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A Novel Lag Window for Spectrum Estimation of the Ornstein-Uhlenbeck Process

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Abstract. In order to forecast and process the behavior of noisy data, spectral analysis is a crucial area of study in data analysis and interpretation. The goal of this study is to identify the optimal lag window to estimate the continuous-time Ornstein-Uhlenbeck (OU) process's spectrum. The equivalent difference equation of the OU process was derived, and a consistent estimate of the spectral density function (SDF) was calculated using the most prominent lag window functions in the different parameter cases and time interval segmentation. A parameterized novel lag window (NLW) was proposed. The parameters can be changed to control the kurtosis and skewness of the NLW curve and reduce the influence of the tails of the estimated autocorrelation function on the consistent estimate of the SDF. The simulation results of comparing the SDF and the consistent estimate of the SDF with lag windows showed that the proposed NLW outperformed all other lag windows in estimating the spectrum of the OU process in all parameter cases and in all time-interval segmentation. The promising results of NLW can be used in signal processing and spectral analysis of phenomena subject to the influence of noise.

1. Introduction

Stochastic processes are one of the most important stochastic dynamical systems that describe many engineering, medical, financial, and natural phenomena [4, 6, 11]. An important mean-reverting, continuous-time stochastic process is the Ornstein-Uhlenbeck (OU) process, which describes the velocity of a massive Wiener particle under the influence of friction. Common applications of the OU process include modeling interest rates and volatility in financial mathematics [17], studying the motion of particles under friction in physical systems [10], studying the spread of epidemics in medicine [9], and machine learning, such as data classification [2].

Stochastic process analysis is an introduction to understanding the behavior of the stochastic process, analyzing their data, and thus predicting the future of the stochastic process to which the phenomenon under study is subject. The stochastic process analysis is divided into the time

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domain analysis and the frequency domain or spectral analysis [13]. Spectral analysis is one of the most prominent aspects of stochastic process analysis. It attributes differences or similarities in the phenomenon data to their sources. The spectrum of a stochastic process represents how much different frequencies contribute to the variance of the stochastic process [16].

The main tool for calculating the spectrum of a stochastic process is the spectral density function (SDF), which is defined as the Fourier transform of the autocorrelation function of the stochastic process [1].

In practice, the phenomena described by stochastic processes include noise or disturbances, and often a limited sample of the phenomenon is studied, so the covariance function, as well as the autocorrelation function, are unknown, which makes the spectral density function unknown; it is necessary to find a consistent estimate of the actual spectral density function that describes the behavior of the stochastic process in the frequency domain. One of the most important non-parametric methods for estimating the spectral density function of a stochastic process with equal time divisions is the Fourier transform of the autocorrelation function of the stochastic process [13].

One of the most important aspects on which the quality of the spectral density function estimation depends is the appropriate lag window. The lag window function weights the autocorrelation function. Researchers have proposed important formulas for delay windows and have used them to estimate the spectrum of many phenomena and mathematical models that involve white noise [1,3,12,15].

The manuscript will introduce a new lag window to estimate the spectrum of the OU process. The new lag window wraps the tails of the truncated autocorrelation function toward apeak, which decreases as the lag increases, thus reducing the amount of data lost due to truncation. The new lag window function makes the consistent estimate of the SDF closer to the actual SDF of the Ornstein-Uhlenbeck process. The necessary statistical properties of the spectral density function will be calculated, and the proposed lag window will be compared with the most prominent classical lag window functions. The simulation will be performed in MATLAB, and the results will be presented in tables and figures.

2. Theoretical Aspect

Consider a stochastic process O_t on a complete probability space (Ω, F, P), the stochastic differential equation with respect to a Wiener process W_t of the form

$$dO_t = -\xi O_t dt + \lambda dW_t, t \in [t_0, T] \tag{2.1}$$

with friction parameter $\xi > 0$ and diffusion parameter $\lambda > 0$, is called the Ornstein-Uhlenbeck (OU) equation (or "Langevin equation") [5]. The analytical solution of the OU equation (2.1) is called the Ornstein-Uhlenbeck process, which is given by

$$O_t = O_0 e^{-\xi(t-t_0)} + \lambda \int_{t_0}^{t} e^{-\xi(t-\tau)}\, dW_\tau$$
$$O_0 = O_{t_0} \tag{2.2}$$

The statistical properties of the OU process (2.2) are required to calculate the spectral density function (SDE), the expected is $E(O_t) = O_0 e^{-\xi(t-t_0)}$, and $var(O_t) = \frac{\lambda^2}{2\xi}\left(1 - e^{-2\xi(t-\tau)}\right)$.

The covariance function R_v with a lag $|v| \le T$ of the OU process (2.2) is,

$$R_v = \mathrm{Cov}(O_t, O_{t+v}) = \frac{\lambda^2}{2\xi} e^{-v\xi}\left(1 - e^{-2\xi(t-t_0)}\right) \tag{2.3}$$

The autocorrelation function ρ_v is defined as the normalization of the covariance function R_v, then ρ_v for the OU process (2.2) is

$$\rho_v = \frac{R_v}{R_0} = \frac{\frac{\lambda^2}{2\xi} e^{-v\xi}\left(1 - e^{-2\xi(t-t_0)}\right)}{\frac{\lambda^2}{2\xi}\left(1 - e^{-2\xi(t-t_0)}\right)} = e^{-v\xi} \tag{2.4}$$

And the SDF $f(\omega)$ at each $-\infty < \omega < \infty$ for the OU process (2.2) is

$$f(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \rho_v e^{-i\omega v} dv = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{-v\xi} e^{-i\omega v} dv \tag{2.5}$$

where $i = \sqrt{-1}$. Since the OU process is a real-valued stochastic process, that is $Re(O_t) = O_t$ and the autocorrelation function ρ_v (2.4) is even at all lag values $|v| \le T$, the SDF (2.5) becomes

$$f(\omega) = \frac{1}{\pi}\int_{0}^{\infty} e^{-v\xi}\cos(\omega v)\, dv = \frac{1}{\pi}\,\mathrm{Re}\left\{\int_{0}^{\infty} e^{-v(\xi - i\omega)}\, dv\right\} \tag{2.6}$$

Therefore, the SDF for the stationary OU process (2.2) is

$$f(\omega) = \frac{1}{\pi}\left(\frac{\xi}{\xi^2 + \omega^2}\right), \quad \xi > 0 \tag{2.7}$$

On the other hand, the consistent form to estimate the SDF of the continuous-time stochastic process O_t at the lag $|v| \le T$ is given by

$$\hat{f}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \hat{\rho}_v\, \lambda_T(v)\cos(v\omega)\, dv \tag{2.8}$$

where the angular frequency $\omega = \frac{2\pi v}{T}$, and $\hat{\rho}_v$ is the consistent form to estimate the autocorrelation function of O_t (2.4) on the time interval $[t_0, T]$, which is given by

$$\hat{\rho}_v = \frac{\displaystyle\int_{t_0}^{T-|v|} (O_t - \bar{O}_t)(O_{t+|v|} - \bar{O}_t)\, dt}{\displaystyle\int_{t_0}^{T} (O_t - \bar{O}_t)^2\, dt} \tag{2.9}$$

where $\bar{O}_t$ is the mean value of the data O_t, and $\lambda_T(v)$ is the lag window [13]. The most famous lag window functions with lag $|v| \le T$ are collected in Table 1.

Window	Form of $\lambda_T(v)$	Notes
Bartlett–Hanning	$\alpha_1 - \alpha_2 \left\lvert \frac{v}{T} - \alpha_3 \right\rvert - \alpha_4 \cos\left(\frac{\pi\lvert v\rvert}{T}\right)$	$\alpha_1 = 0.62$, $\alpha_2 = 0.48$, $\alpha_3 = 0.5$, $\alpha_4 = 0.38$
Blackman	$\alpha_1 + \alpha_2 \cos\left(\frac{\pi\lvert v\rvert}{T}\right) + \alpha_3 \cos\left(\frac{2\pi\lvert v\rvert}{T}\right)$	$\alpha_1 = 0.42$, $\alpha_2 = 0.5$, $\alpha_3 = 0.08$
Blackman–Harris	$\alpha_1 - \alpha_2 \cos\left(\frac{\pi\lvert v\rvert}{T}\right) + \alpha_3 \cos\left(\frac{2\pi\lvert v\rvert}{T}\right) - \alpha_4 \cos\left(\frac{3\pi\lvert v\rvert}{T}\right)$	$\alpha_1 = 0.35875$, $\alpha_2 = 0.48829$, $\alpha_3 = 0.14128$, $\alpha_4 = 0.01168$
Bohman	$\left(1 - \frac{\lvert v\rvert}{T}\right) \cos\left(\frac{\pi\lvert v\rvert}{T}\right) + \frac{1}{\pi} \sin\left(\frac{\pi\lvert v\rvert}{T}\right)$	
Cauchy	$\frac{1}{1+(\alpha v/T)^2}$	$\alpha = 3.0$
Cosine	$\sin\left(\frac{\pi\lvert v\rvert}{T}\right)$	
Exponential	$0.1^{\lvert v\rvert/T}$	
Flat-top	$\alpha_1 - \alpha_2 \cos\left(\frac{\pi\lvert v\rvert}{T}\right) + \alpha_3 \cos\left(\frac{2\pi\lvert v\rvert}{T}\right) - \alpha_4 \cos\left(\frac{3\pi\lvert v\rvert}{T}\right) + \alpha_5 \cos\left(\frac{4\pi\lvert v\rvert}{T}\right)$	$\alpha_1 = 0.21557895$, $\alpha_2 = 0.41663158$, $\alpha_3 = 0.27726316$, $\alpha_4 = 0.08357895$, $\alpha_5 = 0.00694737$
Gaussian	$e^{-\frac{1}{2}\left(\frac{\alpha v}{T}\right)^2}$	$\alpha = 2.5$
Hamming	$\alpha_1 + \alpha_2 \cos\left(\frac{\pi\lvert v\rvert}{T}\right)$	$\alpha_1 = 0.54$, $\alpha_2 = 1 - \alpha_1$
Hanning	$\alpha + \alpha \cos\left(\frac{\pi\lvert v\rvert}{T}\right)$	$\alpha = 0.5$
Hanning–Poisson	$0.5[1 + \cos\left(\frac{\pi v}{T}\right)]e^{-\frac{\alpha\lvert v\rvert}{T}}$	$\alpha = 2$
Parzen	$\begin{cases} 1 - 6\left(\frac{\lvert v\rvert}{T}\right)^2 \left(1 - \frac{\lvert v\rvert}{T}\right), & \lvert v\rvert \le \frac{T}{2} \\ 2\left(1 - \frac{\lvert v\rvert}{T}\right)^3, & \frac{T}{2} < \lvert v\rvert \le T \end{cases}$	
Poisson	$e^{-\alpha \frac{v}{T}}$	$\alpha = 2$, $e = 2.71828$
Fejer	$1 - \frac{\lvert v\rvert}{T}$	
Riesz	$1 - (\frac{\lvert v\rvert}{T})^2$	

Table 1. Most Popular Lag Window Functions [1,7,8,13,16].

3. Novel Lag Window (NLW)

The lag window function $\lambda_T(v)$ is recognized to weight the estimated autocorrelation function $\hat{\rho}_v$ [16] in the consistent estimate of the SDF form $\hat{f}(\omega)$ (4.2).

The integration in the estimated form of the SDF (4.2) at all lag values $\lvert v\rvert \le T$ produces an inconsistent estimate of the SDF, and when $\lvert v\rvert \to T$, then $\hat{\rho}_v$ is not a good estimate to ρ_v, and thus the influence of the tails of the estimated autocorrelation function ρ_v from the data increases, and the estimated autocorrelation function is truncated. A good lag window $\lambda_T(v)$ weights the estimated autocorrelation function $\hat{\rho}_v$ to include the truncated data, resulting in a consistent estimate of the SDF.

The autocorrelation curve has a unique maximum value at lag $v = 0$, then it slopes downward as the lag values v move towards T, meaning that the amount of variation in the phenomenon data increases. More precisely, increasing the lag v leads to decreasing values of the autocorrelation function.

We will propose a lag window that vanishes the effect of the tails of the estimated autocorrelation function $\hat{\rho}_v$ that move away from the peak $\hat{\rho}_0$, called a novel lag window (NLW), and is defined by the following form.

$$\lambda_T(v) = \begin{cases} \frac{\alpha}{2\sqrt{\pi\beta}} \exp\left[-\left(\frac{\pi\beta v}{T}\right)^2\right], & |v| \le T \\ 0, & |v| > T \end{cases} \tag{3.1}$$

where the parameters $\alpha, \beta \ge 0$ control the amount of kurtosis and skewness in the NLW curve along $|v| \le T$. Fig. 1 illustrates the NLW curves at different values of the parameters α, β.

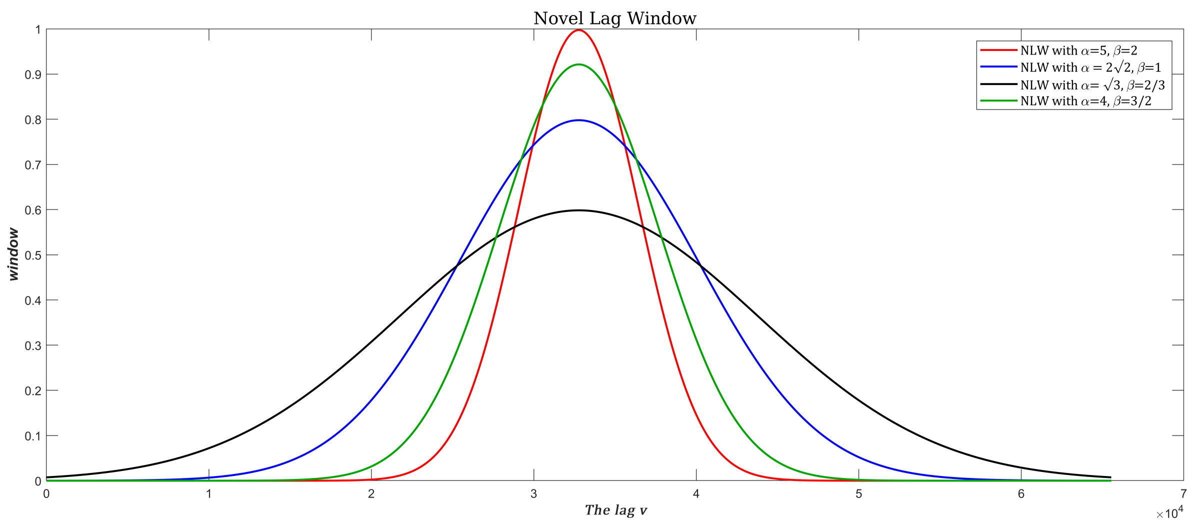

Figure 1. NLW curves with different control parameters

4. Methodology

Spectral analysis of stochastic processes falls within computational techniques that deal with discrete-time mathematical processes. The simulation of spectral analysis techniques for continuous-time stochastic processes is increasingly complex because it requires segmentation of the time interval $[t_0, T]$ of the process and transforming the parameters from the continuous to discrete state. Continuous-time processes can be treated by their equivalent difference equations on sufficiently small time sub-intervals.

To evaluate the stochastic integral in the OU process (2.2), the statistical properties are $E(\lambda \int_{t_0}^{t} e^{-\xi(t-\tau)}\, dW_\tau) = 0$ and $var(\lambda \int_{t_0}^{t} e^{-\xi(t-\tau)}\, dW_\tau) = \frac{\lambda^2}{2\xi}\,(1 - e^{-2\xi(t-\tau)})$

For a sufficiently small $\Delta t \to 0$, the OU process (2.2) at each sub-interval $[t, t + \Delta t]$ is given by,

$$O_{t+\Delta t} = O_t e^{-\xi \Delta t} + \lambda \sqrt{\frac{1 - e^{-\xi \Delta t}}{2\xi}}\, N[0, 1]$$

Therefore, the equivalent OU difference equation of the OU process (2.2) is given by the form

$$\begin{aligned} O_{t_i} &= O_{t_{i-1}} e^{-\xi \Delta t} + \lambda \sqrt{\frac{1 - e^{-\xi \Delta t}}{2\xi}}\, \epsilon_{t_i} \\ O_{t_0} &= O_0 \end{aligned} \tag{4.1}$$

where ϵ_t is the White noise with $N[0, 1]$.

The difference equation form (4.1) represents a linear Markov process that is described by one-step dependence, that is, the value of the process O_{t_i} at time t_i depends at least linearly on the value of $O_{t_{i-1}}$ at time t_{i-1}. If we choose the truncated point N, the consistent form to estimate the SDF of the discrete-time OU process (4.1) at the lag $v = 0, \pm 1, \ldots, N$ is given by,

$$\hat{f}(\omega) = \frac{1}{2\pi} \sum_{v=-N}^{N} \hat{\rho}_v\, \lambda_T(v) \cos(v\omega) \tag{4.2}$$

where $\omega = \frac{2\pi v}{N}$ and $\hat{\rho}_v$ is the consistent form to estimate the autocorrelation function of the discrete-time OU process O_t (4.1) which given by,

$$\hat{\rho}_v = \frac{\sum_{t=t_0}^{T-|v|} (O_t - \bar{O}_t)(O_{t+|v|} - \bar{O}_t)}{\sum_{t=t_0}^{T} (O_t - \bar{O}_t)^2} \tag{4.3}$$

where $O_t = \sum_1^n \frac{O_t}{n}$ is the mean of observations.

5. Numerical Aspect

To achieve the manuscript goals, which are to determine the best lag window to estimate the spectrum of the OU process, the simulation will be performed in MATLAB and includes the following.

- Generate White noise ϵ_t with mean *zero* and variance 1, and calculate the discrete form of the OU process (4.1) with an initial value $O_0 = O_{t_0}$. To determine the effect of the parameters ξ, λ on the spectrum of the OU process, different values of the parameters will be considered as in Table 2.

ξ	1	1	1.5	2
λ	1	1.5	1	2

Table 2. Different Values of the OU Process Parameters

- To determine the effect of the time interval segmentation $[t_0, T]$ on the calculation of the estimated SDF, the time step size Δt will be studied with values $2^{-7}, 2^{-9}, 2^{-11}$.
- To test the stability of the simulation results, the number of simulation generations (number of simulation iterations) $R = 1000$ was chosen.
- The range of values for ω is $[-\pi, \pi]$, where ω is divided by the step size $\Delta\omega = 2^{-11}$.

- The autocorrelation function ρ_v and the SDF $f(\omega)$ for the OU process (2.2) are calculated according to the formulas (2.4) and (2.7), respectively.
- The estimated formulas of the autocorrelation function $\hat{\rho}_v$ and the SDE $\hat{f}(\omega)$ are calculated according to the formulas (4.3) and (4.2), respectively.
- In the consistent estimate form of SDF $\hat{f}(\omega)$, the lag window functions $\lambda_T(v)$ that weighted the estimated autocorrelation values are given in Table 1, in addition to the proposed NLW (3.1) with parameters $\alpha = \sqrt{3}, \beta = \frac{2}{3}$.
- The criterion used to evaluate the performance of lag window functions is the mean square error (MSE) of the formula [14],

$$\text{MSE} = \frac{\sum_{j=1}^{R}\sum_{i=1}^{k}\left(f(\omega_i) - \hat{f}_j(\omega_i)\right)^2}{R\,k} \tag{5.1}$$

where R is the number of simulation generations and k is the segmentation size of ω.

6. Results

The simulation of the spectrum of the OU process was performed using 16 lag window functions in addition to the suggested NLW, at different values of the OU process parameters ξ, λ and different time interval segmentation. The simulation results are rounded to the sixth decimal place.

The MSE values between the SDF $f(\omega)$ and its consistent estimate $\hat{f}(\omega)$ for various parameter values ξ, λ with a time step size $\Delta t = 2^{-7}$, are presented in Table 3.

Window	ξ=1 λ=1	ξ=1 λ=1.5	ξ=1.5 λ=1	ξ=2 λ=2
Bartlett–Hanning	0.026883	0.026875	0.018332	0.013780
Blackman	0.064466	0.068588	0.066233	0.065410
Blackman–Harris	0.030959	0.031309	0.022946	0.017850
Bohman	0.021168	0.022315	0.017913	0.015639
Cauchy	0.028472	0.030174	0.026162	0.023025
Cosine	0.024833	0.024833	0.016163	0.011671
Exponential	0.082525	0.085210	0.084423	0.079461
Flat-top	0.029030	0.029265	0.020845	0.015881
Gaussian	0.019910	0.020921	0.016511	0.014362
Hamming	0.023911	0.025328	0.021016	0.018420
Hanning	0.029518	0.029490	0.021093	0.016534
Hanning–Poisson	0.017303	0.017930	0.013657	0.011731
Parzen	0.020785	0.021790	0.017520	0.015155
Poisson	0.018927	0.019709	0.015510	0.013366
Fejer	0.021700	0.022827	0.018585	0.016192
Riesz	0.027074	0.028755	0.024586	0.021643
NLW	**0.014049**	**0.014312**	**0.008330**	**0.005329**
Best lag window	NLW	NLW	NLW	NLW

Table 3. MSE values between SDF and the consistent estimate of SDF at time step size $\Delta T = 2^{-7}$.

At the time step size $\Delta t = 2^{-9}$, the MSE values between the SDF $f(\omega)$ and the consistent estimate of the SDF $\hat{f}(\omega)$ are given in Table 4.

Window	ξ=1 λ=1	ξ=1 λ=1.5	ξ=1.5 λ=1	ξ=2 λ=2
Bartlett–Hanning	0.026916	0.026908	0.017979	0.013522
Blackman	0.067274	0.066802	0.065934	0.069005
Blackman–Harris	0.031543	0.031558	0.022563	0.018260
Bohman	0.021832	0.021692	0.017654	0.017151
Cauchy	0.029670	0.029509	0.025163	0.024868
Cosine	0.024974	0.024974	0.016249	0.011728
Exponential	0.080735	0.080287	0.069363	0.070660
Flat-top	0.029451	0.029471	0.020504	0.016155
Gaussian	0.020383	0.020251	0.016218	0.015667
Hamming	0.024918	0.024768	0.020682	0.020269
Hanning	0.029330	0.029307	0.020052	0.015678
Hanning–Poisson	0.017449	0.017345	0.013313	0.012649
Parzen	0.021289	0.021155	0.017075	0.016535
Poisson	0.019253	0.019142	0.015073	0.014457
Fejer	0.022392	0.022261	0.018170	0.017663
Riesz	0.028353	0.028194	0.023993	0.023675
NLW	**0.014138**	**0.014092**	**0.008165**	**0.005950**
Best lag window	NLW	NLW	NLW	NLW

TABLE 4. MSE values between SDF and the consistent estimate of SDF at time step size $\Delta T = 2^{-9}$.

At the time step size $\Delta t = 2^{-11}$, the MSE values between the SDF $f(\omega)$ and the consistent estimate of the SDF $\hat{f}(\omega)$ are given in Table 5.

Window	ξ=1 λ=1	ξ=1 λ=1.5	ξ=1.5 λ=1	ξ=2 λ=2
Bartlett–Hanning	0.026895	0.026927	0.018159	0.013643
Blackman	0.067918	0.067624	0.065966	0.068383
Blackman–Harris	0.031590	0.031516	0.022769	0.018279
Bohman	0.022185	0.021949	0.017513	0.016880
Cauchy	0.030017	0.029693	0.025244	0.024653
Cosine	0.025100	0.025100	0.016271	0.011742
Exponential	0.078876	0.079444	0.074436	0.074244
Flat-top	0.029500	0.029452	0.020705	0.016199
Gaussian	0.020722	0.020510	0.016078	0.015428
Hamming	0.025293	0.025006	0.020567	0.019960
Hanning	0.029201	0.029295	0.020485	0.015989
Hanning–Poisson	0.017772	0.017565	0.013158	0.012483
Parzen	0.021621	0.021382	0.016974	0.016314
Poisson	0.019587	0.019350	0.014943	0.014280
Fejer	0.022745	0.022480	0.018055	0.017423
Riesz	0.028732	0.028406	0.023956	0.023373
NLW	**0.014325**	**0.014190**	**0.008066**	**0.005839**
Best lag window	NLW	NLW	NLW	NLW

TABLE 5. MSE values between SDF and the consistent estimate of SDF at time step size $\Delta T = 2^{-11}$.

According to the simulation results, Table 6 shows the order of lag window functions based on the average mean square error (AMSE) between $f(\omega)$ and $\hat{f}(\omega)$ at the segmentation $\Delta t = 2^{-7}, 2^{-9}, 2^{-11}$.

No.	ξ=1 λ=1	ξ=1 λ=1.5	ξ=1.5 λ=1	ξ=2 λ=2
1	**NLW**	**NLW**	**NLW**	**NLW**
2	Hanning-Poisson	Hanning-Poisson	Hanning-Poisson	Cosine
3	Poisson	Poisson	Poisson	Hanning-Poisson
4	Gaussian	Gaussian	Cosine	Bartlett–Hanning
5	Parzen	Parzen	Gaussian	Poisson
6	Bohman	Bohman	Parzen	Gaussian
7	Fejer	Fejer	Bohman	Parzen
8	Hamming	Cosine	Bartlett–Hanning	Hanning
9	Cosine	Hamming	Fejer	Flat-top
10	Bartlett–Hanning	Bartlett–Hanning	Hanning	Bohman
11	Riesz	Riesz	Flat-top	Fejer
12	Flat-top	Hanning	Hamming	Blackman–Harris
13	Hanning	Flat-top	Blackman–Harris	Hamming
14	Cauchy	Cauchy	Riesz	Riesz
15	Blackman–Harris	Blackman–Harris	Cauchy	Cauchy
16	Blackman	Blackman	Blackman	Blackman
17	Exponential	Exponential	Exponential	Exponential

Table 6. Ranking of lag window functions based on their performance in estimating the OU process spectrum at different parameter values.

The stability of the OU process (2.2) is satisfied when the parameter values are strictly positive $\xi, \lambda > 0$. The SDF of the OU process (2.7) is affected only by the friction parameter ξ, while the consistent estimate of the SDF of the OU process (4.2) is affected by the friction parameter ξ and the diffusion parameter λ. On the other hand, choosing the values of the parameter $\xi > 2$ makes the SDF $f(\omega)$ (2.7) very small (SDF< 0.07), and therefore it is difficult to determine the best lag window to estimate the spectrum of the OU process. Fig. 2 shows the SDF curves of the OU process (2.7) at increasing values of the friction parameter ξ.

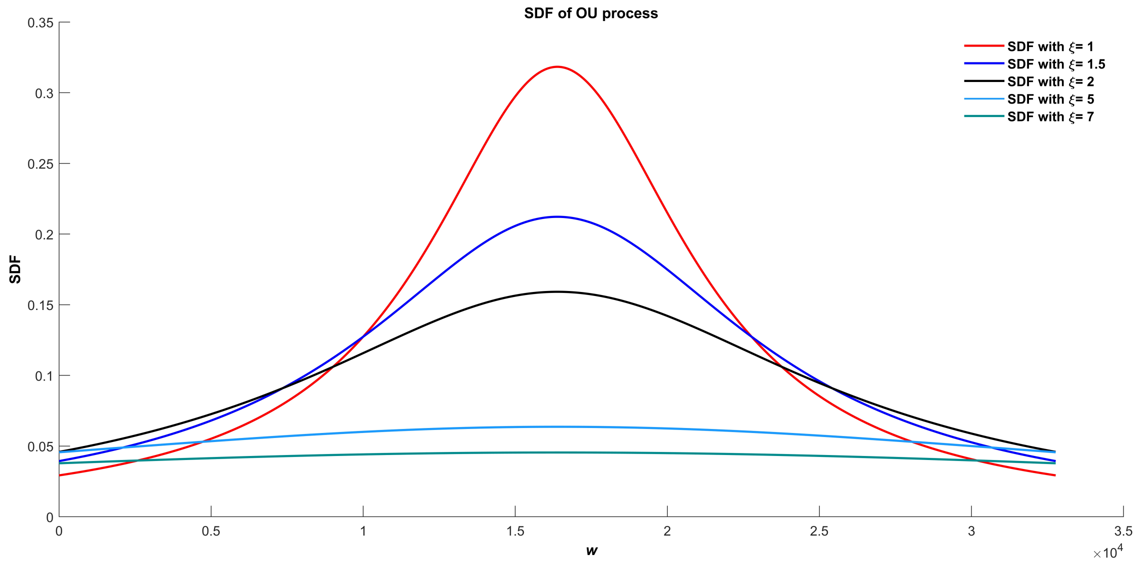

Figure 2. SDF curves of OU process with different parameters

7. Conclusions

In this paper, a novel lag window is proposed and the 16 most popular classical lag windows in spectral analysis are presented. The lag windows are compared to estimate the spectrum of the OU process, which is a continuous-time stochastic process, at different values of the process parameters and different time interval segmentation. The most prominent conclusion is that the suggested lag window outperforms all other lag windows in all parameter cases of the OU process, as well as in all time interval segmentation $\Delta t = 2^{-7}, 2^{-9}, 2^{-11}$.

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