17672^{\{5\}}$	$0.17756^{\{6\}}$
	MSE($\hat{\Phi}$)	$1.10423^{\{1\}}$	$1.13778^{\{2\}}$	$1.1583^{\{4\}}$	$1.21496^{\{8\}}$	$1.19208^{\{6\}}$	$1.17271^{\{5\}}$	$1.15378^{\{3\}}$	$1.33095^{\{10\}}$	$1.20538^{\{7\}}$	$1.32434^{\{9\}}$
	MSE($\hat{\chi}$)	$0.0518^{\{9\}}$	$0.04774^{\{7\}}$	$0.05382^{\{10\}}$	$0.04246^{\{1\}}$	$0.04827^{\{8\}}$	$0.04598^{\{2\}}$	$0.0464^{\{3\}}$	$0.04685^{\{5\}}$	$0.04727^{\{6\}}$	$0.04664^{\{4\}}$
	MRE($\hat{\Phi}$)	$0.361^{\{2\}}$	$0.36225^{\{3\}}$	$0.36934^{\{5\}}$	$0.3733^{\{7\}}$	$0.37174^{\{6\}}$	$0.36477^{\{4\}}$	$0.36075^{\{1\}}$	$0.39728^{\{10\}}$	$0.37496^{\{8\}}$	$0.39491^{\{9\}}$
	MRE($\hat{\chi}$)	$0.36926^{\{9\}}$	$0.35333^{\{4\}}$	$0.37972^{\{10\}}$	$0.33645^{\{1\}}$	$0.35876^{\{8\}}$	$0.34663^{\{2\}}$	$0.34689^{\{3\}}$	$0.35763^{\{7\}}$	$0.35344^{\{5\}}$	$0.35512^{\{6\}}$
	D_{abs}	$0.02956^{\{1\}}$	$0.03032^{\{2\}}$	$0.03139^{\{3\}}$	$0.03141^{\{4\}}$	$0.03284^{\{5\}}$	$0.04384^{\{8\}}$	$0.03854^{\{7\}}$	$0.05677^{\{9\}}$	$0.03527^{\{6\}}$	$0.05773^{\{10\}}$
	D_{max}	$0.04797^{\{2\}}$	$0.04783^{\{1\}}$	$0.05038^{\{4\}}$	$0.04877^{\{3\}}$	$0.05106^{\{5\}}$	$0.0673^{\{8\}}$	$0.05936^{\{7\}}$	$0.0844^{\{9\}}$	$0.05484^{\{6\}}$	$0.08557^{\{10\}}$
	ASAE	$0.04427^{\{2\}}$	$0.03898^{\{1\}}$	$0.03879^{\{4\}}$	$0.03973^{\{3\}}$	$0.03761^{\{5\}}$	$0.05384^{\{8\}}$	$0.05008^{\{7\}}$	$0.06335^{\{9\}}$	$0.04756^{\{6\}}$	$0.06414^{\{10\}}$
	$\sum Ranks$	$40^{\{4\}}$	$29^{\{1\}}$	$53^{\{6.5\}}$	$36^{\{3\}}$	$53^{\{6.5\}}$	$43^{\{5\}}$	$35^{\{2\}}$	$76^{\{10\}}$	$57^{\{8\}}$	$73^{\{9\}}$
60	BIAS($\hat{\Phi}$)	$0.79034^{\{1\}}$	$0.83877^{\{2\}}$	$0.86394^{\{4\}}$	$0.86421^{\{5\}}$	$0.86492^{\{6\}}$	$0.85975^{\{3\}}$	$0.88614^{\{8\}}$	$0.9237^{\{9\}}$	$0.86861^{\{7\}}$	$0.92416^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.14782^{\{3\}}$	$0.15293^{\{6\}}$	$0.17238^{\{10\}}$	$0.13743^{\{1\}}$	$0.16181^{\{9\}}$	$0.15015^{\{5\}}$	$0.14904^{\{4\}}$	$0.1566^{\{7\}}$	$0.14676^{\{2\}}$	$0.15757^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.90077^{\{1\}}$	$1.01466^{\{2\}}$	$1.03913^{\{3\}}$	$1.10889^{\{7\}}$	$1.06645^{\{4\}}$	$1.07081^{\{5\}}$	$1.13568^{\{8\}}$	$1.20437^{\{9\}}$	$1.09272^{\{6\}}$	$1.20792^{\{10\}}$
	MSE($\hat{\chi}$)	$0.03458^{\{4\}}$	$0.03618^{\{6\}}$	$0.04548^{\{10\}}$	$0.0284^{\{1\}}$	$0.04024^{\{9\}}$	$0.03525^{\{5\}}$	$0.03427^{\{3\}}$	$0.03687^{\{7\}}$	$0.03347^{\{2\}}$	$0.03722^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.31614^{\{1\}}$	$0.33551^{\{2\}}$	$0.34558^{\{4\}}$	$0.34568^{\{5\}}$	$0.34597^{\{6\}}$	$0.3439^{\{3\}}$	$0.35446^{\{8\}}$	$0.36948^{\{9\}}$	$0.34744^{\{7\}}$	$0.36967^{\{10\}}$
	MRE($\hat{\chi}$)	$0.29565^{\{3\}}$	$0.30586^{\{6\}}$	$0.34476^{\{10\}}$	$0.27485^{\{1\}}$	$0.32362^{\{9\}}$	$0.3003^{\{5\}}$	$0.29808^{\{4\}}$	$0.31321^{\{7\}}$	$0.29351^{\{2\}}$	$0.31515^{\{8\}}$
	D_{abs}	$0.01831^{\{2\}}$	$0.01849^{\{3\}}$	$0.01908^{\{4.5\}}$	$0.01826^{\{1\}}$	$0.01908^{\{4.5\}}$	$0.02751^{\{8\}}$	$0.024^{\{7\}}$	$0.0328^{\{10\}}$	$0.02183^{\{6\}}$	$0.03272^{\{9\}}$
	D_{max}	$0.03047^{\{2\}}$	$0.03073^{\{3\}}$	$0.03241^{\{5\}}$	$0.03001^{\{1\}}$	$0.03197^{\{4\}}$	$0.0437^{\{8\}}$	$0.03856^{\{7\}}$	$0.05125^{\{10\}}$	$0.03534^{\{6\}}$	$0.05119^{\{9\}}$
	ASAE	$0.02272^{\{2\}}$	$0.02061^{\{3\}}$	$0.021^{\{5\}}$	$0.02174^{\{1\}}$	$0.02056^{\{4\}}$	$0.03079^{\{8\}}$	$0.02853^{\{7\}}$	$0.0346^{\{10\}}$	$0.02603^{\{6\}}$	$0.03475^{\{9\}}$
	$\sum Ranks$	$22^{\{1\}}$	$32^{\{3\}}$	$53.5^{\{7\}}$	$26^{\{2\}}$	$52.5^{\{6\}}$	$50^{\{5\}}$	$56^{\{8\}}$	$77^{\{9\}}$	$44^{\{4\}}$	$82^{\{10\}}$
100	BIAS($\hat{\Phi}$)	$0.71293^{\{1\}}$	$0.78179^{\{2\}}$	$0.8305^{\{7\}}$	$0.81798^{\{4\}}$	$0.82864^{\{6\}}$	$0.82249^{\{5\}}$	$0.83295^{\{8\}}$	$0.90207^{\{10\}}$	$0.79134^{\{3\}}$	$0.89781^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.12537^{\{2\}}$	$0.13707^{\{6\}}$	$0.15689^{\{10\}}$	$0.12102^{\{1\}}$	$0.14905^{\{9\}}$	$0.1339^{\{5\}}$	$0.13216^{\{4\}}$	$0.14526^{\{8\}}$	$0.12698^{\{3\}}$	$0.14408^{\{7\}}$
	MSE($\hat{\Phi}$)	$0.7687^{\{1\}}$	$0.91396^{\{2\}}$	$0.99198^{\{4\}}$	$1.03416^{\{7\}}$	$1.00493^{\{5\}}$	$1.01789^{\{6\}}$	$1.04202^{\{8\}}$	$1.18131^{\{10\}}$	$0.94997^{\{3\}}$	$1.16687^{\{9\}}$
	MSE($\hat{\chi}$)	$0.02499^{\{2\}}$	$0.02937^{\{6\}}$	$0.03799^{\{10\}}$	$0.02192^{\{1\}}$	$0.03432^{\{9\}}$	$0.02813^{\{5\}}$	$0.02707^{\{4\}}$	$0.03166^{\{8\}}$	$0.02525^{\{3\}}$	$0.03136^{\{7\}}$
	MRE($\hat{\Phi}$)	$0.28517^{\{1\}}$	$0.31272^{\{2\}}$	$0.3322^{\{7\}}$	$0.32719^{\{4\}}$	$0.33146^{\{6\}}$	$0.32899^{\{5\}}$	$0.33318^{\{8\}}$	$0.36083^{\{10\}}$	$0.31653^{\{3\}}$	$0.35913^{\{9\}}$
	MRE($\hat{\chi}$)	$0.25073^{\{2\}}$	$0.27415^{\{6\}}$	$0.31378^{\{10\}}$	$0.24204^{\{1\}}$	$0.29809^{\{9\}}$	$0.2678^{\{5\}}$	$0.26432^{\{4\}}$	$0.29053^{\{8\}}$	$0.25397^{\{3\}}$	$0.28817^{\{7\}}$
	D_{abs}	$0.01431^{\{2\}}$	$0.01466^{\{3\}}$	$0.01539^{\{5\}}$	$0.0141^{\{1\}}$	$0.01507^{\{4\}}$	$0.02136^{\{8\}}$	$0.01915^{\{7\}}$	$0.02512^{\{9\}}$	$0.01765^{\{6\}}$	$0.02529^{\{10\}}$
	D_{max}	$0.02403^{\{2\}}$	$0.02488^{\{3\}}$	$0.02664^{\{5\}}$	$0.02376^{\{1\}}$	$0.02601^{\{4\}}$	$0.03446^{\{8\}}$	$0.03122^{\{7\}}$	$0.04025^{\{9\}}$	$0.02884^{\{6\}}$	$0.04045^{\{10\}}$
	ASAE	$0.01695^{\{2\}}$	$0.0156^{\{3\}}$	$0.0158^{\{5\}}$	$0.01651^{\{1\}}$	$0.01572^{\{4\}}$	$0.02346^{\{8\}}$	$0.0215^{\{7\}}$	$0.02616^{\{9\}}$	$0.0197^{\{6\}}$	$0.02626^{\{10\}}$
	$\sum Ranks$	$18^{\{1\}}$	$31^{\{3\}}$	$61^{\{8\}}$	$24^{\{2\}}$	$54^{\{5\}}$	$55^{\{6\}}$	$57^{\{7\}}$	$81^{\{10\}}$	$36^{\{4\}}$	$78^{\{9\}}$
150	BIAS($\hat{\Phi}$)	$0.63832^{\{1\}}$	$0.71292^{\{2\}}$	$0.77005^{\{5\}}$	$0.75914^{\{4\}}$	$0.7848^{\{7\}}$	$0.78124^{\{6\}}$	$0.78987^{\{8\}}$	$0.8523^{\{9\}}$	$0.75081^{\{3\}}$	$0.86473^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.10741^{\{1\}}$	$0.11832^{\{4\}}$	$0.14051^{\{10\}}$	$0.1081^{\{2\}}$	$0.13535^{\{9\}}$	$0.12087^{\{6\}}$	$0.11891^{\{5\}}$	$0.13205^{\{7\}}$	$0.11424^{\{3\}}$	$0.13348^{\{8\}}$
	MSE($\hat{\Phi}$)	$0.64023^{\{1\}}$	$0.79346^{\{2\}}$	$0.88272^{\{3\}}$	$0.92869^{\{5\}}$	$0.93747^{\{6\}}$	$0.94735^{\{7\}}$	$0.97247^{\{8\}}$	$1.09529^{\{9\}}$	$0.89023^{\{4\}}$	$1.10622^{\{10\}}$
	MSE($\hat{\chi}$)	$0.01855^{\{2\}}$	$0.02217^{\{5\}}$	$0.03112^{\{10\}}$	$0.01743^{\{1\}}$	$0.02875^{\{9\}}$	$0.02296^{\{6\}}$	$0.02172^{\{4\}}$	$0.02668^{\{7\}}$	$0.02034^{\{3\}}$	$0.02695^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.25533^{\{1\}}$	$0.28517^{\{2\}}$	$0.30802^{\{5\}}$	$0.30366^{\{4\}}$	$0.31392^{\{7\}}$	$0.3125^{\{6\}}$	$0.31595^{\{8\}}$	$0.34092^{\{9\}}$	$0.30032^{\{3\}}$	$0.34589^{\{10\}}$
	MRE($\hat{\chi}$)	$0.21482^{\{1\}}$	$0.23663^{\{4\}}$	$0.28103^{\{10\}}$	$0.2162^{\{2\}}$	$0.2707^{\{9\}}$	$0.24174^{\{6\}}$	$0.23783^{\{5\}}$	$0.2641^{\{7\}}$	$0.22849^{\{3\}}$	$0.26696^{\{8\}}$
	D_{abs}	$0.01212^{\{2\}}$	$0.01222^{\{3\}}$	$0.01284^{\{5\}}$	$0.01174^{\{1\}}$	$0.01275^{\{4\}}$	$0.0181^{\{8\}}$	$0.01601^{\{7\}}$	$0.02045^{\{9\}}$	$0.01491^{\{6\}}$	$0.02058^{\{10\}}$
	D_{max}	$0.02033^{\{2\}}$	$0.02084^{\{3\}}$	$0.0225^{\{5\}}$	$0.01996^{\{1\}}$	$0.02226^{\{4\}}$	$0.0294^{\{8\}}$	$0.02627^{\{7\}}$	$0.03327^{\{9\}}$	$0.02453^{\{6\}}$	$0.03348^{\{10\}}$
	ASAE	$0.0135^{\{2\}}$	$0.01252^{\{3\}}$	$0.01277^{\{5\}}$	$0.01325^{\{1\}}$	$0.01272^{\{4\}}$	$0.01885^{\{8\}}$	$0.01734^{\{7\}}$	$0.02092^{\{9\}}$	$0.01595^{\{6\}}$	$0.02109^{\{10\}}$
	$\sum Ranks$	$16^{\{1\}}$	$26^{\{3\}}$	$56^{\{5\}}$	$24^{\{2\}}$	$57^{\{6\}}$	$61^{\{8\}}$	$59^{\{7\}}$	$75^{\{9\}}$	$37^{\{4\}}$	$84^{\{10\}}$
200	BIAS($\hat{\Phi}$)	$0.57698^{\{1\}}$	$0.66924^{\{2\}}$	$0.73936^{\{5\}}$	$0.71371^{\{4\}}$	$0.74061^{\{7\}}$	$0.7429^{\{8\}}$	$0.74011^{\{6\}}$	$0.82177^{\{9\}}$	$0.70358^{\{3\}}$	$0.83371^{\{10\}}$
	BIAS($\hat{\chi}$)	$0.09332^{\{1\}}$	$0.10738^{\{5\}}$	$0.12913^{\{10\}}$	$0.09816^{\{2\}}$	$0.124^{\{8\}}$	$0.11229^{\{6\}}$	$0.10712^{\{4\}}$	$0.12371^{\{7\}}$	$0.10312^{\{3\}}$	$0.12534^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.53871^{\{1\}}$	$0.72291^{\{2\}}$	$0.83589^{\{4\}}$	$0.83842^{\{5\}}$	$0.85188^{\{6\}}$	$0.87334^{\{7\}}$	$0.88004^{\{8\}}$	$1.02895^{\{9\}}$	$0.79676^{\{3\}}$	$1.05883^{\{10\}}$
	MSE($\hat{\chi}$)	$0.01382^{\{1\}}$	$0.01815^{\{5\}}$	$0.02625^{\{10\}}$	$0.01412^{\{2\}}$	$0.02394^{\{9\}}$	$0.01974^{\{6\}}$	$0.01739^{\{4\}}$	$0.02329^{\{7\}}$	$0.0164^{\{3\}}$	$0.02385^{\{8\}}$
	MRE($\hat{\Phi}$)	$0.23079^{\{1\}}$	$0.26769^{\{2\}}$	$0.29574^{\{5\}}$	$0.28548^{\{4\}}$	$0.29624^{\{7\}}$	$0.29716^{\{8\}}$	$0.29604^{\{6\}}$	$0.32871^{\{9\}}$	$0.28143^{\{3\}}$	$0.33349^{\{10\}}$
	MRE($\hat{\chi}$)	$0.18663^{\{1\}}$	$0.21477^{\{5\}}$	$0.25826^{\{10\}}$	$0.19633^{\{2\}}$	$0.248^{\{8\}}$	$0.22459^{\{6\}}$	$0.21423^{\{4\}}$	$0.24742^{\{7\}}$	$0.20624^{\{3\}}$	$0.25067^{\{9\}}$
	D_{abs}	$0.01041^{\{2\}}$	$0.01068^{\{3\}}$	$0.01135^{\{5\}}$	$0.01034^{\{1\}}$	$0.01115^{\{4\}}$	$0.01558^{\{8\}}$	$0.01385^{\{7\}}$	$0.01791^{\{10\}}$	$0.01303^{\{6\}}$	$0.01785^{\{9\}}$
	D_{max}	$0.01745^{\{1\}}$	$0.01828^{\{3\}}$	$0.02003^{\{5\}}$	$0.01761^{\{2\}}$	$0.01961^{\{4\}}$	$0.02555^{\{8\}}$	$0.02294^{\{7\}}$	$0.02939^{\{10\}}$	$0.02151^{\{6\}}$	$0.02932^{\{9\}}$
	ASAE	$0.0115^{\{1\}}$	$0.01075^{\{3\}}$	$0.01097^{\{5\}}$	$0.0114^{\{2\}}$	$0.01087^{\{4\}}$	$0.01606^{\{8\}}$	$0.01475^{\{7\}}$	$0.01787^{\{10\}}$	$0.01367^{\{6\}}$	$0.01796^{\{9\}}$
	$\sum Ranks$	$14^{\{1\}}$	$28^{\{3\}}$	$57^{\{7\}}$	$26^{\{2\}}$	$55^{\{6\}}$	$65^{\{8\}}$	$53^{\{5\}}$	$77^{\{9\}}$	$36^{\{4\}}$	$84^{\{10\}}$
300	BIAS($\hat{\Phi}$)	$0.48099^{\{1\}}$	$0.59354^{\{2\}}$	$0.66141^{\{5\}}$	$0.6158^{\{3\}}$	$0.67556^{\{6\}}$	$0.69042^{\{8\}}$	$0.68391^{\{7\}}$	$0.76732^{\{10\}}$	$0.62404^{\{4\}}$	$0.76618^{\{9\}}$
	BIAS($\hat{\chi}$)	$0.07682^{\{1\}}$	$0.092^{\{4\}}$	$0.11205^{\{10\}}$	$0.08458^{\{2\}}$	$0.10861^{\{7\}}$	$0.09991^{\{6\}}$	$0.0955^{\{5\}}$	$0.11118^{\{8\}}$	$0.08984^{\{3\}}$	$0.11174^{\{9\}}$
	MSE($\hat{\Phi}$)	$0.38576^{\{1\}}$	$0.58507^{\{2\}}$	$0.69658^{\{5\}}$	$0.65017^{\{3\}}$	$0.73251^{\{6\}}$	$0.7783^{\{8\}}$	$0.7725^{\{7\}}$	$0.93305^{\{10\}}$	$0.65568^{\{4\}}$	$0.92073^{\{9\}}$
	MSE($\hat{\chi}$)	$0.00944^{\{1\}}$	$0.01303^{\{4\}}$	$0.0197^{\{10\}}$	$0.01063^{\{2\}}$	$0.01823^{\{7\}}$	$0.0153^{\{6\}}$	$0.01364^{\{5\}}$	$0.0188^{\{8\}}$	$0.01225^{\{3\}}$	$0.01886^{\{9\}}$
	MRE($\hat{\Phi}$)	$0.1924^{\{1\}}$	$0.23741^{\{2\}}$	$0.26456^{\{5\}}$	$0.24632^{\{3\}}$	$0.27022^{\{6\}}$	$0.27617^{\{8\}}$	$0.27356^{\{7\}}$	$0.30693^{\{10\}}$	$0.24962^{\{4\}}$	$0.30647^{\{9\}}$
	MRE($\hat{\chi}$)	$0.15364^{\{1\}}$	$0.184^{\{4\}}$	$0.22411^{\{10\}}$	$0.16917^{\{2\}}$	$0.21722^{\{7\}}$	$0.19983^{\{6\}}$	$0.19099^{\{5\}}$	$0.22237^{\{8\}}$	$0.17967^{\{3\}}$	$0.22348^{\{9\}}$
	D_{abs}	$0.00852^{\{1\}}$	$0.00887^{\{3\}}$	$0.00943^{\{5\}}$	$0.00865^{\{2\}}$	$0.00932^{\{4\}}$	$0.01311^{\{8\}}$	$0.01204^{\{7\}}$	$0.01494^{\{10\}}$	$0.01096^{\{6\}}$	$0.01484^{\{9\}}$
	D_{max}	$0.01432^{\{1\}}$	$0.01526^{\{3\}}$	$0.01676^{\{5\}}$	$0.01478^{\{2\}}$	$0.01649^{\{4\}}$	$0.02161^{\{8\}}$	$0.01991^{\{7\}}$	$0.02472^{\{10\}}$	$0.01819^{\{6\}}$	$0.02459^{\{9\}}$
	ASAE	$0.0092^{\{1\}}$	$0.00863^{\{3\}}$	$0.00889^{\{5\}}$	$0.00917^{\{2\}}$	$0.00881^{\{4\}}$	$0.01294^{\{8\}}$	$0.01205^{\{7\}}$	$0.01458^{\{10\}}$	$0.01104^{\{6\}}$	$0.0146^{\{9\}}$
	$\sum Ranks$	$13^{\{1\}}$	$25^{\{3\}}$	$58^{\{7\}}$	$23^{\{2\}}$	$49^{\{5\}}$	$66^{\{8\}}$	$57^{\{6\}}$	$83^{\{10\}}$	$39^{\{4\}}$	$82^{\{9\}}$

Table 11. Partial and overall ranks for all estimation methods of our proposed model by SRS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
$\Phi = 0.8,\ \chi = 0.2$	20	6.0	2.5	8.0	1.0	7.0	2.5	5.0	9.0	4.0	10.0
	60	2.0	3.5	8.0	1.0	7.0	6.0	5.0	10.0	3.5	9.0
	100	2.0	4.0	8.0	1.0	5.0	6.0	7.0	9.0	3.0	10.0
	150	1.5	3.0	7.0	1.5	5.0	8.0	6.0	10.0	4.0	9.0
	200	1.0	3.5	6.0	2.0	7.0	8.0	5.0	9.0	3.5	10.0
	300	1.0	3.0	7.0	2.0	6.0	8.0	5.0	9.0	4.0	10.0
$\Phi = 2.0,\ \chi = 3.0$	20	6.0	3.0	8.0	1.0	7.0	5.0	4.0	10.0	2.0	9.0
	60	2.0	3.5	7.0	1.0	8.0	6.0	5.0	10.0	3.5	9.0
	100	2.5	2.5	8.0	1.0	7.0	6.0	5.0	9.0	4.0	10.0
	150	2.0	4.0	8.0	1.0	7.0	6.0	5.0	10.0	3.0	9.0
	200	2.0	4.0	7.0	1.0	8.0	6.0	5.0	10.0	3.0	9.0
	300	1.0	4.0	7.0	2.0	8.0	6.0	5.0	10.0	3.0	9.0
$\Phi = 1.5,\ \chi = 0.7$	20	4.0	5.0	8.0	1.0	7.0	3.0	2.0	10.0	6.0	9.0
	60	2.0	3.0	8.0	1.0	7.0	6.0	4.0	9.0	5.0	10.0
	100	2.0	3.5	8.0	1.0	7.0	5.5	5.5	10.0	3.5	9.0
	150	1.5	3.0	8.0	1.5	7.0	6.0	5.0	9.0	4.0	10.0
	200	1.0	3.0	8.0	2.0	7.0	6.0	5.0	9.0	4.0	10.0
	300	1.0	3.0	7.5	2.0	7.5	6.0	5.0	9.0	4.0	10.0
$\Phi = 0.5,\ \chi = 2.0$	20	6.0	4.5	9.5	1.0	7.0	2.5	4.5	9.5	2.5	8.0
	60	2.0	3.0	7.0	1.0	6.0	8.0	5.0	10.0	4.0	9.0
	100	2.0	3.0	7.0	1.0	5.0	8.0	6.0	9.0	4.0	10.0
	150	2.0	3.0	5.0	1.0	7.0	8.0	6.0	9.0	4.0	10.0
	200	1.0	3.0	5.0	2.0	6.5	8.0	6.5	10.0	4.0	9.0
	300	1.0	3.0	7.0	2.0	5.5	8.0	5.5	10.0	4.0	9.0
$\Phi = 2.5,\ \chi = 0.5$	20	4.5	2.5	8.0	2.5	7.0	1.0	4.5	9.0	6.0	10.0
	60	2.0	4.0	8.0	1.0	7.0	5.0	6.0	9.0	3.0	10.0
	100	2.0	3.0	8.0	1.0	7.0	5.0	6.0	10.0	4.0	9.0
	150	2.0	3.0	8.0	1.0	7.0	6.0	5.0	9.0	4.0	10.0
	200	2.0	3.0	8.0	1.0	7.0	6.0	5.0	10.0	4.0	9.0
	300	1.0	4.0	8.0	2.0	7.0	6.0	5.0	9.0	3.0	10.0
$\sum$ Ranks		68.0	100.0	225.0	40.5	203.5	177.5	153.5	284.5	113.5	284.0
Overall Rank		2	3	8	1	7	6	5	10	4	9

Table 12. Partial and overall ranks for all estimation methods of our proposed model by RSS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSDE	MSSLDE	MSLNDE
$\Phi = 0.8,\ \chi = 0.2$	20	3.0	2.0	8.0	1.0	4.0	7.0	5.0	10.0	6.0	9.0
	60	1.0	2.0	5.5	3.0	5.5	8.0	7.0	9.0	4.0	10.0
	100	1.0	2.5	6.0	2.5	5.0	8.0	7.0	10.0	4.0	9.0
	150	1.0	3.0	5.5	2.0	5.5	8.0	7.0	9.0	4.0	10.0
	200	1.0	2.5	5.0	2.5	6.0	8.0	7.0	9.0	4.0	10.0
	300	1.0	2.0	4.0	3.0	5.0	8.0	7.0	10.0	6.0	9.0
$\Phi = 2.0,\ \chi = 3.0$	20	3.0	2.0	8.0	1.0	5.0	7.0	4.0	9.0	6.0	10.0
	60	1.5	3.0	7.0	1.5	5.0	8.0	6.0	10.0	4.0	9.0
	100	1.0	3.0	6.0	2.0	5.0	8.0	7.0	9.0	4.0	10.0
	150	1.0	3.0	6.0	2.0	5.0	8.0	7.0	9.0	4.0	10.0
	200	1.0	2.5	7.0	2.5	6.0	8.0	5.0	9.0	4.0	10.0
	300	1.0	3.0	6.0	2.0	5.0	8.0	7.0	10.0	4.0	9.0
$\Phi = 1.5,\ \chi = 0.7$	20	3.0	1.0	7.0	2.0	8.0	6.0	5.0	9.0	4.0	10.0
	60	1.0	3.0	5.5	2.0	5.5	8.0	7.0	10.0	4.0	9.0
	100	1.0	3.0	6.0	2.0	5.0	8.0	7.0	9.0	4.0	10.0
	150	1.0	2.0	5.5	3.0	5.5	7.5	7.5	10.0	4.0	9.0
	200	1.0	3.0	6.0	2.0	5.0	8.0	7.0	9.0	4.0	10.0
	300	1.0	3.0	6.0	2.0	5.0	8.0	7.0	10.0	4.0	9.0
$\Phi = 0.5,\ \chi = 2.0$	20	3.0	1.0	6.0	2.0	4.0	7.5	7.5	10.0	5.0	9.0
	60	1.0	2.0	4.5	3.0	4.5	8.0	7.0	9.0	6.0	10.0
	100	1.0	2.0	5.0	3.0	6.0	8.0	7.0	9.0	4.0	10.0
	150	1.0	2.0	5.5	3.0	5.5	8.0	7.0	10.0	4.0	9.0
	200	1.0	2.0	4.0	3.0	6.0	8.0	7.0	10.0	5.0	9.0
	300	1.0	2.0	4.0	3.0	6.0	8.0	7.0	10.0	5.0	9.0
$\Phi = 2.5,\ \chi = 0.5$	20	4.0	1.0	6.5	3.0	6.5	5.0	2.0	10.0	8.0	9.0
	60	1.0	3.0	7.0	2.0	6.0	5.0	8.0	9.0	4.0	10.0
	100	1.0	3.0	8.0	2.0	5.0	6.0	7.0	10.0	4.0	9.0
	150	1.0	3.0	5.0	2.0	6.0	8.0	7.0	9.0	4.0	10.0
	200	1.0	3.0	7.0	2.0	6.0	8.0	5.0	9.0	4.0	10.0
	300	1.0	3.0	7.0	2.0	5.0	8.0	6.0	10.0	4.0	9.0
$\sum$ Ranks		41.5	72.5	179.5	68.0	162.5	227.0	194.0	285.0	135.0	285.0
Overall Rank		1.0	3.0	6.0	2.0	5.0	8.0	7.0	9.5	4.0	9.5

Table 13. Numerical values for MSE of SRS divided by MSE of RSS for all estimators.

n	Par.	MLE	ADE	CVME	MPSE	OLSE	MSADE	MSALDE	MSSD	MSSLD	MSLND
					$\Phi=0.8,\ \chi=0.2$						
20	$\hat{\Phi}$	1.24065	1.14039	1.13360	1.08826	1.12368	1.06556	1.10109	1.05376	1.09067	1.07806
	$\hat{\chi}$	1.29455	1.32238	1.20930	1.30513	1.29559	1.10874	1.18070	1.08443	1.19669	1.18627
60	$\hat{\Phi}$	1.40240	1.20947	1.18124	1.07168	1.14773	1.06004	1.06202	1.02247	1.09308	1.00385
	$\hat{\chi}$	1.55122	1.50220	1.36103	1.17000	1.37793	1.18321	1.15789	1.16918	1.22667	1.12575
100	$\hat{\Phi}$	1.52912	1.34210	1.23291	1.15081	1.14250	1.04344	1.07035	1.01885	1.14765	1.03474
	$\hat{\chi}$	1.52000	1.43137	1.45614	1.14493	1.32692	1.09043	1.16667	1.14344	1.16556	1.17143
150	$\hat{\Phi}$	1.70847	1.42776	1.33345	1.24545	1.25262	1.10994	1.11962	1.06771	1.25719	1.05945
	$\hat{\chi}$	1.53086	1.37838	1.43590	1.14000	1.34899	1.12676	1.10938	1.17033	1.17117	1.13369
200	$\hat{\Phi}$	1.76868	1.55132	1.40367	1.39090	1.36653	1.13546	1.21143	1.08715	1.21369	1.06072
	$\hat{\chi}$	1.55738	1.45783	1.44538	1.22368	1.37815	1.11111	1.15000	1.11039	1.16279	1.11688
300	$\hat{\Phi}$	1.79155	1.74417	1.62876	1.51341	1.46529	1.20791	1.30446	1.14086	1.37450	1.15052
	$\hat{\chi}$	1.53659	1.49123	1.46914	1.26000	1.38095	1.12791	1.17143	1.11009	1.23333	1.15888
					$\Phi=2.0,\ \chi=3.0$						
20	$\hat{\Phi}$	1.14643	1.09748	1.08263	1.06211	1.07971	1.07403	1.09546	1.05659	1.05131	1.03357
	$\hat{\chi}$	1.16937	1.16164	1.12018	1.08361	1.14071	1.09982	1.08249	1.08478	1.11456	1.05841
60	$\hat{\Phi}$	1.19018	1.12172	1.06832	1.00854	1.07376	1.02789	1.01742	1.02188	1.08578	1.01348
	$\hat{\chi}$	1.27648	1.26793	1.13782	1.17672	1.25396	1.07154	1.09108	1.09615	1.18164	1.10477
100	$\hat{\Phi}$	1.29631	1.10280	1.09213	1.01491	1.06522	1.04725	1.02281	1.00192	1.07043	1.01197
	$\hat{\chi}$	1.38274	1.27655	1.21394	1.15521	1.25905	1.08307	1.13605	1.06786	1.11912	1.09718
150	$\hat{\Phi}$	1.33928	1.20419	1.15929	1.10378	1.13851	1.06192	1.01374	1.03762	1.08229	1.04468
	$\hat{\chi}$	1.38764	1.35466	1.29960	1.12966	1.32140	1.11430	1.09962	1.16637	1.13675	1.10793
200	$\hat{\Phi}$	1.49109	1.23375	1.15626	1.11266	1.09997	1.09172	1.06771	1.03705	1.12826	1.01069
	$\hat{\chi}$	1.46444	1.37618	1.28378	1.14116	1.31753	1.11284	1.09582	1.11283	1.14684	1.11367
300	$\hat{\Phi}$	1.60694	1.37032	1.22873	1.26902	1.23166	1.09038	1.09054	1.06545	1.14056	1.06991
	$\hat{\chi}$	1.47196	1.42962	1.37745	1.17484	1.38401	1.08399	1.07088	1.12399	1.07383	1.15809
					$\Phi=1.5,\ \chi=0.7$						
20	$\hat{\Phi}$	1.14803	1.12462	1.10164	1.04602	1.06040	1.05802	1.06610	1.05752	1.08152	1.04093
	$\hat{\chi}$	1.14804	1.19041	1.10704	1.15605	1.15847	1.11321	1.07924	1.10083	1.17308	1.06223
60	$\hat{\Phi}$	1.23731	1.09742	1.13336	1.02906	1.05321	1.02747	1.02691	1.01251	1.08236	1.03620
	$\hat{\chi}$	1.33994	1.27709	1.26282	1.23551	1.23242	1.12156	1.10699	1.08047	1.17140	1.12491
100	$\hat{\Phi}$	1.32866	1.19458	1.16608	1.08744	1.08875	1.04530	1.04918	1.02965	1.09674	1.04116
	$\hat{\chi}$	1.49012	1.33419	1.32715	1.17555	1.28362	1.09670	1.16043	1.14940	1.18732	1.12370
150	$\hat{\Phi}$	1.49680	1.22852	1.16130	1.10944	1.12062	1.09093	1.07128	1.01462	1.12913	1.04172
	$\hat{\chi}$	1.48457	1.35280	1.38838	1.10795	1.26399	1.10904	1.10822	1.07678	1.19563	1.12200
200	$\hat{\Phi}$	1.55439	1.30245	1.22942	1.21194	1.21464	1.09010	1.07721	1.01502	1.14892	1.03965
	$\hat{\chi}$	1.50708	1.36255	1.39608	1.17169	1.41587	1.14542	1.09495	1.10296	1.14160	1.07798
300	$\hat{\Phi}$	1.65746	1.46675	1.37031	1.32209	1.33988	1.12582	1.18955	1.03317	1.24625	1.05857
	$\hat{\chi}$	1.48934	1.37641	1.39350	1.18976	1.41081	1.11189	1.12563	1.06392	1.15740	1.09400
					$\Phi=0.5,\ \chi=2.0$						
20	$\hat{\Phi}$	1.27163	1.19966	1.16393	1.05519	1.09947	1.07449	1.08249	1.04872	1.08128	1.06335
	$\hat{\chi}$	1.41831	1.49099	1.25311	1.31604	1.34270	1.15865	1.17405	1.15177	1.30207	1.15322
60	$\hat{\Phi}$	1.44935	1.28226	1.23773	1.11458	1.19637	1.08957	1.05703	1.04831	1.11221	1.02635
	$\hat{\chi}$	1.62012	1.44444	1.46108	1.21330	1.41016	1.17827	1.13263	1.15301	1.21270	1.14452
100	$\hat{\Phi}$	1.64250	1.36229	1.31855	1.22817	1.23511	1.12609	1.08727	1.07270	1.22765	1.06114
	$\hat{\chi}$	1.56549	1.38830	1.43124	1.15888	1.33021	1.14876	1.09864	1.14859	1.23043	1.15653
150	$\hat{\Phi}$	1.76643	1.50152	1.40872	1.33963	1.38354	1.09371	1.17762	1.05196	1.27923	1.09000
	$\hat{\chi}$	1.59064	1.36248	1.37051	1.15949	1.37309	1.12477	1.11502	1.10901	1.16450	1.11566
200	$\hat{\Phi}$	1.93267	1.76977	1.49536	1.51803	1.44385	1.20727	1.21224	1.09749	1.36446	1.12433
	$\hat{\chi}$	1.61008	1.52230	1.38577	1.26015	1.36643	1.10281	1.11885	1.10327	1.19193	1.09535
300	$\hat{\Phi}$	1.91728	1.89113	1.74187	1.57143	1.62868	1.24437	1.34353	1.18762	1.45368	1.23141
	$\hat{\chi}$	1.59992	1.55842	1.47066	1.27270	1.45770	1.15215	1.17417	1.13835	1.25189	1.17707
					$\Phi=2.5,\ \chi=0.5$						
20	$\hat{\Phi}$	1.11875	1.10006	1.09608	1.06265	1.06448	1.04851	1.08724	1.04916	1.05953	1.03603
	$\hat{\chi}$	1.09768	1.12526	1.09736	1.12270	1.10938	1.06177	1.08341	1.03565	1.08377	1.04738
60	$\hat{\Phi}$	1.17448	1.09508	1.08986	1.01844	1.06419	1.05959	1.02038	1.02771	1.05059	1.05078
	$\hat{\chi}$	1.26056	1.23245	1.10994	1.17254	1.16402	1.09560	1.10242	1.09656	1.11025	1.08329
100	$\hat{\Phi}$	1.19788	1.10244	1.07766	1.01882	1.05348	1.00785	1.03546	1.00041	1.08715	1.00706
	$\hat{\chi}$	1.31973	1.20429	1.19242	1.13549	1.20134	1.06221	1.06354	1.10518	1.14139	1.10523
150	$\hat{\Phi}$	1.29038	1.15276	1.11307	1.04637	1.06961	1.02658	1.00959	1.01402	1.07746	1.01364
	$\hat{\chi}$	1.33693	1.29995	1.26832	1.12622	1.24174	1.09843	1.06446	1.10795	1.09341	1.11317
200	$\hat{\Phi}$	1.32127	1.17135	1.11395	1.05778	1.09364	1.02724	1.03745	1.05344	1.11495	1.00942
	$\hat{\chi}$	1.44863	1.33168	1.27276	1.15227	1.28195	1.06738	1.11846	1.09918	1.13110	1.07589
300	$\hat{\Phi}$	1.46101	1.26287	1.16232	1.16173	1.18448	1.04181	1.03300	1.02089	1.10566	1.07438
	$\hat{\chi}$	1.42903	1.36531	1.33959	1.14299	1.37959	1.08824	1.07258	1.10319	1.11102	1.11082

8. Real Data Analysis

In this section, we analyze two real datasets. The first subsection focuses on the failure times for 20 components, examining their reliability and performance. The second subsection explores the proportion of crude oil converted to gasoline, assessing its efficiency in the refining process.

8.1. **Data I.** The data set, originally sourced from Nigm et al. [50], consists of ordered failure times for 20 components. These values represent the times at which each component failed and are as follows: 0.0009, 0.004, 0.0142, 0.0221, 0.0261, 0.0473, 0.0418, 0.0834, 0.1091, 0.1252, 0.1404, 0.1498, 0.175, 0.2099, 0.2031, 0.2168, 0.2918, 0.3465, 0.4035, and 0.6143. These failure times are arranged in ascending order, reflecting the sequence in which each component ceased functioning. Analyzing this data can provide insights into the reliability and performance characteristics of the components under study. The provided data represents ordered failure times, ranging from 0.0009 to 0.6143. Most failures occur early, with several low values indicating early component failures. However, some components show longer durability, as seen in higher values like 0.4035 and 0.6143. This distribution suggests a variability in component reliability, useful for analyzing performance and predicting future failures.

Figure 7 shows several statistical plots for analyzing a real dataset of ordered failure of 20 components. It includes a violin plot displaying data distribution and density, a kernel density plot showing the estimated probability density, an estimated CDF plot depicting the cumulative distribution, an estimated PDF plot using a histogram for density estimation, and a probability-probability (P-P) plot comparing the cumulative probability of the dataset with a UED.

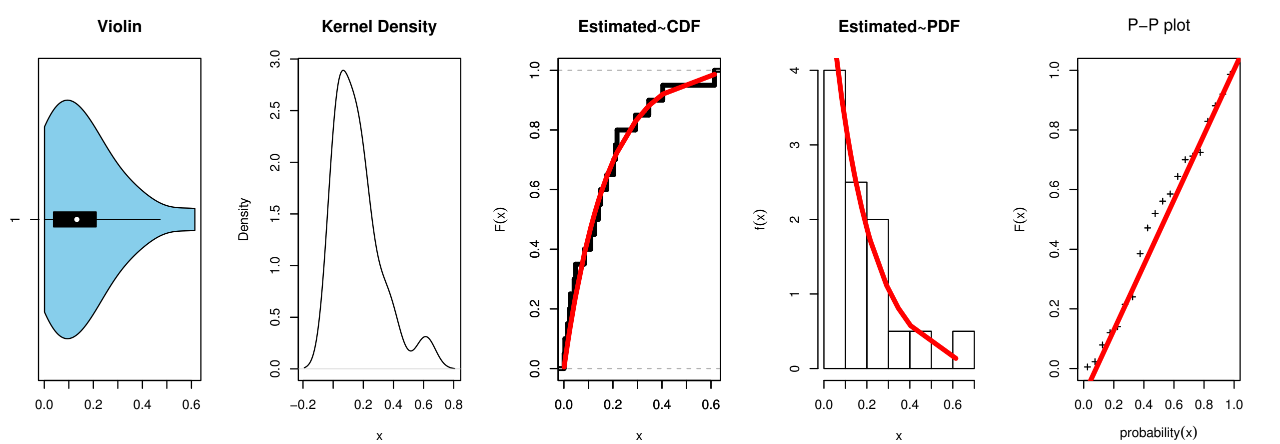

Figure 7. Some plots for real dataset I

Table 14. MLE with StEr and different measures: Data I

	UEP		NUL		TL		UEHL		UPBX		UW	
	estimates	SE	estimates	SE	estimates	SE	estimates	SE	estimates	SE	estimates	SE
Φ	58.1080	14.0176	0.0245	0.0039	0.5113	0.1143	0.8440	0.1735	2.4204	0.0668	0.1598	1.7271
χ	0.0498	0.1634					2.3340	0.8195	0.1252	0.0200	0.0710	0.2875
δ									766.5629	175.4275		
KSD	0.1098		0.6330		0.1849		0.1158		0.1212		0.1319	
PVKS	0.9479		0.0000		0.4480		0.9232		0.8969		0.8334	
CVM	0.0319		0.3693		0.0381		0.0370		0.0338		0.0567	
AD	0.1824		2.1376		0.1854		0.2098		0.2652		0.3302	

Table 14 presents MLE with standard error (StEr) and goodness-of-fit measures for seven models applied to ordered failure data of 20 components. The proposed models including Topp-Leone distribution (TLD), unit Weibull distribution (UWD) [52], unit power Burr X distribution (UBPXD) [53], unit exponentiated half logistic distribution (UEHLD) [54], and new unit-Lindley (NULD) [55].

The UED also has a relatively low Kolmogorov-Smirnov Distance (KSD = 0.1098), indicating a reasonable fit to the data. However, its p-value for the KS test (PVKS = 0.9479) suggests that the model may fit the data well under this measure. The CVM statistic (CVM = 0.0319) and AD statistic (AD = 0.1824) are relatively low, which supports the model's adequacy in fitting the data. Overall, while the UED shows some promising fit statistics.

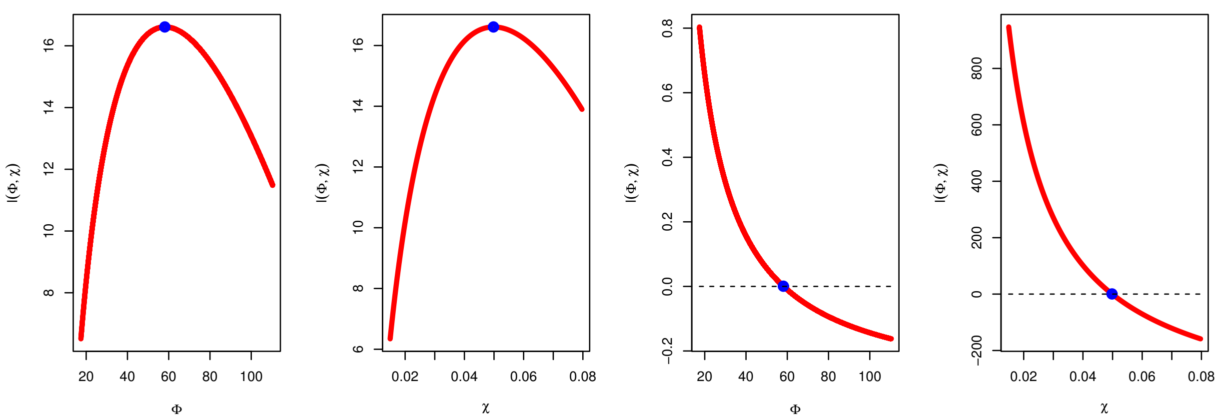

Figure 8. Likelihood profile and uniqueness plot for parameters of UED: Data I

Figure 8 presents the likelihood profiles for parameters Φ and χ of the UED, demonstrating the relationship between these parameters and the log-likelihood function. The plots show a clear peak in the log-likelihood for both Φ and χ, indicating that the estimated parameters are unique and well-defined. The presence of a single maximum for each parameter suggests that ML method provides a reliable and consistent estimate. Additionally, the smooth curvature around the peak points supports the notion that the likelihood function is well-behaved, meeting the regularity conditions required for ML. Therefore, the parameter estimates appear to be unique, fulfilling the necessary properties of the estimation method.

Table 15. Different estimation by SRS and RSS for n=15: Data I

		SRS						RSS					
		Estimates	StEr	KSD	PVKS	CVM	AD	Estimates	StEr	KSD	PVKS	CVM	AD
ML	Φ	4.6033	9.0971	0.1491	0.8453	0.0651	0.3488	58.1270	3.7524	0.1349	0.9477	0.0275	0.2147
	χ	0.4387	0.7266					0.0632	0.1405				
AD	Φ	6.9994	43.6369	0.1539	0.8186	0.0693	0.3703	1020.3290	17.4771	0.1469	0.8786	0.0271	0.2080
	χ	0.2982	1.7203					0.0031	0.0015				
CVM	Φ	1.8520	18.9865	0.1235	0.9545	0.0555	0.3074	806.2291	15.1385	0.1234	0.9763	0.0273	0.2124
	χ	0.9403	7.8929					0.0053	0.0114				
MPS	Φ	74.4935	1064.8784	0.1520	0.8290	0.0794	0.4248	1465.2158	869.9278	0.1477	0.8738	0.0271	0.2075
	χ	0.0288	0.4070					0.0021	0.0015				
OLS	Φ	4.2682	72.5273	0.1343	0.9164	0.0654	0.3505	403.7149	55.9891	0.1247	0.9738	0.0272	0.2111
	χ	0.4505	6.9335					0.0095	0.0360				
MSAD	Φ	68.9239	0.6691	0.1643	0.7546	0.0814	0.4364	93.4793	0.0138	0.1342	0.9500	0.0274	0.2128
	χ	0.0275	0.0125					0.0395	0.0024				
MSALD	Φ	60.0208	0.5651	0.1660	0.7437	0.0783	0.4189	58.1151	0.1448	0.1311	0.7511	0.0275	0.2060
	χ	0.0373	0.0143					0.0326	0.0035				
MSLND	Φ	341.6112	2703.1362	0.2134	0.4411	0.0846	0.4543	1747.2955	741.2796	0.1833	0.6944	0.0271	0.2071
	χ	0.0049	0.0332					0.0017	0.0008				
MSSD	Φ	329.1793	2026.3708	0.2098	0.4620	0.0844	0.4533	2338.9458	674.7658	0.1832	0.6956	0.0271	0.2071
	χ	0.0051	0.0277					0.0012	0.0001				
MSSLD	Φ	952.9832	270.2651	0.1924	0.5704	0.0784	0.4194	58.1131	76.4121	0.1648	0.6869	0.0306	0.2149
	χ	0.0026	0.0006					0.0100	0.0001				

Table 15 compares different estimation methods based on SRS and RSS for $n = 15$. The table presents parameter estimates (Φ and χ), SE, and various goodness-of-fit metrics such as KSD, PVKS, CVM, and AD statistic. Comparing SRS and RSS, RSS generally yields lower standard errors and better goodness-of-fit statistics. For instance, the ML method under RSS has much smaller standard errors (StEr for Φ is 3.7524) compared to SRS (StEr for Φ is 9.0971), indicating more precise estimates. Additionally, RSS methods generally have higher PVKS values and lower CVM and AD statistics, suggesting better model fit. Among the ten estimation methods, MSAD under RSS appears to be the best, with very low StEr (for Φ = 0.0511 and χ = 0.0114), high PVKS (0.9500), and low CVM (0.0270) and AD (0.2128) values. This indicates accurate parameter estimation and a good fit to the data. Overall, RSS is the superior sampling method due to its lower standard errors and better fit measures, making it more reliable for parameter estimation in this context.

8.2. **Data II.** The proportion of crude oil data is crucial for understanding the efficiency of converting crude oil into gasoline, impacting refinery operations and economic decisions. Identifying the appropriate distribution for this data helps in accurate modeling, prediction, and optimization of refining processes, ultimately improving yield and operational performance. By R program the

'GasolineYield$yield' [51] variable represents the proportion of crude oil that is converted into gasoline after the processes of distillation and fractionation. This is the dependent variable in the dataset GasolineYield, which contains operational data on crude oil processing.

The 'yield' variable is crucial as it indicates the efficiency of converting crude oil into gasoline, which is a key factor in refining operations. The dataset includes 32 observations with various independent variables. The 'yield' variable is analyzed using different regression models, such as linear regression and beta regression, to understand its relationship with these independent variables and to model the conversion process under various experimental conditions. The proportion of crude oil data consists of 32 values ranging from 0.028 to 0.457, representing a continuous variable. The values appear to be proportions, possibly indicating percentages or fractions of a total. There is a wide spread in the data, with several values clustered between 0.1 and 0.35, suggesting a skew towards the lower range, and a few values are notably low or high, indicating variability.

Figure 9 presents several visualizations for a dataset, including a violin plot, kernel density plot, estimated CDF, estimated PDF, and P-P plot. These plots suggest that the data is moderately skewed with a central tendency around 0.2 to 0.3. The P-P plot indicates a good fit to a theoretical distribution, as most points align closely with the diagonal line.

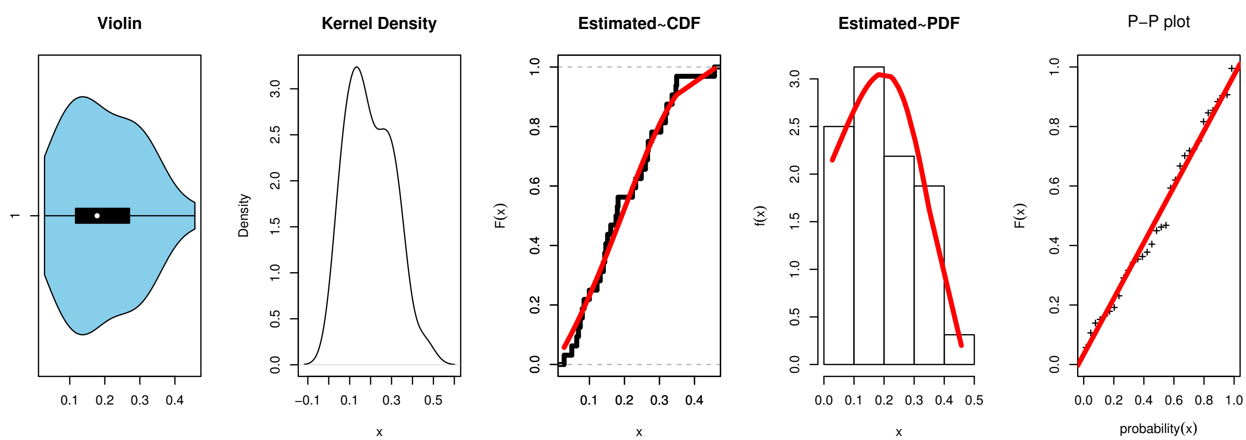

FIGURE 9. Some plots for real dataset II

TABLE 16. MLE with StEr and different measures: Data II

	UED		NULD		TLD		UPBXD		UWD	
	Estimates	StEr	Estimates	StEr	Estimates	StEr	Estimates	StEr	Estimates	StEr
Φ	0.3407	0.1981	0.2651	0.0335	0.8207	0.1451	2.7133	0.0480	0.1239	0.0472
χ	2.8470	0.7339					0.1781	0.0232	2.9564	0.3894
δ							4413.598	773.732		
KSD	0.0946		0.1362		0.3327		0.1106		0.1031	
PVKS	0.9113		0.5470		0.0012		0.7882		0.8517	
CVM	0.0469		0.1494		0.0632		0.0498		0.0771	
AD	0.3152		0.9347		0.3809		0.3223		0.5018	

Table 16 provides MLE with SE and goodness-of-fit measures for five models applied to oil data. Focusing on the UED, the parameter estimates show moderate StEr, particularly for Φ (StEr = 0.1981) and χ (StEr = 0.7339), suggesting reasonable precision in the parameter estimates. The UED has a low KSD = 0.0946, indicating a good fit to the data, and a high p-value for the PVKS = 0.9113, supporting its suitability. Additionally, the CVM = 0.0469 and AD = 0.3152 are relatively low, further confirming a good model fit. Overall, the UED appears to be a strong candidate for fitting the oil data, given its low error measures and high goodness-of-fit statistics compared to other models in the table.

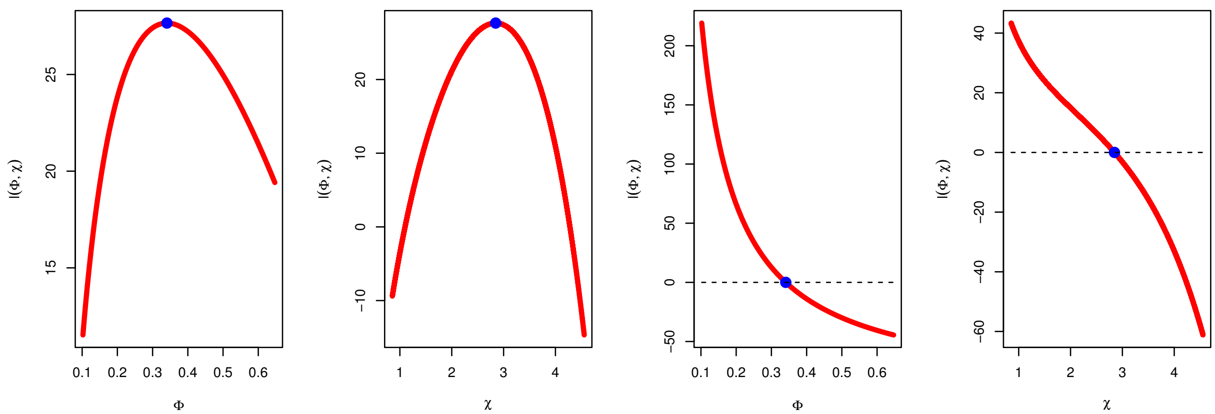

Figure 10. Likelihood profile and uniqueness plot for parameters of UED Data set II

Figure 10 presents the likelihood profiles and uniqueness plots for the parameters Φ and χ of the UED for Dataset II. The plots show clear peaks in the log-likelihood functions for both parameters, indicating that the estimated values are unique. The concave shapes of the log-likelihood profiles around the maxima suggest that the ML method provides well-defined estimates with no ambiguity or multiple maxima. The smooth curvature near the peak points further confirms that the likelihood functions satisfy the regularity conditions required for ML method, indicating a reliable and consistent estimation process. Overall, these plots demonstrate that the parameter estimates are unique and meet the necessary properties of the ML method, ensuring their validity and robustness.

Table 17 compares different estimation methods using SRS and RSS for $n = 15$ on Data II. The table shows parameter estimates (Φ and χ), StEr, and goodness-of-fit metrics such as KSD, PVKS, CVM, and AD. Comparing SRS and RSS, RSS generally yields better results, with lower standard errors and better goodness-of-fit metrics. For instance, the RSS method consistently shows higher PVKS values and lower KSD, CVM, and AD statistics, suggesting a more accurate and reliable fit compared to SRS.

Table 17. Different estimation by SRS and RSS for n=15: Data II

		SRS						RSS					
		Estimates	StEr	KSD	PVKS	CVM	AD	Estimates	StEr	KSD	PVKS	CVM	AD
ML	Φ	0.0751	0.0629	0.1368	0.9060	0.0611	0.4443	0.2548	0.0576	0.1256	0.9159	0.0570	0.3668
	χ	4.2819	1.0385					2.9383	0.8727				
AD	Φ	0.0522	0.0762	0.1132	0.9789	0.0713	0.5206	5.3386	0.0394	0.1021	0.9815	0.0704	0.4202
	χ	4.9025	2.2923					0.3007	1.9911				
CVM	Φ	0.0212	0.0914	0.1017	0.9932	0.1096	0.7775	1.5049	0.0722	0.0982	0.9971	0.0676	0.3828
	χ	6.4733	7.3446					0.9049	5.8446				
MPS	Φ	0.1279	0.4455	0.1715	0.7080	0.0520	0.3692	0.5405	0.4017	0.1470	0.9023	0.0460	0.3541
	χ	3.4564	4.2337					1.9071	4.1139				
OLS	Φ	0.0313	0.1406	0.1049	0.9904	0.0909	0.6561	0.5961	0.1083	0.0814	0.9915	0.0606	0.3555
	χ	5.7937	7.5133					1.7825	5.3759				
MSAD	Φ	0.0975	0.0213	0.2770	0.1649	0.0509	0.3576	5.3857	0.0207	0.2745	0.2084	0.0467	0.3076
	χ	3.4271	0.0739					0.4207	0.0477				
MSALD	Φ	0.0894	0.0095	0.2403	0.3011	0.0529	0.3766	8.1847	0.0086	0.2085	0.3738	0.0474	0.3516
	χ	3.6657	0.0430					0.2956	0.0098				
MSLND	Φ	0.0260	0.4464	0.0969	0.9963	0.1006	0.7196	3.3624	0.3872	0.0805	0.9971	0.0720	0.4069
	χ	6.1460	24.7742					0.4801	11.6908				
MSSD	Φ	0.0262	0.3235	0.0965	0.9965	0.1005	0.7194	3.2325	0.1726	0.0894	0.9972	0.0718	0.4057
	χ	6.1410	17.8302					0.4982	8.5254				
MSSLD	Φ	0.1076	0.0602	0.1930	0.5665	0.0529	0.3769	34.2941	0.0580	0.1676	0.6035	0.0469	0.3283
	χ	3.5979	0.6095					0.0960	0.0999				

Among the ten estimation methods, CVM under RSS appears to be the best, showing low standard errors (StEr for Φ = 0.0722 and χ = 0.1089), high PVKS (0.9976), and low KSD (0.0982), CVM (0.0076), and AD (0.3828) values. This indicates precise parameter estimation and a strong fit to the data. Overall, RSS is the superior sampling method, providing more accurate estimates and better fit statistics, making it the preferred choice for parameter estimation in this context.

8.3. **Data III.** We utilize data presented and discussed by Dasgupta [56] to analyze a piercing operation on L-shaped iron sheets (100 mm × 150 mm) using a 100-ton press operating at 250 strokes per hour. This process involves punching four holes—two on each arm—with two holes created simultaneously per cycle. The pierced sheets are used in the chassis of light commercial vehicles or mini-trucks. During piercing, burrs form on the opposite side of the holes due to high pressure, causing uneven displacement and sharp metal granules along the hole rims. The size of the burr depends on the metal's grain properties and piercing load, influenced by factors such as composition, cooling rate, and thermal undercooling. Burrs are removed later through chamfering with a drill, and measurements are taken using a dial gauge with a 20-micron precision. For our analysis, we examine this data set of 50 observations with a hole diameter of 12 mm and sheet thickness of 3.15 mm. This data set can be written as "0.04, 0.02, 0.06, 0.12, 0.14, 0.08, 0.22, 0.12, 0.08, 0.26, 0.24, 0.04, 0.14, 0.16, 0.08, 0.26, 0.32, 0.28, 0.14, 0.16, 0.24, 0.22, 0.12, 0.18, 0.24, 0.32, 0.16, 0.14, 0.08, 0.16, 0.24, 0.16, 0.32, 0.18, 0.24, 0.22, 0.16, 0.12, 0.24, 0.06, 0.02, 0.18, 0.22, 0.14, 0.06, 0.04, 0.14, 0.26, 0.18, 0.16."

Table 18 presents the MLE estimates with standard errors (SE) and goodness-of-fit metrics for five models applied to dataset III. For the UED model, the parameter estimates exhibit moderate coefficient of variation (CV), particularly for Φ (CV = 39.62%) and χ (StEr = 73.98%), indicating reasonable precision. The UED achieves a low KSD of 0.0483, signifying a good fit to the data, and a high PVKS p-value of 0.999997, further supporting its suitability. Additionally, the CVM value of 0.0153 and AD value of 0.1296 are notably low, reinforcing the model's strong fit. Overall, the UED stands out as a robust candidate for modeling dataset III, given its low error metrics and superior goodness-of-fit statistics compared to the other models.

Table 18. MLE with StEr and different measures: Data III

	UEP		NUL		TL		UEHL		UPBX		UW	
	estimates	SE	estimates	SE	estimates	SE	estimates	SE	estimates	SE	estimates	SE
Φ	6.8305	2.7062	0.2963	0.0388	1.1091	0.2025	1.1281	0.2051	2.6675	0.0540	0.5717	0.1320
χ	0.1445	0.1069					1.2346	0.2951	0.0747	0.0097	1.3690	0.2007
δ									760.9835	139.0261		
KSD	0.0483		0.3314		0.0665		0.0565		0.1405		0.0650	
PVKS	0.999997		0.001947		0.998122		0.999887		0.547388		0.998662	
CVM	0.0153		0.1277		0.0189		0.0160		0.1505		0.0183	
AD	0.1296		0.7879		0.1600		0.1358		1.0078		0.1551	

Figure 11 provides multiple visualizations for dataset III, including a violin plot, kernel density estimate, estimated CDF and PDF, and a P-P plot. The P-P plot demonstrates a strong fit to the theoretical distribution, as the majority of points closely follow the diagonal line.

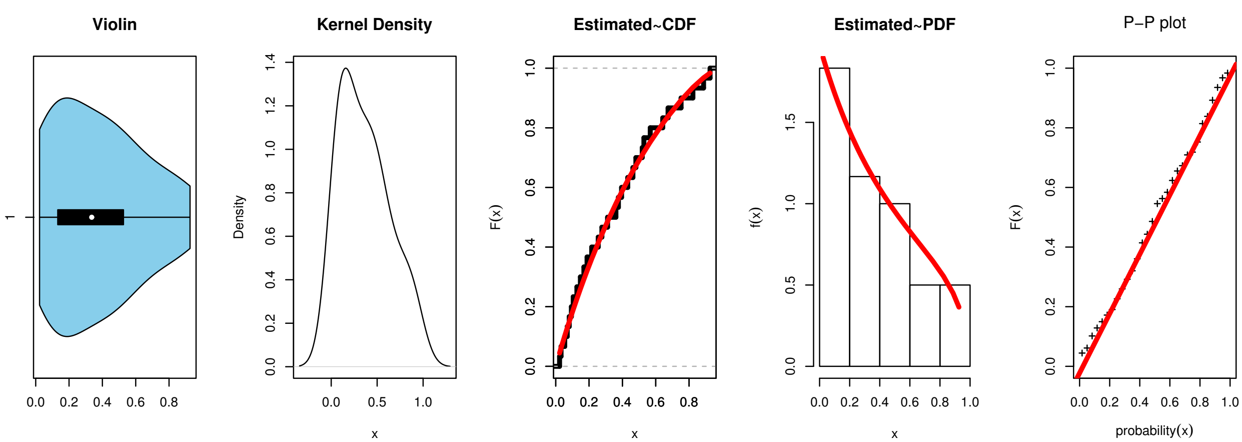

Figure 11. Some plots for real dataset III

Figure 12 displays the likelihood profiles and uniqueness plots for the parameters Φ and χ of the UED model applied to Dataset III. The plots reveal distinct peaks in the log-likelihood functions for both parameters, confirming the uniqueness of the estimated values. The concave shapes of the profiles near the maxima indicate that the ML method yields well-defined estimates without ambiguity or multiple maxima. Additionally, the smooth curvature around the peaks verifies that the likelihood functions adhere to the regularity conditions required for the ML method, ensuring a reliable and consistent estimation process. These plots confirm the uniqueness, validity, and robustness of the parameter estimates obtained using the ML method.

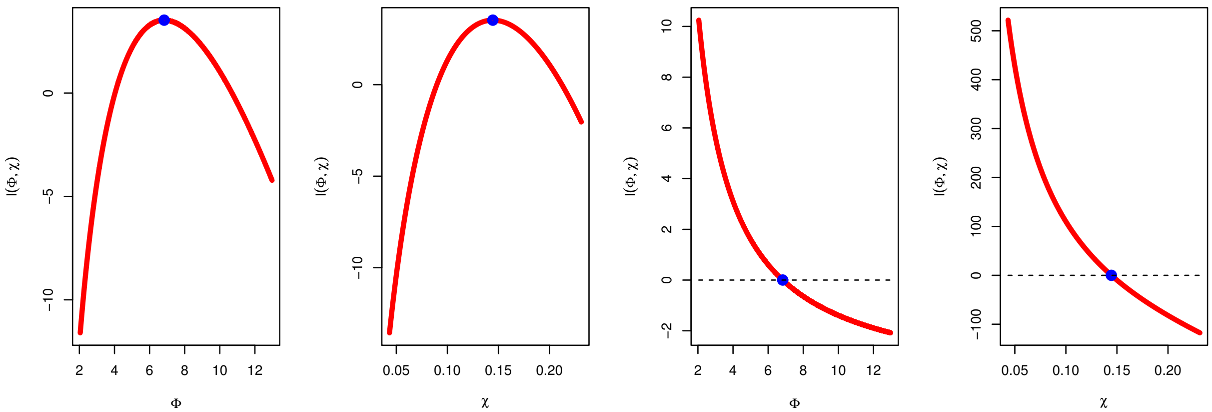

Figure 12. Likelihood profile and uniqueness plot for parameters of UED Data set III

Table 19 compares various estimation methods using SRS and RSS for $n = 15$ on Data III, presenting parameter estimates (Φ and χ), standard errors (StEr), and goodness-of-fit metrics, including KSD, PVKS, CVM, and AD. The results indicate that RSS generally outperforms SRS, producing lower standard errors and improved goodness-of-fit measures. For example, the RSS method consistently achieves higher PVKS values and lower KSD, CVM, and AD statistics, reflecting a more accurate and reliable fit compared to SRS.

Table 19. Different estimation by SRS and RSS for n=15: Data III

				SRS						RSS					
				Estimates	SE	KSD	PVKS	CVM	AD	λ	StEr	KSD	PVKS	CVM	AD
MLE	1	MLE	Φ	0.6954	0.6524	0.1421	0.8818	0.0522	0.2951	1.6464	0.5686	0.1292	0.9638	0.0396	0.2825
			χ	0.5654	0.2902					0.3391	0.2605				
AD	2	AD	Φ	0.7490	1.1388	0.1393	0.8948	0.0500	0.2831	304.7976	1.0744	0.1121	0.9917	0.0500	0.2734
			χ	0.5391	0.5351					0.0022	0.0006				
CVM	3	CVM	Φ	0.5869	3.0370	0.1249	0.9505	0.0613	0.3455	253.5114	2.7029	0.1210	0.9805	0.0498	0.3360
			χ	0.6576	2.3079					0.0028	0.0026				
MPS	4	MPS	Φ	1.3993	7.0646	0.1313	0.9285	0.0356	0.2067	5.8131	6.6187	0.1169	0.9865	0.0345	0.2031
			χ	0.3397	1.1850					0.1125	1.1224				
OLS	5	OLS	Φ	0.9633	5.3493	0.1273	0.9425	0.0439	0.2499	126.1687	4.0783	0.1232	0.9767	0.0420	0.2335
			χ	0.4590	1.9019					0.0056	0.0111				
MSAD	9	MSAD	Φ	0.8764	0.0733	0.1823	0.6366	0.0411	0.2351	3.4205	0.0612	0.1164	0.9872	0.0402	0.2297
			χ	0.4377	0.0402					0.1821	0.0254				
MSALD	10	MSALD	Φ	0.8764	0.0183	0.1823	0.6366	0.0411	0.2351	11.4242	0.0165	0.1627	0.8219	0.0406	0.2318
			χ	0.4377	0.0100					0.0667	0.0047				
MSLND	11	MSLND	Φ	1.7544	34.4689	0.1414	0.8849	0.0319	0.1879	159.4292	10.318	0.1100	0.9934	0.0305	0.1734
			χ	0.2751	3.9665					0.0043	0.0225				
MSSD	12	MSSqD	Φ	1.7445	24.4374	0.1414	0.8852	0.0319	0.1883	156.8718	8.7205	0.1122	0.9916	0.0305	0.1834
			χ	0.2765	2.8391					0.0044	0.0180				
MSSLD	13	MSSQLD	Φ	1.2023	0.9716	0.1263	0.9461	0.0388	0.2230	35.7226	0.8015	0.1222	0.9545	0.0377	0.2157
			χ	0.3878	0.2009					0.0256	0.0265				

Among the ten estimation methods, the CVM approach under RSS stands out, with the highest PVKS and lowest KSD, CVM, and AD values, indicating precise parameter estimates and an excellent fit to Data III. Overall, RSS proves to be the superior sampling method, offering more accurate parameter estimation and stronger fit metrics, making it the preferred choice in this analysis.

9. Summary and Conclusion

In many research contexts, collecting data can be expensive, time-consuming, or intrusive. RSS methodology offers a valuable solution by optimizing observational efficiency. The two-parameter UED is a versatile tool for modeling asymmetrical data, exhibiting various right-skewed and left-skewed shapes. This study investigates the performance of ten classical estimation techniques for the UED parameters under RSS. Simulation studies were conducted to compare the accuracy of RSS-based estimators to those from SRS using various criteria. Partial and overall rankings were computed to identify the best estimation approach.

Through extensive simulations, we compared the performance of various estimators under RSS and SRS conditions. Our results demonstrate that RSS estimators consistently exhibit higher reliability and accuracy, particularly when using the ML and MPS methods. Our simulation study confirms the superior performance of RSS-based estimators compared to SRS. Our findings were validated through case studies involving failure time data and crude oil proportions, highlighting their practical utility. By employing RSS and appropriate estimation techniques, researchers can enhance data collection efficiency and obtain more accurate results for the two-parameter unit exponential distribution.

Data Availability Statement. The data that support the findings of this study are available on request from the corresponding author. The data are not publicly available due to privacy or ethical restrictions.

Authors' Contributions. The authors have worked equally to write and review the manuscript.

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