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A Proposed Method for Establishing Five Algebraic Substructures in UP-Algebras in View of Generalized Neutrosophic Structures

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Abstract. The main goal of this paper is to utilize the notion of generalized neutrosophic structures (GNSs) to five types of algebraic substructures in UP-algebras. We present the concepts of generalized neutrosophic UP-subalgebras (GNUP-Ss), generalized neutrosophic near UP-filters (GNNUP-Fs), generalized neutrosophic UP-filters (GNUP-Fs), generalized neutrosophic UP-ideals (GNUP-Is) and generalized neutrosophic strong UP-ideals (GNUP-SIs) in UP-algebras and investigate some related properties. Furthermore, the relationship between these five types of algebraic substructures in UP-algebras is discussed. After that, the conditions under which GNUP-S can be GNNUP-F, and the condition under which GNUP-F can be GNUP-I in UP-algebra are discovered. At last, a number of characterizations theorems of our concepts are presented and proved.

1. Introduction

In pure mathematics, there are many kinds of algebraic structures, such as BCK/BCI/KU-algebras, see [1,2]. A UP-algebra is one of algebraic structures presented by Iampan [3]. Based on this structure, he presented the idea of UP-subalgebra and UP-ideals. In [4], Iampan proved that the concept of UP-subalgebras is an extension of near UP-filters, near UP-filters is an extension of UP-filters, UP-filters is an extension of UP-ideals, and UP-ideals is an extension of strong UP-ideals. The research of UP-algebras offers a rich and fascinating area of inquiry for mathematicians

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and scientists alike, providing a powerful framework for understanding the underlying structures and behaviour of a wide range of mathematical systems.

In the field of uncertainty mathematics, Zadeh [5] implemented the notion of fuzziness structures as a generalization of classical (crisp) sets. This concept was merged with a crisp UP-algebra by Songsaeng et al. [6] to deal with uncertainty and fuzziness more accurately. In other words, the theory of fuzziness sets can be used within the framework of UP-algebras to explore fuzziness and imprecision in algebraic structures in UP-algebras. The conceptions of fuzzy UP-ideals, fuzzy UP-subalgebras and fuzzy UP-filters of UP-algebras were considered by Somjanta et al. [6]. Fuzzy translations of a fuzzy set in UP-algebras were studied by Guntasow et al. [7]. Also, Kaijae et al. [8] studied and investigated anti-fuzzy UP-subalgebras and anti-fuzzy UP-ideals in UP-algebras.

Atanassov [9] presented intuitionistic fuzzy sets which include the uncertainty degree called uncertainty margin. The uncertainty margin is defined as one minus the sum of membership and non-membership. Therefore, the intuitionistic fuzzy set is characterized by a membership function and non-membership function with a range $[0,1]$ and it is an extension of both classical (crisp) and fuzzy sets. The concept intuitionistic fuzzy sets applied to UP-algebras in several works, see [10,11].

Smarandache [12] presented neutrosophic set theory that studies the origin, nature, and scope of neutralities and engagements with distinct ideational spectra. A neutrosophic set involves truth, indeterminacy and falsity based on three valued logics. Neutrosophic set is a powerful mathematical framework which extensions the perception of classical sets and (intuitionistic) fuzzy sets. As a modification of neutrosophic sets, Wang et al. [13] defined single valued neutrosophic set as an instance of neutrosophic set which can be used in real scientific and engineering applications. In the context of neutrosophic UP-algebras, Songsaeng and Iampan [14] presented the concepts of neutrosophic UP-subalgebras, neutrosophic near UP-filters, neutrosophic UP-filters, neutrosophic UP ideals, and neutrosophic strongly UP-ideals of UP-algebras, and investigated many properties. For more works on the connection between neutrosophic sets and UP-algebras, see [15,16]. Song et al. [17] modified the notion of a neutrosophic set by divide the role of the indeterministic membership function to two membership functions and they presented the concept of GNS. After that, this concept applied to ideals in BCK-algebras [18].

In this paper, we introduce the notions of GNUP-Ss, GNNUP-Fs, GNUP-Fs, GNUP-Is and GNUP-SIs in UP-algebras and prove their generalizations. Furthermore, the relationship between GNUP-Ss (resp., GNNUP-Fs, GNUP-Fs, GNUP-Is and GNUP-SIs) in UP-algebras is discussed. After that, the conditions under which GNUP-S can be GNNUP-F, and the condition under which GNUP-F can be GNUP-I in UP-algebra are established. At last, some characterizations results of our notions are discussed and proved.

2. Preliminaries

In this section, a brief summary of some basic definitions related to this article, such as UP-algebras, subalgebras, ideals and filters in UP-algebras are presented. Thereafter, the main notions related to (intuitionistic) fuzzy sets and (generalized) neutrosophic sets with some results and properties that will be of value for our later pursuits are mentioned.

Throughout this article, Ξ (universe set) denotes a UP-algebra, unless otherwise specified.

2.1. **UP-Algebras and Some UP-Algebraic Substructures.**

Definition 2.1 ([3]). *An algebra $\Xi = (\Xi, \diamond, 0)$ of type $(2,0)$ is said to be a UP-algebra, where "Ξ" is a nonempty set, " $\diamond$ " is a binary operation on Ξ, and "0" is a fixed element of Ξ if it satisfies the following postulates $(\forall h, j, k \in \Xi)$:*

(1) $(j \diamond k) \diamond ((h \diamond j) \diamond (h \diamond k)) = 0$,
(2) $0 \diamond h = h$,
(3) $h \diamond 0 = 0$,
(4) $h \diamond j = j \diamond h = 0 \Rightarrow h = j$.

The following two Propositions are mentioned by [3,19].

Proposition 2.1. *Let Ξ be a UP-algebra. Then, the following assertions are valid $(\forall h, j, k, c \in \Xi)$:*

(1) $h \diamond h = 0$,
(2) $h \diamond (j \diamond h) = 0$,
(3) $h \diamond (j \diamond j) = 0$,
(4) $(h \diamond (j \diamond k)) \diamond (h \diamond ((c \diamond j) \diamond (c \diamond k))) = 0$,
(5) $(((c \diamond h) \diamond (c \diamond j)) \diamond k) \diamond ((h \diamond j) \diamond k) = 0$,
(6) $((h \diamond j) \diamond k) \diamond (j \diamond k) = 0$,
(7) $((h \diamond j) \diamond k) \diamond (h \diamond (j \diamond k)) = 0$,
(8) $((h \diamond j) \diamond k) \diamond (j \diamond (c \diamond k)) = 0$.

Proposition 2.2. *Let Ξ be a UP-algebra.Then, we have the following:*

(1) *If $h \diamond j = 0$ and $j \diamond k = 0$, then $h \diamond k = 0$.*
(2) *If $h \diamond j = 0$, then $(k \diamond h) \diamond (k \diamond j) = 0$,*
(3) *If $h \diamond j = 0$, then $(j \diamond k) \diamond (h \diamond k) = 0$,*
(4) *$(j \diamond h) \diamond h = 0$ if and only if $h = j \diamond h$,*
(5) *If $h \diamond j = 0$, then $h \diamond (k \diamond j) = 0$.*

for all $h, j, k \in \Xi$.

Example 2.1. [3] *Let $\Xi = \{0, v, b, n\}$ be a set with a binary operation "$\diamond$" defined by the following Cayley table:*

Table 1. A UP-algebra $\Xi = \{0, v, b, n\}$ of Example 2.1

$\diamond$	0	v	b	n
0	0	v	b	n
v	0	0	0	0
b	0	v	0	n
n	0	v	b	0

Then, $(\Xi, \diamond, 0)$ *is a UP-algebra.*

In a UP-algebra Ξ, five types of special subalgebraic structures are defined as follows.

Definition 2.2 ([3,6]). *A nonempty subset S of a UP-algebra Ξ $(\forall h, j, k \in \Xi)$ is called:*

(1) *a UP-subalgebra of Ξ if $h \diamond j \in S$ $\forall h, j \in S$.*
(2) *a near UP-filter of Ξ if*
  (i) $0 \in S$,
  (ii) $j \in S \Rightarrow h \diamond j \in S$.
(3) *a UP-filter of $\diamond$ if*
  (i) $0 \in S$,
  (ii) $h \diamond j \in S, h \in S \Rightarrow j \in S$.
(4) *a UP-ideal of Ξ if*
  (i) $0 \in S$,
  (ii) $h \diamond (j \diamond k) \in S, j \in S \Rightarrow h \diamond k \in S$.
(5) *a strong UP-ideal of Ξ if*
  (i) $0 \in S$,
  (ii) $(k \diamond j) \diamond (k \diamond h) \in S, j \in S \Rightarrow h \in S$.

2.2. Some Uncertainty Structures.

Definition 2.3 ([5]). *A fuzzy structure Q in $\Xi \neq \phi$ (universe set) is a structure of the form:*

$$Q = \left\{ \left\langle h, \mu_Q(h) \right\rangle : h \in \Xi \right\},$$

where $\mu_Q : \Xi \to [0,1]$ is the degree of membership function of the element $h \in \Xi$.

Definition 2.4 ([9]). *An intuitionistic fuzzy structure B in $\Xi \neq \phi$ (universe set) is a structure of the form:*

$$B = \{\langle h, \mu_B(h), \xi_B(h) \rangle \mid h \in \Xi\},$$

where the functions $\mu_B : \Xi \to [0,1]$ and $\xi_B : \Xi \to [0,1]$ are the degree of membership and the degree of non-membership of the element $h \in \Xi$, respectively, and $0 \leq \mu_B(h) + \xi_B(h) \leq 1$ $\forall h \in \Xi$.

Definition 2.5 ([12]). *A neutrosophic structure Λ in $\Xi \neq \phi$ (universe set) is a structure of the form:*

$$\Lambda = \{\langle h, \lambda_T(h), \lambda_I(h), \lambda_F(h) \rangle \mid h \in \Xi\},$$

where $\lambda_T : \Xi \to [0,1]$ is a truth, $\lambda_I : \Xi \to [0,1]$ is an indeterminate and $\lambda_F(h) : \Xi \to [0,1]$ is a false membership functions, and $(\forall h \in \Xi)$:

$$0 \leq \lambda_T(h) + \lambda_I(h) + \lambda_F(h) \leq 3.$$

Definition 2.6 ([17]). *A GNS P in Ξ (universe set) is a structure of the form:*

$$P = \{\langle h, P_T(h), P_{IT}(h), P_{IF}(h), P_F(h)\rangle \mid h \in \Xi\},$$

where, $P_T : \Xi \to [0,1]$, $P_{IT} : \Xi \to [0,1]$, $P_{IF} : \Xi \to [0,1]$ and $P_F : \Xi \to [0,1]$ such that $0 \leq P_{IT}(h) + P_{IF}(h) \leq 1$ $\forall h \in \Xi$.

In this article, we use the symbol $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ for the GNS

$$P = \{\langle h, P_T(h), P_{IT}(h), P_{IF}(h), P_F(h)\rangle \mid h \in \Xi\}.$$

Example 2.2. *Let $\Xi = \{0, v, b, n\}$ be a UP-algebra with a binary operation "$\diamond$" defined by the following Cayley table:*

TABLE 2. A UP-algebra $\Xi = \{0, v, b, n\}$ of Example 2.2

$\diamond$	0	v	b	n
0	0	v	b	n
v	0	0	0	0
b	0	v	0	n
n	0	v	b	0

Then,

$$P = \begin{pmatrix} \Xi & 0 & v & b & n \\ P_T & 0.2 & 0.7 & 0.4 & 0.2 \\ P_{IT} & 0.3 & 0.1 & 0.2 & 0.3 \\ P_{IF} & 0.5 & 0.2 & 0.1 & 0.4 \\ P_F & 0.2 & 0.3 & 0.6 & 0.5 \end{pmatrix}$$

is a GNS of Ξ.

2.3. Neutrosophic Algebraic Substructures in UP-Algebras.

In this part, all next definitions related to the connection between neutrosophic structure and some UP-algebraic substructures are mentioned by [14].

Definition 2.7. *Let Λ be a neutrosophic structure of Ξ. Then, Λ is called a neutrosophic UP-subalgebra of Ξ if the following postulates are satisfied ($\forall h, j \in \Xi$):*

(1) $\lambda_T(h \diamond j) \geq \min\{\lambda_T(h), \lambda_T(j)\}$,
(2) $\lambda_I(h \diamond j) \leq \max\{\lambda_I(h), \lambda_I(j)\}$,
(3) $\lambda_F(h \diamond j) \geq \min\{\lambda_F(h), \lambda_F(j)\}$.

Definition 2.8. *Let Λ be a neutrosophic set of Ξ. Then, Λ is called a neutrosophic near UP-filter of Ξ if the condition $(\boldsymbol{K})$, where*

$$(\boldsymbol{K}) \quad (\forall h \in \Xi)\begin{pmatrix} \lambda_T(0) \geq \lambda_T(h), \\ \lambda_I(0) \leq \lambda_I(h), \\ \lambda_F(0) \geq \lambda_F(h) \end{pmatrix},$$

and following postulates are valid $(\forall h, j \in \Xi)$*:*

(1) $\lambda_T(h \diamond j) \geq \lambda_T(j)$,
(2) $\lambda_I(h \diamond j) \leq \lambda_I(j)$,
(3) $\lambda_F(h \diamond j) \geq \lambda_F(j)$.

Definition 2.9. *Let* Λ *be a neutrosophic structure in* Ξ*. Then,* Λ *is called a neutrosophic UP-ideal of* Ξ *if the condition* $(\boldsymbol{K})$ *of Definition 2.8 and the following postulates are valid* $(\forall h, j, k \in \Xi)$*:*

(1) $\lambda_T(h \diamond k) \geq \min\{\lambda_T(h \diamond (j \diamond k)), \lambda_T(j)\}$,
(2) $\lambda_I(h \diamond k) \leq \max\{\lambda_I(h \diamond (j \diamond k)), \lambda_I(j)\}$,
(3) $\lambda_F(h \diamond k) \geq \min\{\lambda_F(h \diamond (j \diamond k)), \lambda_F(j)\}$.

Definition 2.10. *Let* Λ *be a neutrosophic structure in* Ξ*. Then,* Λ *is called a neutrosophic strong UP-ideal of* Ξ *if the condition* $(\boldsymbol{K})$ *of Definition 2.8 and the following postulates are valid* $(\forall h, j, k \in \Xi)$*:*

(1) $\lambda_T(h) \geq \min\{\lambda_T((k \diamond j) \diamond (k \diamond h)), \lambda_T(j)\}$,
(2) $\lambda_I(h) \leq \max\{\lambda_I((k \diamond j) \diamond (k \diamond h)), \lambda_I(j)\}$,
(3) $\lambda_F(h) \geq \min\{\lambda_F((k \diamond j) \diamond (k \diamond h)), \lambda_F(j)\}$.

3. Five Types of Generalized Neutrosophic UP-Algebraic Substructures

In this section, we introduce the notions of generalized neutrosophic algebraic substructures (UP-subalgebras, near UP-filters, UP-filters, UP-ideals, and strong UP-ideals) in UP-algebras. Based on these notions, certain necessary examples and properties with their generalizations are provided and discussed.

Definition 3.1. *A GNS* $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ *is called a GNUP-S of* Ξ *if the following postulates are valid* $(\forall h, j \in \Xi)$:

(1) $P_T(h \diamond j) \geq \min\{P_T(h), P_T(j)\}$,
(2) $P_{IT}(h \diamond j) \geq \min\{P_{IT}(h), P_{IT}(j)\}$,
(3) $P_{IF}(h \diamond j) \leq \max\{P_{IF}(h), P_{IF}(j)\}$,
(4) $P_F(h \diamond j) \leq \max\{P_F(h), P_F(j)\}$.

Example 3.1. *Let* $\Xi = \{0, v, b, n, l\}$ *be a UP-algebra with a binary operation "* $\diamond$ *" defined by the following Cayley table:*

Table 3. A UP-algebra $\Xi = \{0, v, b, n, l\}$ of Example 3.1

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	v	n	l
b	0	0	0	n	l
n	0	0	0	0	l
l	0	0	0	0	0

Define a GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ in Ξ as follows:

$$P = \begin{pmatrix} \Xi & 0 & v & b & n & l \\ P_T & 0.7 & 0.4 & 0.5 & 0.2 & 0.6 \\ P_{IT} & 1 & 0.5 & 0.7 & 0.3 & 0.7 \\ P_{IF} & 0 & 0.4 & 0.1 & 0.8 & 0.2 \\ P_F & 0.3 & 0.5 & 0.4 & 0.9 & 0.4 \end{pmatrix}.$$

Then, P is a GNUP-S of Ξ.

Proposition 3.1. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-S of Ξ, then the condition $(\boldsymbol{P})$ is valid, where*

$$(\boldsymbol{P}) \quad \left(\forall h \in \Xi\right)\begin{pmatrix} P_T(0) \geq P_T(h), \\ P_{IT}(0) \geq P_{IT}(h), \\ P_{IF}(0) \leq P_{IF}(h), \\ P_F(0) \leq P_F(h) \end{pmatrix}.$$

Proof. Let $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ be a GNUP-S of Ξ. Using (1) of Proposition 2.1, we have

$$\begin{aligned} P_T(0) = P_T(h \diamond h) &\geq \min\{P_T(h), P_T(h)\} = P_T(h), \\ P_{IT}(0) = P_{IT}(h \diamond h) &\geq \min\{P_{IT}(h), P_{IT}(h)\} = P_{IT}(h), \\ P_{IF}(0) = P_{IF}(h \diamond h) &\leq \max\{P_{IF}(h), P_{IF}(h)\} = P_{IF}(h), \\ P_F(0) = P_F(h \diamond h) &\geq \min\{P_F(h), P_F(h)\} = P_F(h) \end{aligned}$$

for all $h \in \Xi$. □

Definition 3.2. *A GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is called a GNNUP-F of Ξ if the condition $(\boldsymbol{P})$ of Proposition 3.1 and the following postulates are valid $(\forall h, j, \in \Xi)$:*

$$\begin{pmatrix} P_T(h \diamond j) \geq P_T(j), \\ P_{IT}(h \diamond j) \geq P_{IT}(j), \\ P_{IF}(h \diamond j) \leq P_{IF}(j), \\ P_F(h \diamond j) \leq P_F(j) \end{pmatrix}.$$

Example 3.2. *Let $\Xi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation " $\diamond$ " defined by the following Cayley table:*

Table 4. A UP-algebra $\Xi = \{0, v, b, n, l\}$ of Example 3.2

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	v	b	l
b	0	0	0	v	l
n	0	0	0	0	l
l	0	v	b	n	0

Define a GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ in Ξ as follows:

$$P = \begin{pmatrix} \Xi & 0 & v & b & n & l \\ P_T & 1 & 0.6 & 0.5 & 0.4 & 0.1 \\ P_{IT} & 1 & 0.8 & 0.7 & 0.6 & 0.3 \\ P_{IF} & 0 & 0.1 & 0.3 & 0.5 & 0.8 \\ P_F & 0.1 & 0.3 & 0.4 & 0.6 & 0.9 \end{pmatrix}.$$

Then, P is a GNNUP-F of Ξ.

Definition 3.3. *A GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is called a GNUP-F of Ξ if the condition $(\boldsymbol{P})$ of Proposition 3.1 and the following postulates are valid $(\forall h, j \in \Xi)$:*

$$\begin{pmatrix} P_T(j) \geq \min\{P_T(h \diamond j), P_T(h)\}, \\ P_{IT}(j) \geq \min\{P_{IT}(h \diamond j), P_{IT}(h)\}, \\ P_{IF}(j) \leq \max\{P_{IF}(h \diamond j), P_{IF}(h)\}, \\ P_F(j) \leq \max\{P_F(h \diamond j), P_F(h)\} \end{pmatrix}.$$

Example 3.3. *Let $\Xi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation " $\diamond$ " defined by the following Cayley table:*

Table 5. A UP-algebra $\Xi = \{0, v, b, n, l\}$ of Example 3.3

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	0	0	n	n
n	0	v	b	0	n
l	0	v	b	0	0

Define a GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ in Ξ as follows:

$$P = \begin{pmatrix} \Xi & 0 & v & b & n & l \\ P_T & 0.8 & 0.6 & 0.5 & 0.2 & 0.2 \\ P_{IT} & 0.9 & 0.7 & 0.6 & 0.4 & 0.4 \\ P_{IF} & 0 & 0.2 & 0.4 & 0.7 & 0.7 \\ P_F & 0.1 & 0.3 & 0.5 & 0.9 & 0.9 \end{pmatrix}.$$

Then, P is a GNUP-F of Ξ.

Definition 3.4. *A GNS is called a GNUP-I of Ξ if the condition $(\boldsymbol{P})$ of Proposition 3.1 and the following postulates are valid $(\forall h, j, k \in \Xi)$:*

$$\begin{pmatrix} P_T(h \diamond k) \geq \min\{P_T(h \diamond (j \diamond k)), P_T(j)\}, \\ P_{IT}(h \diamond k) \geq \min\{P_{IT}(h \diamond (j \diamond k)), P_{IT}(j)\}, \\ P_{IF}(h \diamond k) \leq \max\{P_{IF}(h \diamond (j \diamond k)), P_{IF}(j)\}, \\ P_F(h \diamond k) \leq \max\{P_F(h \diamond (j \diamond k)), P_F(j)\} \end{pmatrix}.$$

Example 3.4. *Let $\Xi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation " $\diamond$ " defined by the following Cayley table:*

Table 6. A UP-algebra $\Xi = \{0, v, b, n, l\}$ of Example 3.4

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	0	0	0	l
n	0	0	b	0	l
l	0	0	0	0	0

Define a GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ as follows:

$$P = \begin{pmatrix} \Xi & 0 & v & b & n & l \\ P_T & 0.8 & 0.5 & 0.4 & 0.5 & 0.4 \\ P_{IT} & 0.9 & 0.9 & 0.6 & 0.8 & 0.5 \\ P_{IF} & 0.1 & 0.3 & 0.4 & 0.3 & 0.5 \\ P_F & 0.3 & 0.5 & 0.7 & 0.6 & 0.9 \end{pmatrix}.$$

Then, P is a GNUP-I of Ξ.

Definition 3.5. *A GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is called a GNUP-SI of Ξ if the condition $(\boldsymbol{P})$ of Proposition 3.1 and the following postulates are valid $(\forall h, j, k \in \Xi)$:*

$$\begin{pmatrix} P_T(h) \geq \min\{P_T((k \diamond j) \diamond (k \diamond h)), P_T(j)\}, \\ P_{IT}(h) \geq \min\{P_{IT}((k \diamond j) \diamond (k \diamond h)), P_{IT}(j)\}, \\ P_{IF}(h) \leq \max\{P_{IF}((k \diamond j) \diamond (k \diamond h)), P_{IF}(j)\}, \\ P_F(h) \leq \max\{P_F((k \diamond j) \diamond (k \diamond h)), P_F(j)\} \end{pmatrix}.$$

Example 3.5. *Let $\Xi = \{0, v, b, n, l\}$ be a UP-algebra with a binary operation " $\diamond$ " defined by the following Cayley table:*

Table 7. A UP-algebra $\Xi = \{0, v, b, n, l\}$ of Example 3.5

$\diamond$	0	v	b	n	l
0	0	v	b	n	l
v	0	0	b	n	l
b	0	v	0	n	l
n	0	v	0	0	l
l	0	v	0	n	0

Define a GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ in Ξ as follows:

$$P = \begin{pmatrix} \Xi & 0 & v & b & n & l \\ P_T & 0.4 & 0.4 & 0.4 & 0.4 & 0.4 \\ P_{IT} & 0.5 & 0.5 & 0.5 & 0.5 & 0.5 \\ P_{IF} & 0.3 & 0.3 & 0.3 & 0.3 & 0.3 \\ P_F & 0.9 & 0.9 & 0.9 & 0.9 & 0.9 \end{pmatrix}.$$

Then, P is a GNUP-SI of Ξ.

Definition 3.6. *Let $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ be a GNS of Ξ. Then, P is said to be a constant GNS in Ξ if $P_T(h) = P_T(0)$, $P_{IT}(h) = P_{IT}(0)$, $P_{IF}(h) = P_{IF}(0)$ and $P_F(h) = P_F(0)$ $\forall h \in \Xi$.*

Theorem 3.1. *A GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ in Ξ is constant if and only if it is a GNUP-SI of Ξ.*

Proof. Assume that P is a constant GNS in Ξ. Then, $P_T(h) = P_T(0)$, $P_{IT}(h) = P_{IT}(0)$, $P_{IF}(h) = P_{IF}(0)$ and $P_F(h) = P_F(0)$ for all $h \in \Xi$. This implies that, $P_T(0) \geq P_T(h)$, $P_{IT}(0) \geq P_{IT}(h)$, $P_{IF}(0) \leq P_{IF}(h)$ and $P_F(0) \leq P_F(h)$. Also, for all $h, j, k \in \Xi$, we get

$$\min\{P_T((k \diamond j) \diamond (k \diamond h)), P_T(j)\} = \min\{P_T(0), P_T(0)\} = P_T(0) = P_T(h),$$
$$\min\{P_{IT}((k \diamond j) \diamond (k \diamond h)), P_{IT}(j)\} = \min\{P_{IT}(0), P_{IT}(0)\} = P_{IT}(0) = P_{IT}(h),$$
$$\max\{P_{IF}((k \diamond j) \diamond (k \diamond h)), P_{IF}(j)\} = \max\{P_{IF}(0), P_{IF}(0)\} = P_{IF}(0) = P_{IF}(h),$$
$$\max\{P_F((k \diamond j) \diamond (k \diamond h)), P_F(j)\} = \max\{P_F(0), P_F(0)\} = P_F(0) = P_F(h).$$

Hence, P is a GNUP-SI of Ξ.

Conversely, assume that P is a GNUP-SI of Ξ. Then, for all $h, j, k \in \Xi$, we have

$$\begin{aligned} P_T(h) &\geq \min\{P_T((h \diamond 0) \diamond (h \diamond h)), P_T(0)\} \\ &= \min\{P_T(h \diamond h), P_T(0)\} \\ &= \min\{P_T(0), P_T(0)\} \\ &= P_T(0) \\ &\geq P_T(h), \end{aligned}$$

$$\begin{aligned} P_{IT}(h) &\geq \min\{P_{IT}((h \diamond 0) \diamond (h \diamond h)), P_{IT}(0)\} \\ &= \min\{P_{IT}(0 \diamond (h \diamond h)), P_{IT}(0)\} \\ &= \min\{P_{IT}(h \diamond h), P_{IT}(0)\} \\ &= \min\{P_{IT}(0), P_{IT}(0)\} \\ &= P_{IT}(0) \\ &\geq P_{IT}(h), \end{aligned}$$

$$
\begin{aligned}
P_{IF}(h) &\leq \max\{P_{IF}((h \diamond 0) \diamond (h \diamond h)), P_{IF}(0)\} \\
&= \max\{P_{IF}(0 \diamond (h \diamond h)), P_{IF}(0)\} \\
&= \max\{P_{IF}(h \diamond h), P_{IF}(0)\} \\
&= \max\{P_{IF}(0), P_{IF}(0)\} \\
&= P_{IF}(0) \\
&\leq P_{IF}(h),
\end{aligned}
$$

$$
\begin{aligned}
P_{F}(h) &\leq \max\{P_{F}((h \diamond 0) \diamond (h \diamond h)), P_{F}(0)\} \\
&= \max\{P_{F}(0 \diamond (h \diamond h)), P_{F}(0)\} \\
&= \max\{P_{F}(h \diamond h), P_{F}(0)\} \\
&= \max\{P_{F}(0), P_{F}(0)\} \\
&= P_{F}(0) \\
&\leq P_{F}(h).
\end{aligned}
$$

Thus, $P_T(0) = P_T(h), P_{IT}(0) = P_{IT}(h), P_{IF}(0) = P_{IF}(h)$ and $P_F(0) = P_F(h)$ $(\forall h \in \Xi)$. Hence, P is a constant. □

4. Some Relations of Generalized Neutrosophic UP-Algebraic Substructures

This section discusses the relations between GNUP-Ss (resp., GNNUP-Fs, GNUP-Fs, GNUP-Is and GNUP-SIs) in UP-algebras P.

Theorem 4.1. Every GNNUP-F of Ξ is a GNUP-S.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNNUP-F of Ξ. Then, $P_T(0) \geq P_T(h)$, $P_{IT}(0) \geq P_{IT}(h)$, $P_{IF}(0) \leq P_{IF}(h)$ and $P_F(0) \leq P_F(h)$. $\forall h \in \Xi$. Now, let $h, j \in \Xi$. Then,

$$
\begin{aligned}
P_T(h \diamond j) \geq P_T(j) \geq \min\{P_T(h), P_T(j)\}, \\
P_{IT}(h \diamond j) \geq P_{IT}(j) \geq \min\{P_{IT}(h), P_{IT}(j)\}, \\
P_{IF}(h \diamond j) \leq P_{IF}(j) \leq \max\{P_{IF}(h), P_{IF}(j)\}, \\
P_{F}(h \diamond j) \leq P_{F}(j) \leq \max\{P_{F}(h), P_{F}(j)\}.
\end{aligned}
$$

Hence, P is a GNUP-S of Ξ. □

The following example shows that the converse of Theorem 4.1 is not true.

Example 4.1. *From Example 3.1, P is a GNUP-S of Ξ. Since*

$$P_{IF}(v \diamond b) = 0.4 \nleq P_{IF}(b) = 0.1,$$

then P is not a GNNUP-F of Ξ.

Theorem 4.2. *Every GNUP-F of Ξ is a GNNUP-F.*

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-F of Ξ. Then, $P_T(0) \geq P_T(h)$, $P_{IT}(0) \geq P_{IT}(h)$, $P_{IF}(0) \leq P_{IF}(h)$ and $P_F(0) \leq P_F(h)$ $\forall h \in \Xi$. Now, let $h, j \in \Xi$. Then,

$$\begin{aligned}
P_T(h \diamond j) &\geq \min\{P_T(j \diamond (h \diamond j)), P_T(j)\} \\
&= \min\{P_T(0), P_T(j)\} \\
&= P_T(j), \\
P_{IT}(h \diamond j) &\geq \min\{P_{IT}(j \diamond (h \diamond j)), P_{IT}(j)\} \\
&= \min\{P_{IT}(0), P_{IT}(j)\} \\
&= P_{IT}(j), \\
P_{IF}(h \diamond j) &\leq \max\{P_{IF}(j \diamond (h \diamond j)), P_{IF}(j)\} \\
&= \max\{P_{IF}(0), P_{IF}(j)\} \\
&= P_{IF}(j), \\
P_F(h \diamond j) &\leq \max\{P_F(j \diamond (h \diamond j)), P_F(j)\} \\
&= \max\{P_F(0), P_F(j)\} \\
&= P_F(j).
\end{aligned}$$

Hence, P is a GNNUP-F of Ξ. □

The following example shows that the converse of Theorem 4.2 is not true.

Example 4.2. *From Example 3.2, P is a GNNUP-F of Ξ. Since*

$$P_T(b) = 0.5 \ngeq 0.6 = \min\{P_T(v \diamond b), P_T(v)\},$$

we have P is not a GNUP-F of Ξ.

Theorem 4.3. Every GNUP-I of Ξ is a GNUP-F.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-I of Ξ. Then, $P_T(0) \geq P_T(h)$, $P_{IT}(0) \geq P_{IT}(h)$, $P_{IF}(0) \leq P_{IF}(h)$ and $P_F(0) \leq P_F(h)$. $\forall h \in \Xi$. Now, let $h, j \in \Xi$. Then,

$$\begin{aligned}
P_T(j) &= P_T(0 \diamond j) \\
&\geq \min\{P_T(0 \diamond (h \diamond j)), P_T(h)\} \\
&= \min\{P_T(h \diamond j), P_T(h)\}, \\
P_{IT}(j) &= P_{IT}(0 \diamond j) \\
&\geq \min\{P_{IT}(0 \diamond (h \diamond j)), P_{IT}(h)\} \\
&= \min\{P_{IT}(h \diamond j), P_{IT}(h)\}, \\
P_{IF}(j) &= P_{IF}(0 \diamond j) \\
&\leq \max\{P_{IF}(0 \diamond (h \diamond j)), P_{IF}(h)\} \\
&= \max\{P_{IF}(h \diamond j), P_{IF}(h)\},
\end{aligned}$$

$$\begin{aligned} P_F(j) &= P_F(0 \diamond j) \\ &\leq \max\{P_F(0 \diamond (h \diamond j)), P_F(h)\} \\ &= \max\{P_F(h \diamond j), P_F(h)\}. \end{aligned}$$

Hence, P is a GNUP-F of Ξ. □

The following example shows that the converse of Theorem 4.3 is not true.

Example 4.3. *From Example 3.3, we have P is a GNUP-F of Ξ. Since*

$$P_T(n \diamond l) = 0.2 \ngeq 0.5 = \min\{P_T(n \diamond (b \diamond l)), P_T(b)\},$$

we have P is not a GNUP-I of Ξ.

Theorem 4.4. *Every GNUP-SI of Ξ is a GNUP-I.*

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNUP-SI of Ξ. Then, $P_T(0) \geq P_T(h)$, $P_{IT}(0) \geq P_{IT}(h)$, $P_{IF}(0) \leq P_{IF}(h)$ and $P_F(0) \leq P_F(h)$ $\forall h \in \Xi$. Now, let $h, j, k \in \Xi$. Then,

$$\begin{aligned} P_T(h \diamond k) = P_T(j) &\geq \min\{P_T(h \diamond (j \diamond k)), P_T(j)\}, \\ P_{IT}(h \diamond k) = P_{IT}(j) &\geq \min\{P_{IT}(h \diamond (j \diamond k)), P_{IT}(j)\}, \\ P_{IF}(h \diamond k) = P_{IF}(j) &\leq \max\{P_{IF}(h \diamond (j \diamond k)), P_{IF}(j)\}, \\ P_F(h \diamond k) = P_F(j) &\leq \max\{P_F(h \diamond (j \diamond k)), P_F(j)\}. \end{aligned}$$

Hence, P is a GNUP-I of Ξ. □

The following example shows that the converse of Theorem 4.4 is not true.

Example 4.4. *From Example 3.4, P is a GNUP-I of Ξ. Since*

$$P_T(n) = 0.5 < 0.8 = \min\{P_T((b \diamond 0) \diamond (b \diamond n)), P_T(0)\},$$

then P is not a GNUP-SI of Ξ.

Remark 4.1. *Using Theorems 4.1, 4.2, 4.3 and 4.4; and Examples 4.1, 4.2, 4.3 and 4.4 show the following:*

- *A GNUP-S is an extension of a GNNUP-F.*
- *A GNNUP-F is an extension of a GNUP-F.*
- *A GNUP-F is an extension of a GNUP-I.*
- *A GNUP-I is an extension of GNUP-SIs.*
- *Theorem 3.1 obtains that a GNUP-SI and a constant GNS are coincided.*

Theorem 4.5. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNUP-S of Ξ satisfying the following condition $(\forall h, j \in \Xi)$:*

$$(h \diamond j \neq 0) \Rightarrow \begin{pmatrix} P_T(h) \geq P_T(j), \\ P_{IT}(h) \geq P_{IT}(j), \\ P_{IF}(h) \leq P_{IF}(j), \\ P_F(h) \leq P_F(j) \end{pmatrix},$$

then P is a GNNUP-F of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-S of Ξ satisfying the assumption. This implies that P satisfies the conditions in Proposition 3.1. Now, let $h, j \in \Xi$. Then, we have the following two cases:

Case (1). If $(h \diamond j = 0)$, then

$$\begin{aligned} P_T(h \diamond j) &= P_T(0) \geq P_T(j), \\ P_{IT}(h \diamond j) &= P_{IT}(0) \geq P_{IT}(j), \\ P_{IF}(h \diamond j) &= P_{IF}(0) \leq P_{IF}(j), \\ P_F(h \diamond j) &= P_F(0) \leq P_F(j). \end{aligned}$$

Case (2). If $(h \diamond j \neq 0)$, then

$$\begin{aligned} P_T(h \diamond j) &\geq \min\{P_T(h), P_T(j)\} = P_T(j), \\ P_{IT}(h \diamond j) &\geq \min\{P_{IT}(h), P_{IT}(j)\} = P_{IT}(j), \\ P_{IF}(h \diamond j) &\leq \max\{P_{IF}(h), P_{IF}(j)\} = P_{IF}(j), \\ P_F(h \diamond j) &\leq \max\{P_F(h), P_F(j)\} = P_F(j). \end{aligned}$$

Thus, P is a GNNUP-F of Ξ. □

Theorem 4.6. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-F of Ξ satisfying the following condition $(\forall h, j, k \in \Xi)$:*

$$\begin{pmatrix} P_T(j \diamond (h \diamond k)) = P_T(h \diamond (j \diamond k)) \\ P_{IT}(j \diamond (h \diamond k)) = P_{IT}(h \diamond (j \diamond k)) \\ P_{IF}(j \diamond (h \diamond k)) = P_{IF}(h \diamond (j \diamond k)) \\ P_F(j \diamond (h \diamond k)) = P_F(h \diamond (j \diamond k)) \end{pmatrix},$$

then P is a GNUP-I of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNUP-F of Ξ satisfying the assumption. Then, P satisfies the conditions in Proposition 3.1. Now, let $h, j, k \in \Xi$. Then,

$$\begin{aligned} P_T(h \diamond k) &\geq \min\{P_T(j \diamond (h \diamond k)), P_T(j)\} \\ &= \min\{P_T(h \diamond (j \diamond k)), P_T(j)\}, \\ P_{IT}(h \diamond k) &\geq \min\{P_{IT}(j \diamond (h \diamond k)), P_{IT}(j)\} \\ &= \min\{P_{IT}(h \diamond (j \diamond k)), P_{IT}(j)\}, \\ P_{IF}(h \diamond k) &\leq \max\{P_{IF}(j \diamond (h \diamond k)), P_{IF}(j)\} \\ &= \max\{P_{IF}(h \diamond (j \diamond k)), P_{IF}(j)\}, \\ P_F(h \diamond k) &\leq \max\{P_F(j \diamond (h \diamond k)), P_F(j)\} \\ &= \max\{P_F(h \diamond (j \diamond k)), P_F(j)\}. \end{aligned}$$

Therefore, P is a GNUP-I of Ξ. □

5. UP-Algebraic Substructures and Generalized Neutrosophic Structures

This section investigates some results on certain types of UP-algebraic substructures in view of GNSs.

Theorem 5.1. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNS of Ξ satisfying the condition* $(\mathbf{S1})$, *where*

$$(\mathbf{S1}) \quad (\forall h, j, k \in \Xi)(k \leq h \diamond j) \Rightarrow \begin{pmatrix} P_T(k) \geq \min\{P_T(h), P_T(j)\}, \\ P_{IT}(k) \geq \min\{P_{IT}(h), P_{IT}(j)\}, \\ P_{IF}(k) \leq \max\{P_{IF}(h), P_{IF}(j)\}, \\ P_F(k) \leq \max\{P_F(h), P_F(j)\} \end{pmatrix},$$

then P is a GNUP-S of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNS of Ξ satisfying the condition $(\mathbf{S1})$. Let $h, j \in \Xi$. Then, by (1) of Proposition 2.1, $(h \diamond j) \diamond (h \diamond j) = 0$, that is $h \diamond j \geq h \diamond j$. It follows from $(\mathbf{S1})$ that

$$\begin{aligned} P_T(h \diamond j) &\geq \min\{P_T(h), P_T(j)\}, \\ P_{IT}(h \diamond j) &\geq \min\{P_{IT}(h), P_{IT}(j)\}, \\ P_{IF}(h \diamond j) &\leq \max\{P_{IF}(h), P_{IF}(j)\}, \\ P_F(h \diamond j) &\leq \max\{P_F(h), P_F(j)\}. \end{aligned}$$

Hence, P is a GNUP-S of Ξ. □

Theorem 5.2. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNS of Ξ satisfying the condition* $(\mathbf{S2})$, *where*

$$(\mathbf{S2}) \quad (\forall h, j, k \in \Xi)(k \leq h \diamond j) \Rightarrow \begin{pmatrix} P_T(j) \geq \min\{P_T(k), P_T(h)\}, \\ P_{IT}(j) \geq \min\{P_{IT}(k), P_{IT}(h)\}, \\ P_{IF}(j) \leq \max\{P_{IF}(k), P_{IF}(h)\}, \\ P_F(j) \leq \max\{P_F(k), P_F(h)\} \end{pmatrix},$$

then P is a GNUP-F of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F \rangle$ is a GNS of Ξ satisfying the condition $(\mathbf{S2})$. Let $h \in \Xi$. Then, by (3) of Definition 2.1, $h \diamond (h \diamond 0) = 0$, that is $(h \leq h \diamond 0)$. It follows that from $(\mathbf{S2})$ that

$$\begin{aligned} P_T(0) &\geq \min\{P_T(h), P_T(h)\} = P_T(h), \\ P_{IT}(0) &\geq \min\{P_{IT}(h), P_{IT}(h)\} = P_{IT}(h), \\ P_{IF}(0) &\leq \max\{P_{IF}(h), P_{IF}(h)\} = P_{IF}(h), \\ P_F(0) &\leq \max\{P_F(h), P_F(h)\} = P_F(h). \end{aligned}$$

Next, let $h, j \in \Xi$. Then, by (1) of Proposition 2.1, we have $(h \diamond j) \diamond (h \diamond j) = 0$, that is $h \diamond j \geq h \diamond j$. This implies that

$$\begin{aligned} P_T(j) &\geq \min\{P_T(h \diamond j), P_T(h)\}, \\ P_{IT}(j) &\geq \min\{P_{IT}(h \diamond j), P_{IT}(h)\}, \\ P_{IF}(j) &\leq \max\{P_{IF}(h \diamond j), P_{IF}(h)\}, \\ P_F(j) &\leq \max\{P_F(h \diamond j), P_F(h)\}. \end{aligned}$$

Thus, P is a GNUP-F of Ξ. □

Theorem 5.3. *If $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNS of Ξ satisfying the condition $(\mathbf{S3})$, where $(\forall a, h, j, k \in \Xi)$*

$$(\mathbf{S3}) \quad (a \leq h \diamond (j \diamond k)) \Rightarrow \begin{pmatrix} P_T(h \diamond k) \geq \min\{P_T(a), P_T(j)\}, \\ P_{IT}(h \diamond k) \geq \min\{P_{IT}(a), P_{IT}(j)\}, \\ P_{IF}(h \diamond k) \leq \max\{P_{IF}(a), P_{IF}(j)\}, \\ P_F(h \diamond k) \leq \max\{P_F(a), P_F(j)\} \end{pmatrix},$$

then P is a GNUP-I of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNS of Ξ satisfying the condition $(\mathbf{S3})$. Let $h \in \Xi$. Then, by (3) of Definition 2.1, $h \diamond (0 \diamond (h \diamond 0)) = 0$, that is $h \leq 0 \diamond (h \diamond 0)$. It follows that

$$\begin{aligned} P_T(0) = P_T(0 \diamond 0) &\geq \min\{P_T(h), P_T(h)\} = P_T(h), \\ P_{IT}(0) = P_{IT}(0 \diamond 0) &\geq \min\{P_{IT}(h), P_{IT}(h)\} = P_{IT}(h), \\ P_{IF}(0) = P_{IF}(0 \diamond 0) &\leq \max\{P_{IF}(h), P_{IF}(h)\} = P_{IF}(h), \\ P_F(0) = P_F(0 \diamond 0) &\leq \max\{P_F(h), P_F(h)\} = P_F(h). \end{aligned}$$

Next, let $h, j, k \in \Xi$. Then, by (1) of Definition 2.1, we have $(h \diamond (j \diamond k)) \diamond (h \diamond (j \diamond k)) = 0$, that is $h \diamond (j \diamond k) \geq h \diamond (j \diamond k)$. It follows that

$$\begin{aligned} P_T(h \diamond k) &\geq \min\{P_T(h \diamond (j \diamond k)), P_T(j)\}, \\ P_{IT}(h \diamond k) &\geq \min\{P_{IT}(h \diamond (j \diamond k)), P_{IT}(j)\}, \\ P_{IF}(h \diamond k) &\leq \max\{P_{IF}(h \diamond (j \diamond k)), P_{IF}(j)\}, \\ P_F(h \diamond k) &\leq \max\{P_F(h \diamond (j \diamond k)), P_F(j)\}. \end{aligned}$$

Hence, P is a GNUP-I of Ξ. □

Theorem 5.4. *A GNS $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ satisfies the condition $(\mathbf{S4})$, where*

$$(\mathbf{S4}) \quad (\forall h, j, k \in \Xi)(k \leq h \diamond j) \Rightarrow \begin{pmatrix} P_T(k) \geq P_T(j), \\ P_{IT}(k) \geq P_{IT}(j), \\ P_{IF}(k) \leq P_{IF}(j), \\ P_F(k) \leq P_F(j) \end{pmatrix}$$

if and only if P is a GNUP-SI of Ξ.

Proof. Assume that $P = \langle P_T, P_{IT}, P_{IF}, P_F\rangle$ is a GNS of Ξ satisfying the condition $(\mathbf{S4})$. Let $h, j \in \Xi$. Then, By (3) of Definition 2.1 and (1) of Definition 2.1, $(h \diamond 0 = 0)$, that is $(h \leq 0 = j \diamond j)$. It follows from $(\mathbf{S4})$ that

$$P_T(h) \geq P_T(j), P_{IT}(h) \geq P_{IT}(j), P_{IF}(h) \leq P_{IF}(j) \text{ and } P_F(h) \leq P_F(j).$$

Similarly,

$$P_T(j) \geq P_T(h), P_{IT}(j) \geq P_{IT}(h), P_{IF}(j) \leq P_{IF}(h) \text{ and } P_F(j) \leq P_F(h).$$

Then,

$$P_T(h) = P_T(j), P_{IT}(h) = P_{IT}(j), P_{IF}(h) = P_{IF}(j) \text{ and } P_F(h) = P_F(j).$$

Thus, P is constant. Hence, by Theorem 3.1, P is a GNUP-SI of Ξ. □

6. Conclusions

In this paper, we introduced the notions of GNUP-Ss, GNNUP-Fs, GNUP-Fs, GNUP-Is and GNUP-SIs in UP-algebras and proved their generalizations. Furthermore, we discussed the relationship between GNUP-Ss (resp., GNNUP-Fs, GNUP-Fs, GNUP-Is and GNUP-SIs) in UP-algebras. After that, the conditions under which GNUP-S can be GNNUP-F, and the condition under which GNUP-F can be GNUP-I in UP-algebra were discovered. At last, we presented and proved some characterizations theorems of GNSs in connection with UP-subalgebraic structures. In the future work, we will use the idea and results in this paper to study other algebraic structures, for example, KU-algebras, hoop algebras, MV-algebra and equality algebra.

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