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Effective Estimation Methods Through Ranked Set Sampling for Mixture Model: Industrial and Survival Applications

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Abstract. The sampling strategy has a considerable impact on the representativeness of the sampled data and can lead to incorrect estimates if not carefully chosen. An improved method over more conventional simple random sampling (SRS) is ranked set sampling (RSS). The RSS is more efficient, reducing the number of measurements needed for a desired level of precision, especially in challenging data collection scenarios. The Monsef distribution is a recent mixture lifetime model that has demonstrated effectiveness in modeling various real-world datasets. Several mathematical aspects of the Monsef distribution include quantiles, upper incomplete moments, lower incomplete moments, stochastic ordering, and extropy measures. This work investigates the use of RSS in conjunction with several traditional estimation techniques to estimate the parameters of the Monsef distribution. Fifteen different estimation procedures are investigated, including maximum product spacing, some minimum spacing distance methods, the Kolmogorov method, ordinary least squares, maximum likelihood, and weighted least squares. To assess the performance of the estimation techniques for a range of sample sizes under perfect ranking conditions and both sampling techniques, a simulation scenario is conducted. The partial and total ranks of numerous estimates are displayed to determine the best estimation approach. According to simulation results, the maximum likelihood and maximum product spacing approaches consistently outperform other methods in evaluating the estimated quality for both RSS and SRS. To demonstrate the feasibility of the different methods, three authentic datasets from various fields are examined.

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1. Introduction

In order to handle complicated datasets, researchers and practitioners look for efficient data modeling methods. Conventional models frequently show themselves to be inadequate, resulting in shaky or false conclusions. Flexible models that can faithfully represent the subtleties of various datasets are desperately needed. Models with flexible density and hazard rate shapes, enhanced parameter performance, rigorous mathematical properties, and higher goodness-of-fit values are being proposed by statisticians. These improvements are made possible by expanded or altered models that have strong parameters, specialized functions, and appealing functional forms.
As an extension of the exponential distribution, the Erlang distribution is a flexible statistical distribution. The definition of it is the total of several independent exponential random variables. This distribution was first used by Erlang in the early 20th century to simulate the quantity of phone calls that would arrive in a given amount of time. But since then, its uses have spread to a number of industries, such as telecommunications and queuing theory. More specifically, the time delay between an event and the kth consecutive event is commonly described using the Erlang distribution. Because of this, it is very useful for examining wait and service times in queuing systems. The probability density function (PDF) of the Erlang distribution is defined by:

$$h(v;\delta) = \frac{\delta^m v^{m-1} e^{-\delta v}}{\Gamma(m)}; \qquad v;\delta > 0,\, m = 1,2,3 \tag{1.1}$$

where, parameter m is called the shape parameter, and the parameter $\delta > 0$ is called the rate parameter.

In numerous domains, such as engineering and medicine, finite mixture distributions have been shown to be a strong and adaptable method for simulating complicated datasets. They come in especially handy when related continuous variables are accessible but a categorical variable is absent. Mixtures can be classified as finite, discrete, or continuous. Pearson [1] introduced the idea of mixture distributions by suggesting that a dataset may be fitted using a mixture of two normal distributions with distinct means and variances. Since then, a great deal of research has been done on mixture models, and they have been used to solve a variety of data analysis issues. The mixing methodology is used by many researchers to explore novel statistical distributions. Some of the mixture distributions including Lindley distribution [2], xgamma distribution [3], Erlang-Pareto I distribution [4], Shanker distribution [5], Rayleigh-half normal distribution [6], Rama distribution [7], Burr XII and Burr X distribution [8], Chris-Jerry distribution [9], mixed Erlang distribution [10], among others.

As an original sub-model of the mixed Erlang distribution, the Monsef distribution (MoD) stands out among the recent mixture distributions. Interestingly, there is just one scale parameter on the MoD. The MoD is the primary subject of this investigation. It is particularly effective in modeling data with a V-shaped hazard rate function (HRF), owing to its flexibility in capturing various HRF curve shapes. The MoD's ability to provide a superior fit to COVID-19, breast cancer, and infected guinea pig data compared to other distributions demonstrates its potential for more accurate predictions and informed decision-making [10]. However, due to its recent introduction, there is a limited body of research on this distribution. Reference [11] looked into the weighted form of MoD and applied it to data on carbon fibers and daily ozone measurements. The unit form of the MoD was previously introduced by [12], along with its regression model. Abd El-Monsef et al. [13] investigated the estimation of stress-strength reliability for MoD.

For several reasons, including financial and timing limitations, sampling is an essential procedure in order to make conclusions about the population from the sample's data. Enhancing the efficacy of estimators for population parameters while minimizing sampling errors is the aim of estimating processes in sampling theory. Enhancements in estimator efficiency, cost-effectiveness, simplicity, and time-saving are always sought after in sampling endeavors. In several domains, such as medicine, agriculture, earth sciences, statistics, and mathematics, ranked set sampling (RSS) provides a better option than simple random sampling (SRS). The RSS is more economical and effective because it can lower the number of measured observations required to get a certain level of precision, which is especially useful when gathering data is challenging. McIntyre [14] first outlined the RSS technique for population mean estimate, and Takahashi and Wakimoto [15] presented the mathematical theory behind RSS. The RSS has been explored in various fields, including environmental studies [16], medicine study [17], information theory [18], engineering study [19], quality control [20], and reliability studies [20, 21].

The RSS-based estimation for a range of parametric models has been thoroughly studied in recent research. These investigations have continuously demonstrated RSS's superior efficiency over SRS and other conventional sampling methods. For instance; He et al. [22] for log-logistic distribution; Adatia [23] for half logistic distribution; Khamnei and Abusaleh [24] for generalized logistic distribution; Fei et al. [25] for Weibull distribution; Esemen and Gurler [26] for generalized Rayleigh distribution; Akgül et al. [27] for Lindley distribution; Bantan et al. [28] for half logistic xgamma distribution; Pedroso et al. [29] for Birnbaum-Saunders distribution; Yousef et al. [30] for inverted Topp-Leone; Hassan et al. [31] for generalized exponential distribution; Nagy et al. [32] for inverted Kumaraswamy distribution, and for recent studies [33–37].
Although the MoD is flexible and useful for modeling lifetime data, it has not received much attention for parameter estimation under RSS. Consequently, the following are this article's main goals:

(1) To examine some of the MoD's statistical properties that haven't been covered previously and to emphasize their significance and adaptability in lifetime applications.
(2) To investigate fifteen distinct estimators of the MoD parameter based on RSS design. We specifically compare left tail Anderson-Darling estimator (LTADE), minimum spacing (MS) square distance estimator (MSSDE), ordinary least squares estimator (OLSE), maximum likelihood estimator (MLE), MS absolute-log distance (MSALD), Kolmogorov estimator (KE), Cramér von Mises estimator (CVME), MS square log distance estimator (MSSLDE), Anderson-Darling estimator (ADE), maximum product spacing estimator

(MPSE), Anderson-Darling left tail second order estimator (ADSOE), weighted least squares estimator (WLSE), MS Linex distance estimator (MSLNDE), MS absolute distance estimator (MSADE), right tail Anderson-Darling estimator (RTADE).

(3) To assess the efficiency of various derived estimators under SRS and RSS designs, we will conduct a simulation study using six distinct accuracy metrics. Also, analyzing the partial and total rankings of various estimates to identify the optimal estimation approach.

(4) Analyzing three real-world datasets will help to highlight the research's practical relevance.

The configuration of this article is as follows: The statistical characteristics and structural behavior of the MoD probability density function (PDF) and hazard rate function (HRF) are examined in Section 2. Section 3 explains RSS structure and derives MPSE and MLE for the MoD. Different MS estimators under RSS are shown in Section 4, and alternative estimators are examined in Section 5. In Section 6, we use Monte Carlo simulation to assess these estimators' effectiveness and performance under SRS and SRS. Additional insights based on actual data are presented in Section 7, and concluding observations are presented in Section 8.

2. Overview of the MoD

In this section an overview of the MoD along with its graphical representation of its PDF and HRF is provided. In addition to, a few of the useful distribution properties have been investigated which does not discussed before. These statistical properties including quantile function (QF), lower incomplete moments (LIMs), upper IMs (UIMs), mean residual lifetime function (MRL), and mean inactivity time function (MIT), stochastic ordering (SO), and extropy measures.

2.1. Model Description.

In this part, the description of the MoD is given. It's important to note that the MoD can be considered a special case within the broader family of m-Erlang mixture distributions (see [10]). Abd El-Monsef [10] introduced a mixture of m-Erlang distributions with the following PDF:

$$h(v;\delta) = \frac{\delta^m (v+1)^{m-1} e^{-\delta(v+1)}}{\Gamma(m,\delta)}; \qquad v,\delta > 0,\ m = 1,2,... \tag{2.1}$$

where $\Gamma(m,\delta)$ is the upper incomplete gamma function (IGF). Reference [10] mentioned that:

(1) For $m = 1$, then the PDF (2.1) gives exponential distribution with scale parameter δ. Also, it similar to the PDF (1.1) which has Erlang distribution with parameters $(1,\delta)$.

(2) For $m = 2$, then the PDF (2.1) provides Lindley distribution with scale parameter δ, which is considered as mixture of Erlang $(1,\delta)$ and Erlang $(2,\delta)$.

(3) For $m = 3$, then the PDF (2.1) gives the new one-parameter distribution called MoD.

Our interest here with one-parameter MoD which has the following PDF:

$$h(v;\delta) = \frac{\delta^3 (v+1)^2 e^{-\delta v}}{\delta(\delta+2)+2}; \qquad v;\delta > 0. \tag{2.2}$$

The cumulative distribution function (CDF) of the MoD is given by:

$$H(v;\delta) = 1 - e^{-\delta v}\left[\frac{\{\delta(v+1)+1\}^2+1}{(\delta+1)^2+1}\right]; \qquad v;\delta > 0. \tag{2.3}$$

The HRF of the MoD is given by

$$\hbar(v;\delta) = \frac{\delta^3\left[(\delta+1)^2+1\right](v+1)^2}{[\delta(\delta+2)+2]\left[\{\delta(v+1)+1\}^2+1\right]}.$$

Figure (1) illustrates how the behavior of the PDF and HRF of the MoD is influenced by varying the values of the parameter δ. he figures demonstrate the MoD's versatility in modeling various data patterns, including right-skewed, reversed J-shaped, and increasing-shaped.

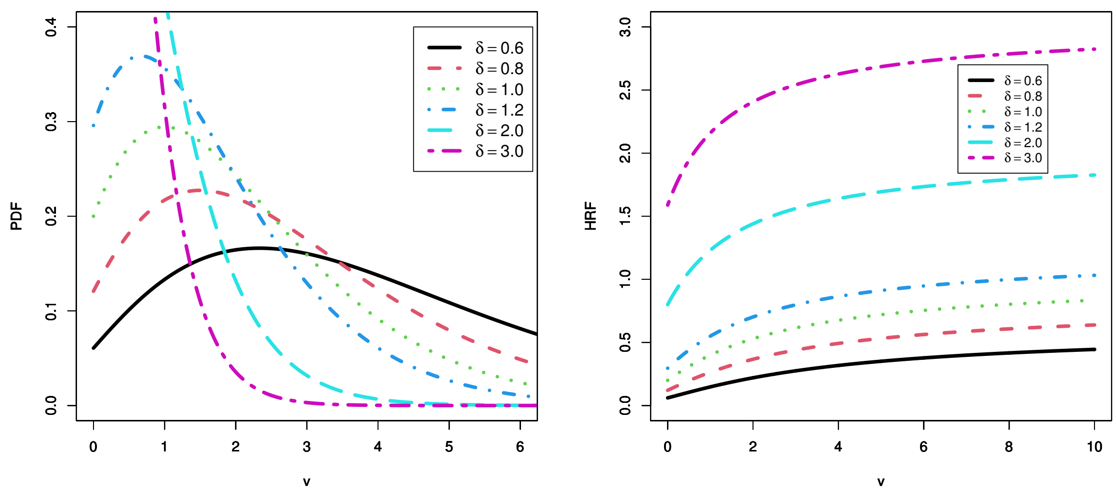

Figure 1. Curves of the PDF and HRF for the MoD

2.2. **Quantile Function.** The expression for the quantile function, say $Q(\mathrm{u})$, $0 < \mathrm{u} < 1$, of the MoD can be obtained by taking the inverse of the CDF provided in Equation (2.3) as follows:

$$\ln(1-u) + \ln\left[(\delta+1)^2+1\right] - \delta Q(u) + \ln\left[\{\delta(Q(u)+1)+1\}^2+1\right] = 0. \tag{2.4}$$

The generated random number of MoD can be created numerically by solving the nonlinear Equation (2.4).

2.3. **Conditional Moments.** For lifetime models, the IMs, MRL, and MIT are valuable characteristics. The Bonferroni and Lorenz curves are prominent applications of the first IM, with significant utility in economics, dependability, demographics, insurance, and medical fields. In lifetime models, the LIMs and UIMs of V, denoted by $\mu_m(t)$ and $\mu_m^\bullet(t)$, are defined, respectively, as follows: $\mu_m(t) = \int_0^t v^m h(v;\delta)dv$and $\mu_m^\bullet(t) = \int_t^\infty v^m h(v;\delta)dv$. The mth LIM for the MoD is given by:

$$\mu_m(z) = \int_0^t v^m h(v;\delta)dv = \frac{\delta^3}{[\delta(\delta+2)+2]} \int_0^t (v^{m+2} + 2v^{m+1} + v^m)e^{-\delta v}dv$$
$$= \frac{\delta^{-m}}{[\delta(\delta+2)+2]} \left[\gamma(m+3,\delta t) + 2\delta\gamma(m+2,\delta t) + \delta^2\gamma(m+1,\delta t)\right], \tag{2.5}$$

where, $\gamma(.,x)$is the lower IGF. The first LIM of the MoD is determined by setting $m = 1$ in Equation (2.5).

The mth UIM for the MoD is given by:

$$\mu^\bullet_m(z) = \int_t^\infty v^m h(v;\delta)dv = \frac{\delta^3}{[\delta(\delta+2)+2]} \int_t^\infty (v^{m+2} + 2v^{m+1} + v^m)e^{-\delta v}dv$$
$$= \frac{\delta^{-m}}{[\delta(\delta+2)+2]} \left[\Gamma(m+3,\delta t) + 2\delta\,\Gamma(m+2,\delta t) + \delta^2\Gamma(m+1,\delta t)\right], \tag{2.6}$$

where, $\Gamma(.,x)$is the upper IGF. The first UIM of the MoD is determined by setting $m = 1$ in Equation (2.6). The MRL function, also known as life expectancy at age t, represents the expected remaining lifespan of a unit that has survived to age t. For the MoD, the MRL function of V is given by:

$$\tau_V(t) = E\left(V\,|V > t\,\right) = \frac{1}{\bar{H}(t;\delta)} \int_t^\infty vh(v)dv - t = \frac{\mu^\bullet_1(t)}{\bar{H}(t;\delta)} - t$$
$$= \frac{\delta^{-1}}{\bar{H}(t;\delta)\,[\delta(\delta+2)+2]} \left[\Gamma(4,\delta t) + 2\delta\Gamma(3,\delta t) + \delta^2\Gamma(2,\delta t)\right] - t,$$

where $\bar{H}(t) = 1 - H(t)$ is the survival function of the MoD. Furthermore, if an item fails, the MIT shows the amount of time that has passed since the failure (0;t). The MIT of random variable V is given by:

$$\tau^*_V(t) = E\left(V\,|V < t\,\right) = t - \frac{1}{H(t)} \int_0^t vh(v;\delta)dv = \; t - \frac{\mu_1(t)}{H(t)}$$
$$= t - \frac{\delta^{-1}}{H(t)\,[\delta(\delta+2)+2]} \left[\gamma(4,\delta t) + 2\delta\,\gamma(3,\delta t) + \delta^2\gamma(2,\delta t)\right].$$

2.4. **Stochastic Ordering.** The SO is a fundamental concept in probability theory with broad applications in various fields. It provides a framework for comparing the behavior of non-negative continuous random variables, offering insights into the efficiency and reliability of system components. Suppose that V_j has the MoD with parameterδ_j for j= 1, 2. Let $H_j(v;\delta_j)$be the CDF and $h_j(v;\delta_j)$be the PDF of V_j. Then V_1 is said to be stochastically smaller than V_2 with respect to likelihood ratio order (say$V_1 \leq_{lr} V_2$), if $\frac{h_1(v;\delta_1)}{h_2(v;\delta_2)}$is a decreasing function $\forall$ v values. The likelihood ratio ordering is as follows

$$\frac{h_1(v;\delta_1)}{h_2(v;\delta_2)} = \frac{\delta_1^3(v+1)^2e^{-\delta_1 v}}{\delta_1(\delta_1+2)+2} \frac{\delta_2(\delta_2+2)+2}{\delta_2^3(v+1)^2e^{-\delta_2 v}}.$$

,

Then, for $\delta_2 < \delta_1$, we obtain $\frac{d}{dv}\log\left[\frac{h_1(v;\delta_1)}{h_2(v;\delta_2)}\right] < 0 \;\forall v \geq 0$. So, $\frac{d}{dv}\log\left[\frac{h_1(v;\delta_1)}{h_2(v;\delta_2)}\right]$ is decreasing in v and consequently $V_1 \leq_{lr} V_2$. Furthermore, V_1 is said to be smaller than V_2 in other different orderings as (SO, represented as $V_1 \leq_{so} V_2$), (hazard rate order; represented by$V_1 \leq_{hr} V_2$), and (reversed hazard rate order; represented by $V_1 \leq_{rhr} V_2$) (see Shaked and Shanthikumar [39]).

2.5. **Extropy Measures.** Extropy (Ex) measure is a quantity that goes hand in hand with entropy to indicate uncertainty or disorder in a system. The Ex quantifies the degree of disorder or unpredictability in a system, whereas entropy quantifies the degree of order or predictability [40]. The Ex is widely used to evaluate systems with complicated behaviors and investigate how order and disorder change over time in disciplines including information theory, physics, and economics. The Ex measure of the MoD is given as below:

$$\psi = \frac{-1}{2}\int_0^{\infty} h^2(v;\delta)dv = \frac{-\delta^6}{2[\delta(\delta+2)+2]^2}\int_0^{\infty}(v^2+2v+1)e^{-2\delta v}dv$$
$$= \frac{-\delta^5}{4[\delta(\delta+2)+2]^2}\left[\frac{1}{2\delta^2}+\frac{1}{\delta}+1\right].$$

Balakrishnan et al. [41] developed the concept of weighted Ex (WEx) measure, which provides a fresh interpretation of classical entropy.Tthe WEx measure gives bigger values more weight than Ex, which considers all values inside a probability distribution equally. Because of this contrast, it is especially helpful in situations where higher values have greater meaning. The WEx measure of the MoD is given by:

$$\psi_w = \frac{-1}{2}\int_0^{\infty} vh^2(v;\delta)dv = \frac{-\delta^6}{2[\delta(\delta+2)+2]^2}\int_0^{\infty}(v^3+2v^2+v)e^{-2\delta v}dv$$
$$= \frac{-\delta^4}{8[\delta(\delta+2)+2]^2}\left[\frac{3}{2\delta^2}+\frac{2}{\delta}+1\right].$$

The residual Ex (REx) measure at time t of residual lifespan V_t is given by

$$\psi_R = \frac{-1}{2\bar{H}^2(t;\delta)}\int_t^{\infty} h^2(v;\delta)dv = \frac{-\delta^6}{2\bar{H}^2(t;\delta)[\delta(\delta+2)+2]^2}\int_t^{\infty}(v^2+2v+1)e^{-2\delta v}dv$$
$$= \frac{-\delta^5}{4\bar{H}^2(t;\delta)[\delta(\delta+2)+2]^2}\left[\frac{\Gamma(3,2\delta t)}{4\delta^2}+\frac{\Gamma(2,2\delta t)}{\delta}+\Gamma(1,2\delta t)\right].$$

3. Maximum Likelihood and Maximum Product Estimators

This section provides the MLE $\hat{\delta}_1$ of the parameter δ and MPSE $\hat{\delta}_2$ of δ for the MoD based on RSS. Firstly, description of the RSS procedure is explained.

3.1. **RSS Description.** In comparison to SRS with an identical number of observations, the RSS scheme offers a sampling process that frequently produces a more representative sample of population data. To extract a sample of size n from a population, RSS uses the following procedures:

(1) The process of random selection involves taking k^2 units at random from the population and dividing them into k sets, each of which has k units.
(2) Using a cost-effective technique or visual inspection, order the k units within each set according to the variable of interest.

(3) Measure the lowest-ranked unit from the first set, then the next-smallest-ranked unit from the second set, and so on, until the kth-smallest unit from the last set is measured. The following matrix notation is considered to express the RSS design.

$$\overset{\textit{Observations}}{\begin{pmatrix} V_{(1:1)} & V_{(1:2)} & \dots & V_{(1:k)} \\ V_{(2:1)} & V_{(2:2)} & \dots & V_{(2:k)} \\ \vdots & \vdots & \ddots & \vdots \\ V_{(k:1)} & V_{(k:2)} & . & V_{(k:k)} \end{pmatrix}} \Rightarrow \overset{\textit{RSS Selection}}{\begin{pmatrix} \boxed{V_{11}} & V_{21} & \dots & V_{k1} \\ V_{12} & \boxed{V_{22}} & \dots & V_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ V_{1k} & V_{2k} & . & \boxed{V_{kk}} \end{pmatrix}} \Rightarrow \begin{matrix} RSS\ (c=1) \\ \boxed{V_{11}, V_{22}, ..., V_{kk}} \end{matrix}$$

(4) Continue this procedure c times to raise the sample size to be $n = k \times c$, where k is the set size and c is the cycle number. For sample size $n = k \times c$, the RSS is represented as $V_{(s:s)i}$ ($s = 1, 2,..., k$, and $i = 1,..., c$. For simplified form at rest of article the RSS is written as V_{si}, instead of writing $V_{(s:s)i}$.

According to Wolf [42], set sizes (k) larger than five would undoubtedly result in an excessive number of ranking errors and so could not likely considerably increase the efficacy of the RSS. Let V_{si} represent the order statistics of s-th sample, with $s = 1,..., k$ in the c cycle. Assuming perfect ranking, the PDF of V_{si} (Arnold et al. [43]) is given by:

$$h(v_{si}) = \frac{1}{B(s, k-s+1)} \left[H(v_{si})\right]^{s-1} \left[1 - H(v_{si})\right]^{k-s} h(v_{si}), \quad -\infty < v_{si} < \infty. \tag{3.1}$$

3.2. **Maximum Likelihood Estimator.** Suppose $V_{si} = (V_{si},\ s = 1, ..., k,\ \ i = 1, 2, ..., c)$ are the RSS of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. The likelihood function (LF) of δ is given by:

$$L_1(\delta) \propto \prod_{i=1}^{c} \prod_{s=1}^{k} \left[1 - e^{-\delta v_{si}} \left[\frac{\{\delta(v_{si}+1)+1\}^2+1}{(\delta+1)^2+1}\right]\right]^{s-1} \left[e^{-\delta v_{si}} \left[\frac{\{\delta(v_{si}+1)+1\}^2+1}{(\delta+1)^2+1}\right]\right]^{k-s} \frac{\delta^3 (v_{si}+1)^2 e^{-\delta v_{si}}}{\delta(\delta+2)+2}. \tag{3.2}$$

The log-LF of Equation (3.2) is obtained as

$$\begin{aligned} \log L_1(\delta) \propto\ & 3n \log \delta - n \log\left[\delta(\delta+2)+2\right] + 2\sum_{i=1}^{c}\sum_{s=1}^{k} \log(v_{si}+1) + \sum_{i=1}^{c}\sum_{s=1}^{k} (s-1) \log\left[1 - D(v_{si}, \delta) e^{-\delta v_{si}}\right] \\ & + \sum_{i=1}^{c}\sum_{s=1}^{k} (k-s) \log\left[D(v_{si}, \delta)\right] - \sum_{i=1}^{c}\sum_{s=1}^{k} \delta(k-s+1) v_{si}, \end{aligned} \tag{3.3}$$

where $D(v_{si}, \delta) = \frac{\{\delta(v_{si}+1)+1\}^2+1}{(\delta+1)^2+1}$.

Maximizing Equation (3.3) with respect to δ provides the MLE $\hat{\delta}_1$ of δ as shown below:

$$\begin{aligned} \log L_1(\delta) \propto\ & \tfrac{3n}{\delta} - \tfrac{2n(\delta+1)}{[\delta(\delta+2)+2]} + \textstyle\sum_{i=1}^{c}\sum_{s=1}^{k} \frac{(s-1)e^{-\delta v_{si}}\left[v_{si} D(v_{si},\delta) - D'(v_{si},\delta)\right]}{\left[1 - D(v_{si},\delta) e^{-\delta v_{si}}\right]} - \textstyle\sum_{i=1}^{c}\sum_{s=1}^{k} (k-s+1) v_{si} \\ & + \textstyle\sum_{i=1}^{c}\sum_{s=1}^{k} \frac{(k-s) D'(v_{si},\delta)}{D(v_{si},\delta)} = 0, \end{aligned} \tag{3.4}$$

where $D'(v_{si}, \delta) = \frac{\partial}{\partial \delta} D(v_{si}, \delta) = \frac{2[\delta(v_{si}+1)+1](v_{si}+1)}{(\delta+1)^2+1} - \frac{2(\delta+1)\left[\{\delta(v_{si}+1)+1\}^2+1\right]}{\left[(\delta+1)^2+1\right]^2}$.

Given the complexity of Equation (3.4), we will employ numerical optimization techniques, specifically the Newton-Raphson method implemented in Mathematica, to determine the solution.

3.3. **Maximum Product Spacing Estimator.** Cheng and Amin [44] proposed a method that aims to optimize data spacing by maximizing its geometric mean. By examining the variations in the CDF values at successive data points, MPSE for the unknown parameter δ of the MoD is provided. Suppose $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$ to be an ordered sample forming RSS of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. To obtain the MPSE $\hat{\delta}_2$ of δ, the function below is maximized with respect to δ

$$L_2(\delta) = \frac{1}{n+1} \sum_{i_1=1}^{n+1} \log\left[\Lambda_l(\delta)\right], \tag{3.5}$$

where $\Lambda_l(\delta)$is uniform spacings and defined as follows:

$$\Lambda_l(\delta) = H\left(v_{(l:n)}; \delta\right) - H\left(v_{(l-1:n)}; \delta\right), \quad l = 1, 2, ..., n+1,$$

where $H\left(v_{(0:n)}; \delta\right) = 0, \quad H\left(v_{(n+1:n)}; \delta\right) = 1$ and $\sum_{l=1}^{n+1} \Lambda_l(\delta) = 1$.
The MPSE $\hat{\delta}_2$ can be generated through the numerical calculation of the following equation:

$$\frac{\partial L_2(\delta)}{\partial \delta} = \frac{1}{n+1} \sum_{i_1=1}^{n+1} \frac{1}{\left[\Lambda_l(\delta)\right]} \left[\hbar\left(v_{(l:n)}; \delta\right) - \hbar\left(v_{(l-1:n)}; \delta\right)\right] = 0,$$

where

$$\begin{aligned} \hbar\left(v_{(l:n)}; \delta\right) &= \frac{\partial}{\partial \delta} H(v_{(l:n)}; \delta) = \frac{\partial}{\partial \delta}\left\{1 - e^{-\delta v_{(l:n)}} D(v_{(l:n)}; \delta)\right\} \\ &= D(v_{(l:n)}; \delta) v_{(l:n)} e^{-\delta v_{(l:n)}} - e^{-\delta v_{(l:n)}} D'(v_{(l:n)}; \delta), \end{aligned}$$

$$\tag{3.6}$$

$$\begin{aligned} \hbar\left(v_{(l-1:n)}; \delta\right) &= \frac{\partial}{\partial \delta} H(v_{(l-1:n)}; \delta) = \left\{1 - e^{-\delta v_{(l-1:n)}} D(v_{(l-1:n)}; \delta)\right\} \\ &= D(v_{(l-1:n)}; \delta) v_{(l-1:n)} e^{-\delta v_{(l:-1n)}} - e^{-\delta v_{(l-1:n)}} D'(v_{(l-1:n)}; \delta). \end{aligned}$$

4. Minimum Distance Estimation Methods

Estimation methods based on minimizing well-known goodness-of-fit statistics are valuable tools in various statistical applications. This section introduces five different estimators: CVME, ADE, LTAD, RTAD, and ADSOE. These estimators are derived by minimizing test statistics that measure the discrepancy between empirical and theoretical CDFs.

4.1. **Cramér-von Mises Estimators.** Suppose $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$to be an ordered sample forming RSS of size of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. Minimizing the following function, the CVME $\hat{\delta}_3$ of δ for the MoD is generated

$$L_3(\delta) = \frac{1}{12n} + \sum_{l=1}^{n} \left\{H(v_{(l:n)}; \delta) - \frac{2l-1}{2n}\right\}^2. \tag{4.1}$$

Rather than using Equation (4.1), CVME $\hat{\delta}_3$ of δ for the MoD is generated by solving the following nonlinear equation:

$$\frac{\partial L_3(\delta)}{\partial \delta} = \sum_{l=1}^{n} \left\{ H(v_{(l:n)};\delta) - \frac{2l-1}{2n} \right\} \hbar(v_{(l:n)};\delta) = 0,$$

where, $\hbar(.;\delta)$is defined in Equation (3.6).

4.2. **Anderson Darling Estimators.** Suppose $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$to be an ordered sample forming RSS of size of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. Here, ADE, LTAD, RTAD, and ADSOE are determined, respectively, by minimizing the following functions:

$$L_4(\delta) = -n - \frac{1}{n} \sum_{l=1}^{n} (2l-1) \left\{ \log\left[H\left(v_{(l:n)};\delta\right)\right] + \log\left[\bar{H}\left(v_{(n+1-l:n)};\delta\right)\right] \right\},$$

$$L_5(\delta) = \frac{-3n}{2} + 2\sum_{l=1}^{n} H\left(v_{(l:n)};\delta\right) - \frac{1}{n}\sum_{l=1}^{n}(2l-1)\log\left[H\left(v_{(l:n)};\delta\right)\right],$$

$$L_6(\delta) = \frac{n}{2} - 2\sum_{l=1}^{n} H\left(v_{(l:n)};\delta\right) - \frac{1}{n}\sum_{l=1}^{n}(2l-1)\log\left[\bar{H}\left(v_{(1+n-l:n)};\delta\right)\right],$$

and

$$L_7(\delta) = 2\sum_{l=1}^{n} \log\left[H\left(v_{(l:n)};\delta\right)\right] + \frac{1}{n}\sum_{l=1}^{n} \frac{(2l-1)}{H\left(v_{(l:n)};\delta\right)}.$$

The ADE $\hat{\delta}_4$ of parameter δ, LTAD $\hat{\delta}_5$ of parameter δ, RTAD $\hat{\delta}_6$ of parameter δ, and ADSOE $\hat{\delta}_7$ of parameter δ, based on the MoD, can be determined numerically by solving the following non-linear equations

$$\frac{\partial L_4(\delta)}{\partial \delta} = \sum_{l=1}^{n}(2l-1)\left\{ \frac{\hbar(v_{(l:n)};\delta)}{H\left(v_{(l:n)};\delta\right)} - \frac{\hbar(v_{(l+1-n:n)};\delta)}{\bar{H}\left(v_{(l+1-n:n)};\delta\right)} \right\} = 0,$$

$$\frac{\partial L_5(\delta)}{\partial \delta} = \frac{-3n}{2} + 2\sum_{l=1}^{n} \hbar\left(v_{(l:n)};\delta\right) - \frac{1}{n}\sum_{l=1}^{n} \frac{(2l-1)\hbar\left(v_{(l:n)};\delta\right)}{H\left(v_{(l:n)};\delta\right)} = 0,$$

$$\frac{\partial L_6(\delta)}{\partial \delta} = \frac{n}{2} - 2\sum_{l=1}^{n} \hbar\left(v_{(l:n)};\delta\right) + \frac{1}{n}\sum_{l=1}^{n} \frac{(2l-1)\hbar\left(v_{(l:n)};\delta\right)}{\bar{H}\left(v_{(1+n-l:n)};\delta\right)} = 0,$$

and

$$\frac{\partial L_7(\delta)}{\partial \delta} = 2\sum_{l=1}^{n} \frac{\hbar\left(v_{(l:n)};\delta\right)}{H\left(v_{(l:n)};\delta\right)} - \frac{1}{n}\sum_{l=1}^{n} \frac{(2l-1)\hbar\left(v_{(l:n)};\delta\right)}{H^2\left(v_{(l:n)};\delta\right)} = 0,$$

where, $\hbar(.;\delta)$is defined in Equation (3.6).

5. Other Estimation Techniques

This section presents several MoD estimators, including MSSDE, MSALD, MSSLDE, and MSLDE, for the parameterδ. Additionally, the OLSE, KE, and WLSE of the parameter δ are provided for the MoD.

5.1. **Minimum Spacing Estimators.** Suppose $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$to be an ordered sample forming RSS of size of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. Here, MSSDE, MSSLDE, MSADE, MSALDE and MSLNDE. are determined, respectively, by minimizing the following functions

$$L_8(\delta) = \sum_{l=1}^{n+1}\left(\Lambda_l - \frac{1}{n+1}\right)^2, \ L_9(\delta) = \sum_{l=1}^{n+1}\left(\log \Lambda_l - \log \frac{1}{n+1}\right)^2,$$
$$L_{10}(\delta) = \sum_{l=1}^{n+1}\left|\Lambda_l - \frac{1}{n+1}\right|, \quad L_{11}(\delta) = \sum_{l=1}^{n+1}\left|\log \Lambda_l - \log \frac{1}{n+1}\right|, \tag{5.1}$$
$$L_{12}(\delta) = \sum_{l=1}^{n+1}\left(e^{\Lambda_l - \frac{1}{n+1}} - \left(\Lambda_l - \frac{1}{n+1}\right) - 1\right).$$

Instead of employing Equation (5.1), the MSSDE $\hat{\delta}_8$ of parameter δ, MSSLDE $\hat{\delta}_9$ of parameter δ, MSADE $\hat{\delta}_{10}$ of parameter δ, MSALDE $\hat{\delta}_{11}$ of parameter δ, and MSLNDE $\hat{\delta}_{12}$ of parameter δ, are given by analytically solving the following non-linear equations:

$$\frac{\partial L_8(\delta)}{\partial \delta} = \sum_{l=1}^{n+1}\left(\Lambda_l - \frac{1}{n+1}\right)\left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right)\right] = 0,$$
$$\frac{\partial L_9(\delta)}{\partial \delta} = \sum_{l=1}^{n+1}\left(\log \Lambda_l - \log \frac{1}{n+1}\right)\frac{1}{\Lambda_l}\left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right)\right] = 0,$$
$$\frac{\partial L_{10}(\delta)}{\partial \delta} = \sum_{l=1}^{n+1}\frac{\Lambda_l - \frac{1}{n+1}}{\left|\Lambda_l - \frac{1}{n+1}\right|}\left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right)\right] = 0,$$
$$\frac{\partial L_{11}(\delta)}{\partial \delta} = \sum_{l=1}^{n+1}\frac{\log \Lambda_l - \log \frac{1}{n+1}}{\left|\log \Lambda_l - \log \frac{1}{n+1}\right|}\frac{1}{\Lambda_l}\left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right)\right] = 0,$$
$$\frac{\partial L_{12}(\delta)}{\partial \delta} = \sum_{l=1}^{n+1}\left(e^{\Lambda_l - \frac{1}{n+1}} - 1\right)\left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right)\right] = 0,$$

where, $\hbar(.;\delta)$is defined in Equation (3.6).

5.2. **Ordinary and Weighted Least Squares Estimators.** Let $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$to be an ordered sample forming RSS of size of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. The OLSE and WLSE of parameter δ are provided by minimizing the following function with respect to δ

$$L_{13}(\delta) = \sum_{l=1}^{n} W_l\left[H\left(v_{(l:n)};\delta\right) - \frac{l}{n+1}\right]^2,$$

where W_l =1 in case of LSE, while $W_l = \frac{(n+1)^2(n+2)}{l(n-l+1)}$ in case of the WLSE. An alternative way, the OLSE $\hat{\delta}_{13}$ and WLSE $\hat{\delta}_{14}$ are obtained by solving the following non-linear equation:

$$\frac{\partial L_{13}(\delta)}{\partial \delta} = \sum_{l=1}^{n} W_l \left[H\left(v_{(l:n)};\delta\right) - \frac{l}{n+1} \right] \left[\hbar\left(v_{(l:n)};\delta\right) - \hbar\left(v_{(l-1:n)};\delta\right) \right] = 0,$$

where, $\hbar(.;\delta)$is defined in Equation (3.6).

5.3. **Kolmogorov Method.** Suppose $V_{(1:n)}, V_{(2:n)}, ..., V_{(n:n)}$to be an ordered sample forming RSS of size $n = k \times c$, where k is the set size and c is the cycle number drawn from the MoD. The following function must be minimized to obtain KE $\tilde{\delta}_{15}$ with regard to δ

$$L_{15}(\delta) = \underset{1 \le l \le n}{Max} \sum_{l=1}^{n} \left[\frac{l}{n} - H(v_{(l:n)};\delta), H(v_{(l:n)};\delta) - \frac{l-1}{n} \right]^2 .$$

6. Numerical Simulation

This section compares the efficiency of several estimation methods for the MoD. We will create randomly generated datasets before selecting the best-suited one. In addition, the datasets will be ranked, and estimation techniques will be utilized to select the best alternative. The simulation will be performed using the following steps:

- SRSs of varying sample sizes (n = 15, 45, 90, 180, 250, and 350) were created using the proposed model.
- An RSS with a fixed set size (k = 5) and varying cycle numbers (c = 3, 9, 18, 36, 50, and 70) was generated using the suggested distribution.
- Obtain MoD estimates ($\hat{\delta}$) for both processes (RSS & SRS). Six different metrics were used to assess the estimation procedures as follows:
  (1) The absolute bias average (BIAS), calculated using the following formula: $|Bias(\widehat{\boldsymbol{\delta}})| = \frac{1}{M}\sum_{i=1}^{M} |\widehat{\boldsymbol{\delta}} - \boldsymbol{\delta}|$.
  (2) The following formula yielded the mean squared error (MSE): $MSE = \frac{1}{M}\sum_{i=1}^{M}(\widehat{\boldsymbol{\delta}} - \boldsymbol{\delta})^2$.
  (3) The following formula is used to get the mean absolute relative error (MRE): $MRE = \frac{1}{M}\sum_{i=1}^{M} |\widehat{\boldsymbol{\delta}} - \boldsymbol{\delta}|/\boldsymbol{\delta}$.
  (4) The average absolute difference, D_{abs}, is computed as follows: $D_{abs} = \frac{1}{nB}\sum_{i=1}^{B}\sum_{j=1}^{n} |H(v_{ij};\boldsymbol{\delta}) - H(v_{ij};\widehat{\boldsymbol{\delta}})|$, where $H(v;\boldsymbol{\delta}) = H(v)$ and v_{ij} represents values obtained at the i-th iteration sample and j-th component of this sample.
  (5) The maximum absolute difference, D_{max}, is computed as follows: $D_{max} = \frac{1}{B}\sum_{i=1}^{B} \max_{j=1,\dots,n} |H(v_{ij};\boldsymbol{\delta}) - H(v_{ij};\widehat{\boldsymbol{\delta}})|$.
  (6) The average squared absolute error (ASAE), determined by: $ASAE = \frac{1}{B}\sum_{i=1}^{H} \frac{|v_{(i)} - \hat{v}(i)|}{v(n) - v_{(1)}}$, where $v_{(i)}$ denotes the ascending ordered observations.
- The results of the evaluation metrics are presented in Tables 1 through 10.
- Figures 2 to 6 provide visual representations of the numerical data in Tables 1 and 2.

- Tables 11 and 12 provide a performance analysis by displaying the partial and total ranks of estimations for SRS and RSS.
- Table 13 shows the ratio of SRS to RSS MSE, allowing for comparison of sampling methodologies' performance.

Based on the simulation results, the following conclusions can be drawn:

- The initial model estimates show consistency across both the SRS and RSS datasets, indicating convergence to true parameter values as sample sizes increase.
- The MPSE and MLE techniques are effective for assessing the estimated quality of RSS and SRS, respectively.
- RSS is a more efficient sampling approach than SRS, resulting in lower MSE and other metrics.

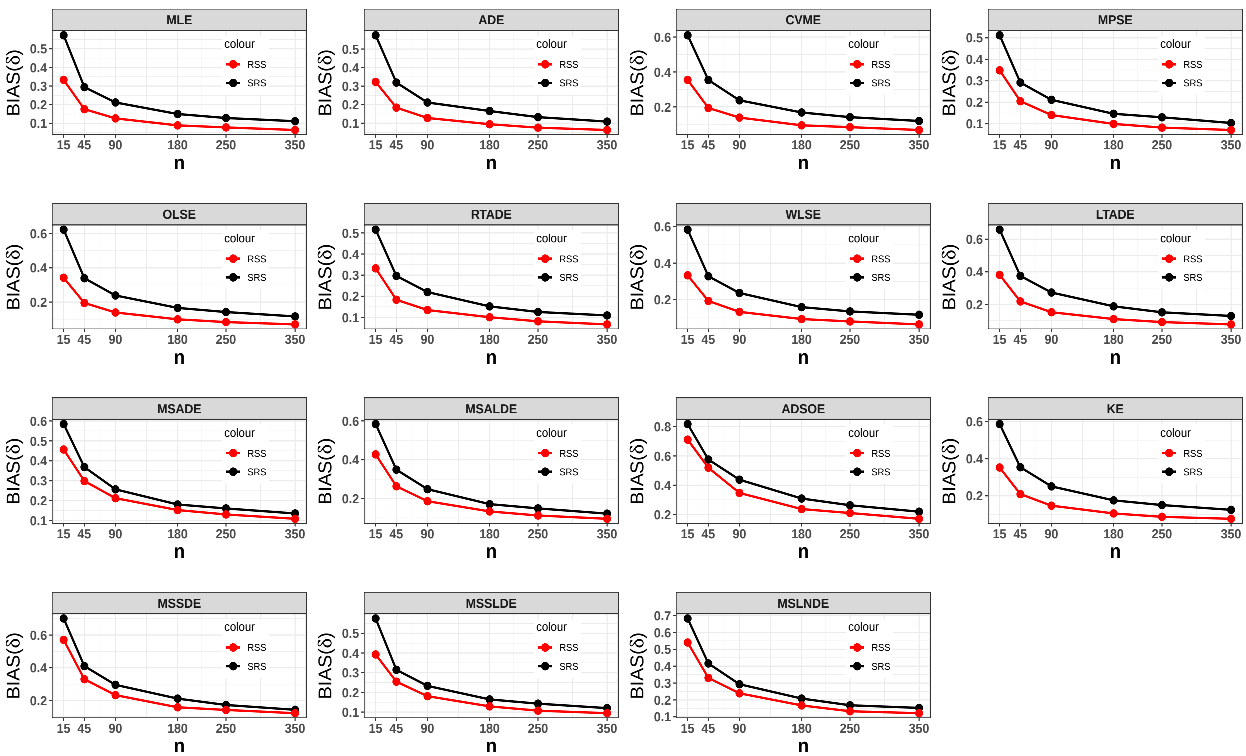

FIGURE 2. A Graphical representation for the numerical values of BIAS presented in Tables 1 and 2.

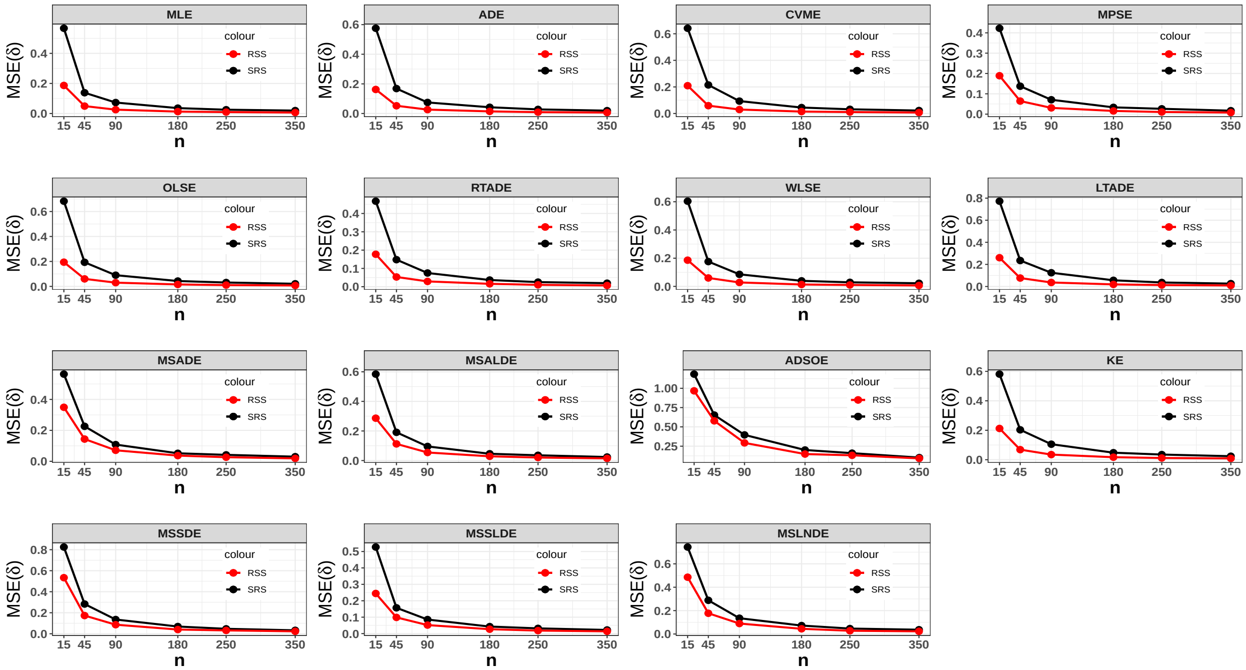

FIGURE 3. A Graphical representation for the numerical values of MSE presented in Tables 1 and 2.

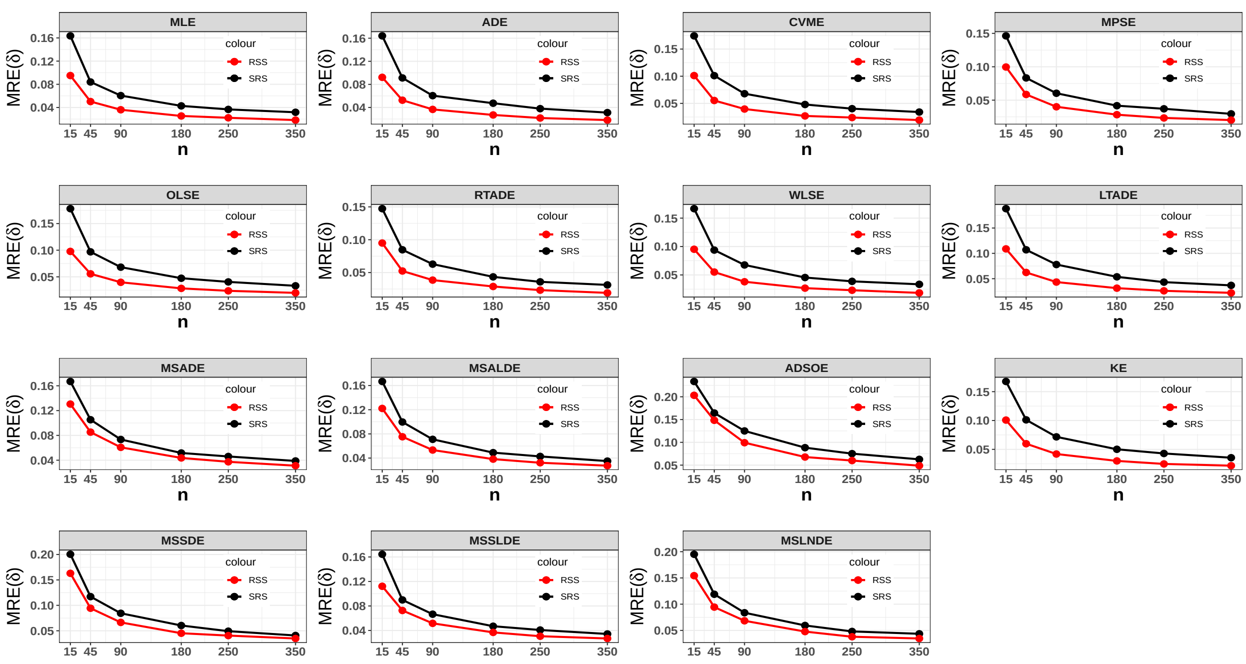

FIGURE 4. A Graphical representation for the numerical values of MRE presented in Tables 1 and 2.

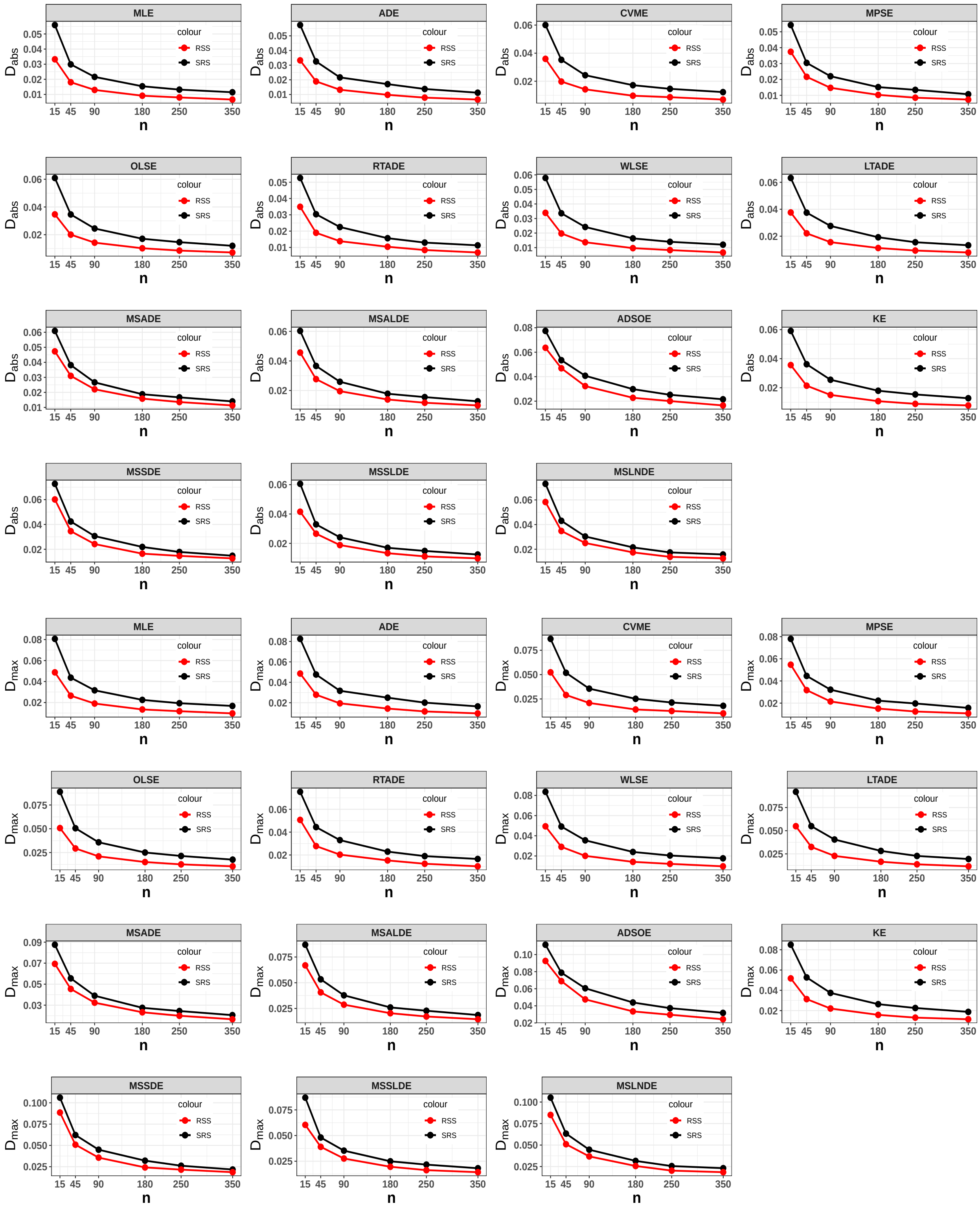

FIGURE 5. A Graphical representation for the numerical values of D_{abs} and D_{max} presented in Tables 1 and 2.

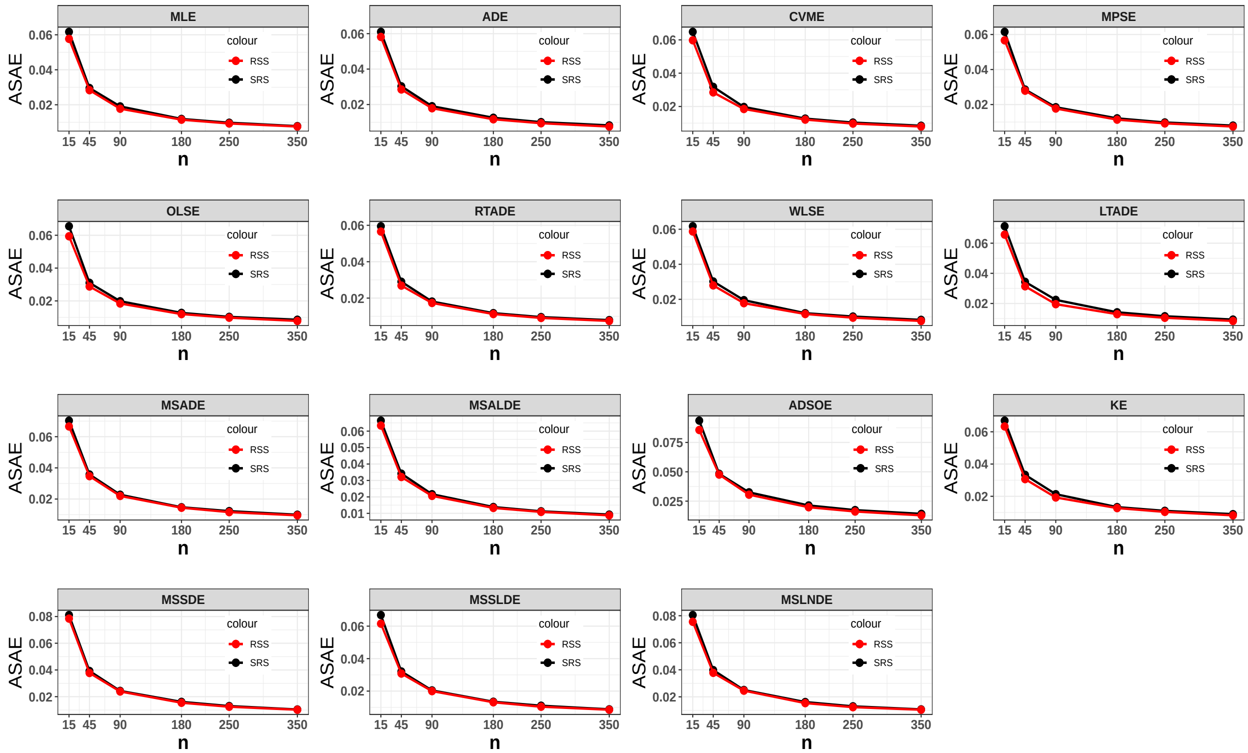

Figure 6. A Graphical representation for the numerical values of ASAE presented in Tables 1 and 2.

Table 1. Numerical values of simulation measures for $\delta = 3.5$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.5725[3]	0.57434[4]	0.61018[10]	0.51202[1]	0.62306[11]	0.51544[2]	0.58339[6]	0.65879[12]	0.58441[8]	0.58364[7]	0.8176[15]	0.58692[9]	0.70156[14]	0.57547[5]	0.68288[13]
	MSE	0.56713[5]	0.57556[6]	0.64156[10]	0.42257[1]	0.68285[11]	0.46728[2]	0.60417[9]	0.77234[13]	0.56366[4]	0.58456[8]	1.18357[15]	0.58145[7]	0.82528[14]	0.52725[3]	0.74378[12]
	MRE	0.16357[3]	0.1641[4]	0.17434[10]	0.14629[1]	0.17802[11]	0.14727[2]	0.16668[6]	0.18823[12]	0.16697[8]	0.16675[7]	0.2336[15]	0.16769[9]	0.20045[14]	0.16442[5]	0.19511[13]
	D_{abs}	0.05591[3]	0.05738[4]	0.06004[7]	0.05427[2]	0.06094[11]	0.05261[1]	0.05787[5]	0.06321[12]	0.06092[10]	0.06032[8]	0.07742[15]	0.05913[6]	0.07272[13]	0.06056[9]	0.07306[14]
	D_{max}	0.08068[3]	0.08255[4]	0.08672[7]	0.07801[2]	0.0889[11]	0.0753[1]	0.08361[5]	0.09199[12]	0.08756[10]	0.08694[9]	0.11155[15]	0.08496[6]	0.10607[14]	0.08691[8]	0.10511[13]
	ASAE	0.0617[4]	0.06103[2]	0.06483[6]	0.06154[3]	0.06552[7]	0.05951[1]	0.06174[5]	0.07127[12]	0.07032[11]	0.0664[8]	0.09361[15]	0.06693[10]	0.08114[14]	0.06686[9]	0.08053[13]
	$\sum$ *Ranks*	21[3]	24[4]	50[9]	10[2]	62[11]	9[1]	36[5]	73[12]	51[10]	47[7.5]	90[15]	47[7.5]	83[14]	39[6]	78[13]
45	BIAS	0.29408[2]	0.31925[5]	0.35386[9]	0.29158[1]	0.33929[7]	0.29643[3]	0.32821[6]	0.37475[12]	0.36851[11]	0.34888[8]	0.57555[15]	0.35404[10]	0.41002[13]	0.31453[4]	0.41618[14]
	MSE	0.13822[2]	0.16835[5]	0.21488[10]	0.13749[1]	0.19317[8]	0.14785[3]	0.17592[6]	0.23522[12]	0.22613[11]	0.19087[7]	0.6531[15]	0.20315[9]	0.28284[13]	0.15708[4]	0.28822[14]
	MRE	0.08402[2]	0.09121[5]	0.1011[9]	0.08331[1]	0.09694[7]	0.08469[3]	0.09377[6]	0.10707[12]	0.10529[11]	0.09968[8]	0.16444[15]	0.10115[10]	0.11715[13]	0.08987[4]	0.11891[14]
	D_{abs}	0.02988[1]	0.03254[4]	0.03526[8]	0.03042[3]	0.03461[7]	0.03036[2]	0.0336[6]	0.03749[11]	0.03815[12]	0.03661[10]	0.05348[15]	0.03618[9]	0.04237[13]	0.03293[5]	0.04307[14]
	D_{max}	0.04372[1]	0.04763[4]	0.05185[8]	0.04456[3]	0.05056[7]	0.0444[2]	0.04917[6]	0.05493[11]	0.05553[12]	0.05334[10]	0.07883[15]	0.05272[9]	0.06219[13]	0.0481[5]	0.06331[14]
	ASAE	0.02958[3]	0.0303[5]	0.03178[7]	0.02857[1]	0.03103[6]	0.02901[2]	0.03021[4]	0.03436[11]	0.03577[12]	0.03416[10]	0.04831[15]	0.03332[9]	0.03924[13]	0.03209[8]	0.03979[14]
	$\sum$ *Ranks*	11[2]	28[4]	51[8]	10[1]	42[7]	15[3]	34[6]	69[11.5]	69[11.5]	53[9]	90[15]	56[10]	78[13]	30[5]	84[14]
90	BIAS	0.21191[3]	0.2118[2]	0.23757[7]	0.21129[1]	0.23859[8]	0.21994[4]	0.23657[6]	0.27283[12]	0.25698[11]	0.24853[9]	0.43775[15]	0.2509[10]	0.29542[14]	0.23304[5]	0.29305[13]
	MSE	0.07293[2]	0.07449[3]	0.09316[8]	0.07077[1]	0.08982[7]	0.07505[4]	0.08553[5]	0.12458[12]	0.10804[11]	0.0954[9]	0.39622[15]	0.10547[10]	0.13557[14]	0.08562[6]	0.1351[13]
	MRE	0.06055[3]	0.06052[2]	0.06788[7]	0.06037[1]	0.06817[8]	0.06284[4]	0.06759[6]	0.07795[12]	0.07342[11]	0.07101[9]	0.12507[15]	0.07169[10]	0.08441[14]	0.06658[5]	0.08373[13]
	D_{abs}	0.02159[1]	0.02165[2]	0.02428[7]	0.02201[3]	0.02447[8]	0.0225[4]	0.02417[6]	0.0276[12]	0.02668[11]	0.02591[10]	0.04076[15]	0.0255[9]	0.03062[14]	0.02412[5]	0.03034[13]
	D_{max}	0.03164[1]	0.0317[2]	0.03559[7]	0.03215[3]	0.03576[8]	0.033[4]	0.0355[6]	0.04051[12]	0.039[11]	0.03779[10]	0.06056[15]	0.03745[9]	0.04497[14]	0.0353[5]	0.04462[13]
	ASAE	0.01907[3.5]	0.01907[3.5]	0.01972[6]	0.01848[2]	0.01982[7]	0.01811[1]	0.01954[5]	0.0224[11]	0.02278[12]	0.02165[10]	0.03253[15]	0.02132[9]	0.02432[13]	0.02042[8]	0.02506[14]
	$\sum$ *Ranks*	13.5[2]	14.5[3]	42[7]	11[1]	46[8]	21[4]	34[5.5]	71[12]	67[11]	57[9.5]	90[15]	57[9.5]	83[14]	34[5.5]	79[13]
180	BIAS	0.14984[2]	0.16585[6.5]	0.16785[8]	0.14611[1]	0.16585[6.5]	0.15217[3]	0.15914[4]	0.18803[12]	0.18087[11]	0.17149[9]	0.30931[15]	0.17549[10]	0.21127[14]	0.16428[5]	0.2082[13]
	MSE	0.03589[2]	0.04216[5]	0.04475[8]	0.03338[1]	0.04312[7]	0.03662[3]	0.03947[4]	0.05617[12]	0.05177[11]	0.04588[9]	0.20064[15]	0.04775[10]	0.06883[13]	0.04272[6]	0.07219[14]
	MRE	0.04281[2]	0.04739[6.5]	0.04796[8]	0.04175[1]	0.04739[6.5]	0.04348[3]	0.04547[4]	0.05372[12]	0.05168[11]	0.049[9]	0.08837[15]	0.05014[10]	0.06036[14]	0.04694[5]	0.05949[13]
	D_{abs}	0.01539[2]	0.01706[6]	0.01718[8]	0.01516[1]	0.01707[7]	0.01566[3]	0.01635[4]	0.01922[12]	0.01876[11]	0.0178[9]	0.02982[15]	0.01794[10]	0.02188[14]	0.01697[5]	0.0215[13]
	D_{max}	0.02253[2]	0.02494[6]	0.02517[8]	0.02222[1]	0.02502[7]	0.02293[3]	0.02399[4]	0.02821[12]	0.02748[11]	0.02601[9]	0.04392[15]	0.02633[10]	0.03209[14]	0.02493[5]	0.03157[13]
	ASAE	0.0119[2]	0.01254[5]	0.01274[7]	0.01213[4]	0.01271[6]	0.01174[1]	0.0121[3]	0.01417[11]	0.01474[12]	0.01386[10]	0.02135[15]	0.01332[8]	0.01611[13]	0.01345[9]	0.01616[14]
	$\sum$ *Ranks*	12[2]	35[5.5]	47[8]	9[1]	40[7]	16[3]	23[4]	71[12]	67[11]	55[9]	90[15]	58[10]	82[14]	35[5.5]	80[13]
250	BIAS	0.1284[2]	0.13288[4]	0.14108[6]	0.12981[3]	0.14167[7]	0.12565[1]	0.13559[5]	0.15133[11]	0.16114[12]	0.14957[9]	0.26329[15]	0.15007[10]	0.17222[14]	0.14266[8]	0.16832[13]
	MSE	0.02515[1]	0.0276[4]	0.03156[7]	0.02627[3]	0.03118[6]	0.02535[2]	0.02888[5]	0.03644[11]	0.04166[12]	0.03555[10]	0.15981[15]	0.03506[9]	0.04719[14]	0.03186[8]	0.04649[13]
	MRE	0.03669[2]	0.03797[4]	0.04031[6]	0.03709[3]	0.04048[7]	0.0359[1]	0.03874[5]	0.04324[11]	0.04604[12]	0.04274[9]	0.07523[15]	0.04288[10]	0.04921[14]	0.04076[8]	0.04809[13]
	D_{abs}	0.01318[2]	0.01374[4]	0.01458[6]	0.01343[3]	0.0146[7]	0.01296[1]	0.01398[5]	0.01553[10]	0.01669[12]	0.01554[11]	0.0252[15]	0.0154[9]	0.01782[14]	0.01483[8]	0.01741[13]
	D_{max}	0.01935[2]	0.02011[4]	0.02135[6]	0.01968[3]	0.02139[7]	0.019[1]	0.02046[5]	0.02278[10.5]	0.02442[12]	0.02278[10.5]	0.03728[15]	0.02256[9]	0.02614[14]	0.02169[8]	0.02556[13]
	ASAE	0.00974[2]	0.01008[4]	0.01036[7]	0.00977[3]	0.01029[6]	0.00957[1]	0.01019[5]	0.01154[11]	0.01226[12]	0.01126[10]	0.01739[15]	0.011[8]	0.01299[13.5]	0.01101[9]	0.01299[13.5]
	$\sum$ *Ranks*	11[2]	24[4]	38[6]	18[3]	40[7]	7[1]	30[5]	64.5[11]	72[12]	59.5[10]	90[15]	55[9]	83.5[14]	49[8]	78.5[13]
350	BIAS	0.11121[4]	0.10891[2]	0.11924[7]	0.10335[1]	0.11605[5]	0.10924[3]	0.11728[6]	0.12893[11]	0.13592[12]	0.12262[9]	0.21946[15]	0.12435[10]	0.14267[13]	0.12003[8]	0.1526[14]
	MSE	0.01946[4]	0.01921[2]	0.02221[7]	0.017[1]	0.02108[5]	0.01938[3]	0.0222[6]	0.02595[11]	0.02994[12]	0.024[10]	0.10228[15]	0.02381[9]	0.03281[13]	0.02222[8]	0.03668[14]
	MRE	0.03177[4]	0.03112[2]	0.03407[7]	0.02953[1]	0.03316[5]	0.03121[3]	0.03351[6]	0.03684[11]	0.03883[12]	0.03503[9]	0.0627[15]	0.03553[10]	0.04076[13]	0.0343[8]	0.0436[14]
	D_{abs}	0.01143[4]	0.01121[2]	0.01226[7]	0.01068[1]	0.01199[5]	0.01127[3]	0.01203[6]	0.01329[11]	0.014[12]	0.0127[9]	0.02153[15]	0.01279[10]	0.0148[13]	0.0124[8]	0.01574[14]
	D_{max}	0.01676[4]	0.01642[2]	0.01798[7]	0.01567[1]	0.01754[5]	0.01651[3]	0.01766[6]	0.01947[11]	0.02052[12]	0.01863[9]	0.03173[15]	0.01874[10]	0.02169[13]	0.01816[8]	0.02311[14]
	ASAE	0.00779[1]	0.00824[4]	0.00845[6]	0.00799[3]	0.00854[7]	0.0079[2]	0.00829[5]	0.00934[11]	0.00995[12]	0.00926[10]	0.01419[15]	0.00894[9]	0.01042[13]	0.00877[8]	0.01073[14]
	$\sum$ *Ranks*	21[4]	14[2]	41[7]	8[1]	32[5]	17[3]	35[6]	66[11]	72[12]	56[9]	90[15]	58[10]	78[13]	48[8]	84[14]

TABLE 2. Numerical values of simulation measures for $\delta = 3.5$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.33293[3]	0.32286[1]	0.35431[8]	0.34869[6]	0.34213[5]	0.3323[2]	0.33334[4]	0.38109[9]	0.45675[12]	0.42748[11]	0.71127[15]	0.35269[7]	0.57039[14]	0.39261[10]	0.54028[13]
	MSE	0.18664[4]	0.16225[1]	0.20926[7]	0.18904[5]	0.19419[6]	0.17746[2]	0.18596[3]	0.26066[10]	0.34922[12]	0.28649[11]	0.967[15]	0.21237[8]	0.53376[14]	0.24444[9]	0.48585[13]
	MRE	0.09512[3]	0.09224[1]	0.10123[8]	0.09963[6]	0.09775[5]	0.09494[2]	0.09524[4]	0.10888[9]	0.1305[12]	0.12214[11]	0.20322[15]	0.10077[7]	0.16297[14]	0.11217[10]	0.15436[13]
	D_{abs}	0.03324[1]	0.03328[2]	0.03595[7]	0.03742[8]	0.03468[4]	0.03492[5]	0.0339[3]	0.03762[9]	0.04731[12]	0.04566[11]	0.06362[15]	0.03562[6]	0.06014[14]	0.04155[10]	0.05832[13]
	D_{max}	0.04876[2]	0.04857[1]	0.05236[7]	0.05469[8]	0.05074[5]	0.0507[4]	0.04939[3]	0.05489[9]	0.06932[12]	0.06687[11]	0.09256[15]	0.05178[6]	0.08859[14]	0.06039[10]	0.08502[13]
	ASAE	0.05768[3]	0.05816[4]	0.05975[7]	0.05667[2]	0.05939[6]	0.05656[1]	0.05872[5]	0.06569[11]	0.06657[12]	0.0634[10]	0.08563[15]	0.06331[9]	0.07846[14]	0.06152[8]	0.07551[13]
	$\sum Ranks$	16[2.5]	10[1]	44[8]	35[6]	31[5]	16[2.5]	22[4]	57[9.5]	72[12]	65[11]	90[15]	43[7]	84[14]	57[9.5]	78[13]
45	BIAS	0.17595[1]	0.18407[3]	0.19376[5]	0.20494[7]	0.19482[6]	0.18338[2]	0.19294[4]	0.21814[9]	0.29853[12]	0.26344[11]	0.51932[15]	0.20925[8]	0.33007[13]	0.25427[10]	0.33053[14]
	MSE	0.04933[1]	0.05221[2]	0.05886[4]	0.06438[7]	0.06033[6]	0.05329[3]	0.05986[5]	0.07664[9]	0.14377[12]	0.11258[11]	0.57817[15]	0.06852[8]	0.17321[13]	0.0986[10]	0.17731[14]
	MRE	0.05027[1]	0.05259[3]	0.05536[5]	0.05855[7]	0.05566[6]	0.05239[2]	0.05512[4]	0.06233[9]	0.08529[12]	0.07527[11]	0.14838[15]	0.05979[8]	0.09431[13]	0.07265[10]	0.09444[14]
	D_{abs}	0.01806[1]	0.01893[2]	0.01976[5]	0.02164[8]	0.02005[6]	0.01897[3]	0.01971[4]	0.02214[9]	0.031[12]	0.0277[11]	0.04688[15]	0.02142[7]	0.03459[13]	0.02659[10]	0.03482[14]
	D_{max}	0.02653[1]	0.02785[3]	0.02907[5]	0.03173[8]	0.0293[6]	0.02779[2]	0.02904[4]	0.03251[9]	0.04545[12]	0.04067[11]	0.06893[15]	0.03133[7]	0.05084[13]	0.03901[10]	0.05097[14]
	ASAE	0.02834[4]	0.02851[6]	0.02846[5]	0.02792[2]	0.02879[7]	0.02687[1]	0.02796[3]	0.0314[10]	0.03469[12]	0.03212[11]	0.04763[15]	0.03066[8]	0.03774[13]	0.03081[9]	0.03776[14]
	$\sum Ranks$	9[1]	19[3]	29[5]	39[7]	37[6]	13[2]	24[4]	55[9]	72[12]	66[11]	90[15]	46[8]	78[13]	59[10]	84[14]
90	BIAS	0.12639[1]	0.12829[2]	0.1387[5]	0.14035[7]	0.13914[6]	0.13484[4]	0.13315[3]	0.1518[9]	0.2127[12]	0.18657[11]	0.34741[15]	0.1468[8]	0.23252[13]	0.18076[10]	0.2393[14]
	MSE	0.02557[1]	0.02618[2]	0.02976[5]	0.03075[7]	0.03006[6]	0.02907[4]	0.02798[3]	0.03636[9]	0.0712[12]	0.05441[11]	0.29276[15]	0.03485[8]	0.08636[13]	0.05186[10]	0.0895[14]
	MRE	0.03611[1]	0.03665[2]	0.03963[5]	0.0401[7]	0.03975[6]	0.03853[4]	0.03804[3]	0.04337[9]	0.06077[12]	0.05331[11]	0.09926[15]	0.04194[8]	0.06643[13]	0.05165[10]	0.06837[14]
	D_{abs}	0.01297[1]	0.01323[2]	0.01423[5]	0.01471[7]	0.01426[6]	0.01382[4]	0.01368[3]	0.01558[9]	0.02207[12]	0.01954[11]	0.03231[15]	0.01505[8]	0.0242[13]	0.01883[10]	0.02504[14]
	D_{max}	0.01904[1]	0.01942[2]	0.02089[5]	0.02152[7]	0.02097[6]	0.02031[4]	0.0201[3]	0.02284[9]	0.03238[12]	0.0287[11]	0.04755[15]	0.02207[8]	0.03564[13]	0.02763[10]	0.03682[14]
	ASAE	0.01775[3]	0.01791[5]	0.0185[7]	0.01772[2]	0.01841[6]	0.01733[1]	0.01781[4]	0.01954[9]	0.022[12]	0.02062[11]	0.03052[15]	0.01928[8]	0.02387[13]	0.01998[10]	0.02458[14]
	$\sum Ranks$	8[1]	15[2]	32[5]	37[7]	36[6]	21[4]	19[3]	54[9]	72[12]	66[11]	90[15]	48[8]	78[13]	60[10]	84[14]
180	BIAS	0.08886[1]	0.09463[4]	0.09426[3]	0.09899[5]	0.09914[6]	0.10082[7]	0.09376[2]	0.10977[9]	0.15295[12]	0.13396[11]	0.23726[15]	0.10501[8]	0.15802[13]	0.12892[10]	0.16739[14]
	MSE	0.01268[1]	0.01422[3]	0.01444[4]	0.01572[5.5]	0.01572[5.5]	0.01585[7]	0.01353[2]	0.01903[9]	0.03687[12]	0.02864[11]	0.14826[15]	0.01707[8]	0.04022[13]	0.02678[10]	0.04449[14]
	MRE	0.02539[1]	0.02704[4]	0.02693[3]	0.02828[5]	0.02833[6]	0.02881[7]	0.02679[2]	0.03136[9]	0.0437[12]	0.03827[11]	0.06779[15]	0.03[8]	0.04515[13]	0.03684[10]	0.04783[14]
	D_{abs}	0.00914[1]	0.00976[4]	0.00968[3]	0.01027[6]	0.01022[5]	0.0104[7]	0.00964[2]	0.01126[9]	0.01585[12]	0.01392[11]	0.02276[15]	0.01076[8]	0.01646[13]	0.01329[10]	0.01745[14]
	D_{max}	0.01341[1]	0.01429[4]	0.0142[3]	0.01507[6]	0.01499[5]	0.01524[7]	0.01416[2]	0.01656[9]	0.02324[12]	0.02039[11]	0.03348[15]	0.0158[8]	0.02416[13]	0.0196[10]	0.02567[14]
	ASAE	0.01147[3]	0.01168[5]	0.01212[7]	0.01137[2]	0.01196[6]	0.01123[1]	0.01161[4]	0.01291[9]	0.01442[12]	0.01329[11]	0.01982[15]	0.0127[8]	0.01544[14]	0.0131[10]	0.01535[13]
	$\sum Ranks$	8[1]	24[4]	23[3]	29.5[5]	33.5[6]	36[7]	14[2]	54[9]	72[12]	66[11]	90[15]	48[8]	79[13]	60[10]	83[14]
250	BIAS	0.07792[2]	0.0761[1]	0.08383[7]	0.0818[5]	0.08304[6]	0.08142[4]	0.08073[3]	0.09127[9]	0.131[12]	0.113[11]	0.21021[15]	0.08666[8]	0.14184[14]	0.10718[10]	0.13288[13]
	MSE	0.00949[2]	0.0091[1]	0.01089[7]	0.01046[5]	0.01072[6]	0.01032[4]	0.01031[3]	0.01327[9]	0.02643[12]	0.02031[11]	0.13193[15]	0.01177[8]	0.0318[14]	0.01827[10]	0.02773[13]
	MRE	0.02226[2]	0.02174[1]	0.02395[7]	0.02337[5]	0.02372[6]	0.02326[4]	0.02307[3]	0.02608[9]	0.03743[12]	0.03228[11]	0.06006[15]	0.02476[8]	0.04053[14]	0.03062[10]	0.03797[13]
	D_{abs}	0.00799[2]	0.00785[1]	0.00863[7]	0.00849[5]	0.00856[6]	0.0084[4]	0.0083[3]	0.00937[9]	0.01357[12]	0.01172[11]	0.02007[15]	0.00892[8]	0.01468[14]	0.01112[10]	0.0138[13]
	D_{max}	0.01175[2]	0.0115[1]	0.01265[7]	0.01244[5]	0.01255[6]	0.01232[4]	0.01217[3]	0.01376[9]	0.01989[12]	0.01719[11]	0.02957[15]	0.01307[8]	0.02155[14]	0.01629[10]	0.02028[13]
	ASAE	0.00929[3]	0.00944[4]	0.00974[6]	0.00922[2]	0.00977[7]	0.00909[1]	0.00952[5]	0.01051[10]	0.0116[12]	0.01088[11]	0.01616[15]	0.01035[8]	0.01244[14]	0.01037[9]	0.01235[13]
	$\sum Ranks$	13[2]	9[1]	41[7]	27[5]	37[6]	21[4]	20[3]	55[9]	72[12]	66[11]	90[15]	48[8]	84[14]	59[10]	78[13]
350	BIAS	0.06403[2]	0.06349[1]	0.06748[5]	0.07064[7]	0.0695[6]	0.06679[4]	0.06447[3]	0.07703[9]	0.10916[12]	0.09598[11]	0.17083[15]	0.07603[8]	0.12191[14]	0.09423[10]	0.12131[13]
	MSE	0.00644[1]	0.00646[2]	0.00719[5]	0.00788[7]	0.00754[6]	0.0069[4]	0.0066[3]	0.00936[9]	0.01879[12]	0.01453[11]	0.09188[15]	0.009[8]	0.02322[14]	0.01385[10]	0.02288[13]
	MRE	0.01829[2]	0.01814[1]	0.01928[5]	0.02018[7]	0.01986[6]	0.01908[4]	0.01842[3]	0.02201[9]	0.03119[12]	0.02742[11]	0.04881[15]	0.02172[8]	0.03483[14]	0.02692[10]	0.03466[13]
	D_{abs}	0.00659[2]	0.00653[1]	0.00694[5]	0.00731[7]	0.00716[6]	0.00688[4]	0.00664[3]	0.00792[9]	0.01132[12]	0.0099[11]	0.01651[15]	0.00783[8]	0.01267[14]	0.00977[10]	0.01259[13]
	D_{max}	0.00967[2]	0.00958[1]	0.01019[5]	0.01074[7]	0.0105[6]	0.01008[4]	0.00974[3]	0.0116[9]	0.01659[12]	0.01453[11]	0.02429[15]	0.01147[8]	0.01856[14]	0.01432[10]	0.01846[13]
	ASAE	0.00754[3]	0.00757[4]	0.00795[7]	0.00742[1]	0.00778[6]	0.00743[2]	0.00765[5]	0.0084[9]	0.00953[12]	0.00878[11]	0.01289[15]	0.00819[8]	0.0102[13]	0.0085[10]	0.01044[14]
	$\sum Ranks$	12[2]	10[1]	32[5]	36[6.5]	36[6.5]	22[4]	20[3]	54[9]	72[12]	66[11]	90[15]	48[8]	83[14]	60[10]	79[13]

Table 3. Numerical values of simulation measures for $\delta = 2.5$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	$0.35856^{\{2\}}$	$0.38959^{\{8\}}$	$0.38526^{\{7\}}$	$0.34093^{\{1\}}$	$0.4016^{\{9\}}$	$0.36599^{\{3\}}$	$0.36924^{\{5\}}$	$0.43152^{\{11\}}$	$0.42411^{\{10\}}$	$0.38029^{\{6\}}$	$0.58452^{\{15\}}$	$0.43274^{\{12\}}$	$0.46587^{\{14\}}$	$0.36673^{\{4\}}$	$0.44064^{\{13\}}$
	MSE	$0.22919^{\{3\}}$	$0.26808^{\{9\}}$	$0.25161^{\{7\}}$	$0.18869^{\{1\}}$	$0.26694^{\{8\}}$	$0.23617^{\{5\}}$	$0.23456^{\{4\}}$	$0.3335^{\{13\}}$	$0.31762^{\{11\}}$	$0.24823^{\{6\}}$	$0.60276^{\{15\}}$	$0.31434^{\{10\}}$	$0.36887^{\{14\}}$	$0.22041^{\{2\}}$	$0.31943^{\{12\}}$
	MRE	$0.14343^{\{2\}}$	$0.15584^{\{8\}}$	$0.1541^{\{7\}}$	$0.13637^{\{1\}}$	$0.16064^{\{9\}}$	$0.1464^{\{3\}}$	$0.1477^{\{5\}}$	$0.17261^{\{11\}}$	$0.16964^{\{10\}}$	$0.15212^{\{6\}}$	$0.23381^{\{15\}}$	$0.17309^{\{12\}}$	$0.18635^{\{14\}}$	$0.14669^{\{4\}}$	$0.17626^{\{13\}}$
	D_{abs}	$0.05301^{\{1\}}$	$0.0592^{\{8\}}$	$0.0589^{\{6\}}$	$0.05449^{\{2\}}$	$0.06209^{\{9\}}$	$0.05646^{\{3\}}$	$0.05652^{\{4\}}$	$0.06248^{\{10\}}$	$0.06416^{\{11\}}$	$0.05912^{\{7\}}$	$0.08085^{\{15\}}$	$0.06435^{\{12\}}$	$0.07257^{\{14\}}$	$0.0572^{\{5\}}$	$0.0693^{\{13\}}$
	D_{max}	$0.07638^{\{1\}}$	$0.08507^{\{8\}}$	$0.08426^{\{6\}}$	$0.07901^{\{2\}}$	$0.08818^{\{9\}}$	$0.07992^{\{3\}}$	$0.0816^{\{4\}}$	$0.09054^{\{10\}}$	$0.09268^{\{11\}}$	$0.08498^{\{7\}}$	$0.11744^{\{15\}}$	$0.09273^{\{12\}}$	$0.10576^{\{14\}}$	$0.082^{\{5\}}$	$0.1009^{\{13\}}$
	ASAE	$0.06109^{\{3\}}$	$0.06408^{\{7\}}$	$0.06269^{\{4\}}$	$0.06061^{\{2\}}$	$0.06329^{\{5\}}$	$0.05964^{\{1\}}$	$0.06355^{\{6\}}$	$0.06977^{\{11\}}$	$0.07305^{\{12\}}$	$0.06767^{\{10\}}$	$0.09275^{\{15\}}$	$0.06632^{\{9\}}$	$0.07743^{\{13\}}$	$0.06538^{\{8\}}$	$0.08183^{\{14\}}$
	$\sum Ranks$	$12^{\{2\}}$	$48^{\{8\}}$	$37^{\{6\}}$	$9^{\{1\}}$	$49^{\{9\}}$	$18^{\{3\}}$	$28^{\{4.5\}}$	$66^{\{11\}}$	$65^{\{10\}}$	$42^{\{7\}}$	$90^{\{15\}}$	$67^{\{12\}}$	$83^{\{14\}}$	$28^{\{4.5\}}$	$78^{\{13\}}$
45	BIAS	$0.20602^{\{3\}}$	$0.21286^{\{5\}}$	$0.21825^{\{6\}}$	$0.20321^{\{2\}}$	$0.22416^{\{8\}}$	$0.21243^{\{4\}}$	$0.19626^{\{1\}}$	$0.24122^{\{11\}}$	$0.24903^{\{12\}}$	$0.23668^{\{10\}}$	$0.40154^{\{15\}}$	$0.22346^{\{7\}}$	$0.29086^{\{14\}}$	$0.22649^{\{9\}}$	$0.26729^{\{13\}}$
	MSE	$0.07023^{\{3\}}$	$0.07339^{\{5\}}$	$0.08031^{\{7\}}$	$0.06216^{\{2\}}$	$0.08382^{\{8.5\}}$	$0.07175^{\{4\}}$	$0.06177^{\{1\}}$	$0.09749^{\{11\}}$	$0.10556^{\{12\}}$	$0.08481^{\{10\}}$	$0.34312^{\{15\}}$	$0.08002^{\{6\}}$	$0.13879^{\{14\}}$	$0.08382^{\{8.5\}}$	$0.11938^{\{13\}}$
	MRE	$0.08241^{\{3\}}$	$0.08514^{\{5\}}$	$0.0873^{\{6\}}$	$0.08128^{\{2\}}$	$0.08967^{\{8\}}$	$0.08497^{\{4\}}$	$0.0785^{\{1\}}$	$0.09649^{\{11\}}$	$0.09961^{\{12\}}$	$0.09467^{\{10\}}$	$0.16062^{\{15\}}$	$0.08939^{\{7\}}$	$0.11634^{\{14\}}$	$0.0906^{\{9\}}$	$0.10692^{\{13\}}$
	D_{abs}	$0.03133^{\{2\}}$	$0.03284^{\{5\}}$	$0.03309^{\{6\}}$	$0.03185^{\{3\}}$	$0.03424^{\{8\}}$	$0.03257^{\{4\}}$	$0.03046^{\{1\}}$	$0.03638^{\{10\}}$	$0.03811^{\{12\}}$	$0.03738^{\{11\}}$	$0.05463^{\{15\}}$	$0.03376^{\{7\}}$	$0.0452^{\{14\}}$	$0.0353^{\{9\}}$	$0.04158^{\{13\}}$
	D_{max}	$0.04573^{\{2\}}$	$0.04759^{\{4\}}$	$0.04846^{\{6\}}$	$0.04644^{\{3\}}$	$0.05007^{\{8\}}$	$0.04782^{\{5\}}$	$0.04434^{\{1\}}$	$0.05307^{\{10\}}$	$0.05554^{\{12\}}$	$0.05423^{\{11\}}$	$0.08094^{\{15\}}$	$0.04949^{\{7\}}$	$0.06589^{\{14\}}$	$0.05147^{\{9\}}$	$0.0609^{\{13\}}$
	ASAE	$0.02988^{\{4\}}$	$0.03027^{\{6\}}$	$0.03098^{\{7\}}$	$0.02916^{\{2\}}$	$0.02982^{\{3\}}$	$0.0285^{\{1\}}$	$0.03005^{\{5\}}$	$0.03431^{\{11\}}$	$0.03746^{\{12\}}$	$0.03424^{\{10\}}$	$0.04938^{\{15\}}$	$0.03308^{\{9\}}$	$0.04031^{\{14\}}$	$0.03292^{\{8\}}$	$0.03945^{\{13\}}$
	$\sum Ranks$	$17^{\{3\}}$	$30^{\{5\}}$	$38^{\{6\}}$	$14^{\{2\}}$	$43.5^{\{8\}}$	$22^{\{4\}}$	$10^{\{1\}}$	$64^{\{11\}}$	$72^{\{12\}}$	$62^{\{10\}}$	$90^{\{15\}}$	$43^{\{7\}}$	$84^{\{14\}}$	$52.5^{\{9\}}$	$78^{\{13\}}$
90	BIAS	$0.14975^{\{3\}}$	$0.14693^{\{2\}}$	$0.16047^{\{6\}}$	$0.1408^{\{1\}}$	$0.1603^{\{5\}}$	$0.15484^{\{4\}}$	$0.16217^{\{7\}}$	$0.17707^{\{11\}}$	$0.18422^{\{12\}}$	$0.16454^{\{8\}}$	$0.27934^{\{15\}}$	$0.16848^{\{10\}}$	$0.20084^{\{13\}}$	$0.16519^{\{9\}}$	$0.20396^{\{14\}}$
	MSE	$0.03564^{\{3\}}$	$0.03356^{\{2\}}$	$0.04186^{\{7\}}$	$0.03153^{\{1\}}$	$0.04031^{\{5\}}$	$0.03737^{\{4\}}$	$0.04119^{\{6\}}$	$0.0526^{\{11\}}$	$0.05495^{\{12\}}$	$0.04274^{\{9\}}$	$0.14572^{\{15\}}$	$0.04598^{\{10\}}$	$0.06083^{\{13\}}$	$0.04242^{\{8\}}$	$0.06764^{\{14\}}$
	MRE	$0.0599^{\{3\}}$	$0.05877^{\{2\}}$	$0.06419^{\{6\}}$	$0.05632^{\{1\}}$	$0.06412^{\{5\}}$	$0.06194^{\{4\}}$	$0.06487^{\{7\}}$	$0.07083^{\{11\}}$	$0.07369^{\{12\}}$	$0.06581^{\{8\}}$	$0.11173^{\{15\}}$	$0.06739^{\{10\}}$	$0.08033^{\{13\}}$	$0.06608^{\{9\}}$	$0.08159^{\{14\}}$
	D_{abs}	$0.02312^{\{3\}}$	$0.02275^{\{2\}}$	$0.02443^{\{5\}}$	$0.02192^{\{1\}}$	$0.02463^{\{6\}}$	$0.02377^{\{4\}}$	$0.02489^{\{7\}}$	$0.02683^{\{11\}}$	$0.0285^{\{12\}}$	$0.02575^{\{9\}}$	$0.04069^{\{15\}}$	$0.02571^{\{8\}}$	$0.03134^{\{13\}}$	$0.0258^{\{10\}}$	$0.03154^{\{14\}}$
	D_{max}	$0.0336^{\{3\}}$	$0.03314^{\{2\}}$	$0.03591^{\{5\}}$	$0.03203^{\{1\}}$	$0.03606^{\{6\}}$	$0.03467^{\{4\}}$	$0.03642^{\{7\}}$	$0.03938^{\{11\}}$	$0.04162^{\{12\}}$	$0.03752^{\{8\}}$	$0.05957^{\{15\}}$	$0.03766^{\{10\}}$	$0.04581^{\{13\}}$	$0.03759^{\{9\}}$	$0.04633^{\{14\}}$
	ASAE	$0.01875^{\{3\}}$	$0.01954^{\{5\}}$	$0.02019^{\{7\}}$	$0.01856^{\{2\}}$	$0.01991^{\{6\}}$	$0.01841^{\{1\}}$	$0.01937^{\{4\}}$	$0.02183^{\{10\}}$	$0.02335^{\{12\}}$	$0.02222^{\{11\}}$	$0.03189^{\{15\}}$	$0.02077^{\{8\}}$	$0.02533^{\{14\}}$	$0.02113^{\{9\}}$	$0.02506^{\{13\}}$
	$\sum Ranks$	$18^{\{3\}}$	$15^{\{2\}}$	$36^{\{6\}}$	$7^{\{1\}}$	$33^{\{5\}}$	$21^{\{4\}}$	$38^{\{7\}}$	$65^{\{11\}}$	$72^{\{12\}}$	$53^{\{8\}}$	$90^{\{15\}}$	$56^{\{10\}}$	$79^{\{13\}}$	$54^{\{9\}}$	$83^{\{14\}}$
180	BIAS	$0.10121^{\{3\}}$	$0.10277^{\{4\}}$	$0.11243^{\{8\}}$	$0.09569^{\{1\}}$	$0.10921^{\{7\}}$	$0.09933^{\{2\}}$	$0.10303^{\{5\}}$	$0.11918^{\{11\}}$	$0.13467^{\{12\}}$	$0.11404^{\{9\}}$	$0.20168^{\{15\}}$	$0.11709^{\{10\}}$	$0.14328^{\{14\}}$	$0.10559^{\{6\}}$	$0.13686^{\{13\}}$
	MSE	$0.01563^{\{3\}}$	$0.01677^{\{4\}}$	$0.01944^{\{8\}}$	$0.01485^{\{1\}}$	$0.019^{\{7\}}$	$0.01503^{\{2\}}$	$0.01715^{\{5\}}$	$0.0233^{\{11\}}$	$0.02736^{\{12\}}$	$0.02155^{\{9\}}$	$0.07459^{\{15\}}$	$0.02215^{\{10\}}$	$0.03203^{\{13\}}$	$0.01804^{\{6\}}$	$0.03205^{\{14\}}$
	MRE	$0.04048^{\{3\}}$	$0.04111^{\{4\}}$	$0.04497^{\{8\}}$	$0.03827^{\{1\}}$	$0.04368^{\{7\}}$	$0.03973^{\{2\}}$	$0.04121^{\{5\}}$	$0.04767^{\{11\}}$	$0.05387^{\{12\}}$	$0.04562^{\{9\}}$	$0.08067^{\{15\}}$	$0.04684^{\{10\}}$	$0.05731^{\{14\}}$	$0.04224^{\{6\}}$	$0.05474^{\{13\}}$
	D_{abs}	$0.01552^{\{3\}}$	$0.01585^{\{4\}}$	$0.01734^{\{8\}}$	$0.01485^{\{1\}}$	$0.01691^{\{7\}}$	$0.01535^{\{2\}}$	$0.01593^{\{5\}}$	$0.0183^{\{11\}}$	$0.02096^{\{12\}}$	$0.01775^{\{9\}}$	$0.0295^{\{15\}}$	$0.01797^{\{10\}}$	$0.02221^{\{14\}}$	$0.01642^{\{6\}}$	$0.02115^{\{13\}}$
	D_{max}	$0.02271^{\{3\}}$	$0.02319^{\{4\}}$	$0.02531^{\{8\}}$	$0.02173^{\{1\}}$	$0.02474^{\{7\}}$	$0.02244^{\{2\}}$	$0.0233^{\{5\}}$	$0.02676^{\{11\}}$	$0.03051^{\{12\}}$	$0.02601^{\{9\}}$	$0.04339^{\{15\}}$	$0.02633^{\{10\}}$	$0.03257^{\{14\}}$	$0.02391^{\{6\}}$	$0.03098^{\{13\}}$
	ASAE	$0.0123^{\{3\}}$	$0.0126^{\{5\}}$	$0.01282^{\{7\}}$	$0.01221^{\{2\}}$	$0.01263^{\{6\}}$	$0.01208^{\{1\}}$	$0.0125^{\{4\}}$	$0.01396^{\{10\}}$	$0.0151^{\{12\}}$	$0.01416^{\{11\}}$	$0.02183^{\{15\}}$	$0.01358^{\{8\}}$	$0.01658^{\{13\}}$	$0.01367^{\{9\}}$	$0.01669^{\{14\}}$
	$\sum Ranks$	$18^{\{3\}}$	$25^{\{4\}}$	$47^{\{8\}}$	$7^{\{1\}}$	$41^{\{7\}}$	$11^{\{2\}}$	$29^{\{5\}}$	$65^{\{11\}}$	$72^{\{12\}}$	$56^{\{9\}}$	$90^{\{15\}}$	$58^{\{10\}}$	$82^{\{14\}}$	$39^{\{6\}}$	$80^{\{13\}}$
250	BIAS	$0.0838^{\{1\}}$	$0.09117^{\{6\}}$	$0.09432^{\{7\}}$	$0.08597^{\{3\}}$	$0.10021^{\{10\}}$	$0.08493^{\{2\}}$	$0.09053^{\{5\}}$	$0.10289^{\{11\}}$	$0.10664^{\{12\}}$	$0.09804^{\{9\}}$	$0.17717^{\{15\}}$	$0.09713^{\{8\}}$	$0.12251^{\{13\}}$	$0.08803^{\{4\}}$	$0.12858^{\{14\}}$
	MSE	$0.0112^{\{1\}}$	$0.01313^{\{5\}}$	$0.01439^{\{7\}}$	$0.01129^{\{3\}}$	$0.01594^{\{10\}}$	$0.01124^{\{2\}}$	$0.01296^{\{4\}}$	$0.01661^{\{11\}}$	$0.01804^{\{12\}}$	$0.01532^{\{9\}}$	$0.06225^{\{15\}}$	$0.01463^{\{8\}}$	$0.02345^{\{13\}}$	$0.01315^{\{6\}}$	$0.02568^{\{14\}}$
	MRE	$0.03352^{\{1\}}$	$0.03647^{\{6\}}$	$0.03773^{\{7\}}$	$0.03439^{\{3\}}$	$0.04008^{\{10\}}$	$0.03397^{\{2\}}$	$0.03621^{\{5\}}$	$0.04116^{\{11\}}$	$0.04266^{\{12\}}$	$0.03921^{\{9\}}$	$0.07087^{\{15\}}$	$0.03885^{\{8\}}$	$0.04901^{\{13\}}$	$0.03521^{\{4\}}$	$0.05143^{\{14\}}$
	D_{abs}	$0.01294^{\{1\}}$	$0.01414^{\{6\}}$	$0.01441^{\{7\}}$	$0.01333^{\{3\}}$	$0.01542^{\{10\}}$	$0.01309^{\{2\}}$	$0.01398^{\{5\}}$	$0.01568^{\{11\}}$	$0.01656^{\{12\}}$	$0.01519^{\{9\}}$	$0.026^{\{15\}}$	$0.01498^{\{8\}}$	$0.01905^{\{13\}}$	$0.01375^{\{4\}}$	$0.0201^{\{14\}}$
	D_{max}	$0.0189^{\{1\}}$	$0.02069^{\{6\}}$	$0.02113^{\{7\}}$	$0.01946^{\{3\}}$	$0.0226^{\{10\}}$	$0.01914^{\{2\}}$	$0.02047^{\{5\}}$	$0.02306^{\{11\}}$	$0.02421^{\{12\}}$	$0.02224^{\{9\}}$	$0.03825^{\{15\}}$	$0.0219^{\{8\}}$	$0.02785^{\{13\}}$	$0.02009^{\{4\}}$	$0.02946^{\{14\}}$
	ASAE	$0.00984^{\{2\}}$	$0.01016^{\{5\}}$	$0.01052^{\{6\}}$	$0.00976^{\{1\}}$	$0.01065^{\{7\}}$	$0.01^{\{3\}}$	$0.01007^{\{4\}}$	$0.01147^{\{11\}}$	$0.01247^{\{12\}}$	$0.01141^{\{10\}}$	$0.0178^{\{15\}}$	$0.01072^{\{8\}}$	$0.01322^{\{13\}}$	$0.01124^{\{9\}}$	$0.01375^{\{14\}}$
	$\sum Ranks$	$7^{\{1\}}$	$34^{\{6\}}$	$41^{\{7\}}$	$16^{\{3\}}$	$57^{\{10\}}$	$13^{\{2\}}$	$28^{\{4\}}$	$66^{\{11\}}$	$72^{\{12\}}$	$55^{\{9\}}$	$90^{\{15\}}$	$48^{\{8\}}$	$78^{\{13\}}$	$31^{\{5\}}$	$84^{\{14\}}$
350	BIAS	$0.06881^{\{1\}}$	$0.07599^{\{5\}}$	$0.07963^{\{8\}}$	$0.07105^{\{3\}}$	$0.07691^{\{6\}}$	$0.06914^{\{2\}}$	$0.07498^{\{4\}}$	$0.08954^{\{11\}}$	$0.09144^{\{12\}}$	$0.08319^{\{10\}}$	$0.14235^{\{15\}}$	$0.0783^{\{7\}}$	$0.10696^{\{14\}}$	$0.08091^{\{9\}}$	$0.09609^{\{13\}}$
	MSE	$0.00754^{\{2\}}$	$0.00929^{\{5\}}$	$0.01013^{\{8\}}$	$0.00769^{\{3\}}$	$0.0095^{\{6\}}$	$0.0074^{\{1\}}$	$0.00902^{\{4\}}$	$0.01238^{\{11\}}$	$0.01339^{\{12\}}$	$0.01103^{\{10\}}$	$0.05011^{\{15\}}$	$0.00976^{\{7\}}$	$0.01808^{\{14\}}$	$0.01062^{\{9\}}$	$0.01432^{\{13\}}$
	MRE	$0.02753^{\{1\}}$	$0.0304^{\{5\}}$	$0.03185^{\{8\}}$	$0.02842^{\{3\}}$	$0.03076^{\{6\}}$	$0.02766^{\{2\}}$	$0.02999^{\{4\}}$	$0.03581^{\{11\}}$	$0.03658^{\{12\}}$	$0.03328^{\{10\}}$	$0.05694^{\{15\}}$	$0.03132^{\{7\}}$	$0.04278^{\{14\}}$	$0.03236^{\{9\}}$	$0.03844^{\{13\}}$
	D_{abs}	$0.01062^{\{1\}}$	$0.01172^{\{5\}}$	$0.01228^{\{8\}}$	$0.01105^{\{3\}}$	$0.01195^{\{6\}}$	$0.01073^{\{2\}}$	$0.01157^{\{4\}}$	$0.01372^{\{11\}}$	$0.01416^{\{12\}}$	$0.01296^{\{10\}}$	$0.02089^{\{15\}}$	$0.01206^{\{7\}}$	$0.01665^{\{14\}}$	$0.01259^{\{9\}}$	$0.01507^{\{13\}}$
	D_{max}	$0.01553^{\{1\}}$	$0.01715^{\{5\}}$	$0.01796^{\{8\}}$	$0.01617^{\{3\}}$	$0.01745^{\{6\}}$	$0.01568^{\{2\}}$	$0.01693^{\{4\}}$	$0.02012^{\{11\}}$	$0.0207^{\{12\}}$	$0.01892^{\{10\}}$	$0.03072^{\{15\}}$	$0.01766^{\{7\}}$	$0.02434^{\{14\}}$	$0.0184^{\{9\}}$	$0.02199^{\{13\}}$
	ASAE	$0.00797^{\{1.5\}}$	$0.00841^{\{6\}}$	$0.00847^{\{7\}}$	$0.00799^{\{3\}}$	$0.00832^{\{5\}}$	$0.00797^{\{1.5\}}$	$0.00815^{\{4\}}$	$0.00923^{\{9\}}$	$0.00977^{\{12\}}$	$0.00936^{\{11\}}$	$0.01392^{\{15\}}$	$0.00863^{\{8\}}$	$0.0112^{\{14\}}$	$0.00924^{\{10\}}$	$0.01076^{\{13\}}$
	$\sum Ranks$	$7.5^{\{1\}}$	$31^{\{5\}}$	$47^{\{8\}}$	$18^{\{3\}}$	$35^{\{6\}}$	$10.5^{\{2\}}$	$24^{\{4\}}$	$64^{\{11\}}$	$72^{\{12\}}$	$61^{\{10\}}$	$90^{\{15\}}$	$43^{\{7\}}$	$84^{\{14\}}$	$55^{\{9\}}$	$78^{\{13\}}$

TABLE 4. Numerical values of simulation measures for $\delta = 2.5$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.21747[2]	0.231[5]	0.23728[6]	0.24367[7]	0.22269[3]	0.21206[1]	0.22912[4]	0.25894[9]	0.30577[12]	0.29714[11]	0.49247[15]	0.24723[8]	0.37452[14]	0.27803[10]	0.3639[13]
	MSE	0.07457[2]	0.0881[5]	0.09064[6]	0.09093[7]	0.07554[3]	0.06858[1]	0.07809[4]	0.11247[9]	0.1581[12]	0.13439[11]	0.48503[15]	0.09859[8]	0.23573[14]	0.12137[10]	0.20379[13]
	MRE	0.08699[2]	0.0924[5]	0.09491[6]	0.09747[7]	0.08908[3]	0.08483[1]	0.09165[4]	0.10358[9]	0.12231[12]	0.11886[11]	0.19699[15]	0.09889[8]	0.14981[14]	0.11121[10]	0.14556[13]
	D_{abs}	0.03302[1]	0.03536[4]	0.03638[6]	0.0387[8]	0.03528[3]	0.03369[2]	0.03554[5]	0.03885[9]	0.0469[11]	0.0483[12]	0.0654[15]	0.03747[7]	0.05899[13]	0.04389[10]	0.05922[14]
	D_{max}	0.04788[1]	0.05151[5]	0.05284[6]	0.05689[9]	0.05043[3]	0.04827[2]	0.0514[4]	0.05647[8]	0.06865[11]	0.07071[12]	0.09598[15]	0.05463[7]	0.08639[14]	0.06403[10]	0.08588[13]
	ASAE	0.05733[4]	0.05807[5]	0.05911[6]	0.0551[2]	0.05991[7]	0.05502[1]	0.05703[3]	0.0649[10]	0.06683[12]	0.06494[11]	0.08694[15]	0.06361[9]	0.07717[14]	0.0626[8]	0.07555[13]
	$\sum$ *Ranks*	12[2]	29[5]	36[6]	40[7]	22[3]	8[1]	24[4]	54[9]	70[12]	68[11]	90[15]	47[8]	83[14]	58[10]	79[13]
45	BIAS	0.12456[3]	0.12006[2]	0.12743[4]	0.13242[8]	0.1311[7]	0.1301[6]	0.11943[1]	0.15175[9]	0.19712[12]	0.17812[11]	0.3298[15]	0.12894[5]	0.22215[14]	0.16453[10]	0.20845[13]
	MSE	0.02505[3]	0.02297[1]	0.0265[5]	0.02789[8]	0.02786[7]	0.02744[6]	0.0231[2]	0.03814[9]	0.06332[12]	0.04914[11]	0.25425[15]	0.02626[4]	0.07719[14]	0.04136[10]	0.06612[13]
	MRE	0.04982[3]	0.04802[2]	0.05097[4]	0.05297[8]	0.05244[7]	0.05204[6]	0.04777[1]	0.0607[9]	0.07885[12]	0.07125[11]	0.13192[15]	0.05158[5]	0.08886[14]	0.06581[10]	0.08338[13]
	D_{abs}	0.01919[3]	0.01862[2]	0.01951[4]	0.02098[8]	0.02003[7]	0.02001[6]	0.01851[1]	0.02304[9]	0.03067[12]	0.02813[11]	0.04496[15]	0.01967[5]	0.03482[14]	0.02576[10]	0.03281[13]
	D_{max}	0.02795[3]	0.02715[2]	0.02858[4]	0.03065[8]	0.02944[7]	0.02942[6]	0.02707[1]	0.03364[9]	0.04503[12]	0.04114[11]	0.0657[15]	0.02887[5]	0.05102[14]	0.03783[10]	0.04811[13]
	ASAE	0.02856[6]	0.02816[2]	0.02894[7]	0.02823[3]	0.0285[5]	0.02767[1]	0.02843[4]	0.0316[10]	0.03452[12]	0.03225[11]	0.04752[15]	0.03124[8]	0.03749[14]	0.0313[9]	0.03728[13]
	$\sum$ *Ranks*	21[3]	11[2]	28[4]	43[8]	40[7]	31[5]	10[1]	55[9]	72[12]	66[11]	90[15]	32[6]	84[14]	59[10]	78[13]
90	BIAS	0.08558[2]	0.08147[1]	0.09698[7]	0.09609[6]	0.0882[3]	0.09248[5]	0.08943[4]	0.0992[8]	0.13963[12]	0.11672[10]	0.22494[15]	0.1039[9]	0.15057[13]	0.11791[11]	0.16055[14]
	MSE	0.01119[2]	0.011[1]	0.01443[6]	0.01453[7]	0.01282[4]	0.01309[5]	0.01261[3]	0.01558[8]	0.03107[12]	0.02287[11]	0.11994[15]	0.01664[9]	0.03804[13]	0.02127[10]	0.04188[14]
	MRE	0.03423[2]	0.03259[1]	0.03879[7]	0.03844[6]	0.03528[3]	0.03699[5]	0.03577[4]	0.03968[8]	0.05585[12]	0.04669[10]	0.08998[15]	0.04156[9]	0.06023[13]	0.04717[11]	0.06422[14]
	D_{abs}	0.01318[2]	0.01262[1]	0.01485[6]	0.0151[7]	0.01359[3]	0.01439[5]	0.01373[4]	0.01526[8]	0.02177[12]	0.01816[10]	0.03176[15]	0.01601[9]	0.02339[13]	0.01838[11]	0.02486[14]
	D_{max}	0.01928[2]	0.01841[1]	0.02186[6]	0.02211[7]	0.01987[3]	0.02098[5]	0.02012[4]	0.02236[8]	0.03175[12]	0.02659[10]	0.04648[15]	0.02341[9]	0.03421[13]	0.0269[11]	0.03655[14]
	ASAE	0.01776[3]	0.01771[2]	0.01848[6]	0.01784[4]	0.01888[7]	0.01756[1]	0.01829[5]	0.01978[9]	0.02281[12]	0.02072[11]	0.0314[15]	0.0198[10]	0.02368[13]	0.01969[8]	0.02431[14]
	$\sum$ *Ranks*	13[2]	7[1]	38[7]	37[6]	23[3]	26[5]	24[4]	49[8]	72[12]	62[10.5]	90[15]	55[9]	78[13]	62[10.5]	84[14]
180	BIAS	0.06547[6]	0.06236[4]	0.06141[2]	0.06264[5]	0.06157[3]	0.06574[7]	0.06091[1]	0.06929[8]	0.10118[12]	0.0898[11]	0.16312[15]	0.07275[9]	0.11249[13]	0.08517[10]	0.11357[14]
	MSE	0.00658[7]	0.00609[5]	0.00574[1]	0.00604[4]	0.00593[3]	0.00655[6]	0.0059[2]	0.00779[8]	0.01519[12]	0.01269[11]	0.0688[15]	0.00846[9]	0.02028[14]	0.01167[10]	0.01999[13]
	MRE	0.02619[6]	0.02495[4]	0.02457[2]	0.02506[5]	0.02463[3]	0.0263[7]	0.02436[1]	0.02772[8]	0.04047[12]	0.03592[11]	0.06525[15]	0.0291[9]	0.04499[13]	0.03407[10]	0.04543[14]
	D_{abs}	0.01011[6]	0.00965[4]	0.00948[2]	0.00982[5]	0.00951[3]	0.01019[7]	0.0094[1]	0.01067[8]	0.01579[12]	0.01392[11]	0.02344[15]	0.01124[9]	0.01746[13]	0.01327[10]	0.01759[14]
	D_{max}	0.01479[6]	0.01412[4]	0.01386[2]	0.01431[5]	0.01394[3]	0.01491[7]	0.01375[1]	0.01559[8]	0.02305[12]	0.0204[11]	0.03443[15]	0.01644[9]	0.02561[13]	0.0194[10]	0.02578[14]
	ASAE	0.01158[3]	0.01178[6]	0.01201[7]	0.01161[4]	0.01163[5]	0.01134[1]	0.01145[2]	0.01272[9]	0.01492[12]	0.01355[11]	0.02007[15]	0.01261[8]	0.01592[14]	0.0132[10]	0.01546[13]
	$\sum$ *Ranks*	34[6]	27[4]	16[2]	28[5]	20[3]	35[7]	8[1]	49[8]	72[12]	66[11]	90[15]	53[9]	80[13]	60[10]	82[14]
250	BIAS	0.05292[3]	0.05169[1]	0.05647[6]	0.05812[7]	0.05186[2]	0.05318[4]	0.05632[5]	0.06111[9]	0.08348[11]	0.0843[12]	0.14257[15]	0.05905[8]	0.09915[14]	0.07316[10]	0.09264[13]
	MSE	0.00433[3]	0.00412[1]	0.00473[5]	0.00514[7]	0.00423[2]	0.00458[4]	0.00493[6]	0.00599[9]	0.01118[12]	0.01099[11]	0.05723[15]	0.00578[8]	0.01488[14]	0.00849[10]	0.01423[13]
	MRE	0.02117[3]	0.02067[1]	0.02259[6]	0.02325[7]	0.02074[2]	0.02127[4]	0.02253[5]	0.02444[9]	0.03339[11]	0.03372[12]	0.05703[15]	0.02362[8]	0.03966[14]	0.02926[10]	0.03705[13]
	D_{abs}	0.00816[3]	0.00799[1]	0.00872[6]	0.00905[7]	0.008[2]	0.00823[4]	0.00868[5]	0.00943[9]	0.01291[11]	0.01315[12]	0.02064[15]	0.00912[8]	0.01545[14]	0.01144[10]	0.01448[13]
	D_{max}	0.01195[3]	0.01169[1]	0.01276[6]	0.01322[7]	0.01171[2]	0.01203[4]	0.01271[5]	0.01382[9]	0.0189[11]	0.01924[12]	0.03026[15]	0.0133[8]	0.0226[14]	0.0167[10]	0.0212[13]
	ASAE	0.00939[2]	0.0094[3]	0.00965[5]	0.00942[4]	0.0098[7]	0.00896[1]	0.00971[6]	0.01034[9]	0.0122[12]	0.01103[11]	0.0167[15]	0.01018[8]	0.01295[14]	0.01066[10]	0.01284[13]
	$\sum$ *Ranks*	17[2.5]	8[1]	34[6]	39[7]	17[2.5]	21[4]	32[5]	54[9]	68[11]	70[12]	90[15]	48[8]	84[14]	60[10]	78[13]
350	BIAS	0.04169[1]	0.04361[3]	0.04562[4]	0.04883[7]	0.04651[5]	0.04294[2]	0.04733[6]	0.05249[9]	0.07358[12]	0.0622[10]	0.11284[15]	0.04943[8]	0.07888[13]	0.06242[11]	0.07994[14]
	MSE	0.00285[1]	0.00299[3]	0.0033[4]	0.00359[7]	0.00336[5]	0.00291[2]	0.0035[6]	0.00436[9]	0.00875[12]	0.00604[11]	0.0367[15]	0.00379[8]	0.00992[13]	0.00585[10]	0.01004[14]
	MRE	0.01668[1]	0.01744[3]	0.01825[4]	0.01953[7]	0.0186[5]	0.01717[2]	0.01893[6]	0.021[9]	0.02943[12]	0.02488[10]	0.04514[15]	0.01977[8]	0.03155[13]	0.02497[11]	0.03197[14]
	D_{abs}	0.00643[1]	0.00672[3]	0.00707[4]	0.0076[7]	0.00719[5]	0.00666[2]	0.00731[6]	0.00812[9]	0.01143[12]	0.00966[10]	0.01651[15]	0.00763[8]	0.01233[13]	0.00967[11]	0.01248[14]
	D_{max}	0.0094[1]	0.00985[3]	0.01034[4]	0.01112[7]	0.01052[5]	0.00972[2]	0.01071[6]	0.01188[9]	0.01671[12]	0.0141[10]	0.02423[15]	0.01118[8]	0.01804[13]	0.01415[11]	0.01822[14]
	ASAE	0.00748[1]	0.00764[4]	0.00806[6]	0.00774[5]	0.00807[7]	0.00752[2]	0.00757[3]	0.00856[10]	0.00963[12]	0.00869[11]	0.01318[15]	0.00836[8]	0.01024[13]	0.00848[9]	0.01035[14]
	$\sum$ *Ranks*	6[1]	19[3]	26[4]	40[7]	32[5]	12[2]	33[6]	55[9]	72[12]	62[10]	90[15]	48[8]	78[13]	63[11]	84[14]

Table 5. Numerical values of simulation measures for $\delta = 1.5$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.20327{3}	0.20039{2}	0.23606{10}	0.18411{1}	0.21485{7}	0.20527{4}	0.21125{5}	0.26622{13}	0.24336{11}	0.21582{8}	0.29288{15}	0.22844{9}	0.28294{14}	0.21325{6}	0.25637{12}
	MSE	0.07211{3}	0.06667{2}	0.09659{10}	0.05194{1}	0.07863{8}	0.07307{4}	0.07698{7}	0.11816{13}	0.10462{11}	0.07507{6}	0.15235{15}	0.0964{9}	0.13567{14}	0.07425{5}	0.11283{12}
	MRE	0.13551{3}	0.13359{2}	0.15737{10}	0.12274{1}	0.14324{7}	0.13685{4}	0.14083{5}	0.17748{13}	0.16224{11}	0.14388{8}	0.19525{15}	0.1523{9}	0.18863{14}	0.14217{6}	0.17092{12}
	D_{abs}	0.05519{2}	0.05589{3}	0.06391{10}	0.05396{1}	0.05951{6}	0.05689{4}	0.05814{5}	0.07261{13}	0.06728{11}	0.06307{9}	0.07539{14}	0.06131{8}	0.08048{15}	0.05991{7}	0.07133{12}
	D_{max}	0.07882{2}	0.07981{3}	0.09206{10}	0.07691{1}	0.08439{6}	0.08145{4}	0.08347{5}	0.10309{12}	0.09627{11}	0.08988{9}	0.10992{14}	0.08771{8}	0.11615{15}	0.08638{7}	0.10315{13}
	ASAE	0.06075{2}	0.06281{7}	0.06271{5}	0.06147{3}	0.06278{6}	0.05808{1}	0.06213{4}	0.07145{11}	0.07302{12}	0.0675{10}	0.09019{15}	0.06603{9}	0.0797{13}	0.06585{8}	0.07972{14}
	$\sum$ *Ranks*	15{2}	19{3}	55{10}	8{1}	40{7}	21{4}	31{5}	75{12.5}	67{11}	50{8}	88{15}	52{9}	85{14}	39{6}	75{12.5}
45	BIAS	0.11203{2}	0.12115{6}	0.11898{5}	0.11587{3}	0.12125{7}	0.1089{1}	0.1184{4}	0.13387{11}	0.14005{12}	0.13344{10}	0.21455{15}	0.12256{8}	0.15235{13}	0.12526{9}	0.15516{14}
	MSE	0.02{2}	0.02339{5}	0.02225{4}	0.02125{3}	0.02459{7}	0.01943{1}	0.02381{6}	0.02837{10}	0.03168{12}	0.02913{11}	0.08596{15}	0.02512{9}	0.0389{13}	0.02461{8}	0.04041{14}
	MRE	0.07469{2}	0.08076{6}	0.07932{5}	0.07725{3}	0.08083{7}	0.0726{1}	0.07893{4}	0.08925{11}	0.09337{12}	0.08896{10}	0.14304{15}	0.08171{8}	0.10157{13}	0.08351{9}	0.10344{14}
	D_{abs}	0.03148{2}	0.03394{7}	0.03349{5}	0.03315{4}	0.03381{6}	0.03064{1}	0.03288{3}	0.03775{10}	0.03983{12}	0.03836{11}	0.05496{15}	0.03463{8}	0.04306{13}	0.0356{9}	0.04355{14}
	D_{max}	0.04546{2}	0.04912{6.5}	0.04844{5}	0.04793{3}	0.04912{6.5}	0.04435{1}	0.04797{4}	0.05447{10}	0.05758{12}	0.05512{11}	0.0808{15}	0.04986{8}	0.06279{13}	0.0515{9}	0.06355{14}
	ASAE	0.02951{2}	0.02992{5}	0.03106{7}	0.03014{6}	0.02989{4}	0.02901{1}	0.02968{3}	0.03384{10}	0.03625{12}	0.03356{9}	0.04952{15}	0.0322{8}	0.03933{13}	0.03438{11}	0.03957{14}
	$\sum$ *Ranks*	12{2}	35.5{6}	31{5}	22{3}	37.5{7}	6{1}	24{4}	62{10.5}	72{12}	62{10.5}	90{15}	49{8}	78{13}	55{9}	84{14}
90	BIAS	0.07701{1}	0.08286{3}	0.08646{6}	0.08483{5}	0.08691{7}	0.079{2}	0.08384{4}	0.09437{11}	0.10378{12}	0.09279{10}	0.15923{15}	0.08891{8}	0.10686{14}	0.09192{9}	0.10475{13}
	MSE	0.00952{1}	0.01097{3}	0.01191{6}	0.01137{5}	0.01215{7}	0.00999{2}	0.01127{4}	0.01408{11}	0.01768{13}	0.01365{10}	0.05216{15}	0.01281{8}	0.01741{12}	0.01297{9}	0.01872{14}
	MRE	0.05134{1}	0.05524{3}	0.05764{6}	0.05655{5}	0.05794{7}	0.05267{2}	0.0559{4}	0.06291{11}	0.06919{12}	0.06186{10}	0.10615{15}	0.05927{8}	0.07124{14}	0.06128{9}	0.06984{13}
	D_{abs}	0.0217{1}	0.02332{3}	0.02435{7}	0.02422{5}	0.0243{6}	0.02231{2}	0.02354{4}	0.02636{11}	0.02948{12}	0.02627{10}	0.04139{15}	0.02479{8}	0.03076{14}	0.02609{9}	0.02972{13}
	D_{max}	0.03142{1}	0.03387{3}	0.03531{6}	0.03511{5}	0.03533{7}	0.0324{2}	0.03418{4}	0.03828{11}	0.04279{12}	0.03809{10}	0.06094{15}	0.03609{8}	0.04445{14}	0.03793{9}	0.0432{13}
	ASAE	0.01894{1}	0.01941{4}	0.01982{6}	0.01935{3}	0.01968{5}	0.01895{2}	0.02005{7}	0.02168{10}	0.02329{12}	0.02211{11}	0.03326{15}	0.02051{8}	0.02513{13}	0.02132{9}	0.02566{14}
	$\sum$ *Ranks*	6{1}	19{3}	37{6}	28{5}	39{7}	12{2}	27{4}	65{11}	73{12}	61{10}	90{15}	48{8}	81{14}	54{9}	80{13}
180	BIAS	0.0618{7}	0.05725{3}	0.06135{6}	0.05509{1}	0.05869{4}	0.05578{2}	0.06086{5}	0.06708{10}	0.0686{12}	0.06796{11}	0.10338{15}	0.06695{9}	0.07802{14}	0.06252{8}	0.0774{13}
	MSE	0.00583{5}	0.00518{3}	0.00591{6}	0.00474{1}	0.00566{4}	0.00477{2}	0.00598{7}	0.00714{10}	0.00751{12}	0.00718{11}	0.02096{15}	0.007{9}	0.00911{13}	0.00616{8}	0.00961{14}
	MRE	0.0412{7}	0.03817{3}	0.0409{6}	0.03673{1}	0.03913{4}	0.03718{2}	0.04057{5}	0.04472{10}	0.04573{12}	0.04531{11}	0.06892{15}	0.04463{9}	0.05201{14}	0.04168{8}	0.0516{13}
	D_{abs}	0.01739{7}	0.01622{3}	0.01733{6}	0.01564{1}	0.01652{4}	0.01574{2}	0.01705{5}	0.01886{9}	0.01943{12}	0.01934{11}	0.0277{15}	0.01894{10}	0.02211{14}	0.01775{8}	0.02201{13}
	D_{max}	0.02535{7}	0.02353{3}	0.02514{6}	0.02276{1}	0.02403{4}	0.02285{2}	0.02485{5}	0.02738{9}	0.02824{12}	0.02808{11}	0.04059{15}	0.02741{10}	0.03223{14}	0.0258{8}	0.03196{13}
	ASAE	0.01252{3}	0.01235{2}	0.01255{4}	0.01278{5}	0.01307{7}	0.01219{1}	0.01285{6}	0.01385{9}	0.0152{12}	0.01431{11}	0.01998{15}	0.01352{8}	0.01653{13}	0.01402{10}	0.01676{14}
	$\sum$ *Ranks*	36{7}	17{3}	34{6}	10{1}	27{4}	11{2}	33{5}	57{10}	72{12}	66{11}	90{15}	55{9}	82{14}	50{8}	80{13}
250	BIAS	0.0502{4}	0.04889{3}	0.0515{6}	0.04623{1}	0.05336{9}	0.04842{2}	0.05043{5}	0.05462{10}	0.05917{12}	0.0558{11}	0.08385{15}	0.05162{7}	0.06638{13}	0.05278{8}	0.06672{14}
	MSE	0.00407{5}	0.0037{2}	0.00412{6}	0.00333{1}	0.00459{9}	0.00374{3}	0.00401{4}	0.00479{10}	0.00559{12}	0.00482{11}	0.01391{15}	0.00431{7}	0.00677{13}	0.00448{8}	0.00748{14}
	MRE	0.03346{4}	0.03259{3}	0.03433{6}	0.03082{1}	0.03557{9}	0.03228{2}	0.03362{5}	0.03641{10}	0.03945{12}	0.0372{11}	0.0559{15}	0.03441{7}	0.04426{13}	0.03519{8}	0.04448{14}
	D_{abs}	0.01414{4}	0.01383{3}	0.01449{7}	0.01316{1}	0.01508{9}	0.01364{2}	0.01423{5}	0.0154{10}	0.0168{12}	0.01578{11}	0.02281{15}	0.01447{6}	0.01897{14}	0.01496{8}	0.01877{13}
	D_{max}	0.02055{4}	0.02011{3}	0.02108{7}	0.01909{1}	0.02193{9}	0.01983{2}	0.02069{5}	0.02239{10}	0.02436{12}	0.02298{11}	0.0333{15}	0.02106{6}	0.02751{14}	0.02174{8}	0.02732{13}
	ASAE	0.01006{1}	0.01028{6}	0.01022{4}	0.01012{2}	0.01056{7}	0.01022{4}	0.01022{4}	0.01123{9}	0.01226{12}	0.01205{11}	0.01664{15}	0.01098{8}	0.01358{13}	0.01124{10}	0.01402{14}
	$\sum$ *Ranks*	22{4}	20{3}	36{6}	7{1}	52{9}	15{2}	28{5}	59{10}	72{12}	66{11}	90{15}	41{7}	80{13}	50{8}	82{14}
350	BIAS	0.03963{1}	0.04186{4}	0.04327{6}	0.03978{2}	0.04707{11}	0.0416{3}	0.04206{5}	0.04503{8}	0.05335{13}	0.04542{10}	0.06621{15}	0.04449{7}	0.05302{12}	0.0452{9}	0.05555{14}
	MSE	0.00249{2}	0.00274{3.5}	0.003{6}	0.00243{1}	0.00361{11}	0.00279{5}	0.00274{3.5}	0.00332{10}	0.00451{12}	0.00324{8}	0.00813{15}	0.00307{7}	0.00457{13}	0.00329{9}	0.00471{14}
	MRE	0.02642{1}	0.02791{4}	0.02884{6}	0.02652{2}	0.03138{11}	0.02773{3}	0.02804{5}	0.03002{8}	0.03557{13}	0.03028{10}	0.04414{15}	0.02966{7}	0.03534{12}	0.03014{9}	0.03703{14}
	D_{abs}	0.01117{1}	0.01178{4}	0.01217{6}	0.01129{2}	0.0131{11}	0.01176{3}	0.01193{5}	0.01273{8}	0.01506{13}	0.01283{10}	0.01824{15}	0.01257{7}	0.01504{12}	0.01279{9}	0.01572{14}
	D_{max}	0.01623{1}	0.01713{4}	0.01769{6}	0.01643{2}	0.01918{11}	0.01712{3}	0.01729{5}	0.01847{8}	0.02191{13}	0.01865{10}	0.02659{15}	0.01824{7}	0.02187{12}	0.01859{9}	0.02292{14}
	ASAE	0.00824{3}	0.00838{5}	0.00862{6.5}	0.0082{2}	0.00862{6.5}	0.00788{1}	0.00832{4}	0.00928{10}	0.01029{12}	0.00946{11}	0.01315{15}	0.00888{8}	0.01107{13}	0.00891{9}	0.0111{14}
	$\sum$ *Ranks*	9{1}	24.5{4}	36.5{6}	11{2}	61.5{11}	18{3}	27.5{5}	52{8}	76{13}	59{10}	90{15}	43{7}	74{12}	54{9}	84{14}

Table 6. Numerical values of simulation measures for $\delta = 1.5$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.12057[4]	0.12009[3]	0.12553[7]	0.12359[5]	0.11964[2]	0.12489[6]	0.10956[1]	0.13917[9]	0.19028[12]	0.17694[11]	0.24845[15]	0.13072[8]	0.20188[13]	0.15067[10]	0.21136[14]
	MSE	0.02253[3]	0.02227[2]	0.02686[7]	0.02379[5]	0.02276[4]	0.0245[6]	0.01979[1]	0.03372[9]	0.06013[12]	0.04771[11]	0.11472[15]	0.02883[8]	0.06838[13]	0.0352[10]	0.07698[14]
	MRE	0.08038[4]	0.08006[3]	0.08369[7]	0.08239[5]	0.07976[2]	0.08326[6]	0.07304[1]	0.09278[9]	0.12685[12]	0.11796[11]	0.16563[15]	0.08715[8]	0.13459[13]	0.10045[10]	0.14091[14]
	D_{abs}	0.03374[2]	0.03394[3]	0.03507[5]	0.03612[8]	0.03404[4]	0.03549[6]	0.03096[1]	0.03822[9]	0.05294[12]	0.05147[11]	0.06163[15]	0.03593[7]	0.0585[13]	0.04297[10]	0.06026[14]
	D_{max}	0.04852[2]	0.04883[4]	0.05063[5]	0.05202[7]	0.04869[3]	0.05165[6]	0.04472[1]	0.05506[9]	0.07692[12]	0.07432[11]	0.09004[15]	0.05239[8]	0.08493[13]	0.06244[10]	0.08735[14]
	ASAE	0.05722[2]	0.05908[7]	0.0588[6]	0.05718[1]	0.05815[5]	0.05725[3]	0.05796[4]	0.06351[9]	0.06944[12]	0.06393[11]	0.08409[15]	0.06133[8]	0.07522[13]	0.06391[10]	0.07695[14]
	$\sum$ *Ranks*	17[2]	22[4]	37[7]	31[5]	20[3]	33[6]	9[1]	54[9]	72[12]	66[11]	90[15]	47[8]	78[13]	60[10]	84[14]
45	BIAS	0.06659[1]	0.06703[2]	0.07212[6]	0.07471[7]	0.07078[3]	0.07191[5]	0.07123[4]	0.07621[8]	0.10173[12]	0.09671[11]	0.18527[15]	0.07861[9]	0.12291[14]	0.09279[10]	0.12005[13]
	MSE	0.00684[1]	0.00689[2]	0.00778[4]	0.00864[7]	0.00777[3]	0.00828[6]	0.00818[5]	0.00952[9]	0.01724[12]	0.01442[11]	0.07876[15]	0.00936[8]	0.02509[14]	0.01367[10]	0.02396[13]
	MRE	0.0444[1]	0.04469[2]	0.04808[6]	0.04981[7]	0.04719[3]	0.04794[5]	0.04748[4]	0.0508[8]	0.06782[12]	0.06447[11]	0.12351[15]	0.0524[9]	0.08194[14]	0.06186[10]	0.08004[13]
	D_{abs}	0.01873[1]	0.01901[2]	0.02037[6]	0.02147[8]	0.01975[3]	0.02034[5]	0.01995[4]	0.02136[7]	0.02918[12]	0.02769[11]	0.04637[15]	0.02226[9]	0.03433[14]	0.02669[10]	0.03409[13]
	D_{max}	0.02728[1]	0.02753[2]	0.02961[6]	0.03114[8]	0.02884[3]	0.02952[5]	0.02905[4]	0.03103[7]	0.04223[12]	0.04011[11]	0.06775[15]	0.03214[9]	0.0502[14]	0.03865[10]	0.04989[13]
	ASAE	0.02738[1]	0.02829[4]	0.02885[7]	0.02858[5]	0.02879[6]	0.02768[2]	0.02811[3]	0.03147[9]	0.03365[12]	0.03243[11]	0.04762[15]	0.03039[8]	0.03779[14]	0.03222[10]	0.03745[13]
	$\sum$ *Ranks*	6[1]	14[2]	35[6]	42[7]	21[3]	28[5]	24[4]	48[8]	72[12]	66[11]	90[15]	52[9]	84[14]	60[10]	78[13]
90	BIAS	0.04731[2]	0.04816[3]	0.05288[7]	0.05187[6]	0.04832[4]	0.04878[5]	0.04675[1]	0.05347[8]	0.07924[12]	0.07176[11]	0.1262[15]	0.05524[9]	0.08416[13]	0.06861[10]	0.08513[14]
	MSE	0.00364[4]	0.00362[3]	0.00416[6]	0.00426[7]	0.0037[5]	0.00361[2]	0.00351[1]	0.00459[8]	0.00988[12]	0.00803[11]	0.03856[15]	0.00463[9]	0.01178[14]	0.00759[10]	0.01149[13]
	MRE	0.03154[2]	0.0321[3]	0.03525[7]	0.03458[6]	0.03222[4]	0.03252[5]	0.03117[1]	0.03565[8]	0.05283[12]	0.04784[11]	0.08413[15]	0.03682[9]	0.0561[13]	0.04574[10]	0.05676[14]
	D_{abs}	0.01339[2]	0.01362[3.5]	0.01493[7]	0.01488[6]	0.01362[3.5]	0.01384[5]	0.01317[1]	0.01505[8]	0.02251[12]	0.02051[11]	0.03277[15]	0.01544[9]	0.0241[13]	0.01968[10]	0.02425[14]
	D_{max}	0.01938[2]	0.01983[3]	0.0217[7]	0.0216[6]	0.01985[4]	0.02011[5]	0.01915[1]	0.02191[8]	0.03282[12]	0.02977[11]	0.04782[15]	0.02255[9]	0.03496[13]	0.02866[10]	0.03534[14]
	ASAE	0.01859[7]	0.01812[4]	0.01829[6]	0.01802[2]	0.01809[3]	0.01782[1]	0.01819[5]	0.0199[8]	0.02324[12]	0.02097[11]	0.03079[15]	0.0201[9]	0.02454[14]	0.0202[10]	0.02432[13]
	$\sum$ *Ranks*	19[2]	19.5[3]	40[7]	33[6]	23.5[5]	23[4]	10[1]	48[8]	72[12]	66[11]	90[15]	54[9]	80[13]	60[10]	82[14]
180	BIAS	0.03411[3]	0.03229[1]	0.03615[6]	0.03649[7]	0.03568[5]	0.03486[4]	0.0338[2]	0.03781[9]	0.0537[12]	0.05281[11]	0.08641[15]	0.03661[8]	0.0614[13]	0.04783[10]	0.06511[14]
	MSE	0.00178[2]	0.00165[1]	0.0021[7]	0.00204[6]	0.00199[5]	0.00188[4]	0.0018[3]	0.00229[9]	0.00448[12]	0.00433[11]	0.01655[15]	0.00213[8]	0.00596[13]	0.00349[10]	0.00669[14]
	MRE	0.02274[3]	0.02153[1]	0.0241[6]	0.02433[7]	0.02379[5]	0.02324[4]	0.02253[2]	0.02521[9]	0.0358[12]	0.03521[11]	0.0576[15]	0.0244[8]	0.04093[13]	0.03189[10]	0.04341[14]
	D_{abs}	0.00959[3]	0.00912[1]	0.01022[6]	0.01037[8]	0.01008[5]	0.00987[4]	0.00953[2]	0.01064[9]	0.01521[12]	0.01506[11]	0.02315[15]	0.01036[7]	0.01743[13]	0.01356[10]	0.01859[14]
	D_{max}	0.014[3]	0.01325[1]	0.01482[6]	0.01509[8]	0.01464[5]	0.01433[4]	0.01391[2]	0.0155[9]	0.02216[12]	0.02185[11]	0.03371[15]	0.01502[7]	0.02536[13]	0.01974[10]	0.02696[14]
	ASAE	0.0116[1.5]	0.01224[7]	0.01194[6]	0.0116[1.5]	0.01182[5]	0.01175[3]	0.01177[4]	0.01302[9]	0.01466[12]	0.01387[11]	0.01976[15]	0.01245[8]	0.01583[14]	0.01359[10]	0.01577[13]
	$\sum$ *Ranks*	15.5[3]	12[1]	37[6]	37.5[7]	30[5]	23[4]	15[2]	54[9]	72[12]	66[11]	90[15]	46[8]	79[13]	60[10]	83[14]
250	BIAS	0.02878[2]	0.03061[6]	0.0295[4]	0.02945[3]	0.0326[7]	0.03003[5]	0.02831[1]	0.03324[8]	0.0488[12]	0.04006[11]	0.06501[15]	0.03386[9]	0.05315[13]	0.03668[10]	0.05489[14]
	MSE	0.00127[2]	0.00147[6]	0.00138[4]	0.00134[3]	0.00161[7]	0.00141[5]	0.00126[1]	0.00172[8]	0.00366[12]	0.00251[11]	0.00977[15]	0.00184[9]	0.00463[13]	0.0021[10]	0.0048[14]
	MRE	0.01919[2]	0.02041[6]	0.01966[4]	0.01964[3]	0.02173[7]	0.02002[5]	0.01887[1]	0.02216[8]	0.03253[12]	0.0267[11]	0.04334[15]	0.02258[9]	0.03543[13]	0.02445[10]	0.03659[14]
	D_{abs}	0.00814[2]	0.00865[6]	0.0083[3]	0.00838[4]	0.0092[7]	0.00849[5]	0.00799[1]	0.00937[8]	0.01387[12]	0.01133[11]	0.01764[15]	0.00951[9]	0.01514[13]	0.01042[10]	0.01558[14]
	D_{max}	0.01182[2]	0.01257[6]	0.01207[3]	0.01215[4]	0.01336[7]	0.01235[5]	0.01161[1]	0.01362[8]	0.02011[12]	0.01651[11]	0.02564[15]	0.01385[9]	0.02201[13]	0.01515[10]	0.02264[14]
	ASAE	0.00969[5]	0.00969[5]	0.01015[7]	0.00969[5]	0.00958[2.5]	0.0095[1]	0.00958[2.5]	0.01066[9]	0.01226[12]	0.01109[11]	0.01555[15]	0.01028[8]	0.01351[14]	0.011[10]	0.01326[13]
	$\sum$ *Ranks*	15[2]	35[6]	25[4]	22[3]	37.5[7]	26[5]	7.5[1]	49[8]	72[12]	66[11]	90[15]	53[9]	79[13]	60[10]	83[14]
350	BIAS	0.02387[1]	0.02557[5]	0.02431[2]	0.02585[6]	0.02484[3]	0.02651[8]	0.02555[4]	0.02685[9]	0.04109[12]	0.03617[11]	0.05791[15]	0.02621[7]	0.04611[14]	0.03299[10]	0.04457[13]
	MSE	0.00089[1]	0.00103[5]	0.00092[2]	0.00104[6]	0.00097[3]	0.00109[8]	0.00102[4]	0.00117[9]	0.00276[12]	0.00197[11]	0.00757[15]	0.00107[7]	0.00323[13.5]	0.00175[10]	0.00323[13.5]
	MRE	0.01591[1]	0.01705[5]	0.01621[2]	0.01723[6]	0.01656[3]	0.01767[8]	0.01703[4]	0.0179[9]	0.0274[12]	0.02412[11]	0.03861[15]	0.01747[7]	0.03074[14]	0.022[10]	0.02971[13]
	D_{abs}	0.00673[1]	0.00722[5]	0.00688[2]	0.00731[6]	0.00698[3]	0.00746[8]	0.00721[4]	0.00757[9]	0.01161[12]	0.01026[11]	0.01577[15]	0.00741[7]	0.0131[14]	0.00935[10]	0.01263[13]
	D_{max}	0.0098[1]	0.0105[5]	0.00998[2]	0.01065[6]	0.0102[3]	0.01085[8]	0.01046[4]	0.01101[9]	0.01689[12]	0.01493[11]	0.02301[15]	0.01077[7]	0.01906[14]	0.01359[10]	0.01837[13]
	ASAE	0.00788[4]	0.00797[6.5]	0.00797[6.5]	0.00786[3]	0.00796[5]	0.00769[1]	0.00783[2]	0.00841[9]	0.01002[12]	0.00908[11]	0.01293[15]	0.00826[8]	0.01033[13]	0.00879[10]	0.01067[14]
	$\sum$ *Ranks*	9[1]	31.5[5]	16.5[2]	33[6]	20[3]	41[7]	22[4]	54[9]	72[12]	66[11]	90[15]	43[8]	82.5[14]	60[10]	79.5[13]

Table 7. Numerical values of simulation measures for $\delta = 0.9$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.10646{1}	0.11327{3}	0.11968{5}	0.11132{2}	0.12963{8}	0.12255{7}	0.1177{4}	0.14151{12}	0.13686{11}	0.13038{9}	0.16919{15}	0.12216{6}	0.15772{14}	0.13109{10}	0.14381{13}
	MSE	0.01873{1}	0.02131{3}	0.02381{5}	0.02092{2}	0.02987{10}	0.02403{6}	0.02354{4}	0.03526{13}	0.03096{11}	0.02662{8}	0.0514{15}	0.02633{7}	0.04094{14}	0.02743{9}	0.03288{12}
	MRE	0.11829{1}	0.12586{3}	0.13298{5}	0.12369{2}	0.14403{8}	0.13617{7}	0.13078{4}	0.15724{12}	0.15206{11}	0.14486{9}	0.18799{15}	0.13574{6}	0.17525{14}	0.14565{10}	0.15979{13}
	D_{abs}	0.05242{1}	0.05552{2}	0.05934{5}	0.05594{3}	0.06234{8}	0.06057{6.5}	0.05847{4}	0.06626{11}	0.06967{12}	0.06577{9}	0.07662{14}	0.06057{6.5}	0.0782{15}	0.06578{10}	0.07294{13}
	D_{max}	0.07492{1}	0.07913{2}	0.08361{5}	0.07981{3}	0.08924{8}	0.08615{7}	0.08308{4}	0.09559{11}	0.09749{12}	0.09321{9}	0.11072{14}	0.08569{6}	0.11203{15}	0.09403{10}	0.10488{13}
	ASAE	0.06192{5}	0.05943{1}	0.06144{3}	0.06191{4}	0.06531{7}	0.0602{2}	0.06362{6}	0.06895{10}	0.07216{12}	0.0702{11}	0.08715{15}	0.06794{9}	0.08069{14}	0.06696{8}	0.07929{13}
	$\sum$ *Ranks*	10{1}	14{2}	28{5}	16{3}	49{8}	35.5{6}	26{4}	69{11.5}	69{11.5}	55{9}	88{15}	40.5{7}	86{14}	57{10}	77{13}
45	BIAS	0.06542{2}	0.07064{9}	0.06654{4}	0.06385{1}	0.06806{6}	0.06543{3}	0.06796{5}	0.07212{10}	0.08262{12}	0.06995{8}	0.11519{15}	0.06837{7}	0.08395{13}	0.07274{11}	0.08979{14}
	MSE	0.00673{2}	0.00748{7}	0.00708{3}	0.00625{1}	0.00734{6}	0.0072{4}	0.00727{5}	0.0083{11}	0.01088{12}	0.0082{10}	0.02447{15}	0.00753{8}	0.01198{13}	0.00816{9}	0.01287{14}
	MRE	0.07269{2}	0.07848{9}	0.07393{4}	0.07094{1}	0.07562{6}	0.0727{3}	0.07551{5}	0.08013{10}	0.0918{12}	0.07772{8}	0.12799{15}	0.07597{7}	0.09328{13}	0.08082{11}	0.09977{14}
	D_{abs}	0.03223{1}	0.03551{10}	0.03289{4}	0.03245{2}	0.03407{7}	0.03261{3}	0.03382{5}	0.03527{9}	0.04156{12}	0.03511{8}	0.05328{15}	0.03396{6}	0.04196{13}	0.03677{11}	0.04503{14}
	D_{max}	0.04655{1.5}	0.05079{9}	0.0475{4}	0.04655{1.5}	0.0489{7}	0.0469{3}	0.04856{5}	0.05122{10}	0.05987{12}	0.05015{8}	0.07755{15}	0.04886{6}	0.06059{13}	0.05273{11}	0.06469{14}
	ASAE	0.03053{5}	0.0304{4}	0.03108{7}	0.02985{2}	0.03107{6}	0.0295{1}	0.03033{3}	0.03339{10}	0.03665{12}	0.03486{11}	0.04809{15}	0.03187{8}	0.03867{14}	0.0331{9}	0.03792{13}
	$\sum$ *Ranks*	13.5{2}	48{8}	26{4}	8.5{1}	38{6}	17{3}	28{5}	60{10}	72{12}	53{9}	90{15}	42{7}	79{13}	62{11}	83{14}
90	BIAS	0.04501{2}	0.05195{9}	0.04891{5}	0.04442{1}	0.04911{6}	0.04951{7}	0.04746{4}	0.05209{10}	0.05951{12}	0.0526{11}	0.08899{15}	0.04618{3}	0.0627{13}	0.05186{8}	0.06418{14}
	MSE	0.00318{2}	0.00419{9}	0.00396{6}	0.00315{1}	0.00397{7}	0.00362{5}	0.00354{4}	0.00429{10}	0.00546{12}	0.0045{11}	0.01484{15}	0.00328{3}	0.00641{14}	0.00418{8}	0.00624{13}
	MRE	0.05001{2}	0.05772{9}	0.05435{5}	0.04936{1}	0.05456{6}	0.05501{7}	0.05273{4}	0.05788{10}	0.06612{12}	0.05845{11}	0.09887{15}	0.05131{3}	0.06967{13}	0.05762{8}	0.07131{14}
	D_{abs}	0.0225{2}	0.02588{9}	0.02424{5}	0.02226{1}	0.02438{6}	0.02477{7}	0.0237{4}	0.02575{8}	0.02973{12}	0.02644{11}	0.04129{15}	0.02298{3}	0.03168{13}	0.02609{10}	0.0324{14}
	D_{max}	0.03236{2}	0.03731{9}	0.03504{5}	0.03202{1}	0.03521{6}	0.03574{7}	0.03416{4}	0.03726{8}	0.04295{12}	0.03812{11}	0.06037{15}	0.03317{3}	0.04559{13}	0.03758{10}	0.04675{14}
	ASAE	0.01977{5}	0.01964{4}	0.0202{7}	0.01914{2}	0.02013{6}	0.01854{1}	0.0193{3}	0.02144{9}	0.02388{12}	0.02225{11}	0.03405{15}	0.02046{8}	0.0265{14}	0.02173{10}	0.02626{13}
	$\sum$ *Ranks*	15{2}	49{8}	33{5}	7{1}	37{7}	34{6}	23{3.5}	55{10}	72{12}	66{11}	90{15}	23{3.5}	80{13}	54{9}	82{14}
180	BIAS	0.03174{1}	0.03433{5}	0.03455{6}	0.03289{4}	0.03624{9}	0.03251{3}	0.03191{2}	0.03636{10}	0.04064{12}	0.03743{11}	0.05898{15}	0.03489{7}	0.04233{13}	0.03585{8}	0.04368{14}
	MSE	0.00158{1}	0.00178{5}	0.0019{6}	0.00172{4}	0.00203{9}	0.0017{3}	0.00163{2}	0.00206{10}	0.00268{12}	0.00219{11}	0.00679{15}	0.00195{7}	0.00284{13}	0.00199{8}	0.00285{14}
	MRE	0.03527{1}	0.03814{5}	0.03839{6}	0.03654{4}	0.04026{9}	0.03612{3}	0.03546{2}	0.0404{10}	0.04515{12}	0.04159{11}	0.06553{15}	0.03877{7}	0.04703{13}	0.03984{8}	0.04853{14}
	D_{abs}	0.01579{1}	0.01726{6}	0.01712{5}	0.01654{4}	0.018{8.5}	0.01618{3}	0.01595{2}	0.01805{10}	0.02013{12}	0.01882{11}	0.02825{15}	0.01735{7}	0.02125{13}	0.018{8.5}	0.02184{14}
	D_{max}	0.02277{1}	0.02483{6}	0.02475{5}	0.0239{4}	0.02606{9}	0.02332{3}	0.02301{2}	0.02608{10}	0.02918{12}	0.02713{11}	0.04096{15}	0.02505{7}	0.0308{13}	0.02592{8}	0.03163{14}
	ASAE	0.01252{3}	0.01256{4.5}	0.01269{6}	0.01244{2}	0.01323{7}	0.01227{1}	0.01256{4.5}	0.01349{8}	0.01563{12}	0.01453{11}	0.02097{15}	0.01351{9}	0.01679{14}	0.0142{10}	0.0163{13}
	$\sum$ *Ranks*	8{1}	31.5{5}	34{6}	22{4}	51.5{9}	16{3}	14.5{2}	58{10}	72{12}	66{11}	90{15}	44{7}	79{13}	50.5{8}	83{14}
250	BIAS	0.02632{1}	0.02635{2}	0.0295{7}	0.02699{3}	0.02999{8}	0.02845{4}	0.02928{6}	0.03151{10}	0.03463{12}	0.03263{11}	0.04858{15}	0.02885{5}	0.03765{14}	0.03098{9}	0.03617{13}
	MSE	0.00109{1}	0.00112{2}	0.00136{7}	0.00115{3}	0.00139{8}	0.00122{4}	0.0013{5.5}	0.00149{10}	0.00193{12}	0.00167{11}	0.0044{15}	0.0013{5.5}	0.00225{14}	0.00148{9}	0.00196{13}
	MRE	0.02924{1}	0.02928{2}	0.03277{7}	0.02998{3}	0.03332{8}	0.03161{4}	0.03253{6}	0.03502{10}	0.03848{12}	0.03625{11}	0.05398{15}	0.03205{5}	0.04183{14}	0.03443{9}	0.04019{13}
	D_{abs}	0.01316{1}	0.01319{2}	0.01472{7}	0.01349{3}	0.015{8}	0.01422{4}	0.01458{6}	0.01565{10}	0.01726{12}	0.01632{11}	0.02346{15}	0.0144{5}	0.01889{14}	0.01557{9}	0.01809{13}
	D_{max}	0.01897{1}	0.01899{2}	0.02126{7}	0.01948{3}	0.02161{8}	0.02049{4}	0.02104{6}	0.02261{10}	0.02491{12}	0.02355{11}	0.03407{15}	0.02077{5}	0.02728{14}	0.02243{9}	0.02608{13}
	ASAE	0.01037{4}	0.01035{3}	0.01039{5}	0.01028{2}	0.01081{7}	0.00996{1}	0.01051{6}	0.01143{9}	0.0126{12}	0.01179{11}	0.01618{15}	0.01088{8}	0.01382{14}	0.01163{10}	0.01345{13}
	$\sum$ *Ranks*	9{1}	13{2}	40{7}	17{3}	47{8}	21{4}	35.5{6}	59{10}	72{12}	66{11}	90{15}	33.5{5}	84{14}	55{9}	78{13}
350	BIAS	0.02297{2}	0.02389{4.5}	0.02602{9}	0.02364{3}	0.02597{8}	0.02282{1}	0.02389{4.5}	0.02732{11}	0.02965{12}	0.02702{10}	0.04254{15}	0.02406{6}	0.03302{13}	0.02568{7}	0.03313{14}
	MSE	0.00084{2}	0.00087{4}	0.00109{9}	0.00085{3}	0.00108{8}	0.00082{1}	$9e-04${5}	0.00116{11}	0.00142{12}	0.00112{10}	0.00416{15}	0.00093{6}	0.00175{13}	0.00105{7}	0.00177{14}
	MRE	0.02553{2}	0.02655{5}	0.02891{9}	0.02626{3}	0.02886{8}	0.02535{1}	0.02654{4}	0.03035{11}	0.03295{12}	0.03002{10}	0.04727{15}	0.02673{6}	0.03668{13}	0.02853{7}	0.03681{14}
	D_{abs}	0.01143{2}	0.01196{5}	0.01297{9}	0.01187{3}	0.01291{8}	0.01133{1}	0.01192{4}	0.01362{11}	0.01479{12}	0.01352{10}	0.0205{15}	0.01197{6}	0.01652{13}	0.01283{7}	0.01653{14}
	D_{max}	0.01649{2}	0.01721{4}	0.01871{9}	0.0171{3}	0.01866{8}	0.01641{1}	0.01723{5}	0.01968{11}	0.02137{12}	0.01951{10}	0.0297{15}	0.01728{6}	0.02388{14}	0.01856{7}	0.02385{13}
	ASAE	0.00808{1}	0.00833{4}	0.00865{7}	0.00827{3}	0.00848{5.5}	0.00814{2}	0.00848{5.5}	0.00932{9}	0.01016{12}	0.00951{11}	0.01388{15}	0.00896{8}	0.01131{14}	0.00947{10}	0.01098{13}
	$\sum$ *Ranks*	11{2}	26.5{4}	51{9}	18{3}	44.5{8}	7{1}	38{6}	63{11}	71{12}	60{10}	89{15}	37{5}	79{13}	44{7}	81{14}

TABLE 8. Numerical values of simulation measures for $\delta = 0.9$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.06589[1]	0.06591[2]	0.07084[6]	0.07014[5]	0.07168[7]	0.06816[4]	0.06682[3]	0.07695[9]	0.10438[12]	0.09182[11]	0.14087[15]	0.07241[8]	0.11785[14]	0.08404[10]	0.11737[13]
	MSE	0.00702[3]	0.00673[1]	0.00842[7]	0.00772[5]	0.00801[6]	0.00735[4]	0.00698[2]	0.01008[9]	0.01905[12]	0.01268[11]	0.03751[15]	0.00844[8]	0.02291[14]	0.01097[10]	0.02254[13]
	MRE	0.07321[1]	0.07323[2]	0.07871[6]	0.07793[5]	0.07965[7]	0.07573[4]	0.07424[3]	0.0855[9]	0.11598[12]	0.10202[11]	0.15653[15]	0.08046[8]	0.13094[14]	0.09338[10]	0.13041[13]
	D_{abs}	0.03198[1]	0.03297[2]	0.03462[5]	0.03594[8]	0.03559[6]	0.03384[4]	0.03306[3]	0.03748[9]	0.05156[12]	0.0472[11]	0.06295[15]	0.03582[7]	0.05855[13]	0.04238[10]	0.05982[14]
	D_{max}	0.04615[1]	0.04709[2]	0.04989[5]	0.05158[8]	0.05142[7]	0.04878[4]	0.04782[3]	0.05383[9]	0.07404[12]	0.06809[11]	0.09079[15]	0.05139[6]	0.08475[13]	0.06162[10]	0.08634[14]
	ASAE	0.05719[3]	0.05822[5]	0.05912[7]	0.05676[1]	0.05845[6]	0.05698[2]	0.05736[4]	0.06365[9]	0.06989[12]	0.06467[10]	0.08525[15]	0.06096[8]	0.07751[14]	0.0653[11]	0.07626[13]
	$\sum Ranks$	10[1]	14[2]	36[6]	32[5]	39[7]	22[4]	18[3]	54[9]	72[12]	65[11]	90[15]	45[8]	82[14]	61[10]	80[13]
45	BIAS	0.03682[1]	0.0413[7]	0.03757[2]	0.04171[8]	0.03999[5]	0.03994[4]	0.0396[3]	0.0423[9]	0.06374[12]	0.05596[11]	0.09653[15]	0.0405[6]	0.07069[14]	0.05066[10]	0.06506[13]
	MSE	0.00211[1]	0.00262[7]	0.00227[2]	0.00272[8]	0.00254[5]	0.00249[3.5]	0.00249[3.5]	0.00278[9]	0.00653[12]	0.00486[11]	0.01941[15]	0.0026[6]	0.0078[14]	0.0041[10]	0.00681[13]
	MRE	0.04091[1]	0.04589[7]	0.04174[2]	0.04634[8]	0.04444[5]	0.04438[4]	0.044[3]	0.047[9]	0.07083[12]	0.06218[11]	0.10725[15]	0.045[6]	0.07855[14]	0.05629[10]	0.07229[13]
	D_{abs}	0.01832[1]	0.02077[7]	0.01859[2]	0.02118[9]	0.01997[4.5]	0.02015[6]	0.01977[3]	0.02083[8]	0.03177[12]	0.02859[11]	0.0443[15]	0.01997[4.5]	0.03597[14]	0.02544[10]	0.03301[13]
	D_{max}	0.02645[1]	0.02978[7]	0.0269[2]	0.03054[9]	0.02874[4]	0.02896[5]	0.02856[3]	0.03026[8]	0.04595[12]	0.04106[11]	0.06408[15]	0.02897[6]	0.0516[14]	0.03675[10]	0.04774[13]
	ASAE	0.02828[2]	0.02849[5]	0.02878[6]	0.02844[4]	0.02879[7]	0.02832[3]	0.02786[1]	0.03112[9]	0.03553[12]	0.03317[11]	0.04711[15]	0.03064[8]	0.03845[14]	0.03195[10]	0.0375[13]
	$\sum Ranks$	7[1]	40[7]	16[2]	46[8]	30.5[5]	25.5[4]	16.5[3]	52[9]	72[12]	66[11]	90[15]	36.5[6]	84[14]	60[10]	78[13]
90	BIAS	0.02676[2]	0.02767[4]	0.02731[3]	0.02982[7]	0.02932[6]	0.02868[5]	0.02671[1]	0.03055[8]	0.04632[12]	0.04103[11]	0.06227[15]	0.03077[9]	0.04978[13]	0.03735[10]	0.05124[14]
	MSE	0.00114[2]	0.00119[3]	0.00122[4]	0.00136[7]	0.00133[5.5]	0.00133[5.5]	0.00113[1]	0.00145[8]	0.00343[12]	0.00257[11]	0.00903[15]	0.00151[9]	0.00394[13]	0.00223[10]	0.0042[14]
	MRE	0.02973[2]	0.03074[4]	0.03034[3]	0.03313[7]	0.03258[6]	0.03187[5]	0.02968[1]	0.03395[8]	0.05146[12]	0.04559[11]	0.06919[15]	0.03419[9]	0.05531[13]	0.0415[10]	0.05693[14]
	D_{abs}	0.0133[1]	0.01387[4]	0.01361[3]	0.01497[7]	0.01463[6]	0.01425[5]	0.01338[2]	0.01515[8]	0.02327[12]	0.02064[11]	0.02909[15]	0.01528[9]	0.02487[13]	0.01876[10]	0.02569[14]
	D_{max}	0.01923[1]	0.01998[4]	0.01965[3]	0.02168[7]	0.02114[6]	0.02064[5]	0.01931[2]	0.02194[8]	0.03374[12]	0.02982[11]	0.04219[15]	0.0221[9]	0.03597[13]	0.02708[10]	0.03705[14]
	ASAE	0.01795[1]	0.01811[3]	0.01856[5]	0.01819[4]	0.01906[7]	0.01808[2]	0.01871[6]	0.01972[9]	0.02295[12]	0.02146[11]	0.02924[15]	0.01946[8]	0.02459[14]	0.0205[10]	0.02449[13]
	$\sum Ranks$	9[1]	22[4]	21[3]	39[7]	36.5[6]	27.5[5]	13[2]	49[8]	72[12]	66[11]	90[15]	53[9]	79[13]	60[10]	83[14]
180	BIAS	0.01899[2]	0.01971[3]	0.02008[4]	0.02164[9]	0.02053[6]	0.02037[5]	0.01883[1]	0.02104[7]	0.03268[12]	0.02767[11]	0.04857[15]	0.02141[8]	0.03563[14]	0.02553[10]	0.03503[13]
	MSE	0.00058[2]	0.00061[3]	0.00064[4.5]	0.00074[9]	0.00065[6]	0.00064[4.5]	0.00056[1]	0.00068[7]	0.00168[12]	0.0012[11]	0.00608[15]	0.00073[8]	0.00201[14]	0.00099[10]	0.00196[13]
	MRE	0.0211[2]	0.0219[3]	0.02231[4]	0.02404[9]	0.02281[6]	0.02264[5]	0.02093[1]	0.02337[7]	0.03631[12]	0.03075[11]	0.05396[15]	0.02379[8]	0.03958[14]	0.02837[10]	0.03893[13]
	D_{abs}	0.00943[2]	0.00989[3]	0.01001[4]	0.01086[9]	0.0102[6]	0.01017[5]	0.00942[1]	0.01049[7]	0.01628[12]	0.01389[11]	0.02289[15]	0.01067[8]	0.01794[14]	0.01278[10]	0.01755[13]
	D_{max}	0.01366[2]	0.01423[3]	0.01449[4]	0.0157[9]	0.01481[6]	0.01469[5]	0.01358[1]	0.01513[7]	0.02362[12]	0.02006[11]	0.03318[15]	0.01542[8]	0.02595[14]	0.01844[10]	0.0254[13]
	ASAE	0.01199[3]	0.012[4.5]	0.01238[7]	0.012[4.5]	0.01225[6]	0.01178[2]	0.01176[1]	0.01289[8]	0.01471[12]	0.01369[11]	0.01978[15]	0.01312[9]	0.0161[14]	0.01313[10]	0.01576[13]
	$\sum Ranks$	13[2]	19.5[3]	27.5[5]	49.5[9]	36[6]	26.5[4]	6[1]	43[7]	72[12]	66[11]	90[15]	49[8]	84[14]	60[10]	78[13]
250	BIAS	0.01699[4]	0.01753[6]	0.01707[5]	0.01657[1]	0.01788[7]	0.01673[2]	0.01679[3]	0.01842[9]	0.0284[12]	0.02422[11]	0.03949[15]	0.01822[8]	0.03184[14]	0.02127[10]	0.03103[13]
	MSE	0.00045[3]	0.00047[5.5]	0.00047[5.5]	0.00041[1]	$5e-04$[7]	0.00044[2]	0.00046[4]	0.00055[9]	0.00126[12]	0.00091[11]	0.00367[15]	0.00051[8]	0.0016[14]	0.00075[10]	0.0015[13]
	MRE	0.01888[4]	0.01948[6]	0.01896[5]	0.01841[1]	0.01986[7]	0.01858[2]	0.01866[3]	0.02047[9]	0.03155[12]	0.02691[11]	0.04388[15]	0.02025[8]	0.03538[14]	0.02363[10]	0.03448[13]
	D_{abs}	0.00846[4]	0.00876[6]	0.0085[5]	0.0083[1]	0.00892[7]	0.00834[2]	0.00839[3]	0.00918[9]	0.01418[12]	0.01215[11]	0.01886[15]	0.00909[8]	0.016[14]	0.01062[10]	0.01557[13]
	D_{max}	0.01221[4]	0.01264[6]	0.01229[5]	0.01201[1]	0.01285[7]	0.01206[2]	0.01211[3]	0.01328[9]	0.02048[12]	0.01754[11]	0.02736[15]	0.01312[8]	0.02309[14]	0.01535[10]	0.02248[13]
	ASAE	0.00958[3]	0.0095[1]	0.00988[7]	0.0096[5]	0.00977[6]	0.00958[3]	0.00958[3]	0.01037[8.5]	0.0123[12]	0.01132[11]	0.01596[15]	0.01037[8.5]	0.01318[13]	0.01088[10]	0.01326[14]
	$\sum Ranks$	22[4]	30.5[5]	32.5[6]	10[1]	49[8]	13[2]	19[3]	52.5[9]	71[12]	65[11]	89[15]	47.5[7]	82[14]	59[10]	78[13]
350	BIAS	0.01285[1]	0.01444[4]	0.01431[3]	0.01501[6]	0.01559[8]	0.01445[5]	0.01346[2]	0.01575[9]	0.02433[12]	0.02102[11]	0.03231[15]	0.01522[7]	0.02576[13]	0.01972[10]	0.02678[14]
	MSE	0.00026[1]	0.00033[5]	0.00032[3.5]	0.00035[6]	0.00039[8]	0.00032[3.5]	0.00028[2]	$4e-04$[9]	0.00094[12]	0.00069[11]	0.00293[15]	0.00038[7]	0.00101[13]	0.00061[10]	0.00119[14]
	MRE	0.01427[1]	0.01604[4]	0.0159[3]	0.01668[6]	0.01732[8]	0.01606[5]	0.01496[2]	0.0175[9]	0.02703[12]	0.02336[11]	0.0359[15]	0.01691[7]	0.02862[13]	0.02191[10]	0.02976[14]
	D_{abs}	0.00641[1]	0.00721[5]	0.00715[3]	0.00753[6]	0.00776[8]	0.0072[4]	0.00672[2]	0.00786[9]	0.01217[12]	0.01051[11]	0.01551[15]	0.00759[7]	0.01287[13]	0.00988[10]	0.0134[14]
	D_{max}	0.00925[1]	0.01041[4]	0.01032[3]	0.01086[6]	0.01122[8]	0.01042[5]	0.0097[2]	0.01134[9]	0.01756[12]	0.0152[11]	0.02246[15]	0.01096[7]	0.01862[13]	0.01428[10]	0.01934[14]
	ASAE	0.00781[1]	0.00789[3]	0.00796[5.5]	0.00791[4]	0.008[7]	0.00783[2]	0.00796[5.5]	0.00868[9]	0.01006[12]	0.00922[11]	0.01277[15]	0.00839[8]	0.01074[13]	0.0089[10]	0.01095[14]
	$\sum Ranks$	6[1]	25[5]	21[3]	34[6]	47[8]	24.5[4]	15.5[2]	60[10]	71[12]	65[11]	89[15]	43[7]	77[13]	59[9]	83[14]

Table 9. Numerical values of simulation measures for $\delta = 0.3$ under SRS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.03796[5]	0.0392[8]	0.03849[6]	0.03708[3]	0.03871[7]	0.03691[2]	0.03583[1]	0.04376[11]	0.04481[12]	0.04029[9]	0.05387[15]	0.03792[4]	0.05017[13]	0.04032[10]	0.05101[14]
	MSE	0.00237[4]	0.00245[6]	0.00247[8]	0.0021[1]	0.00246[7]	0.00227[3]	0.00216[2]	0.00317[11]	0.00334[12]	0.00259[9]	0.00473[15]	0.00243[5]	0.00447[13]	0.00261[10]	0.00457[14]
	MRE	0.12654[5]	0.13068[8]	0.12829[6]	0.1236[3]	0.12903[7]	0.12302[2]	0.11944[1]	0.14586[11]	0.14936[12]	0.13429[9]	0.17957[15]	0.12638[4]	0.16724[13]	0.13438[10]	0.17005[14]
	D_{abs}	0.05852[3]	0.05996[7]	0.0593[5]	0.05887[4]	0.06016[8]	0.05708[2]	0.05566[1]	0.06672[11]	0.06935[12]	0.06377[10]	0.07947[15]	0.05965[6]	0.07762[14]	0.06269[9]	0.07682[13]
	D_{max}	0.08233[3]	0.08454[8]	0.08343[5]	0.08356[6]	0.08444[7]	0.07999[2]	0.07802[1]	0.09457[11]	0.09883[12]	0.09067[10]	0.1123[15]	0.08313[4]	0.11109[14]	0.08855[9]	0.11052[13]
	ASAE	0.2551[3]	0.26729[4]	0.29215[5]	0.32269[8]	0.3179[7]	0.23456[1]	0.24997[2]	0.37131[10]	0.44854[12]	0.40258[11]	0.4567[13]	0.32818[9]	0.63053[15]	0.31033[6]	0.54302[14]
	$\sum$ *Ranks*	23[3]	41[7]	35[6]	25[4]	43[8]	12[2]	8[1]	65[11]	72[12]	58[10]	88[15]	32[5]	82[13.5]	54[9]	82[13.5]
45	BIAS	0.02079[1.5]	0.02129[5]	0.02242[7]	0.02079[1.5]	0.02261[8]	0.02105[3]	0.02116[4]	0.02366[10]	0.0268[12]	0.02415[11]	0.03637[15]	0.02266[9]	0.03011[13]	0.02229[6]	0.03014[14]
	MSE	0.00071[4.5]	$7e-04$[2.5]	0.00084[9]	$7e-04$[2.5]	0.00081[7]	0.00067[1]	0.00071[4.5]	$9e-04$[10]	0.00115[12]	0.00092[11]	0.00235[15]	0.00082[8]	0.00154[14]	0.00077[6]	0.00143[13]
	MRE	0.06928[1]	0.07096[5]	0.07474[7]	0.06929[2]	0.07535[8]	0.07018[3]	0.07052[4]	0.07885[10]	0.08934[12]	0.0805[11]	0.12124[15]	0.07555[9]	0.10038[13]	0.07431[6]	0.10048[14]
	D_{abs}	0.03193[1]	0.03325[5]	0.03465[6]	0.03268[2]	0.03519[8]	0.0329[3]	0.03317[4]	0.0368[10]	0.04166[12]	0.03758[11]	0.05276[15]	0.03484[7]	0.04639[13]	0.03528[9]	0.04675[14]
	D_{max}	0.04574[1]	0.04751[5]	0.04962[6]	0.04672[2]	0.05013[8]	0.04715[4]	0.0471[3]	0.05259[10]	0.05963[12]	0.05384[11]	0.07653[15]	0.05002[7]	0.06676[13]	0.05021[9]	0.0673[14]
	ASAE	0.06659[1]	0.08676[3]	0.09492[8]	0.08963[7]	0.0895[5]	0.08303[2]	0.08961[6]	0.10962[10]	0.14055[13]	0.11251[11]	0.13555[12]	0.08738[4]	0.18183[14]	0.10716[9]	0.1824[15]
	$\sum$ *Ranks*	8[1]	36.5[5]	41[6]	28[4]	42[7.5]	16[2]	23.5[3]	65[11]	70[12]	63[10]	84[15]	42[7.5]	77[13]	43[9]	81[14]
90	BIAS	0.01499[4]	0.01572[7]	0.01544[6]	0.01423[2]	0.0154[5]	0.014[1]	0.01433[3]	0.01659[9]	0.01897[12]	0.01664[10]	0.02615[15]	0.01649[8]	0.02061[14]	0.0167[11]	0.0194[13]
	MSE	0.00036[4]	0.00039[6]	0.00039[6]	0.00031[1]	0.00039[6]	0.00032[2.5]	0.00032[2.5]	0.00042[8]	0.00057[12]	0.00044[11]	0.00127[15]	0.00043[9.5]	0.00067[14]	0.00043[9.5]	0.00061[13]
	MRE	0.04996[4]	0.0524[7]	0.05147[6]	0.04743[2]	0.05135[5]	0.04667[1]	0.04778[3]	0.0553[9]	0.06324[12]	0.05547[10]	0.08717[15]	0.05497[8]	0.06871[14]	0.05565[11]	0.06466[13]
	D_{abs}	0.02334[4]	0.0245[7]	0.02396[6]	0.02228[2]	0.02387[5]	0.02191[1]	0.0225[3]	0.02581[9]	0.02969[12]	0.02609[10]	0.03857[15]	0.02551[8]	0.03214[14]	0.02612[11]	0.03036[13]
	D_{max}	0.03335[4]	0.03507[7]	0.03429[6]	0.03189[2]	0.03424[5]	0.03131[1]	0.03208[3]	0.03697[9]	0.04236[12]	0.03746[11]	0.05571[15]	0.03667[8]	0.04618[14]	0.03744[10]	0.04362[13]
	ASAE	0.03545[1]	0.04108[7]	0.03783[5]	0.03585[2]	0.0386[6]	0.0364[3]	0.03642[4]	0.04745[9]	0.06498[12]	0.0571[11]	0.06543[13]	0.04129[8]	0.07965[15]	0.04747[10]	0.07961[14]
	$\sum$ *Ranks*	21[4]	41[7]	35[6]	11[2]	32[5]	9.5[1]	18.5[3]	53[9]	72[12]	63[11]	88[15]	49.5[8]	85[14]	62.5[10]	79[13]
180	BIAS	0.01042[1]	0.01114[8]	0.01064[5]	0.01053[2.5]	0.01087[7]	0.01063[4]	0.01053[2.5]	0.01213[11]	0.01343[13]	0.01206[10]	0.01945[15]	0.01155[9]	0.01449[14]	0.01078[6]	0.01341[12]
	MSE	0.00017[1.5]	0.00019[7.5]	0.00018[4.5]	0.00017[1.5]	0.00019[7.5]	0.00018[4.5]	0.00018[4.5]	0.00023[10.5]	0.00028[12]	0.00023[10.5]	$7e-04$[15]	$2e-04$[9]	0.00033[14]	0.00018[4.5]	0.00029[13]
	MRE	0.03472[1]	0.03714[8]	0.03547[5]	0.03509[2.5]	0.03622[7]	0.03542[4]	0.03509[2.5]	0.04045[11]	0.04475[13]	0.0402[10]	0.06484[15]	0.03851[9]	0.04829[14]	0.03592[6]	0.0447[12]
	D_{abs}	0.01624[1]	0.0173[8]	0.01663[5]	0.0164[3]	0.01686[7]	0.01651[4]	0.01638[2]	0.01878[10]	0.02093[13]	0.01882[11]	0.02904[15]	0.01801[9]	0.02267[14]	0.01681[6]	0.02085[12]
	D_{max}	0.02321[1]	0.02483[8]	0.02376[5]	0.02355[3]	0.0242[7]	0.02364[4]	0.02349[2]	0.02696[11]	0.02998[13]	0.02692[10]	0.04197[15]	0.0258[9]	0.03245[14]	0.02414[6]	0.02992[12]
	ASAE	0.01562[2]	0.01776[7]	0.01679[3]	0.01704[5]	0.01681[4]	0.01539[1]	0.01717[6]	0.02077[10]	0.02764[12]	0.02153[11]	0.03402[15]	0.01993[9]	0.03378[14]	0.0184[8]	0.02854[13]
	$\sum$ *Ranks*	7.5[1]	46.5[8]	27.5[5]	17.5[2]	39.5[7]	21.5[4]	19.5[3]	62.5[11]	75[13]	61.5[10]	90[15]	59[9]	83[14]	36.5[6]	73[12]
250	BIAS	0.00908[4]	0.00925[5]	0.00958[6]	0.00894[2]	0.00962[7]	0.0086[1]	0.00905[3]	0.01002[11]	0.01138[12]	0.00978[8]	0.01684[15]	0.00989[9]	0.01257[14]	0.00997[10]	0.01201[13]
	MSE	0.00013[3]	0.00013[3]	0.00014[5.5]	0.00013[3]	0.00015[8]	0.00012[1]	0.00014[5.5]	0.00016[10.5]	$2e-04$[12]	0.00015[8]	0.00054[15]	0.00015[8]	0.00025[14]	0.00016[10.5]	0.00023[13]
	MRE	0.03025[4]	0.03082[5]	0.03192[6]	0.02979[2]	0.03205[7]	0.02868[1]	0.03018[3]	0.03341[11]	0.03792[12]	0.03259[8]	0.05614[15]	0.03296[9]	0.0419[14]	0.03322[10]	0.04002[13]
	D_{abs}	0.01412[4]	0.01442[5]	0.01487[6]	0.01394[2]	0.01491[7]	0.01339[1]	0.01405[3]	0.01564[11]	0.01776[12]	0.01531[8.5]	0.02539[15]	0.01531[8.5]	0.01962[14]	0.01552[10]	0.01878[13]
	D_{max}	0.02024[4]	0.02064[5]	0.02136[6]	0.02001[2]	0.02143[7]	0.01918[1]	0.02014[3]	0.02239[11]	0.02548[12]	0.02192[8]	0.03658[15]	0.02202[9]	0.0282[14]	0.02226[10]	0.027[13]
	ASAE	0.01142[2]	0.01215[4]	0.01278[6]	0.01274[5]	0.01323[7]	0.01126[1]	0.01181[3]	0.01539[10]	0.01974[12]	0.01559[11]	0.0272[15]	0.0136[8]	0.02443[14]	0.01519[9]	0.02401[13]
	$\sum$ *Ranks*	21[4]	27[5]	35.5[6]	16[2]	43[7]	6[1]	20.5[3]	64.5[11]	75[12]	51.5[8.5]	89[15]	51.5[8.5]	83[14]	59.5[10]	77[13]
350	BIAS	0.00731[1]	0.00774[5]	0.0079[7]	0.00739[3]	0.00777[6]	0.00738[2]	0.00754[4]	0.00869[10]	0.00957[12.5]	0.00921[11]	0.01473[15]	0.00827[8]	0.00957[12.5]	0.00843[9]	0.0111[14]
	MSE	$8e-05$[1.5]	$1e-04$[6.5]	$1e-04$[6.5]	$9e-05$[3.5]	$1e-04$[6.5]	$8e-05$[1.5]	$9e-05$[3.5]	0.00012[10]	0.00015[12.5]	0.00013[11]	0.00043[15]	$1e-04$[6.5]	0.00015[12.5]	0.00011[9]	0.00019[14]
	MRE	0.02437[1]	0.02579[5]	0.02633[7]	0.02462[3]	0.0259[6]	0.0246[2]	0.02513[4]	0.02898[10]	0.0319[12]	0.03069[11]	0.04909[15]	0.02756[8]	0.03191[13]	0.02811[9]	0.037[14]
	D_{abs}	0.0114[1]	0.0121[6]	0.01232[7]	0.01155[3]	0.01206[5]	0.01153[2]	0.01174[4]	0.01358[10]	0.0149[12]	0.01443[11]	0.02214[15]	0.01287[8]	0.01494[13]	0.01313[9]	0.01724[14]
	D_{max}	0.01634[1]	0.01732[5.5]	0.01766[7]	0.01653[2.5]	0.01732[5.5]	0.01653[2.5]	0.01683[4]	0.01945[10]	0.02138[12]	0.02065[11]	0.03197[15]	0.01847[8]	0.02139[13]	0.01884[9]	0.02474[14]
	ASAE	0.00893[1]	0.01013[7]	0.01051[8]	0.00906[4]	0.00905[3]	0.00901[2]	0.00915[5]	0.0124[10]	0.01337[12]	0.01244[11]	0.01931[15]	0.00978[6]	0.01407[13]	0.01065[9]	0.01473[14]
	$\sum$ *Ranks*	17.5[1]	38[6]	45.5[7]	30[3]	35[4]	23[2]	35.5[5]	52[10]	65[12]	58[11]	82[15]	47.5[9]	69[13]	46[8]	76[14]

Table 10. Numerical values of simulation measures for $\delta = 0.3$ under RSS.

n	Est.	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
15	BIAS	0.02209[5]	0.0214[1]	0.02201[3]	0.02413[8]	0.02274[7]	0.02188[2]	0.0223[6]	0.02515[9]	0.03379[12]	0.03001[11]	0.04659[15]	0.02202[4]	0.03765[14]	0.02657[10]	0.03707[13]
	MSE	0.00077[4.5]	0.00071[1]	0.00077[4.5]	0.00086[8]	0.00084[7]	0.00076[3]	$8e-04$[6]	0.00104[9]	0.00187[12]	0.00137[11]	0.00407[15]	0.00073[2]	0.00229[13.5]	0.0011[10]	0.00229[13.5]
	MRE	0.07364[5]	0.07133[1]	0.07336[3]	0.08043[8]	0.0758[7]	0.07293[2]	0.07433[6]	0.08383[9]	0.11262[12]	0.10003[11]	0.1553[15]	0.07341[4]	0.12551[14]	0.08856[10]	0.12355[13]
	D_{abs}	0.03435[5]	0.03312[1]	0.034[2]	0.03843[8]	0.0353[7]	0.03417[3]	0.03487[6]	0.03858[9]	0.05228[12]	0.04752[11]	0.06653[15]	0.03427[4]	0.05813[14]	0.04166[10]	0.05804[13]
	D_{max}	0.04874[5]	0.04741[1]	0.04813[2]	0.05484[8]	0.04994[7]	0.0485[3]	0.04952[6]	0.05494[9]	0.07545[12]	0.06868[11]	0.09386[15]	0.04858[4]	0.0837[14]	0.05935[10]	0.08353[13]
	ASAE	0.10282[2]	0.10836[3]	0.09571[1]	0.15415[9]	0.10855[4]	0.10857[5]	0.13278[8]	0.13062[7]	0.30005[13]	0.28545[12]	0.21352[11]	0.11474[6]	0.36986[14]	0.16625[10]	0.40932[15]
	$\sum Ranks$	26.5[5]	8[1]	15.5[2]	48[8]	38[6]	18[3]	47[7]	51[9]	72[12]	66[11]	85[15]	24[4]	82.5[14]	59[10]	79.5[13]
45	BIAS	0.01248[2]	0.01228[1]	0.01325[5]	0.0139[8]	0.01361[6]	0.01307[4]	0.01281[3]	0.01425[9]	0.02002[12]	0.01784[11]	0.02972[15]	0.01386[7]	0.02089[13]	0.0163[10]	0.02273[14]
	MSE	0.00025[2]	0.00024[1]	0.00027[4.5]	0.00029[6.5]	0.00029[6.5]	0.00027[4.5]	0.00026[3]	0.00032[9]	0.00064[12]	0.00047[11]	0.00192[15]	0.00031[8]	0.00073[13]	0.00043[10]	0.00084[14]
	MRE	0.0416[2]	0.04095[1]	0.04416[5]	0.04635[8]	0.04536[6]	0.04357[4]	0.04269[3]	0.0475[9]	0.06674[12]	0.05945[11]	0.09906[15]	0.04619[7]	0.06962[13]	0.05433[10]	0.07576[14]
	D_{abs}	0.01931[2]	0.01901[1]	0.02065[5]	0.02205[9]	0.02127[6]	0.02032[4]	0.0199[3]	0.02193[8]	0.03131[12]	0.02813[11]	0.04265[15]	0.02137[7]	0.03268[13]	0.02586[10]	0.03576[14]
	D_{max}	0.02782[2]	0.02733[1]	0.02963[5]	0.03158[9]	0.0304[6]	0.02917[4]	0.02844[3]	0.03155[8]	0.04494[12]	0.04055[11]	0.06153[15]	0.03081[7]	0.04696[13]	0.03677[10]	0.05145[14]
	ASAE	0.03769[3]	0.03493[1]	0.04216[5]	0.04955[9]	0.0424[6]	0.04013[4]	0.03684[2]	0.04248[7]	0.09808[13]	0.07753[11]	0.08017[12]	0.04372[8]	0.1092[14]	0.0655[10]	0.13489[15]
	$\sum Ranks$	13[2]	6[1]	29.5[5]	49.5[8]	36.5[6]	24.5[4]	17[3]	50[9]	73[12]	66[11]	87[15]	44[7]	79[13]	60[10]	85[14]
90	BIAS	0.00877[3]	0.00844[1]	0.00867[2]	0.00957[7]	0.00898[5]	0.00903[6]	0.00888[4]	0.01014[9]	0.01478[12]	0.01268[11]	0.02104[15]	0.00973[8]	0.01612[14]	0.0122[10]	0.01561[13]
	MSE	0.00012[3]	0.00011[1]	0.00012[3]	0.00014[7]	0.00013[5.5]	0.00013[5.5]	0.00012[3]	0.00016[9]	0.00034[12]	0.00025[11]	0.00094[15]	0.00015[8]	0.00041[14]	0.00023[10]	0.00038[13]
	MRE	0.02922[3]	0.02814[1]	0.02889[2]	0.03189[7]	0.02994[5]	0.03009[6]	0.02959[4]	0.03381[9]	0.04928[12]	0.04227[11]	0.07013[15]	0.03244[8]	0.05373[14]	0.04068[10]	0.05204[13]
	D_{abs}	0.01364[3]	0.0132[1]	0.01348[2]	0.01506[8]	0.014[5]	0.01414[6]	0.0139[4]	0.01574[9]	0.02326[12]	0.01991[11]	0.03108[15]	0.01502[7]	0.02536[14]	0.01915[10]	0.02449[13]
	D_{max}	0.01954[3]	0.01891[1]	0.01933[2]	0.02158[7]	0.02008[5]	0.0202[6]	0.01986[4]	0.02255[9]	0.03329[12]	0.02864[11]	0.0446[15]	0.02164[8]	0.03635[14]	0.0274[10]	0.0352[13]
	ASAE	0.01971[2]	0.01962[1]	0.02123[5]	0.02233[8]	0.02185[6]	0.02009[3]	0.02072[4]	0.02327[9]	0.04753[13]	0.03712[11]	0.03802[12]	0.02188[7]	0.05887[15]	0.02997[10]	0.05309[14]
	$\sum Ranks$	17[3]	6[1]	16[2]	44[7]	31.5[5]	32.5[6]	23[4]	54[9]	73[12]	66[11]	87[15]	46[8]	85[14]	60[10]	79[13]
180	BIAS	0.00633[2]	0.00626[1]	0.00665[5.5]	0.0068[8]	0.00653[4]	0.00669[7]	0.00649[3]	0.00687[9]	0.01049[12]	0.00929[11]	0.01569[15]	0.00665[5.5]	0.01087[13]	0.00842[10]	0.01182[14]
	MSE	$6e-05$[1.5]	$6e-05$[1.5]	$7e-05$[5.5]	$7e-05$[5.5]	$7e-05$[5.5]	$7e-05$[5.5]	$7e-05$[5.5]	$8e-05$[9]	0.00017[12]	0.00014[11]	0.00056[15]	$7e-05$[5.5]	0.00019[13]	0.00011[10]	0.00021[14]
	MRE	0.02111[2]	0.02086[1]	0.02215[5]	0.02266[8]	0.02176[4]	0.02229[7]	0.02162[3]	0.02291[9]	0.03497[12]	0.03098[11]	0.0523[15]	0.02216[6]	0.03625[13]	0.02807[10]	0.0394[14]
	D_{abs}	0.00988[2]	0.00974[1]	0.01034[5.5]	0.01064[8]	0.01016[4]	0.01044[7]	0.01009[3]	0.01066[9]	0.01646[12]	0.01459[11]	0.02323[15]	0.01034[5.5]	0.01703[13]	0.01311[10]	0.01848[14]
	D_{max}	0.01416[2]	0.01399[1]	0.01482[5]	0.01526[8]	0.01457[4]	0.01493[7]	0.01449[3]	0.01529[9]	0.02363[12]	0.02092[11]	0.03356[15]	0.01483[6]	0.02438[13]	0.01886[10]	0.0265[14]
	ASAE	0.01255[7]	0.01235[5]	0.01224[3]	0.01193[1]	0.01225[4]	0.012[2]	0.01252[6]	0.01318[9]	0.02188[13]	0.019[11]	0.02241[14]	0.01282[8]	0.02098[12]	0.01544[10]	0.02458[15]
	$\sum Ranks$	22.5[2]	16.5[1]	35.5[5]	44.5[8]	31.5[4]	41.5[6]	29.5[3]	60[11]	64[12]	57[10]	80[15]	42.5[7]	68[13]	51[9]	76[14]
250	BIAS	0.00537[3]	0.0051[1]	0.00548[5.5]	0.00551[7]	0.00542[4]	0.00548[5.5]	0.00523[2]	0.00594[9]	0.00902[12]	0.00755[10]	0.01314[15]	0.00578[8]	0.00988[13]	0.00759[11]	0.01[14]
	MSE	$4e-05$[2]	$4e-05$[2]	$5e-05$[6]	$5e-05$[6]	$5e-05$[6]	$5e-05$[6]	$4e-05$[2]	$6e-05$[9]	0.00013[12]	$9e-05$[10.5]	0.00042[15]	$5e-05$[6]	0.00016[14]	$9e-05$[10.5]	0.00015[13]
	MRE	0.01789[3]	0.01699[1]	0.01826[5]	0.01838[7]	0.01808[4]	0.01828[6]	0.01743[2]	0.01979[9]	0.03007[12]	0.02518[10]	0.04381[15]	0.01927[8]	0.03293[13]	0.0253[11]	0.03335[14]
	D_{abs}	0.00836[3]	0.00792[1]	0.00853[5]	0.00861[7]	0.00847[4]	0.00854[6]	0.00815[2]	0.00924[9]	0.01406[12]	0.01177[10]	0.01965[15]	0.00898[8]	0.01543[13]	0.01184[11]	0.01566[14]
	D_{max}	0.01198[3]	0.01139[1]	0.01223[5]	0.01236[7]	0.01214[4]	0.01225[6]	0.01167[2]	0.01324[9]	0.02018[12]	0.01689[10]	0.02832[15]	0.01292[8]	0.02218[13]	0.017[11]	0.02249[14]
	ASAE	0.00978[5]	0.00976[3]	0.0099[6]	0.00963[2]	0.01007[7]	0.00962[1]	0.00977[4]	0.01072[9]	0.01459[12]	0.01221[11]	0.0173[13]	0.01029[8]	0.01909[14]	0.01197[10]	0.01961[15]
	$\sum Ranks$	23[3]	13[1]	36.5[6]	40[7]	33[4]	34.5[5]	18[2]	58[9]	61[10]	65.5[11]	77[15]	50[8]	69[13]	68.5[12]	73[14]
350	BIAS	0.00447[3.5]	0.00461[6.5]	0.00447[3.5]	0.00458[5]	0.00427[1]	0.00475[8]	0.00446[2]	0.00478[9]	0.00745[12]	0.00613[10]	0.00995[15]	0.00461[6.5]	0.00843[13]	0.00619[11]	0.00856[14]
	MSE	$3e-05$[4]	$3e-05$[4]	$3e-05$[4]	$3e-05$[4]	$3e-05$[4]	$4e-05$[8.5]	$3e-05$[4]	$4e-05$[8.5]	$9e-05$[12]	$6e-05$[10.5]	0.00021[15]	$3e-05$[4]	0.00011[13]	$6e-05$[10.5]	0.00012[14]
	MRE	0.01491[4]	0.01537[7]	0.0149[3]	0.01526[5]	0.01423[1]	0.01583[8]	0.01486[2]	0.01594[9]	0.02483[12]	0.02045[10]	0.03316[15]	0.01536[6]	0.02809[13]	0.02062[11]	0.02852[14]
	D_{abs}	0.00696[3.5]	0.00718[7]	0.00696[3.5]	0.00713[5]	0.00665[1]	0.00739[8]	0.00695[2]	0.00744[9]	0.0116[12]	0.00958[10]	0.01508[15]	0.00717[6]	0.01317[13]	0.00968[11]	0.01334[14]
	D_{max}	0.00999[4]	0.01029[6]	0.00998[3]	0.01024[5]	0.00954[1]	0.01061[8]	0.00996[2]	0.01068[9]	0.01667[12]	0.01376[10]	0.0217[15]	0.0103[7]	0.01887[13]	0.0139[11]	0.01918[14]
	ASAE	0.00785[3]	0.00787[4]	0.00808[7]	0.0078[2]	0.00803[6]	0.00778[1]	0.00793[5]	0.00857[9]	0.01061[12]	0.00936[11]	0.01231[14]	0.00836[8]	0.01215[13]	0.00918[10]	0.01343[15]
	$\sum Ranks$	25[3]	37.5[6]	27[4]	29[5]	17[1]	44.5[8]	20[2]	56.5[9]	75[14]	64.5[10]	77[15]	40.5[7]	66[11]	67.5[12]	73[13]

Table 11. Partial and overall ranks for all estimation methods of our proposed model by SRS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
$\delta = 3.5$	15	3.0	4.0	9.0	2.0	11.0	1.0	5.0	12.0	10.0	7.5	15.0	7.5	14.0	6.0	13.0
	45	2.0	4.0	8.0	1.0	7.0	3.0	6.0	11.5	11.5	9.0	15.0	10.0	13.0	5.0	14.0
	90	2.0	3.0	7.0	1.0	8.0	4.0	5.5	12.0	11.0	9.5	15.0	9.5	14.0	5.5	13.0
	180	2.0	5.5	8.0	1.0	7.0	3.0	4.0	12.0	11.0	9.0	15.0	10.0	14.0	5.5	13.0
	250	2.0	4.0	6.0	3.0	7.0	1.0	5.0	11.0	12.0	10.0	15.0	9.0	14.0	8.0	13.0
	350	4.0	2.0	7.0	1.0	5.0	3.0	6.0	11.0	12.0	9.0	15.0	10.0	13.0	8.0	14.0
$\delta = 2.5$	15	2.0	8.0	6.0	1.0	9.0	3.0	4.5	11.0	10.0	7.0	15.0	12.0	14.0	4.5	13.0
	45	3.0	5.0	6.0	2.0	8.0	4.0	1.0	11.0	12.0	10.0	15.0	7.0	14.0	9.0	13.0
	90	3.0	2.0	6.0	1.0	5.0	4.0	7.0	11.0	12.0	8.0	15.0	10.0	13.0	9.0	14.0
	180	3.0	4.0	8.0	1.0	7.0	2.0	5.0	11.0	12.0	9.0	15.0	10.0	14.0	6.0	13.0
	250	1.0	6.0	7.0	3.0	10.0	2.0	4.0	11.0	12.0	9.0	15.0	8.0	13.0	5.0	14.0
	350	1.0	5.0	8.0	3.0	6.0	2.0	4.0	11.0	12.0	10.0	15.0	7.0	14.0	9.0	13.0
$\delta = 1.5$	15	2.0	3.0	10.0	1.0	7.0	4.0	5.0	12.5	11.0	8.0	15.0	9.0	14.0	6.0	12.5
	45	2.0	6.0	5.0	3.0	7.0	1.0	4.0	10.5	12.0	10.5	15.0	8.0	13.0	9.0	14.0
	90	1.0	3.0	6.0	5.0	7.0	2.0	4.0	11.0	12.0	10.0	15.0	8.0	14.0	9.0	13.0
	180	7.0	3.0	6.0	1.0	4.0	2.0	5.0	10.0	12.0	11.0	15.0	9.0	14.0	8.0	13.0
	250	4.0	3.0	6.0	1.0	9.0	2.0	5.0	10.0	12.0	11.0	15.0	7.0	13.0	8.0	14.0
	350	1.0	4.0	6.0	2.0	11.0	3.0	5.0	8.0	13.0	10.0	15.0	7.0	12.0	9.0	14.0
$\delta = 0.9$	15	1.0	2.0	5.0	3.0	8.0	6.0	4.0	11.5	11.5	9.0	15.0	7.0	14.0	10.0	13.0
	45	2.0	8.0	4.0	1.0	6.0	3.0	5.0	10.0	12.0	9.0	15.0	7.0	13.0	11.0	14.0
	90	2.0	8.0	5.0	1.0	7.0	6.0	3.5	10.0	12.0	11.0	15.0	3.5	13.0	9.0	14.0
	180	1.0	5.0	6.0	4.0	9.0	3.0	2.0	10.0	12.0	11.0	15.0	7.0	13.0	8.0	14.0
	250	1.0	2.0	7.0	3.0	8.0	4.0	6.0	10.0	12.0	11.0	15.0	5.0	14.0	9.0	13.0
	350	2.0	4.0	9.0	3.0	8.0	1.0	6.0	11.0	12.0	10.0	15.0	5.0	13.0	7.0	14.0
$\delta = 0.3$	15	3.0	7.0	6.0	4.0	8.0	2.0	1.0	11.0	12.0	10.0	15.0	5.0	13.5	9.0	13.5
	45	1.0	5.0	6.0	4.0	7.5	2.0	3.0	11.0	12.0	10.0	15.0	7.5	13.0	9.0	14.0
	90	4.0	7.0	6.0	2.0	5.0	1.0	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	180	1.0	8.0	5.0	2.0	7.0	4.0	3.0	11.0	13.0	10.0	15.0	9.0	14.0	6.0	12.0
	250	4.0	5.0	6.0	2.0	7.0	1.0	3.0	11.0	12.0	8.5	15.0	8.5	14.0	10.0	13.0
	350	1.0	6.0	7.0	3.0	4.0	2.0	5.0	10.0	12.0	11.0	15.0	9.0	13.0	8.0	14.0
$\sum$ Ranks		68.0	141.5	197.0	65.0	219.5	81.0	129.5	323.0	354.0	289.0	450.0	239.5	405.5	235.5	402.0
Overall Rank		2	5	6	1	7	3	4	11	12	10	15	9	14	8	13

7. Application of Real Data

We utilized data from Nigm et al. [45], which consists of the ordered failure times of 20 components. The data points are as follows: "0.0009, 0.004, 0.0142, 0.0221, 0.0261, 0.0418, 0.0473, 0.0834, 0.1091, 0.1252, 0.1404, 0.1498, 0.175, 0.2031, 0.2099, 0.2168, 0.2918, 0.3465, 0.4035, 0.6143". Table 14 compares several estimation methods using SRS and RSS for Data I. The estimates of different methods are evaluated using different metrics such as mean of the estimates, standard error (StEr), Kolmogorov-Smirnov distance (KSD), P-value for the KS test (PVKS), Cramer-von Mises statistic (CVMS), and Anderson-Darling statistic (ADS). Generally, the RSS estimates are higher than SRS for most methods, and RSS tends to have lower variability, indicated by smaller standard errors. Notably, MLE under RSS yields a much higher estimate (9.4622) compared to SRS (6.5493) with a lower KSD value, indicating a better fit. The KO exhibits a significant difference between SRS and RSS, especially in KSD and PVKS, suggesting variability in performance across the two sampling schemes. Additionally, MSLND has high variability under SRS compared to RSS, reflecting

Table 12. Partial and overall ranks for all estimation methods of our proposed model by RSS.

Parameter	n	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
$\delta = 3.5$	15	2.5	1.0	8.0	6.0	5.0	2.5	4.0	9.5	12.0	11.0	15.0	7.0	14.0	9.5	13.0
	45	1.0	3.0	5.0	7.0	6.0	2.0	4.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	90	1.0	2.0	5.0	7.0	6.0	4.0	3.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	180	1.0	4.0	3.0	5.0	6.0	7.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	250	2.0	1.0	7.0	5.0	6.0	4.0	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	350	2.0	1.0	5.0	6.5	6.5	4.0	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
$\delta = 2.5$	15	2.0	5.0	6.0	7.0	3.0	1.0	4.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	45	3.0	2.0	4.0	8.0	7.0	5.0	1.0	9.0	12.0	11.0	15.0	6.0	14.0	10.0	13.0
	90	2.0	1.0	7.0	6.0	3.0	5.0	4.0	8.0	12.0	10.5	15.0	9.0	13.0	10.5	14.0
	180	6.0	4.0	2.0	5.0	3.0	7.0	1.0	8.0	12.0	11.0	15.0	9.0	13.0	10.0	14.0
	250	2.5	1.0	6.0	7.0	2.5	4.0	5.0	9.0	11.0	12.0	15.0	8.0	14.0	10.0	13.0
	350	1.0	3.0	4.0	7.0	5.0	2.0	6.0	9.0	12.0	10.0	15.0	8.0	13.0	11.0	14.0
$\delta = 1.5$	15	2.0	4.0	7.0	5.0	3.0	6.0	1.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	45	1.0	2.0	6.0	7.0	3.0	5.0	4.0	8.0	12.0	11.0	15.0	9.0	14.0	10.0	13.0
	90	2.0	3.0	7.0	6.0	5.0	4.0	1.0	8.0	12.0	11.0	15.0	9.0	13.0	10.0	14.0
	180	3.0	1.0	6.0	7.0	5.0	4.0	2.0	9.0	12.0	11.0	15.0	8.0	13.0	10.0	14.0
	250	2.0	6.0	4.0	3.0	7.0	5.0	1.0	8.0	12.0	11.0	15.0	9.0	13.0	10.0	14.0
	350	1.0	5.0	2.0	6.0	3.0	7.0	4.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
$\delta = 0.9$	15	1.0	2.0	6.0	5.0	7.0	4.0	3.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	45	1.0	7.0	2.0	8.0	5.0	4.0	3.0	9.0	12.0	11.0	15.0	6.0	14.0	10.0	13.0
	90	1.0	4.0	3.0	7.0	6.0	5.0	2.0	8.0	12.0	11.0	15.0	9.0	13.0	10.0	14.0
	180	2.0	3.0	5.0	9.0	6.0	4.0	1.0	7.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	250	4.0	5.0	6.0	1.0	8.0	2.0	3.0	9.0	12.0	11.0	15.0	7.0	14.0	10.0	13.0
	350	1.0	5.0	3.0	6.0	8.0	4.0	2.0	10.0	12.0	11.0	15.0	7.0	13.0	9.0	14.0
$\delta = 0.3$	15	5.0	1.0	2.0	8.0	6.0	3.0	7.0	9.0	12.0	11.0	15.0	4.0	14.0	10.0	13.0
	45	2.0	1.0	5.0	8.0	6.0	4.0	3.0	9.0	12.0	11.0	15.0	7.0	13.0	10.0	14.0
	90	3.0	1.0	2.0	7.0	5.0	6.0	4.0	9.0	12.0	11.0	15.0	8.0	14.0	10.0	13.0
	180	2.0	1.0	5.0	8.0	4.0	6.0	3.0	11.0	12.0	10.0	15.0	7.0	13.0	9.0	14.0
	250	3.0	1.0	6.0	7.0	4.0	5.0	2.0	9.0	10.0	11.0	15.0	8.0	13.0	12.0	14.0
	350	3.0	6.0	4.0	5.0	1.0	8.0	2.0	9.0	14.0	10.0	15.0	7.0	11.0	12.0	13.0
$\sum$ Ranks		65.0	86.0	143.0	189.5	151.0	133.5	88.0	265.5	359.0	327.5	450.0	232.0	402.0	303.0	405.0
Overall Rank		1	2	5	7	6	4	3	9	12	11	15	8	13	10	14

a better reliability of RSS in this context. The overall comparison shows that RSS often yields higher estimates with improved precision, highlighting its effectiveness for parameter estimation compared to SRS for Data I.

Figure 7 illustrates the ML estimate for the δ parameter in Data I, showing two plots of the likelihood function $l(\delta)$ against δ. The left plot highlights the peak likelihood at approximately δ = 8, suggesting this is the optimal estimate, with the blue dot marking this maximum value. The right plot shows a different perspective, emphasizing the decrease in likelihood as δ moves away from the optimal value, further validating the estimated parameter. Figure 8 provides several graphical representations of the MoD for Data I. The violin plot reveals the spread and density of the data, while the kernel density plot depicts the overall distribution shape. The estimated CDF and PDF plots compare the fitted MoD against the empirical data, showing reasonable alignment. Finally, the P-P plot demonstrates a close match between empirical and theoretical quantiles, with points largely following the diagonal, indicating a good fit of the MoD to Data I.

Table 13. Numerical values for MSE of SRS divided by MSE of RSS for all estimators.

n	MLE	ADE	CVME	MPSE	OLSE	RTADE	WLSE	LTADE	MSADE	MSALDE	ADSOE	KE	MSSDE	MSSLDE	MSLNDE
								$\delta = 3.5$							
15	3.03863	3.54737	3.06585	2.23535	3.51640	2.63316	3.24892	2.96302	1.61405	2.04042	1.22396	2.73791	1.54616	2.15697	1.53088
45	2.80195	3.22448	3.65070	2.13560	3.20189	2.77444	2.93886	3.06915	1.57286	1.69542	1.12960	2.96483	1.63293	1.59310	1.62551
90	2.85217	2.84530	3.13038	2.30146	2.98802	2.58170	3.05683	3.42629	1.51742	1.75335	1.35340	3.02640	1.56982	1.65098	1.50950
180	2.83044	2.96484	3.09903	2.12341	2.74300	2.31041	2.91722	2.95166	1.40412	1.60196	1.35330	2.79731	1.71134	1.59522	1.62261
250	2.83044	2.96484	3.09903	2.12341	2.74300	2.31041	2.91722	2.95166	1.40412	1.60196	1.35330	2.79731	1.71134	1.59522	1.62261
350	3.02174	2.97368	3.08901	2.15736	2.79576	2.80870	3.36364	2.77244	1.59340	1.65175	1.11319	2.64556	1.41301	1.60433	1.60315
								$\delta = 2.5$							
15	3.07349	3.04291	2.77593	2.07511	3.53376	3.44372	3.00371	2.96524	2.00898	1.84709	1.24273	3.18836	1.56480	1.81602	1.56745
45	2.80359	3.19504	3.03057	2.22876	3.00861	2.61480	2.67403	2.55611	1.66709	1.72589	1.34954	3.04722	1.79803	2.02660	1.80551
90	3.18499	3.05091	2.90090	2.16999	3.14431	2.85485	3.26646	3.37612	1.76859	1.86882	1.21494	2.76322	1.59911	1.99436	1.61509
180	2.37538	2.75369	3.38676	2.45861	3.20405	2.29466	2.90678	2.99101	1.80118	1.69819	1.08416	2.61820	1.57939	1.54584	1.60330
250	2.37538	2.75369	3.38676	2.45861	3.20405	2.29466	2.90678	2.99101	1.80118	1.69819	1.08416	2.61820	1.57939	1.54584	1.60330
350	2.64561	3.10702	3.06970	2.14206	2.82738	2.54296	2.57714	2.83945	1.53029	1.82616	1.36540	2.57520	1.82258	1.81538	1.42629
								$\delta = 1.5$							
15	3.20062	2.99371	3.59605	2.18327	3.45475	2.98245	3.88984	3.50415	1.73990	1.57346	1.32802	3.34374	1.98406	2.10937	1.46571
45	2.92398	3.39478	2.85990	2.45949	3.16474	2.34662	2.91076	2.98004	1.83759	2.02011	1.09142	2.68376	1.55042	1.80029	1.68656
90	2.61538	3.03039	2.86298	2.66901	3.28378	2.76731	3.21083	3.06754	1.78947	1.69988	1.35270	2.76674	1.47793	1.70883	1.62924
180	3.27528	3.13939	2.81429	2.32353	2.84422	2.53723	3.32222	3.11790	1.67634	1.65820	1.26647	3.28638	1.52852	1.76504	1.43647
250	3.27528	3.13939	2.81429	2.32353	2.84422	2.53723	3.32222	3.11790	1.67634	1.65820	1.26647	3.28638	1.52852	1.76504	1.43647
350	2.79775	2.66019	3.26087	2.33654	3.72165	2.55963	2.68627	2.83761	1.63406	1.64467	1.07398	2.86916	1.41486	1.88000	1.45820
								$\delta = 0.9$							
15	2.66809	3.16642	2.82779	2.70984	3.72909	3.26939	3.37249	3.49802	1.62520	2.09937	1.37030	3.11967	1.78699	2.50046	1.45874
45	3.18957	2.85496	3.11894	2.29779	2.88976	2.89157	2.91968	2.98561	1.66616	1.68724	1.26069	2.89615	1.53590	1.99024	1.88987
90	2.78947	3.52101	3.24590	2.31618	2.98496	2.72180	3.13274	2.95862	1.59184	1.75097	1.64341	2.17219	1.62690	1.87444	1.48571
180	2.72414	2.91803	2.96875	2.32432	3.12308	2.65625	2.91071	3.02941	1.59524	1.82500	1.11678	2.67123	1.41294	2.01010	1.45408
250	2.72414	2.91803	2.96875	2.32432	3.12308	2.65625	2.91071	3.02941	1.59524	1.82500	1.11678	2.67123	1.41294	2.01010	1.45408
350	3.23077	2.63636	3.40625	2.42857	2.76923	2.56250	3.21429	2.90000	1.51064	1.62319	1.41980	2.44737	1.73267	1.72131	1.48739
								$\delta = 0.3$							
15	3.07792	3.45070	3.20779	2.44186	2.92857	2.98684	2.70000	3.04808	1.78610	1.89051	1.16216	3.32877	1.95197	2.37273	1.99563
45	2.84000	2.91667	3.11111	2.41379	2.79310	2.48148	2.73077	2.81250	1.79687	1.95745	1.22396	2.64516	2.10959	1.79070	1.70238
90	3.00000	3.54545	3.25000	2.21429	3.00000	2.46154	2.66667	2.62500	1.67647	1.76000	1.35106	2.86667	1.63415	1.86957	1.60526
180	2.83333	3.16667	2.57143	2.42857	2.71429	2.57143	2.57143	2.87500	1.64706	1.64286	1.25000	2.85714	1.73684	1.63636	1.38095
250	2.83333	3.16667	2.57143	2.42857	2.71429	2.57143	2.57143	2.87500	1.64706	1.64286	1.25000	2.85714	1.73684	1.63636	1.38095
350	2.66667	3.33333	3.33333	3.00000	3.33333	2.00000	3.00000	3.00000	1.66667	2.16667	2.04762	3.33333	1.36364	1.83333	1.58333

Data II: we apply the real-life test data set, which reported in Ahmed et al. [46]. The data set which contains 25 electrodes (segments cut from bars). This data set is as follows: 2 3 8 21 31 64 69 104 119 144 160 168 191 203 211 221 264 298 303 317 320 348 360 369 446. Table 15 compares various estimation methods using SRS and RSS for Data II. The estimates of different methods are evaluated using StEr, KSD, PVKS, CVMS, and ADS. Generally, RSS yields lower standard errors compared to SRS, indicating improved precision of estimates under RSS. For instance, the ML method has an estimate of 0.0123 for SRS and 0.0133 for RSS, with RSS showing a slightly lower standard error, suggesting a more reliable estimate. Notably, methods such as MSLND and MSSLD show substantial differences between SRS and RSS, particularly in the goodness-of-fit metrics (KSD and PVKS), indicating that the performance of these methods varies significantly depending on the sampling technique. This comparison highlights the effectiveness of RSS in improving the reliability and precision of parameter estimation for most methods.

Table 14. Different estimation methods by SRS, and RSS: Data I

	SRS						RSS					
	Estimates	StEr	KSD	PVKS	CVMS	ADS	Estimates	StEr	KSD	PVKS	CVMS	ADS
ML	6.5493	1.3488	0.1938	0.5617	0.0837	0.4508	9.4622	1.5492	0.1247	0.9737	0.0260	0.1956
AD	6.1856	1.3601	0.1696	0.7202	0.0847	0.4566	8.2334	1.8915	0.1620	0.8259	0.0262	0.1954
CVM	5.9644	2.8595	0.1545	0.8148	0.0853	0.4604	10.2505	6.6478	0.1233	0.9766	0.0259	0.1959
MPS	6.1084	4.9729	0.1644	0.7541	0.0849	0.4579	8.0662	6.9088	0.1637	0.7957	0.0262	0.1954
OLS	5.8958	2.8516	0.1497	0.8419	0.0856	0.4616	9.4618	5.7257	0.1247	0.9737	0.0260	0.1956
WLS	5.9977	0.3099	0.1568	0.8011	0.0852	0.4598	9.4595	0.6457	0.1248	0.9736	0.0260	0.1956
RELD	6.2353	1.7150	0.1730	0.6983	0.0846	0.4558	6.6046	2.2537	0.1602	0.7149	0.0266	0.1960
LTAD	5.8676	0.1641	0.1478	0.8526	0.0856	0.4621	5.8676	0.1642	0.1327	0.8593	0.0269	0.1967
MSAD	5.8676	0.1641	0.1478	0.8526	0.0856	0.4621	9.1929	0.2293	0.1328	0.9541	0.0260	0.1955
MSALD	6.1311	1.8001	0.1659	0.7442	0.0849	0.4576	5.8676	0.0410	0.1603	0.7522	0.0269	0.1967
MSLND	4.9453	11.3540	0.2065	0.4821	0.0887	0.4800	7.1964	27.2472	0.1947	0.6201	0.0264	0.1956
MSSD	4.9828	8.4143	0.2029	0.5042	0.0886	0.4792	7.1933	19.8673	0.1948	0.6194	0.0264	0.1956
MSSLD	6.7948	0.7761	0.2095	0.4639	0.0830	0.4471	2.4287	0.3390	0.1964	0.5053	0.0299	0.2099
ADSO	9.2042	2.5075	0.3428	0.0446	0.0779	0.4173	14.2367	3.8369	0.3352	0.0502	0.0710	0.3784
KO	4.0408	0.0959	0.3011	0.1057	0.0924	0.5016	4.6675	0.1115	0.3008	0.1270	0.0275	0.1989

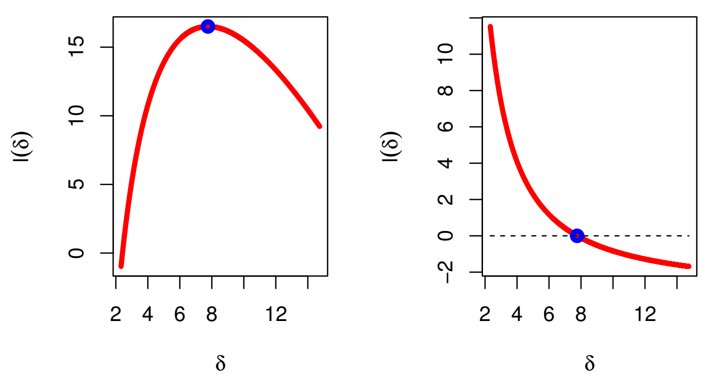

Figure 7. ML estimate checked of δ parameter for Data I

Figure 9 illustrates the ML estimate for the δ parameter in Data II, showing two plots of the likelihood function $l(\delta)$ against δ. The left plot highlights the peak likelihood at approximately $\delta =$ 0.015, suggesting this is the optimal estimate, with the blue dot marking this maximum value. The right plot shows a different perspective, emphasizing the decrease in likelihood as δ moves away from the optimal value, further validating the estimated parameter. Figure 10 provides several graphical representations of the MoD for Data II. The violin plot reveals the spread and density of the data, while the kernel density plot depicts the overall distribution shape. The estimated CDF and PDF plots compare the fitted MoD against the empirical data, showing reasonable

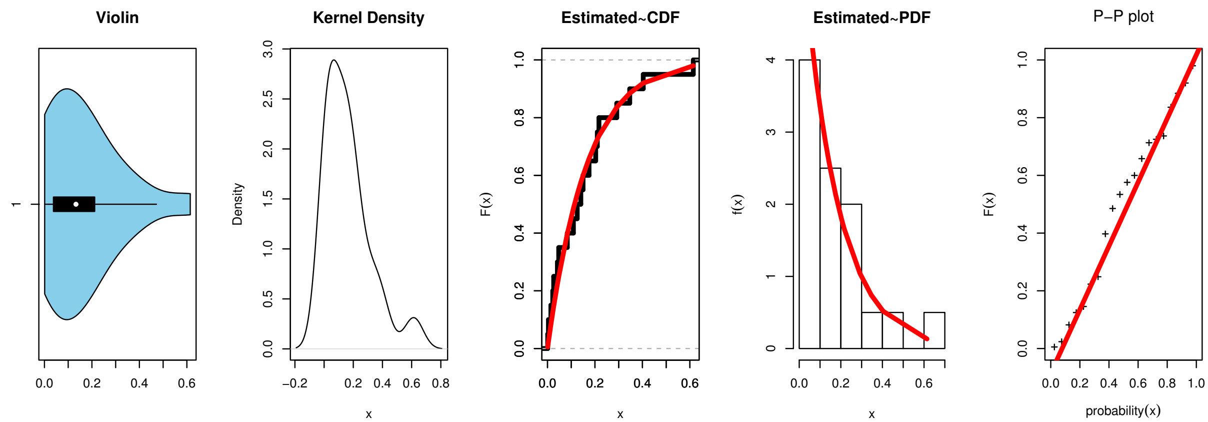

Figure 8. Some plots of MoD for Data I

Table 15. Different estimation methods by SRS, and RSS: Data II

	SRS						RSS					
	Estimates	StEr	KSD	PVKS	CVMS	ADS	λ	StEr	KSD	PVKS	CVMS	ADS
ML	0.0123	0.0018	0.3267	0.0813	0.2456	1.4160	0.0133	0.0015	0.2417	0.2949	0.2268	1.2880
AD	0.0113	0.0017	0.2855	0.1733	0.2463	1.4189	0.0124	0.0019	0.1919	0.5734	0.2285	1.2970
CVM	0.0108	0.0035	0.2612	0.2578	0.2475	1.4239	0.0109	0.0041	0.1706	0.7141	0.2295	1.3024
MPS	0.0119	0.0070	0.3327	0.0722	0.2455	1.4156	0.0134	0.0079	0.2257	0.3728	0.2273	1.2909
OLS	0.0107	0.0035	0.2607	0.2598	0.2472	1.4227	0.0113	0.0042	0.1760	0.6785	0.2297	1.3035
WLS	0.0111	0.0004	0.2743	0.2091	0.2465	1.4196	0.0121	0.0005	0.1805	0.6485	0.2289	1.2990
RELD	0.0112	0.0022	0.2615	0.2564	0.2477	1.4249	0.0107	0.0022	0.1860	0.6123	0.2287	1.2980
LTAD	0.0105	0.0112	0.4967	0.0012	0.2512	1.4396	0.0074	0.0041	0.1866	0.6085	0.2301	1.3056
MSAD	0.0134	0.0054	0.3568	0.0439	0.2491	1.4310	0.0092	0.0073	0.2960	0.1165	0.2249	1.2776
MSALD	0.0134	0.0012	0.3345	0.0697	0.2455	1.4154	0.0135	0.0030	0.2960	0.1165	0.2249	1.2776
MSLND	0.0102	0.0137	0.2983	0.1384	0.2461	1.4180	0.0126	0.0206	0.2075	0.4757	0.2308	1.3096
MSSD	0.0102	0.0103	0.2974	0.1407	0.2461	1.4181	0.0126	0.0153	0.2065	0.4819	0.2308	1.3094
MSSLD	0.0154	0.0009	0.3967	0.0178	0.2443	1.4105	0.0153	0.0008	0.3723	0.0225	0.2218	1.2612
ADLTSO	0.1848	0.0509	0.8523	0.0000	0.1639	0.9182	0.1134	0.0201	0.8067	0.0000	0.1308	0.9075
KO	0.0074	0.0038	0.4336	0.0071	0.2503	1.4357	0.0082	0.0048	0.4140	0.0077	0.2376	1.3460

alignment. Finally, the P-P plot demonstrates a close match between empirical and theoretical quantiles, with points largely following the diagonal, indicating a good fit of the MoD to Data II.

Data III: The breaking strength data of jute fiber with a gauge length of 10 mm shows notable variability, which can be analyzed through measures of central tendency and dispersion. The are as follows: 693.73, 704.66, 323.83, 778.17, 123.06, 637.66, 383.43, 151.48, 108.94, 50.16, 671.49, 183.16, 257.44, 727.23, 291.27, 101.15, 376.42, 163.40, 141.38, 700.74, 262.90, 353.24, 212.13, 303.90, 506.60, 530.55, 177.25, 422.11, 43.93, 590.48". This data set has been discussed by [47]. The average breaking strength provides insight into the typical value, while the median gives a robust central point less affected by outliers. The standard deviation is crucial for understanding the spread

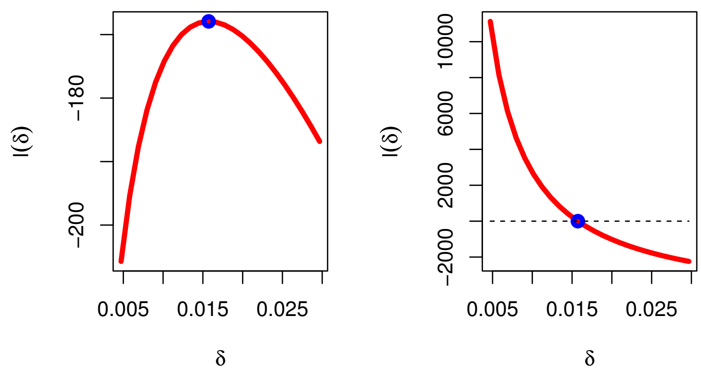

Figure 9. ML estimate checked of δ parameter for Data II

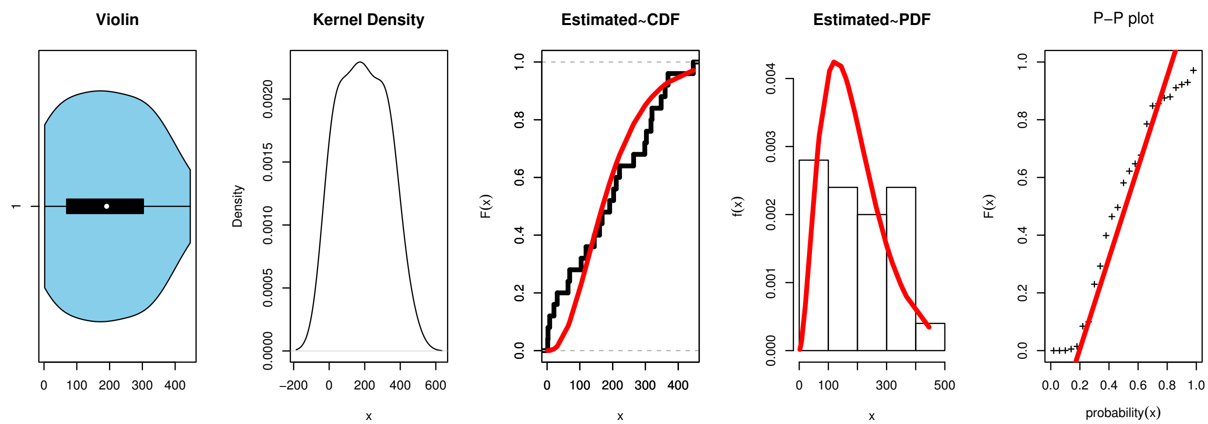

Figure 10. Some plots of MoD for Data II

of the values, indicating how much the individual strengths deviate from the average. A large standard deviation would suggest considerable inconsistency in fiber strength, which could be a concern for reliability in applications. The range, calculated as the difference between the maximum and minimum values, also highlights the extent of variation in fiber strength, giving a sense of the best and worst performance among the samples. This type of analysis is essential for assessing the quality and reliability of jute fibers for practical use, as it helps identify any significant inconsistencies that may impact their performance.

Table 16 provides a comparison of statistical estimation methods under two sampling techniques: SRS and RSS. The methods compared include the fifteen suggested methods with several metrics to assess their performance. These metrics measure estimation accuracy, variability, and goodness-of-fit. From the table, we observe differences in how estimation methods perform under SRS and RSS, with RSS generally resulting in slightly higher parameter estimates and smaller standard errors, which suggests better precision compared to SRS. Notably, the methods ML, OLS, and WLS maintain relatively consistent estimates between SRS and RSS, indicating their robustness across different sampling schemes.

The goodness-of-fit metrics such as KSD, PVKS, CVM, and AD show variation across methods, emphasizing that some estimation methods are more suitable than others depending on the specific sampling approach. For instance, KO has a noticeably smaller estimate compared to other methods but a higher KSD, indicating poorer fit quality. The MSALD method, while showing improved KSD values under RSS, still has substantial differences between SRS and RSS, implying variability in its performance. Methods like RELD and LTAD demonstrate better stability in estimates and good fit metrics across both sampling techniques, making them reliable choices depending on the context. Overall, this comparative analysis highlights the advantages of RSS in terms of reduced standard error and the importance of selecting appropriate estimation methods to achieve optimal statistical efficiency and accuracy.

Table 16. Different estimation methods by SRS, and RSS: Data III

	SRS						RSS					
	Estimates	StEr	KSD	PVKS	CVMS	ADS	Estimates	StEr	KSD	PVKS	CVMS	ADS
ML	0.0059	0.0009	0.1808	0.6467	0.1575	0.9462	0.0064	0.0007	0.1723	0.6540	0.0892	0.5683
AD	0.0056	0.0008	0.1924	0.5703	0.1577	0.9470	0.0061	0.0009	0.1908	0.5974	0.0894	0.5698
CVM	0.0055	0.0017	0.1998	0.5233	0.1579	0.9482	0.0057	0.0021	0.1967	0.6074	0.0895	0.5701
MPS	0.0058	0.0033	0.1686	0.7273	0.1575	0.9462	0.0064	0.0037	0.1523	0.7419	0.0892	0.5687
OLS	0.0055	0.0017	0.2032	0.5020	0.1578	0.9480	0.0058	0.0021	0.1949	0.6191	0.0895	0.5702
WLS	0.0057	0.0002	0.1763	0.6765	0.1576	0.9470	0.0061	0.0002	0.1620	0.6801	0.0893	0.5690
RELD	0.0057	0.0011	0.1791	0.6580	0.1580	0.9486	0.0056	0.0012	0.1620	0.6813	0.0893	0.5692
LTAD	0.0056	0.0038	0.1882	0.5979	0.1568	0.9430	0.0077	0.0059	0.1735	0.6154	0.0894	0.5696
MSAD	0.0053	0.0034	0.2172	0.4192	0.1568	0.9430	0.0077	0.0060	0.2035	0.5394	0.0896	0.5708
MSALD	0.0049	0.0007	0.2667	0.1971	0.1572	0.9450	0.0069	0.0010	0.2552	0.2049	0.0899	0.5728
MSLND	0.0063	0.0092	0.2225	0.3901	0.1580	0.9486	0.0056	0.0075	0.2007	0.5815	0.0890	0.5668
MSSD	0.0063	0.0068	0.2217	0.3944	0.1579	0.9484	0.0057	0.0060	0.1988	0.5935	0.0890	0.5669
MSSLD	0.0054	0.0005	0.2110	0.4549	0.1577	0.9472	0.0060	0.0004	0.1923	0.6360	0.0895	0.5706
ADSO	0.0036	0.0002	0.4685	0.0015	0.0885	0.5637	0.0073	0.0008	0.3138	0.0823	0.0791	0.5580
KO	0.0033	0.0024	0.5254	0.0002	0.1596	0.9567	0.0036	0.0028	0.4673	0.0029	0.0913	0.5821

Figure 11 illustrates the Maximum Likelihood Estimation (MLE) process for the δ parameter of Data III. The left plot shows the likelihood function $l(\delta)$ against δ, with a clear peak at around $\delta = 0.008$, indicated by a blue dot, representing the point of maximum likelihood. The right plot further supports this by showing the behavior of the likelihood function, which decreases significantly away from the estimated value, confirming the optimality of the estimate. In Figure 12, multiple plots assess the MoD for Data III. The violin plot and kernel density estimate provide insights into the data's spread and density. The estimated CDF and PDF plots show the fit of the MoD compared to the empirical data, with a generally good alignment, implying a reasonable fit. Lastly, the P-P plot displays the consistency between theoretical and empirical quantiles, with most points following the 45-degree line, further indicating that the MoD fits the data well.

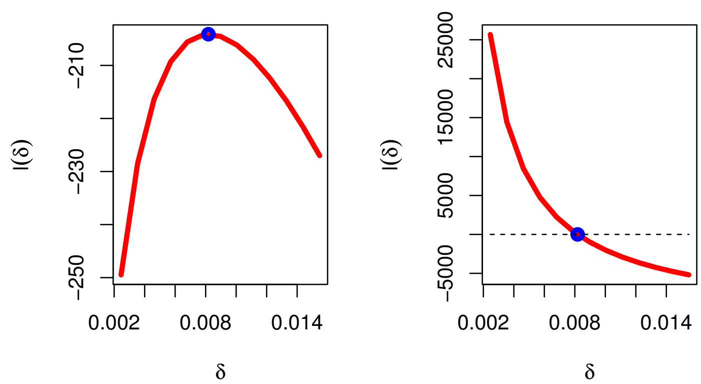

Figure 11. ML estimate checked of δ parameter for Data III

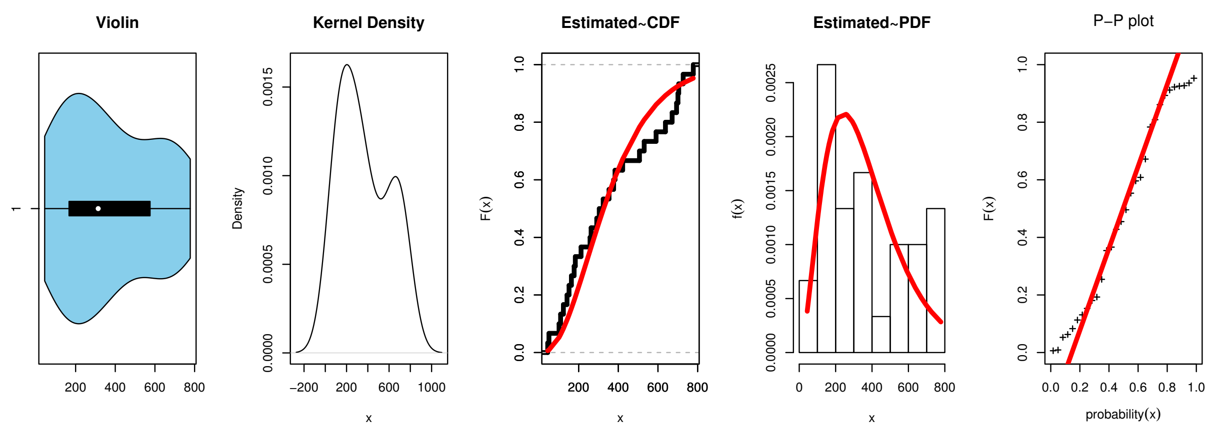

Figure 12. Some plots of MoD for Data III

8. Summary and Conclusion

Inadequate selection of the sampling procedure can lead to inaccurate estimation and significantly affect the statistical validity of the sampled data. Typically, in difficult data gathering situations, the RSS is more effective and requires fewer measurements to provide the required degree of precision. A modern mixed lifetime model that has been shown to be successful in simulating a range of real-world datasets is the MoD. In this study, a number of mathematical characteristics that have not been previously mentioned, such as quantiles, upper and lower IMs, SO, and extropy measurements. This paper explores estimating the parameter of the MoD using RSS in combination with fifteen conventional estimation methods. A simulation scenario is run to evaluate the performance of the estimation strategies for a variety of sample sizes under SRS and RSS in the case of perfect ranking. The optimal estimation strategy is identified by displaying the partial and total ranks of different estimates. Simulation results show that the ML and MSP approaches consistently perform better than other techniques when assessing the estimated quality for both RSS and SRS. The different estimates generated via RSS are more efficient than those derived with SRS, as seen by the lower precision measure values, including MSE and other metrics. One important finding from the simulation results is that the supplied accuracy measures decrease for all parameter combinations as the sample size grows. This indicates the consistency of all estimating techniques, a fundamental property for accurate parameter estimation. The superiority of the RSS design over SRS has been further validated through empirical analyses using three real-world datasets. Across fifteen different estimation techniques, we consistently find that RSS methods produce slightly larger parameter estimates and have smaller standard errors, indicating greater precision. Our research of actual data shows that the variety in metrics indicates that the selection of estimating techniques has a considerable impact on the goodness of fit. This emphasizes how crucial it is to choose the right techniques depending on the particular sampling strategy.

Authors' Contributions: The authors have worked equally to write and review the manuscript.

Data Availability Statement: The data that supports the findings of this study are available within the article.

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References

[1] K. Pearson, III. Contributions to the Mathematical Theory of Evolution, Philos. Trans. R. Soc. Lond. 185 (1894), 71–110. https://doi.org/10.1098/rsta.1894.0003.

[2] D.V. Lindley, Fiducial Distributions and Bayes' Theorem, J. R. Stat. Soc. Ser. B: Stat. Methodol. 20 (1958), 102–107. https://doi.org/10.1111/j.2517-6161.1958.tb00278.x.

[3] S. Sen, S.S. Maiti, N. Chandra, The Xgamma Distribution: Statistical Properties and Application, J. Mod. Appl. Stat. Methods 15 (2016), 774–788. https://doi.org/10.22237/jmasm/1462077420.

[4] P. Jordanova, J. Dušek, M. Stehlík, Microergodicity Effects on Ebullition of Methane Modelled by Mixed Poisson Process with Pareto Mixing Variable, Chemom. Intell. Lab. Syst. 128 (2013), 124–134. https://doi.org/10.1016/j.chemolab.2013.08.006.

[5] R. Shanker, Shanker Distribution and Its Applications, Int. J. Stat. Appl. 5(2015), 338–348.

[6] M.M.E. Abd El-Monsef, M.M. Abd El-Raouf, Rayleigh-Half Normal Distribution for Modeling Tooth Movement Data, J. Biopharm. Stat. 30 (2019), 481–494. https://doi.org/10.1080/10543406.2019.1684312.

[7] R. Shanker, Rama Distribution and Its Application, Int. J. Stat. Appl. 7 (2017), 26–35.

[8] A.S. Hassan, A.M. Abd-Elfattah, M.M. Hassan, Bayesian Analysis for Mixture of Burr XII and Burr X Distributions, Far East J. Math. Sci. 103 (2018), 1031–1041. https://doi.org/10.17654/ms103061031.

[9] C.K. Onyekwere, O.J. Obulezi, Chris-Jerry Distribution and Its Applications, Asian J. Probab. Stat. 20 (2022), 16–30. https://doi.org/10.9734/ajpas/2022/v20i130480.

[10] M.M.E. Abd El-Monsef, Erlang Mixture Distribution with Application on COVID-19 Cases in Egypt, Int. J. Biomath. 14 (2021), 2150015. https://doi.org/10.1142/s1793524521500157.
[11] A. Mustafa, M.M.E. Abd El-Monsef, The Weighted Monsef Distribution, Afr. J. Math. Comput. Sci. Res. 13 (2020), 74–84.
[12] M.M.E. Abd El-Monsef, N.M. Sohsah, W.A. Hassanein, Unit Monsef Distribution with Regression Model, Asian J. Probab. Stat. 15 (2021), 330–340. https://doi.org/10.9734/ajpas/2021/v15i430384.
[13] M.M.E. Abd El-Monsef, E.I. Soliman, M.A. El-Qurashi, Stress-Strength Reliability for Monsef Distribution, J. Stat. Appl. Probab. 13 (2024), 1107–1126. https://doi.org/10.18576/jsap/130320.
[14] G. McIntyre, A Method for Unbiased Selective Sampling, Using Ranked Sets, Aust. J. Agric. Res. 3 (1952), 385–390. https://doi.org/10.1071/ar9520385.
[15] K. Takahasi, K. Wakimoto, On Unbiased Estimates of the Population Mean Based on the Sample Stratified by Means of Ordering, Ann. Inst. Stat. Math. 20 (1968), 1–31. https://doi.org/10.1007/bf02911622.
[16] G.P. Patil, A.K. Sinha, C. Taille, Relative Precision of Ranked Set Sampling: A Comparison with the Regression Estimator, Environmetrics 4 (1993), 399–412. https://doi.org/10.1002/env.3170040404.
[17] M. Mahdizadeh, E. Zamanzade, Efficient Body Fat Estimation Using Multistage Pair Ranked Set Sampling, Stat. Methods Med. Res. 28 (2017), 223–234. https://doi.org/10.1177/0962280217720473.
[18] E.G. Koçyiğit, C. Kadilar, Information Theory Approach to Ranked Set Sampling and New Sub-Ratio Estimators, Commun. Stat. - Theory Methods 53 (2022), 1331–1353. https://doi.org/10.1080/03610926.2022.2100910.
[19] A.S. Hassan, N. Alsadat, M. Elgarhy, H. Ahmad, H.F. Nagy, On Estimating Multi- Stress Strength Reliability for Inverted Kumaraswamy Under Ranked Set Sampling with Application in Engineering, J. Nonlinear Math. Phys. 31 (2024), 30. https://doi.org/10.1007/s44198-024-00196-y.
[20] H. Muttlak, W. Al-Sabah, Statistical Quality Control Based on Ranked Set Sampling, J. Appl. Stat. 30 (2003), 1055–1078. https://doi.org/10.1080/0266476032000076173.
[21] H. NAGY, A. HASSAN, Reliability Estimation in Multicomponent Stress-Strength for Generalized Inverted Exponential Distribution Based on Ranked Set Sampling, Gazi Univ. J. Sci. 35 (2022), 314–331. https://doi.org/10.35378/gujs.760469.
[22] X. He, W. Chen, W. Qian, Maximum Likelihood Estimators of the Parameters of the Log-Logistic Distribution, Stat. Pap. 61 (2018), 1875–1892. https://doi.org/10.1007/s00362-018-1011-3.
[23] A. Adatia, Estimation of Parameters of the Half-Logistic Distribution Using Generalized Ranked Set Sampling, Comput. Stat. Data Anal. 33 (2000), 1–13. https://doi.org/10.1016/s0167-9473(99)00035-3.
[24] H.J. Khamnei, S. Abusaleh, Estimation of Parameters in the Generalized Logistic Distribution Based on Ranked Set Sampling, Int. J. Nonlinear Sci. 24 (2017), 154–160.
[25] H. Fei, B.K. Sinha, Z. Wu, EStimation of Parameters in Two-Parameter Weibull and Extreme-Value Distributions Using a Ranked Set Sample, J. Stat. Res. 28 (1994), 149–161.
[26] M. Esemen, S. Gürler, Parameter Estimation of Generalized Rayleigh Distribution Based on Ranked Set Sample, J. Stat. Comput. Simul. 88 (2017), 615–628. https://doi.org/10.1080/00949655.2017.1398256.
[27] F.G. Akgül, Ş. Acıtaş, B. Şenoğlu, Inferences on Stress–Strength Reliability Based on Ranked Set Sampling Data in Case of Lindley Distribution, J. Stat. Comput. Simul. 88 (2018), 3018–3032. https://doi.org/10.1080/00949655.2018.1498095.
[28] R. Bantan, A.S. Hassan, M. Elsehetry, B.M.G. Kibria, Half-Logistic Xgamma Distribution: Properties and Estimation Under Censored Samples, Discret. Dyn. Nat. Soc. 2020 (2020), 9136513. https://doi.org/10.1155/2020/9136513.
[29] V.C. Pedroso, C.A. Taconeli, S.R. Giolo, Estimation Based on Ranked Set Sampling for the Two-Parameter Birnbaum–Saunders Distribution, J. Stat. Comput. Simul. 91 (2020), 316–333. https://doi.org/10.1080/00949655.2020.1814287.

[30] M.M. Yousef, A.S. Hassan, A.H. Al-Nefaie, E.M. Almetwally, H.M. Almongy, Bayesian Estimation Using MCMC Method of System Reliability for Inverted Topp–Leone Distribution Based on Ranked Set Sampling, Mathematics 10 (2022), 3122. https://doi.org/10.3390/math10173122.

[31] A.S. Hassan, R.S. Elshaarawy, H.F. Nagy, Parameter Estimation of Exponentiated Exponential Distribution Under Selective Ranked Set Sampling, Stat. Transit. New Ser. 23 (2022), 37–58. https://doi.org/10.2478/stattrans-2022-0041.

[32] H.F. Nagy, A.I. Al-Omari, A.S. Hassan, G.A. Alomani, Improved Estimation of the Inverted Kumaraswamy Distribution Parameters Based on Ranked Set Sampling with an Application to Real Data, Mathematics 10 (2022), 4102. https://doi.org/10.3390/math10214102.

[33] N. Alsadat, A.S. Hassan, A.M. Gemeay, C. Chesneau, M. Elgarhy, Different Estimation Methods for the Generalized Unit Half-Logistic Geometric Distribution: Using Ranked Set Sampling, AIP Adv. 13 (2023), 085230. https://doi.org/10.1063/5.0169140.

[34] A. Hassan, R. Elshaarawy, H. Nagy, Estimation Study of Multicomponent Stress-Strength Reliability Using Advanced Sampling Approach, Gazi Univ. J. Sci. 37 (2024), 465–481. https://doi.org/10.35378/gujs.1132770.

[35] A.S. Hassan, N. Alsadat, M. Elgarhy, C. Chesneau, R. Elmorsy Mohamed, Different Classical Estimation Methods Using Ranked Set Sampling and Data Analysis for the Inverse Power Cauchy Distribution, J. Radiat. Res. Appl. Sci. 16 (2023), 100685. https://doi.org/10.1016/j.jrras.2023.100685.

[36] N. Alsadat, A.S. Hassan, A.M. Gemeay, C. Chesneau, M. Elgarhy, Different Estimation Methods for the Generalized Unit Half-Logistic Geometric Distribution: Using Ranked Set Sampling, AIP Adv. 13 (2023), 085230. https://doi.org/10.1063/5.0169140.

[37] S.A. Alyami, A.S. Hassan, I. Elbatal, N. Alotaibi, A.M. Gemeay, et al., Estimation Methods Based on Ranked Set Sampling for the Arctan Uniform Distribution with Application, AIMS Math. 9 (2024), 10304–10332. https://doi.org/10.3934/math.2024504.

[38] N. Alsadat, A.S. Hassan, M. Elgarhy, A. Johannssen, A.M. Gemeay, Estimation Methods Based on Ranked Set Sampling for the Power Logarithmic Distribution, Sci. Rep. 14 (2024), 17652. https://doi.org/10.1038/s41598-024-67693-4.

[39] M. Shaked, J.G. Shanthikumar, Stochastic Orders, Springer, 2007.

[40] F. Lad, G. Sanfilippo, G. Agrò, Extropy: Complementary Dual of Entropy, Stat. Sci. 30 (2015), 40–58. https://doi.org/10.1214/14-sts430.

[41] N. Balakrishnan, F. Buono, M. Longobardi, On Weighted Extropies, Commun. Stat. - Theory Methods 51 (2020), 6250–6267. https://doi.org/10.1080/03610926.2020.1860222.

[42] D.A. Wolfe, Ranked Set Sampling: Its Relevance and Impact on Statistical Inference, ISRN Probab. Stat. 2012 (2012), 568385. https://doi.org/10.5402/2012/568385.

[43] B.C. Arnold, N. Balakrishnan, H.N. Nagaraja, A First Course in Order Statistics, SIAM, (1992).

[44] Cheng, R.C.H.; Amin, N.A.K. Maximum Product-of-Spacings Estimation With Applications to the Log-Normal Distribution, Technical Report, Department of Mathematics, University of Wales, 1979.

[45] A.M. Nigm, E.K. Al-Hussaini, Z.F. Jaheen, Bayesian One-Sample Prediction of Future Observations Under Pareto Distribution, Statistics 37 (2003), 527–536. https://doi.org/10.1080/0233188031000159883 7.

[46] E.A. Ahmed, Z. Ali Alhussain, M.M. Salah, H. Haj Ahmed, M.S. Eliwa, Inference of Progressively Type-Ii Censored Competing Risks Data from Chen Distribution with an Application, J. Appl. Stat. 47 (2020), 2492–2524. https://doi.org/10.1080/02664763.2020.1815670.

[47] B. Saraçoğlu, I. Kinaci, D. Kundu, On Estimation of $R = P(Y < X)$ for Exponential Distribution Under Progressive Type-II Censoring, J. Stat. Comput. Simul. 82 (2012), 729–744. https://doi.org/10.1080/00949655.2010.551772.