Int. J. Anal. Appl. (2026), 24:19

International Journal of Analysis and Applications

Interval Valued Fuzzy Ordered Almost n-Interior-Ideals in Ordered Semigroups

Anothai Phukhaengsi[1], Pannawit Khamrot[2], Aiyared Iampan[1], Thiti Gaketem[1,*]

[1] *Department of Mathematics, School of Science, University of Phayao, Mae Ka, Mueang, Phayao 56000, Thailand*

[2] *Department of Mathematics, Faculty of Science and Agricultural Technology, Rajamangala University Technology Lanna Phitsanulok, Phitsanulok, Thailand*

[*] *Corresponding author:* thiti.ga@up.ac.th

Abstract. An almost ideal in a semigroup is a generalization of the concept of an ideal initiated by Grosek and Stako in 1980. In 2019, S. Suebsung et al. developed almost (m, n)-ideals in semigroups. Later, in 2021, T. Gaketem. introduced interval valued fuzzy almost (m, n)-ideals in semigroups. This paper aims we define interval valued fuzzy ordered almost n-interior ideals ordered semigroups. We prove some basic properties of interval valued fuzzy ordered almost n-interior ideals in ordered semigroups. And, we investigate a bridge between almost n-interior ideals and interval valued fuzzy ordered almost n-interior ideals in ordered semigroups.

1. Introduction

Ordered semigroups are an algebraic structure in a binary operation consisting of the associative property and a partial order, which has been applied in the study of many fields of study, such as coding theory, automata, etc. n-interior ideals in ordered semigroups studied by N. Tiprachot et al. in 2022, [13]. In the late twentieth century, the definition of fuzzy sets was studied by Zadeh 1965, [21]. He later developed the study into interval valued fuzzy sets in 1975, [22]. Both studies of fuzzy sets and interval valued fuzzy set models have been applied to many fields of study, such as medical science, theoretical physics, robotics, computer science, control engineering, information science, measure theory, logic, set theory, and topology. In 2006, A. L. Narayanan and T. Manikantan [15] developed the theory of interval valued fuzzy subsemigroup and studied types of interval valued fuzzy ideals in semigroups. In 1985, Satko and Grosek [18] discussed the concept of an almost-ideal (A-ideal) in semigroups. And S. Bogdanovic [1] gave the concept of

Received: Oct. 29, 2025.

2020 *Mathematics Subject Classification.* 06F05, 06D72, 08A72.

Key words and phrases. almost n-interior ideals, interval valued fuzzy ordered almost n-interior ideals; ordered semigroups.

https://doi.org/10.28924/2291-8639-24-2026-19
ISSN: 2291-8639

almost bi-ideals in semigroups. In 2020, Chinram et al. [8] discussed almost interior ideals and weakly almost interior ideals in semigroups and studied the relationship between almost interior ideals and weakly almost interior ideals in semigroups. In 2019, S. Suebsung et al. [19] studied almost (m,n)-ideals in semigroups. Later, in 2022, S. Suebsung et al. [20] introduced almost ideals in ordered semigroups. This paper aims to define almost (m,n)-ideals and fuzzy almost (m,n)-ideals in ordered semigroups. Meanwhile, T. Gaketem [3–5] studied types interval-valued fuzzy almost (m,n)-ideals in semigroups.

In the same year, T. Gaketem and P. Khamrot [6] explored the concept of almost ideals within the framework of bipolar fuzzy sets, explicitly focusing on bipolar fuzzy almost bi-ideals in semigroups. In 2023, T. Gaketem and P. Khamrot [7] studied bipolar fuzzy almost interior ideals in semigroups. In 2024, T. Gaketem and P. Khamrot [10,11] discussed bipolar fuzzy almost ideals and quasi ideals in semigroups. In addition, almost Ideal's work also has many studies, such as almost ideals in ordered semigroup [2], almost ideals in semirings [16], almost ideals in ternary semiring [17], etc. In 2025, P. Khamrot et al. [12] studied fuzzy (m,n)-ideals and n-interior ideals in ordered semigroups. In the same year, P. Khamrot et al. [13] studied concepts of ordered almost (m,n)-ideals in ordered semigroups.

In this paper, we defined interval valued fuzzy ordered almost n-interior ideals ordered semigroups. We prove some basic properties of interval valued fuzzy ordered almost n-interior ideals in ordered semigroups. And, we investigate a bridge between almost n-interior ideals and interval valued fuzzy ordered almost n-interior ideals in ordered semigroups.

2. Preliminaries

In this section, we will repeat the definitions such as ordered semigroups, fuzzy sets, interval valued fuzzy sets, and almost ideals.

Definition 2.1. *[14]. Let $\ddot{\Omega}$ be a set with a binary operation consisting of $\cdot$ and a binary operation relation $\leq$. Then $(\ddot{\Omega},\cdot,\leq)$ is called an* ordred semigroup *if*

(1) $(\ddot{\Omega},\cdot)$ *is a semigroup,*
(2) $(\ddot{\Omega},\leq)$ *is a partially ordered set,*
(3) *for all* $\ddot{\mathfrak{x}},\ddot{\mathfrak{y}},\ddot{\ddot{\mathfrak{z}}} \in \ddot{\Omega}$, *we have* $\ddot{\mathfrak{x}} \leq \ddot{\mathfrak{y}}$ *then* $\ddot{\mathfrak{x}}\ddot{\ddot{\mathfrak{z}}} \leq \ddot{\mathfrak{y}}\ddot{\ddot{\mathfrak{z}}}$ *and* $\ddot{\ddot{\mathfrak{z}}}\ddot{\mathfrak{x}} \leq \ddot{\ddot{\mathfrak{z}}}\ddot{\mathfrak{y}}$.

For a nonempty subset $\ddot{\Omega}_1$ and $\ddot{\Omega}_2$ of ordered semigroup $\ddot{\Omega}$, we write $\ddot{\Omega}_1 := \{\ddot{\mathfrak{x}} \in \ddot{\Omega}_1 \mid \ddot{\mathfrak{x}} \leq \ddot{\mathfrak{y}}$ for some $\ddot{\mathfrak{x}} \in \ddot{\Omega}\}$ and $\ddot{\Omega}_1\ddot{\Omega}_2 := \{\ddot{\mathfrak{x}}\ddot{\mathfrak{y}} \mid \ddot{\mathfrak{x}} \in \ddot{\Omega}_1 \text{ and } \ddot{\mathfrak{y}} \in \ddot{\Omega}_2\}$.

It is observed that

(1) $\ddot{\Omega}_1 \subseteq (\ddot{\Omega}_1]$,
(2) if $\ddot{\Omega}_1 \subseteq \ddot{\Omega}_2$, then $(\ddot{\Omega}_1] \subseteq (\ddot{\Omega}_2]$,
(3) $((\ddot{\Omega}_1]] = (\ddot{\Omega}_1]$,
(4) $(\ddot{\Omega}_1](\ddot{\Omega}_2] \subseteq (\ddot{\Omega}_1\ddot{\Omega}_2]$,
(5) $((\ddot{\Omega}_1](\ddot{\Omega}_2]] = (\ddot{\Omega}_1\ddot{\Omega}_2]$,

(6) $(\ddot{\Omega}_1 \cup \ddot{\Omega}_2] = (\ddot{\Omega}_1] \cup (\ddot{\Omega}_2]$,
(7) $(\ddot{\Omega}_1 \cap \ddot{\Omega}_2] = (\ddot{\Omega}_1] \cap (\ddot{\Omega}_2]$.

Let $(\emptyset \neq \ddot{\aleph} \subseteq \ddot{\Omega})$ is called a *subsemigroup* (SSG) such that $\ddot{\aleph}^2 \subseteq \ddot{\aleph}$ and $(\ddot{\aleph}]$. A *left (right) ideal* of an ordered semigroup $\ddot{\Omega}$ is a non-empty set $\ddot{\aleph}$ of $\ddot{\Omega}$ such that $\ddot{\aleph}\ddot{\Omega} \subseteq \ddot{\aleph}$ $(\ddot{\Omega}\ddot{\aleph} \subseteq \ddot{\aleph})$ and $(\ddot{\aleph}]$. By an *ideal* of an ordered semigroup $\ddot{\Omega}$, we mean a non-empty set of $\ddot{\Omega}$ which is both a left and a right ideal of $\ddot{\Omega}$. A subsemigroup of an ordered semigroup $\ddot{\Omega}$ is called an *interior ideal* if $\ddot{\Omega}\ddot{\aleph}\ddot{\Omega} \subseteq \ddot{\aleph}$ and $(\ddot{\aleph}]$.

Definition 2.2. *[14] A SSG $\ddot{\aleph}$ of an ordered semigroup $(\ddot{\Omega}, \cdot, \leq)$ is called an (m, n)-*ideal *of $\ddot{\Omega}$ if following conditions:*

(1) $\ddot{\aleph}^m\ddot{\Omega}\ddot{\aleph}^n \subseteq \ddot{\aleph}$.
(2) $\ddot{\aleph} = (\ddot{\aleph}]$, *that is for $\ddot{\mathfrak{x}} \in \ddot{\aleph}$ and $\ddot{\mathfrak{y}} \in \ddot{\Omega}$, $\ddot{\mathfrak{y}} \leq \ddot{\mathfrak{x}}$ implies $\ddot{\mathfrak{y}} \in \ddot{\aleph}$.*

where m, n are non-negative integers.

Definition 2.3. *[20] A subsemigroup $\ddot{\aleph}$ of an ordered semigroup $(\ddot{\Omega}, \cdot, \leq)$ is called an n-*interior ideal *of $\ddot{\Omega}$ if following conditions:*

(1) $(\ddot{\Omega}\ddot{\aleph}^n\ddot{\Omega}] \subseteq \ddot{\aleph}$.
(2) $\ddot{\aleph} = (\ddot{\aleph}]$, *that is for $\ddot{\mathfrak{x}} \in \ddot{\aleph}$ and $\ddot{\mathfrak{y}} \in \ddot{\Omega}$, $\ddot{\mathfrak{y}} \leq \ddot{\mathfrak{x}}$ implies $\ddot{\mathfrak{y}} \in \ddot{\aleph}$.*

where n are non-negative integers.

Definition 2.4. *[20] A nonempty subset of $\ddot{\aleph}$ an ordered semigroup $\ddot{\Omega}$ is called*

(1) *a* left ordered almost ideal *(LOAI) of $\ddot{\Omega}$ if $(\ddot{\mathfrak{s}}\ddot{\aleph}] \cap \ddot{\aleph} \neq \emptyset$ for all $\ddot{\mathfrak{s}} \in \ddot{\Omega}$,*
(2) *a* right ordered almost ideal *(ROAI) of $\ddot{\Omega}$ if $(\ddot{\aleph}\ddot{\mathfrak{s}}] \cap \ddot{\aleph} \neq \emptyset$ for all $\ddot{\mathfrak{s}} \in \ddot{\Omega}$,*
(3) *an* ordered almost ideal *(OAI) of $\ddot{\Omega}$ if it is both LOAT and ROAI of $\ddot{\Omega}$,*
(4) *an* ordered almost n-interior ideal *(OA-n-II) of $\ddot{\Omega}$ if $(\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}] \cap \ddot{\aleph} \neq \emptyset$, for all $\ddot{\mathfrak{s}}, \ddot{\mathfrak{b}} \in \ddot{\Omega}$ and n are non-negative integers [9].*

For any $\ddot{\mathfrak{h}}_i \in [0, 1]$, $i \in \ddot{\mathcal{F}}$, define

$$\bigvee_{i \in \ddot{\mathcal{F}}} \ddot{\mathfrak{h}}_i := \sup_{i \in \ddot{\mathcal{F}}}\{\ddot{\mathfrak{h}}_i\} \quad \text{and} \quad \bigwedge_{i \in \ddot{\mathcal{F}}} \ddot{\mathfrak{h}}_i := \inf_{i \in \ddot{\mathcal{F}}}\{\ddot{\mathfrak{h}}_i\}.$$

We see that for any $\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}} \in [0, 1]$, we have

$$\ddot{\mathfrak{h}} \vee \ddot{\mathfrak{r}} = \max\{\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}}\} \quad \text{and} \quad \ddot{\mathfrak{h}} \wedge \ddot{\mathfrak{r}} = \min\{\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}}\}.$$

A *fuzzy set* $\ddot{\Upsilon}$ in a nonempty set $\ddot{\mathfrak{T}}$ is a function from $\ddot{\mathfrak{T}}$ into the unit closed interval $[0, 1]$ of real numbers, i.e., $\ddot{\Upsilon} : \ddot{\mathfrak{T}} \to [0, 1]$.

Let CS$[0, 1]$ be the set of all closed subintervals of $[0, 1]$, i.e.,

$$\mathrm{CS}[0, 1] = \{\tilde{\ddot{\Upsilon}} = [\ddot{\Upsilon}^-, \ddot{\Upsilon}^+] \mid 0 \leq \ddot{\Upsilon}^- \leq \ddot{\Upsilon}^+ \leq 1\}.$$

We note that $[\ddot{\Upsilon}, \ddot{\Upsilon}] = \{\ddot{\Upsilon}\}$ for all $\ddot{\Upsilon} \in [0, 1]$. For $\ddot{\Upsilon} = 0$ or 1 we shall denote $[0, 0]$ by $\tilde{0}$ and $[1, 1]$ by $\tilde{1}$.

Let $\tilde{\ddot{\Upsilon}} = [\ddot{\Upsilon}^-, \ddot{\Upsilon}^+]$ and $\tilde{\ddot{\rho}} = [\ddot{\rho}^-, \ddot{\rho}^+] \in \mathrm{CS}[0, 1]$. Define the operations $\leq$, $=$, $\wedge$ and $\vee$ as follows:

(1) $\tilde{\ddot{\Upsilon}} \leq \tilde{\rho}$ if and only if $\ddot{\Upsilon}^- \leq \ddot{\rho}^-$ and $\ddot{\Upsilon}^+ \leq \ddot{\rho}^+$
(2) $\tilde{\ddot{\Upsilon}} = \tilde{\rho}$ if and only if $\ddot{\Upsilon}^- = \ddot{\rho}^-$ and $\ddot{\Upsilon}^+ = \ddot{\rho}^+$
(3) $\tilde{\ddot{\Upsilon}} \wedge \tilde{\rho} = [(\ddot{\Upsilon}^- \wedge \ddot{\rho}^-), (\ddot{\Upsilon}^+ \wedge \ddot{\rho}^+)]$
(4) $\tilde{\ddot{\Upsilon}} \vee \tilde{\rho} = [(\ddot{\Upsilon}^- \vee \ddot{\rho}^-), (\ddot{\Upsilon}^+ \vee \ddot{\rho}^+)]$.
If $\tilde{\ddot{\Upsilon}} \geq \tilde{\rho}$, we mean $\tilde{\rho} \leq \tilde{\ddot{\Upsilon}}$.

For each interval $\tilde{\ddot{\Upsilon}}_i = [\ddot{\Upsilon}_i^-, \ddot{\Upsilon}_i^+] \in \mathrm{CS}[0,1]$, $i \in \ddot{\mathcal{A}}$ where $\ddot{\mathcal{A}}$ is an index set, we define

$$\bigwedge_{i\in\ddot{\mathcal{A}}} \tilde{\ddot{\Upsilon}}_i = [\bigwedge_{i\in\ddot{\mathcal{A}}} \ddot{\Upsilon}_i^-, \bigwedge_{i\in\ddot{\mathcal{A}}} \ddot{\Upsilon}_i^+] \quad \text{and} \quad \bigvee_{i\in\ddot{\mathcal{A}}} \tilde{\ddot{\Upsilon}}_i = [\bigvee_{i\in\ddot{\mathcal{A}}} \ddot{\Upsilon}_i^-, \bigvee_{i\in\ddot{\mathcal{A}}} \ddot{\Upsilon}_i^+].$$

Definition 2.5. *[15] Let $\ddot{\mathfrak{T}}$ be a non-empty set. Then the function $\tilde{\ddot{\Upsilon}} : \ddot{\mathfrak{T}} \to CS[0,1]$ is called* interval valued fuzzy set *(shortly, IVF set) of $\ddot{\mathfrak{T}}$.*

Definition 2.6. *[15] Let $\ddot{\mathfrak{L}}$ be a subset of a non-empty set $\ddot{\mathfrak{T}}$. An* interval valued characteristic function *of $\ddot{\mathfrak{L}}$ is defined to be a function $\tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{L}}} : \ddot{\mathfrak{T}} \to CS[0,1]$ by*

$$\tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{L}}}(\ddot{\mathfrak{e}}) = \begin{cases} \tilde{1} & if \quad \ddot{\mathfrak{e}} \in \ddot{\mathfrak{L}}, \\ \tilde{0} & if \quad \ddot{\mathfrak{e}} \notin \ddot{\mathfrak{L}} \end{cases}$$

for all $\ddot{\mathfrak{e}} \in \ddot{\mathfrak{T}}$.

Lemma 2.1. *If $\ddot{\mathfrak{M}}$ and $\ddot{\mathfrak{L}}$ are nonempty subsets of an oredred semigroup $\mathfrak{T}$, then the following are true:*
(1) $\tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{M}}} \wedge \tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{L}}} = \tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{M}} \cap \ddot{\mathfrak{L}}}$.
(2) *If $\ddot{\mathfrak{M}} \subseteq \ddot{\mathfrak{L}}$, then $\tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{M}}} \leq \tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{L}}}$.*
(3) $\tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{M}}} \circ \tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{L}}} = \tilde{\ddot{\gtrdot}}_{\ddot{\mathfrak{M}}\ddot{\mathfrak{L}}}$.

Definition 2.7. *Let $\mathfrak{T}$ be an ordered semigroup and F be a non-empty subset of $\ddot{\Omega}$, we define the set $F_{\ddot{\mathfrak{u}}}$ by*

$$F_{\ddot{\mathfrak{u}}} := \{(\ddot{\mathfrak{x}}, \ddot{\mathfrak{y}}) \in \ddot{\Omega} \times \ddot{\Omega} \mid \ddot{\mathfrak{u}} \leq \ddot{\mathfrak{x}}\ddot{\mathfrak{y}}\}.$$

Definition 2.8. *Let $\tilde{\ddot{\Upsilon}}$ and $\tilde{\ddot{\eta}}$ be IVF sets of an ordered semigroup $\ddot{\Omega}$. The product of IVF sets $\tilde{\ddot{\Upsilon}}$ and $\tilde{\ddot{\eta}}$ of $\ddot{\Omega}$ is defined as follow, for all $\ddot{\mathfrak{x}} \in \ddot{\Omega}$*

$$(\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\eta}})(\ddot{\mathfrak{u}}) = \begin{cases} \bigvee_{(\ddot{\mathfrak{x}},\ddot{\mathfrak{y}})\in F_{\ddot{\mathfrak{u}}}} \{\tilde{\ddot{\Upsilon}}(\ddot{\mathfrak{x}}) \wedge \tilde{\ddot{\eta}}(\ddot{\mathfrak{y}})\} & if \quad F_{\ddot{\mathfrak{u}}} \neq \emptyset, \\ 0 & if \quad F_{\ddot{\mathfrak{u}}} = \emptyset. \end{cases}$$

For $\ddot{\mathfrak{k}} \in \mathbb{N}$, let $\tilde{\ddot{\Upsilon}}^n := \underbrace{\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\Upsilon}} \circ \cdots \circ \tilde{\ddot{\Upsilon}}}_{n\text{-times}}$.

The support of IVF set $\tilde{\ddot{\Upsilon}}$ of a set $\ddot{\Omega}$ is defined by $\mathrm{supp}(\tilde{\ddot{\Upsilon}}) = \{\ddot{\mathfrak{u}} \in \ddot{\Omega} | \tilde{\ddot{\Upsilon}}(\ddot{\mathfrak{u}}) \neq \tilde{0}\}$.

Lemma 2.2. *[2] If $\tilde{\ddot{\Upsilon}}$, $\tilde{\rho}$ and $\tilde{\ddot{\xi}}$ are IVF sets of an ordered semigroup $\ddot{\Omega}$, then the following are true:*
(1) *If $\tilde{\ddot{\Upsilon}} \leq \tilde{\rho}$, then $\tilde{\ddot{\Upsilon}}^n \leq \tilde{\rho}^n$*
(2) *If $\tilde{\ddot{\Upsilon}} \leq \tilde{\rho}$, then $\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\xi}} \leq \tilde{\rho} \circ \tilde{\ddot{\xi}}$.*
(3) *If $\tilde{\ddot{\Upsilon}} \leq \tilde{\rho}$, then $\tilde{\ddot{\Upsilon}} \vee \tilde{\ddot{\xi}} \leq \tilde{\rho} \vee \tilde{\ddot{\xi}}$.*

(4) *If* $\tilde{\ddot{\Upsilon}} \preceq \tilde{\rho}$, *then* $\tilde{\ddot{\Upsilon}} \wedge \tilde{\ddot{\xi}} \preceq \tilde{\rho} \wedge \tilde{\ddot{\xi}}$.
(5) *If* $\tilde{\ddot{\Upsilon}} \preceq \tilde{\rho}$, *then* $\mathrm{supp}(\tilde{\ddot{\Upsilon}}) \subseteq \mathrm{supp}(\tilde{\rho})$.

For a IVF set $\tilde{\ddot{\Upsilon}}$ of an ordered semigroup $\ddot{\Omega}$, we define $(\tilde{\ddot{\Upsilon}}] : \ddot{\Omega} \to \mathrm{CS}[0,1]$ by $(\tilde{\ddot{\Upsilon}}] := \sup_{\ddot{a} \le \ddot{\aleph}} \tilde{\ddot{\Upsilon}}(\ddot{\aleph})$ for all $\ddot{a} \in \ddot{\Omega}$.

Lemma 2.3. *[2] If* $\tilde{\ddot{\Upsilon}}$, $\tilde{\rho}$ *and* $\tilde{\ddot{\xi}}$ *are IVF sets of an ordered semigroup* $\ddot{\Omega}$, *then the following are true:*
(1) $\tilde{\ddot{\Upsilon}} \preceq (\tilde{\ddot{\Upsilon}}]$.
(2) *If* $\tilde{\ddot{\Upsilon}} \preceq \tilde{\rho}$, *then* $(\tilde{\ddot{\Upsilon}}] \preceq (\tilde{\ddot{\xi}}]$.
(3) *If* $\tilde{\ddot{\Upsilon}} \preceq \tilde{\rho}$, *then* $(\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\xi}}] \preceq (\tilde{\rho} \circ \tilde{\ddot{\xi}}]$ *and* $(\tilde{\ddot{\xi}} \circ \tilde{\ddot{\Upsilon}}] \preceq (\tilde{\ddot{\xi}} \circ \tilde{\rho}]$.

Lemma 2.4. *[2] If* $\tilde{\ddot{\Upsilon}}$ *is an IVF set of an ordered semigroup* $\ddot{\Omega}$, *then the following are equivalent.*
(1) *If* $\ddot{a} \le \ddot{b}$, *then* $\tilde{\ddot{\Upsilon}}(\ddot{a}) \succeq \tilde{\ddot{\Upsilon}}(\ddot{b})$.
(2) $(\tilde{\ddot{\Upsilon}}] = \tilde{\ddot{\Upsilon}}$.

Definition 2.9. *A IVF set* $\tilde{\ddot{\Upsilon}}$ *of an ordered semigroup* $\ddot{\Omega}$ *is called*
(1) *a* IVF subsemigroup *(IVFSG) of* $\ddot{\Omega}$ *if* $\tilde{\ddot{\Upsilon}}(\ddot{a}\ddot{b}) \preceq \tilde{\ddot{\Upsilon}}(\ddot{a}) \wedge \tilde{\ddot{\Upsilon}}(\ddot{b})$ *for all* $\ddot{a}, \ddot{b} \in \ddot{\Omega}$,
(2) *a* IVF left ideal *(IVFLI) of* $\ddot{\Omega}$ *if* $\tilde{\ddot{\Upsilon}}(\ddot{a}\ddot{b}) \preceq \tilde{\ddot{\Upsilon}}(\ddot{b})$ *and if* $\ddot{a} \le \ddot{b}$, *then* $\tilde{\ddot{\Upsilon}}(\ddot{a}) \succeq \tilde{\ddot{\Upsilon}}(\ddot{b})$ *for all* $\ddot{a}, \ddot{\aleph} \in \ddot{\Omega}$,
(3) *a* IVF right ideal *(IVFRI) of* $\ddot{\Omega}$ *if* $\tilde{\ddot{\Upsilon}}(\ddot{a}\ddot{b}) \preceq \tilde{\ddot{\Upsilon}}(\ddot{a})$ *and if* $\ddot{a} \le \ddot{b}$, *then* $\tilde{\ddot{\Upsilon}}(\ddot{a}) \succeq \tilde{\ddot{\Upsilon}}(\ddot{b})$ *for all* $\ddot{a}, \ddot{b} \in \ddot{\Omega}$,
(4) *a* TVF ideal *(IVFI) of* $\ddot{\Omega}$ *if it is both a IVFLI and IVFRI of* $\ddot{\Omega}$,
(5) *a* IVF left ordered almost ideal *(IVFLOAI) of* $\ddot{\Omega}$ *if*
$(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}] \wedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$ *where* $\tilde{\ddot{\succ}}_{\ddot{\Omega}}$ *is an IVF set of* $\ddot{\Omega}$ *mapping every element to* $\tilde{1}$,
(6) *a* IVF right ordered almost ideal *(IVFROAI) of* $\ddot{\Omega}$ *if*
$(\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\succ}}_{\ddot{\Omega}}] \wedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$.
(7) *a* IVF ordered almost ideal *(IVFOAI) of* $\ddot{\Omega}$ *if it is both a IVFLOAI and IVFROAI of* $\ddot{\Omega}$,

Definition 2.10. *An IVFSG* $\tilde{\ddot{\Upsilon}}$ *of an ordered semigroup* $\ddot{\Omega}$ *is called an* interval valued fuzzy n-interior ideal *(IVF n-id) of* $\ddot{\Omega}$ *if*
(1) $\tilde{\ddot{\Upsilon}}(\ddot{a}\ddot{r}_1\ddot{r}_2\cdots\ddot{r}_n\ddot{t}) \succeq \tilde{\ddot{\Upsilon}}(\ddot{r}_1) \wedge \tilde{\ddot{\Upsilon}}(\ddot{r}_2) \wedge \cdots \wedge \tilde{\ddot{\Upsilon}}(\ddot{r}_n)$ *for all* $\ddot{a}, \ddot{r}_1, \ddot{r}_2 \ldots, \ddot{r}_n, \ddot{t}, \in \ddot{\Omega}$.
(2) *If* $\ddot{a} \le \ddot{b}$, *then* $\tilde{\ddot{\Upsilon}}(\ddot{a}) \succeq \tilde{\ddot{\Upsilon}}(\ddot{b})$ *for all* $\ddot{a}, \ddot{b} \in \ddot{\Omega}$.

3. Interval Valued Almost n-Interior Ideals

This section defines IVFA-n-interior ideals in an ordered semigroup. We prove the union of IVFA-n-interior ideals is an IVFA-n-interior ideal. And we create a bridge between almost n-interior ideals and IVFA-n-interior ideals.

Definition 3.1. *An IVF set* $\tilde{\ddot{\Upsilon}}$ *of an ordered semigroup* $\ddot{\Omega}$ *is called a* interval valued fuzzy ordered almost n-interior ideal *(IVFOA-n-II) of* $\ddot{\Omega}$ *if* $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \wedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$ *where* $n \in \mathbb{N}_0$.

Lemma 3.1. *Let* $\tilde{\ddot{\Upsilon}}$ *be an IVF set of an ordered semigroup* $\ddot{\Omega}$. *Then every IVF0A-n-II of* $\ddot{\Omega}$ *is an IVFA-n-id of* $\ddot{\Omega}$.

Theorem 3.1. *Let $\tilde{\ddot{\Upsilon}}$ be an IVF0A-n-II and $\tilde{\ddot{\nu}}$ be an IVF set of an ordered semigroup $\ddot{\Omega}$ with $\tilde{\ddot{\Upsilon}} \preceq \tilde{\ddot{\nu}}$. Then $\tilde{\ddot{\nu}}$ is an IVF0A-n-II of $\ddot{\Omega}$.*

Proof. Suppose that $\tilde{\ddot{\Upsilon}}$ is an IVFOA-n-II of $\ddot{\Omega}$ and $\tilde{\ddot{\nu}}$ is an IVF set of $\ddot{\Omega}$ such that $\tilde{\ddot{\Upsilon}} \preceq \tilde{\ddot{\nu}}$. Then we obtain that $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$. Thus,

$$(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}} \preceq (\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\nu}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\nu}} \neq \tilde{0}.$$

Hence, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\nu}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\nu}} \neq \tilde{0}$. Therefore, $\tilde{\ddot{\nu}}$ is an IVFOA-n-II of $\ddot{\Omega}$. □

Theorem 3.2. *Let $\tilde{\ddot{\Upsilon}}$ and $\tilde{\ddot{\nu}}$ be IVFOA-n-IIs of $\ddot{\Omega}$. Then $\tilde{\ddot{\Upsilon}} \curlyvee \tilde{\ddot{\nu}}$ is an IVFOA-n-II of $\ddot{\Omega}$.*

Proof. Since $\tilde{\ddot{\Upsilon}} \preceq \tilde{\ddot{\Upsilon}} \curlyvee \tilde{\ddot{\nu}}$ we have $\tilde{\ddot{\Upsilon}} \curlyvee \tilde{\ddot{\nu}}$ is an IVFOA-n-II of $\ddot{\Omega}$ by Theorem 3.1. □

In the following theorems, we prove the relationship between almost n-interior ideal and IVFOA n-II in ordered semigroups.

Theorem 3.3. *Let $\ddot{\aleph}$ be a nonempty subset of an ordered semigroup $\ddot{\Omega}$. Then $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$ if and only if $\tilde{\chi}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$.*

Proof. Suppose that $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Then $(\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}] \cap \ddot{\aleph} \neq \emptyset$, for all $\ddot{\mathfrak{s}}, \ddot{\mathfrak{b}} \in \ddot{\Omega}$ and $n \in \mathbb{N}_0$. Thus, there exists $\ddot{\mathfrak{p}} \in \ddot{\Omega}$ such that $\ddot{\mathfrak{p}} \in (\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}]$ and $\ddot{\mathfrak{p}} \in \ddot{\aleph}$ for all $n \in \mathbb{N}_0$. So $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}](\ddot{\mathfrak{p}}) \neq \tilde{0}$ and $\tilde{\ddot{\succ}}^n_{\ddot{\aleph}}(\ddot{\mathfrak{p}}) = \tilde{1}$ for all $n \in \mathbb{N}_0$. It implies that $(((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}^n_{\ddot{\aleph}}(\ddot{\mathfrak{p}}) \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. Hence, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}^n_{\ddot{\aleph}}) \neq 0$ for all $n \in \mathbb{N}_0$. Therefore, $\tilde{\chi}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$.

For the converse, assume that $\tilde{\chi}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Then for all $n \in \mathbb{N}_0$, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}^n_{\ddot{\aleph}}) \neq 0$. Thus, there exists $\ddot{\mathfrak{p}} \in \ddot{\Omega}$ with $(((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}^n_{\ddot{\aleph}}(\ddot{\mathfrak{p}}) \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. It implies that $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\ddot{\aleph}} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}](\ddot{\mathfrak{p}}) \neq \tilde{0}$ and $\tilde{\ddot{\succ}}^n_{\ddot{\aleph}}(\ddot{\mathfrak{p}}) = \tilde{1}$ for all $n \in \mathbb{N}_0$. So $\ddot{\mathfrak{p}} \in (\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}]$ and $\ddot{\mathfrak{p}} \in \ddot{\aleph}$ for all $n \in \mathbb{N}_0$. Hence, $\ddot{\mathfrak{p}} \in (\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}] \cap \ddot{\aleph}$ for all $n \in \mathbb{N}_0$. So $(\ddot{\mathfrak{s}}\ddot{\aleph}^n\ddot{\mathfrak{b}}] \cap \ddot{\aleph} \neq \emptyset$ for all $n \in \mathbb{N}_0$. Therefore, $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. □

Theorem 3.4. *Let φ $\tilde{\ddot{\Upsilon}}$ be a nonzero IVF set of an ordered semigroup $\ddot{\Omega}$. Then $\tilde{\ddot{\Upsilon}}$ is an IVFOA-n-II of $\ddot{\Omega}$ if and only if* $\mathrm{supp}(\ddot{\Upsilon})$ *is an OA-n-II of $\ddot{\Omega}$.*

Proof. Suppose that $\tilde{\ddot{\Upsilon}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Then $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. Thus, there exist $\mathfrak{p} \in \ddot{\Omega}$ such that $((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}})(\mathfrak{p}) \neq \tilde{0}$ for all $n \in \mathbb{N}_0$ so $\tilde{\ddot{\Upsilon}}(\mathfrak{p}) \neq 0$ and $\mathfrak{z} = \mathfrak{r}\mathfrak{a}_1\mathfrak{a}_2 \cdots \mathfrak{a}_n\mathfrak{k}$ for some $\mathfrak{a}_1, \mathfrak{a}_2, \cdots, \mathfrak{r}, \mathfrak{k}, \mathfrak{a}_m \in \ddot{\Omega}$ such that $\tilde{\ddot{\Upsilon}}(\mathfrak{a}_1) \neq \tilde{0}$, $\tilde{\ddot{\Upsilon}}(\mathfrak{a}_2) \neq \tilde{0}, \cdots, \tilde{\ddot{\Upsilon}}(\mathfrak{a}_n) \neq \tilde{0}$ Thus, $\mathfrak{a}_1, \mathfrak{a}_2, \cdots \mathfrak{a}_n \in \mathrm{supp}(\tilde{\ddot{\Upsilon}})$. It implies that, $((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})}(\mathfrak{p}) \neq \tilde{0}$. Hence, $((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}})(\mathfrak{p}) \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. Therefore, $\tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})}$ is an IVFOA-n-II of $\ddot{\Omega}$. By Theorem 3.3, $\mathrm{supp}(\ddot{\Upsilon})$ is an OA-n-II of $\ddot{\Omega}$.

For the converse, assume that $\mathrm{supp}(\ddot{\Upsilon})$ is an OA-n-II of $\ddot{\Omega}$. By Theorem 3.3, $\tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})}$ is an IVFOA-n-II of $\ddot{\Omega}$. Thus, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})} \neq \tilde{0}$. So. there exists $\mathfrak{p} \in \ddot{\Omega}$ such that $((\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})}(\mathfrak{p}) \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. Hence, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\succ}}^n_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})} \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}](\mathfrak{p}) \neq \tilde{0}$ and $\tilde{\ddot{\succ}}_{\mathrm{supp}(\tilde{\ddot{\Upsilon}})}(\mathfrak{p}) \neq \tilde{0}$. Then there exists $\mathfrak{p} \in \mathrm{supp}(\tilde{\ddot{\Upsilon}})$ such that $\mathfrak{z} = \mathfrak{r}\mathfrak{a}_1\mathfrak{a}_2 \cdots \mathfrak{a}_n\mathfrak{k}$ for some $\mathfrak{a}_1, \mathfrak{a}_2, \cdots, \mathfrak{r}, \mathfrak{k}, \mathfrak{a}_m \in \ddot{\Omega}$. Thus, $\tilde{\ddot{\Upsilon}}(\mathfrak{a}_1) \neq \tilde{0}$, $\tilde{\ddot{\Upsilon}}(\mathfrak{a}_2) \neq \tilde{0}, \cdots, \tilde{\ddot{\Upsilon}}(\mathfrak{a}_n) \neq \tilde{0}$. Thus, $\mathfrak{a}_1, \mathfrak{a}_2, \cdots \mathfrak{a}_n \in \mathrm{supp}(\tilde{\ddot{\Upsilon}})$. Hence, $(\tilde{\ddot{\succ}}_{\ddot{\Omega}} \circ \tilde{\ddot{\Upsilon}}^n \circ \tilde{\ddot{\succ}}'_{\ddot{\Omega}}] \curlywedge \tilde{\ddot{\Upsilon}} \neq \tilde{0}$ for all $n \in \mathbb{N}_0$. Therefore, $\tilde{\ddot{\Upsilon}}$ is an IVFOA-n-II of $\ddot{\Omega}$. □

Next, we investigate the connection between minimal, prime, semiprime, strongly prime, almost n-interior ideals and minimal, prime, semiprime, strongly prime IVFOA-n-IIs of an ordered semigroup.

Definition 3.2. *An OA-n-II $\ddot{\mathcal{K}}$ of an SG $\ddot{\Omega}$ is called*

(1) minimal *if for any OA-n-II $\ddot{\mathcal{L}}$ of $\ddot{\Omega}$ if whenever $\ddot{\mathcal{L}} \subseteq \ddot{\mathcal{K}}$, then $\ddot{\mathcal{L}} = \ddot{\mathcal{K}}$,*

(2) prime *if for any two OA-n-IIs $\ddot{\mathcal{L}}$ and $\ddot{\mathcal{M}}$ of $\ddot{\Omega}$ such that $\ddot{\mathcal{L}}\ddot{\mathcal{M}} \subseteq \ddot{\mathcal{K}}$ implies that $\ddot{\mathcal{L}} \subseteq \ddot{\mathcal{K}}$ or $\ddot{\mathcal{M}} \subseteq \ddot{\mathcal{K}}$.*

(3) semiprime *if for any OA-n-II $\ddot{\mathcal{L}}$ of $\ddot{\Omega}$ such that $\ddot{\mathcal{L}}^2 \subseteq \ddot{\mathcal{K}}$ implies that $\ddot{\mathcal{L}} \subseteq \ddot{\mathcal{K}}$.*

(4) strongly prime *if for any OA-n-IIs $\ddot{\mathcal{L}}$ and $\ddot{\mathcal{M}}$ of $\ddot{\Omega}$ such that $\ddot{\mathcal{L}}\ddot{\mathcal{M}} \cap \ddot{\mathcal{M}}\ddot{\mathcal{L}} \subseteq \ddot{\mathcal{K}}$ implies that $\ddot{\mathcal{L}} \subseteq \ddot{\mathcal{K}}$ or $\ddot{\mathcal{M}} \subseteq \ddot{\mathcal{K}}$.*

Definition 3.3. *An IVFOA-n-II $\tilde{\ddot{\Upsilon}}$ of an ordered semigroup $\ddot{\Omega}$ is called*

(1) minimal *if for any IVFOA-n-II $\tilde{\ddot{\xi}}$ of $\ddot{\Omega}$ if whenever $\tilde{\ddot{\xi}} \preceq \tilde{\ddot{\Upsilon}}$, then* $\mathrm{supp}(\tilde{\ddot{\xi}}) = \mathrm{supp}(\tilde{\ddot{\Upsilon}})$,

(2) prime *if for any two IVFOA-n-IIs $\tilde{\ddot{\xi}}$ and $\tilde{\ddot{\rho}}$ of $\dot{\Omega}$ such that $\tilde{\ddot{\xi}} \circ \tilde{\ddot{\rho}} \preceq \tilde{\ddot{\Upsilon}}$ implies that $\tilde{\ddot{\xi}} \preceq \tilde{\ddot{\Upsilon}}$ or $\tilde{\ddot{\rho}} \preceq \tilde{\ddot{\Upsilon}}$.*

(3) semiprime *if for any IVFOA-n-II $\tilde{\ddot{\xi}}$ of $\ddot{\Omega}$ such that $\tilde{\ddot{\xi}} \circ \tilde{\ddot{\xi}} \preceq \tilde{\ddot{\xi}}$ implies that $\tilde{\ddot{\xi}} \preceq \tilde{\ddot{\Upsilon}}$.*

(4) strongly prime *if for any two IVFOA-n-IIs $\tilde{\ddot{\xi}}$ and $\tilde{\ddot{\rho}}$ of $\ddot{\Omega}$ such that $(\tilde{\ddot{\xi}} \circ \tilde{\ddot{\rho}}) \wedge (\tilde{\ddot{\rho}} \circ \tilde{\ddot{\xi}}) \preceq \tilde{\ddot{\Upsilon}}$ implies that $\tilde{\ddot{\xi}} \preceq \tilde{\ddot{\Upsilon}}$ or $\tilde{\ddot{\rho}} \preceq \tilde{\ddot{\Upsilon}}$.*

It is clear that every strongly prime IVFOA-n-II of an ordered semigroup is a prime IVFOA-n-II, and every prime IVFOA-n-II of an ordered semigroup is a semiprime IVFOA-n-II.

Theorem 3.5. *Let $\ddot{\aleph}$ be a nonempty subset of an ordered semigroup $\ddot{\Omega}$. Then*

(1) *$\ddot{\aleph}$ is a minimal OA-n-II of $\ddot{\Omega}$ if and only if $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a minimal IVFOA-n-II of $\ddot{\Omega}$.*

(2) *$\ddot{\aleph}$ is a prime OA-n-II of $\ddot{\Omega}$ if and only if $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a prime IVFOA-n-II of $\ddot{\Omega}$.*

(3) *$\ddot{\aleph}$ is a semiprime OA-n-II of $\ddot{\Omega}$ if and only if $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a semiprime IVFOA-n-II of $\ddot{\Omega}$.*

(4) *$\ddot{\aleph}$ is a strongly prime OA-n-II of $\mathfrak{T}$ if and only if $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a strongly prime IVFOA-n-II of $\ddot{\Omega}$.*

Proof. (1) Assume that $\ddot{\aleph}$ is a minimal OA-n-II of $\ddot{\Omega}$. Then $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\tilde{\gtrdot}_{\ddot{\aleph}}$ is an IVFOA-(m,n)-I of $\ddot{\Omega}$. Let $\tilde{\ddot{\xi}}$ be an IVFOA-(m,n)-I of $\ddot{\Omega}$ such that $\tilde{\ddot{\xi}} \preceq \tilde{\gtrdot}_{\ddot{\aleph}}$. Then by Theorem 3.4, $\mathrm{supp}(\tilde{\ddot{\xi}})$ is an OA-n-II of $\ddot{\Omega}$ such that $\mathrm{supp}(\tilde{\ddot{\xi}}) \subseteq \mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}}) = \ddot{\aleph}$. By assumption, $\mathrm{supp}(\tilde{\ddot{\xi}}) = \mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}})$. Therefore, $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a minimal IVFOA-n-II of $\ddot{\Omega}$.

Conversely, suppose that $\tilde{\gtrdot}_{\ddot{\aleph}}$ is a minimal IVFOA-n-II of $\ddot{\Omega}$. Then $\tilde{\gtrdot}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\ddot{\aleph}$ is an OA-n-II of Ω. Let $\ddot{\aleph}_1$ be an OA-n-II of Ω such that $\ddot{\aleph}_1 \subseteq \ddot{\aleph}$. Then $\tilde{\gtrdot}_{\ddot{\aleph}_1}$ is an IVFOA-n-II of Ω such that $\tilde{\gtrdot}_{\ddot{\aleph}_1} \preceq \tilde{\gtrdot}_{\ddot{\aleph}}$. Thus, $\mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}_1}) \subseteq \mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}})$. By assumption, $\ddot{\aleph}_1 = \mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}_1}) = \mathrm{supp}(\tilde{\gtrdot}_{\ddot{\aleph}}) = \ddot{\aleph}$. Therefore, $\ddot{\aleph}$ is a minimal OA-n-II of $\ddot{\Omega}$.

(2) Suppose that $\ddot{\aleph}$ is a prime OA-n-II of $\ddot{\Omega}$. Then $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\tilde{\gtrdot}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Let $\tilde{\ddot{\Upsilon}}$ and $\tilde{\ddot{\xi}}$ be IVFOA-n-IIs of $\ddot{\Omega}$ such that $\tilde{\ddot{\Upsilon}} \circ \tilde{\ddot{\xi}} \preceq \tilde{\gtrdot}_{\ddot{\aleph}}$. Assume that $\tilde{\ddot{\Upsilon}} \npreceq \tilde{\gtrdot}_{\ddot{\aleph}}$ and $\tilde{\ddot{\xi}} \npreceq \tilde{\gtrdot}_{\ddot{\aleph}}$. Then there exist $\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}} \in \ddot{\Omega}$ such that $\tilde{\gtrdot}_{\ddot{\aleph}}(\ddot{\mathfrak{h}}) = \tilde{0}$ and $\tilde{\gtrdot}_{\ddot{\aleph}}(\ddot{\mathfrak{r}}) = 0$. Thus, $\ddot{\mathfrak{h}} \in \mathrm{supp}(\tilde{\ddot{\Upsilon}})$ and $\ddot{\mathfrak{r}} \in \mathrm{supp}(\tilde{\ddot{\xi}})$, but $\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}} \notin \ddot{\aleph}$. So $\mathrm{supp}(\tilde{\ddot{\Upsilon}}) \nsubseteq \ddot{\aleph}$ and $\mathrm{supp}(\tilde{\ddot{\xi}}) \nsubseteq \ddot{\aleph}$. Since $\mathrm{supp}(\tilde{\ddot{\Upsilon}})$ and $\mathrm{supp}(\tilde{\ddot{\xi}})$ are OA-n-IIs of $\ddot{\Omega}$ we have $\mathrm{supp}(\tilde{\ddot{\Upsilon}})\,\mathrm{supp}(\tilde{\ddot{\xi}}) \nsubseteq \ddot{\aleph}$. Thus, there exists $\ddot{\mathfrak{m}} = \ddot{\mathfrak{p}}\ddot{\mathfrak{q}}$ for some $\ddot{\mathfrak{p}} \in \mathrm{supp}(\tilde{\ddot{\Upsilon}})$ and $\ddot{\mathfrak{q}} \in \mathrm{supp}(\tilde{\ddot{\xi}})$ such that $\mathfrak{m} \notin \ddot{\aleph}$. Hence, $\tilde{\gtrdot}_{\ddot{\aleph}}(\ddot{\mathfrak{m}}) = \tilde{0}$ implies that

$(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}})(\ddot{\mathfrak{m}}) = \tilde{0}$. Since $\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}} \leq \tilde{\succ}_{\ddot{\aleph}}$. we have $\ddot{\mathfrak{p}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ and $\ddot{\mathfrak{q}} \in \operatorname{supp}(\tilde{\tilde{\xi}})$. Thus, $\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{p}}) \neq \tilde{0}$, and $\tilde{\tilde{\xi}}(\ddot{\mathfrak{q}}) \neq 0$. It implies that

$$(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}})(\ddot{\mathfrak{m}}) = \bigvee_{(\ddot{\mathfrak{p}},\ddot{\mathfrak{q}}) \in F_{\ddot{\mathfrak{m}}}} \{\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{p}}) \wedge \tilde{\tilde{\xi}}(\ddot{\mathfrak{q}})\} \neq \tilde{0}.$$

It is a contradiction so $\tilde{\tilde{\Upsilon}} \leq \tilde{\succ}_{\ddot{\aleph}}$ or $\tilde{\tilde{\xi}} \leq \tilde{\succ}_{\ddot{\aleph}}$. Therefore, $\tilde{\succ}_{\ddot{\aleph}}$ is a prime IVFOA-n-II of $\ddot{\Omega}$.

Conversely, suppose that $\tilde{\succ}_{\ddot{\aleph}}$ is a prime IVFOA-n-II of $\ddot{\Omega}$. Then $\tilde{\succ}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Let $\ddot{\aleph}_1$ and $\ddot{\aleph}_2$ be OA-(m,n)-Is of $\ddot{\Omega}$ such that $\ddot{\aleph}_1\ddot{\aleph}_2 \subseteq \ddot{\aleph}$. Then $\tilde{\succ}_{\ddot{\aleph}_1}$ and $\tilde{\succ}_{\ddot{\aleph}_2}$ are IVOA-n-IIs of $\ddot{\Omega}$. By Lemma 2.1 $\tilde{\succ}_{\ddot{\aleph}_1} \circ \tilde{\succ}_{\ddot{\aleph}_2} = \tilde{\succ}_{\ddot{\aleph}_1\ddot{\aleph}_2} \leq \tilde{\succ}_{\ddot{\aleph}}$. By assumption, $\tilde{\succ}_{\ddot{\aleph}_1} \leq \tilde{\succ}_{\ddot{\aleph}}$ or $\tilde{\succ}_{\ddot{\aleph}_2} \leq \tilde{\succ}_{\ddot{\aleph}}$. Thus, $\ddot{\aleph}_1 \subseteq \ddot{\aleph}$ or $\ddot{\aleph}_2 \subseteq \ddot{\aleph}$. We conclude that $\ddot{\aleph}$ is a prime OA-n-II of $\ddot{\Omega}$.

(3) Suppose that $\ddot{\aleph}$ is a semiprime OA-n-II of $\ddot{\Omega}$. Then $\ddot{\aleph}$ is an OA-n-II of $\mathfrak{T}$. Thus by Theorem 3.3, $\tilde{\succ}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Let $\tilde{\tilde{\Upsilon}}$ be an IVFOA-n-II of $\ddot{\Omega}$ such that $\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\Upsilon}} \leq \tilde{\succ}_{\ddot{\aleph}}$. Assume that $\tilde{\tilde{\Upsilon}} \not\leq \tilde{\succ}_{\ddot{\aleph}}$. Then there exist $\ddot{\mathfrak{h}} \in \ddot{\Omega}$ such that $\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{h}}) \neq \tilde{0}$. While $\tilde{\succ}_{\ddot{\aleph}}(\ddot{\mathfrak{h}}) = \tilde{0}$. Thus, there exists $\ddot{\mathfrak{m}} = \ddot{\mathfrak{p}}\ddot{\mathfrak{q}}$ for some $\ddot{\mathfrak{p}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ and $\ddot{\mathfrak{q}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ such that $\ddot{\mathfrak{m}} \notin \ddot{\aleph}$. Hence, $\tilde{\succ}_{\ddot{\aleph}}(\ddot{\mathfrak{m}}) = \tilde{0}$ implies that $(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\Upsilon}})(\ddot{\mathfrak{m}}) = \tilde{0}$. Since $\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\Upsilon}} \leq \tilde{\succ}_{\ddot{\aleph}}$ we have $\ddot{\mathfrak{p}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ and $\ddot{\mathfrak{q}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$. Thus, $\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{p}}) \neq \tilde{0}$ and $\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{q}}) \neq \tilde{0}$. It implies that

$$(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\Upsilon}})(\ddot{\mathfrak{m}}) = \bigvee_{(\ddot{\mathfrak{p}},\ddot{\mathfrak{q}}) \in F_{\ddot{\mathfrak{m}}}} \{\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{p}}) \wedge \tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{q}})\} \neq \tilde{0}.$$

It is a contradiction so $\tilde{\tilde{\Upsilon}} \leq \tilde{\succ}_{\ddot{\aleph}}$. Therefore, $\tilde{\succ}_{\ddot{\aleph}}$ is a semiprime IVFOA-n-II of $\ddot{\Omega}$.

Conversely, suppose that $\tilde{\succ}_{\ddot{\aleph}}$ is a semiprime IVFOA-n-II of $\ddot{\Omega}$. Then $\tilde{\succ}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Let $\ddot{\aleph}_1$ be anOA-n-II of $\ddot{\Omega}$ such that $\ddot{\aleph}_1^2 \subseteq \ddot{\aleph}$. Then $\tilde{\succ}_{\ddot{\aleph}_1}$ is an IVFOA-n-II of $\ddot{\Omega}$. By Lemma 2.1 $\tilde{\succ}_{\ddot{\aleph}_1} \circ \tilde{\succ}_{\ddot{\aleph}_1} = \tilde{\succ}_{\ddot{\aleph}_1^2} \leq \tilde{\succ}_{\ddot{\aleph}}$. By assumption, $\tilde{\succ}_{\ddot{\aleph}_1} \leq \tilde{\succ}_{\ddot{\aleph}}$. Thus, $\ddot{\aleph}_1 \subseteq \ddot{\aleph}$. We conclude that $\ddot{\aleph}$ is a semiprime OA-n-II of $\ddot{\Omega}$.

(4) Suppose that $\ddot{\aleph}$ is a strongly prime OA-n-II of $\ddot{\Omega}$. Then $\ddot{\aleph}$ is an OA-n-II of $\ddot{\Omega}$. Thus by Theorem 3.3, $\tilde{\succ}_{\ddot{\aleph}}$ is an IVFOA-n-II of $\ddot{\Omega}$. Let $\tilde{\tilde{\Upsilon}}$ and $\tilde{\tilde{\xi}}$ be IVFOA-n-IIs of $\ddot{\Omega}$ such that $(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}}) \wedge (\tilde{\tilde{\xi}} \circ \tilde{\tilde{\Upsilon}}) \leq \tilde{\succ}_{\ddot{\aleph}}$. Assume that $\tilde{\tilde{\Upsilon}} \not\leq \tilde{\succ}_{\ddot{\aleph}}$ and $\tilde{\tilde{\xi}} \not\leq \tilde{\succ}_{\ddot{\aleph}}$. Then there exist $\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}} \in \ddot{\Omega}$ such that $\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{h}}) \neq \tilde{0}$ and $\tilde{\tilde{\xi}}(\ddot{\mathfrak{r}}) \neq \tilde{0}$. While $\tilde{\succ}_{\ddot{\aleph}}(\ddot{\mathfrak{h}}) = \tilde{0}$ and $\tilde{\succ}_{\ddot{\aleph}}(\ddot{\mathfrak{r}}) = \tilde{0}$. Thus, $\ddot{\mathfrak{h}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ and $\ddot{\mathfrak{r}} \in \operatorname{supp}(\tilde{\tilde{\xi}})$, but $\ddot{\mathfrak{h}}, \ddot{\mathfrak{r}} \notin \ddot{\aleph}$. So, $\operatorname{supp}(\tilde{\tilde{\Upsilon}}) \not\subseteq \ddot{\aleph}$ and $\operatorname{supp}(\tilde{\tilde{\xi}}) \not\subseteq \ddot{\aleph}$. Hence, there exists $\ddot{\mathfrak{m}} \in [\operatorname{supp}(\tilde{\tilde{\Upsilon}}) \operatorname{supp}(\tilde{\tilde{\xi}})] \cap (\operatorname{supp}(\tilde{\tilde{\xi}}) \operatorname{supp}(\tilde{\tilde{\Upsilon}}))$ such that $\ddot{\mathfrak{m}} \notin \ddot{\aleph}$. Thus, $\chi_{\ddot{\aleph}}(\mathfrak{m}) = \tilde{0}$ such that $(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}})(\mathfrak{m}) \wedge (\tilde{\tilde{\xi}} \circ \tilde{\tilde{\Upsilon}})(\mathfrak{m}) = 0$. Since $\ddot{\mathfrak{m}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}}) \operatorname{supp}(\tilde{\tilde{\xi}})$ and $\ddot{\mathfrak{m}} \in \operatorname{supp}(\tilde{\tilde{\xi}}) \operatorname{supp}(\tilde{\tilde{\Upsilon}})$ we have $\ddot{\mathfrak{m}} = \ddot{\mathfrak{d}}\ddot{\mathfrak{k}}$ and $\mathfrak{m} = \mathfrak{g}\ddot{\mathfrak{q}}$ for some $\ddot{\mathfrak{d}}, \ddot{\mathfrak{q}} \in \operatorname{supp}(\tilde{\tilde{\Upsilon}})$, and for some $\ddot{\mathfrak{k}}, \ddot{\mathfrak{g}} \in \operatorname{supp}(\tilde{\tilde{\xi}})$. we have

$$(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}})(\ddot{\mathfrak{m}}) = \bigvee_{(\ddot{\mathfrak{d}},\ddot{\mathfrak{r}}) \in F_{\ddot{\mathfrak{m}}}} \{\tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{d}}) \wedge \tilde{\tilde{\xi}}(\ddot{\mathfrak{k}})\}.$$

Similarly

$$(\tilde{\tilde{\xi}} \circ \tilde{\tilde{\Upsilon}})(\ddot{\mathfrak{m}}) = \bigvee_{(\ddot{\mathfrak{g}},\ddot{\mathfrak{q}}) \in F_{\ddot{\mathfrak{m}}}} \{\tilde{\tilde{\xi}}(\ddot{\mathfrak{g}}) \wedge \tilde{\tilde{\Upsilon}}(\ddot{\mathfrak{q}})\}.$$

So, $(\tilde{\tilde{\Upsilon}} \circ \tilde{\tilde{\xi}})(\ddot{\mathfrak{m}}) \wedge (\tilde{\tilde{\xi}} \circ \tilde{\tilde{\Upsilon}})(\ddot{\mathfrak{m}}) \neq \tilde{0}$. It is a contradiction Hence, $\tilde{\tilde{\Upsilon}} \leq \tilde{\succ}_{\ddot{\aleph}}$ or $\tilde{\tilde{\xi}} \leq \tilde{\succ}_{\ddot{\aleph}}$. Therefore, $\tilde{\succ}_{\ddot{\aleph}}$ is a strongly prime IVFOA-n-II of $\ddot{\Omega}$.

Conversely, suppose that $\ddot{\tilde{\succ}}_{\ddot{\aleph}}$ is a strongly prime IVFOA-n-II of $\ddot{\Omega}$. Then $\ddot{\tilde{\succ}}_{\ddot{\aleph}}$ is an IVOA-n-II of $\ddot{\Omega}$. Thus, by Theorem 3.3, $\ddot{\aleph}$ is an OA-n-IIof $\mathfrak{T}$. Let $\ddot{\aleph}_1$ and $\ddot{\aleph}_2$ be OA-n-IIs of $\ddot{\Omega}$ such that $\ddot{\aleph}_1\ddot{\aleph}_2 \cap \ddot{\aleph}_2\ddot{\aleph}_1 \subseteq \ddot{\aleph}$. Then $\ddot{\tilde{\succ}}_{\ddot{\aleph}_1}$ and $\ddot{\tilde{\succ}}_{\ddot{\aleph}_2}$ are IVFOA (m,n)-Is of $\ddot{\Omega}$. By Lemma 2.1 $\ddot{\tilde{\succ}}_{\ddot{\aleph}_1\ddot{\aleph}_2} = \ddot{\tilde{\succ}}_{\ddot{\aleph}_1} \circ \ddot{\tilde{\succ}}_{\ddot{\aleph}_2}$ and $\ddot{\tilde{\succ}}_{\ddot{\aleph}_2\ddot{\aleph}_1} = \ddot{\tilde{\succ}}_{\ddot{\aleph}_2} \circ \ddot{\tilde{\succ}}_{\ddot{\aleph}_1}$. Thus, $(\ddot{\tilde{\succ}}_{\ddot{\aleph}_1} \circ \ddot{\tilde{\succ}}_{\ddot{\aleph}_2}) \curlywedge (\ddot{\tilde{\succ}}_{\ddot{\aleph}_1} \circ \ddot{\tilde{\succ}}_{\ddot{\aleph}_2}) = \ddot{\tilde{\succ}}_{\ddot{\aleph}_1\ddot{\aleph}_2} \curlywedge \ddot{\tilde{\succ}}_{\ddot{\aleph}_2\ddot{\aleph}_1} = \ddot{\tilde{\succ}}_{\ddot{\aleph}_1\ddot{\aleph}_2\cap\ddot{\aleph}_2\ddot{\aleph}_1} \preceq \ddot{\tilde{\succ}}_{\ddot{\aleph}}$. By assumption, $\ddot{\tilde{\succ}}_{\ddot{\aleph}_1} \preceq \ddot{\tilde{\succ}}_{\ddot{\aleph}}$ and $\ddot{\tilde{\succ}}_{\ddot{\aleph}_2} \preceq \ddot{\tilde{\succ}}_{\ddot{\aleph}}$. Thus, $\ddot{\aleph}_1 \subseteq \ddot{\aleph}$ or $\ddot{\aleph}_2 \subseteq \ddot{\aleph}$. We conclude that $\ddot{\aleph}$ is a strongly prime OA-n-II of $\ddot{\Omega}$.

□

Corollary 3.1. *Let $\ddot{\Omega}$ be an ordered semigroup. Then $\ddot{\Omega}$ has no proper OA-n-II if and only if* $\text{supp}(\ddot{\tilde{\Upsilon}}) = \ddot{\Omega}$ *for every IVFOA-n-II $\ddot{\tilde{\Upsilon}}$ of $\ddot{\Omega}$.*

4. Conclusion

The aim of the paper is to give the concept of IVOA-n-IIs in ordered semigroups. We prove properties IVFOA-n-IIs. In Theorems 3.3, 3.4 and 3.5, we prove the relationship between OA-n-IIs and class fuzzifications. In future work, we can study other kinds of almost ideals and their fuzzifications in an ordered ternary semigroup.

Acknowledgment: This research was supported by University of Phayao and Thailand Science Research and Innovation Fund (Fundamental Fund 2026, Grant No. 2252/2568).

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] S. Bogdanovic, Semigroups in Which Some Bi-Ideal Is a Group, Rev. Res. Fac. Sci. Novi Sad 11 (1981), 261–266.

[2] R. Chinramm, S. Baupradist, A. Iampan, P. Singvananda, Characterizations of Ordered Almost Ideals and Fuzzifications in Partially Ordered Ternary Semigroups, ICIC Express Lett. 17 (2023), 631–639. https://doi.org/10.24507/icicel.17.06.631.

[3] T. Gaketem, On Interval Valued Fuzzy Almost (m,n)-Ideal in Semigroups, J. Discret. Math. Sci. Cryptogr. 25 (2022), 171–180. https://doi.org/10.1080/09720529.2021.1995178.

[4] T. Gaketem, On interval valued fuzzy almost (m,n)-Bi-Ideal in Semigroups, J. Math. Comput. Sci. 11 (2021), 6657–6665.

[5] T. Gaketem, On Interval Valued Fuzzy Almost (m,n)-Quasi-Ideal in Semigroups, ICIC Express Lett. 16 (2022), 235–240. https://doi.org/10.24507/icicel.16.03.235.

[6] T. Gaketem, P. Khamrot, Bipolar Fuzzy Almost Bi-Ideal in Semigroups, Int. J. Math. Comput. Sci. 17 (2022), 345–352.

[7] T. Gaketem, Bipolar Fuzzy Almost Interior Ideals in Semigroups, ICIC Express Lett. 17 (2023), 381–387. https://doi.org/10.24507/icicel.17.04.381.

[8] N. Kaopusek, T. Kaewnoi, R. Chinram, On Almost Interior Ideals and Weakly Almost Interior Ideals of Semigroups, J. Discret. Math. Sci. Cryptogr. 23 (2020), 773–778. https://doi.org/10.1080/09720529.2019.1696917.

[9] P. Khamrot, A. Phukhaengst, T. Gaketem, Ordered Almost n-Interior Ideals in Semigroups Class Fuzzifications, IAENG Int. J. Comput. Sci. 51 (2024), 1711–1719.

[10] P. Khamrot, T. Gaketem, Applications of Bipolar Fuzzy Almost Ideals in Semigroups, Int. J. Anal. Appl. 22 (2024), 8. https://doi.org/10.28924/2291-8639-22-2024-8.

[11] P. Khamrot, T. Gaketem, Bipolar Fuzzy Almost Quasi-Ideals in Semigroups, Int. J. Anal. Appl. 22 (2024), 12. https://doi.org/10.28924/2291-8639-22-2024-12.

[12] P. Khramrot, A. Iampan, T. Gaketem, Fuzzy (m,n)-Ideals and n-Interior Ideals in Ordered Semigroups, Eur. J. Pure Appl. Math. 18 (2025), 5596. https://doi.org/10.29020/nybg.ejpam.v18i1.5596.

[13] P. Khramrot, P. Chaisuwan, P. Keawton, C. Wangsamphao, T. Gaketem, Almost (m,n)-Quasi-Ideals and Fuzzy Almost (m,n)-Quasi-Ideals in Ordered Semigroups, Eur. J. Pure Appl. Math. 18 (2025), 5837. https://doi.org/10.29020/nybg.ejpam.v18i2.5837.

[14] A. Mahboob, M. Al-Tahan, G. Muhiuddin, Characterizations of Ordered Semigroups in Terms of Fuzzy (m,n)-Substructures, Soft Comput. 28 (2024), 10827–10834. https://doi.org/10.1007/s00500-024-09880-z.

[15] A. Narayanan, T. Manikantan, Interval-Valued Fuzzy Ideals Generated by an Interval-Valued Fuzzy Subset in Semigroups, J. Appl. Math. Comput. 20 (2006), 455–464. https://doi.org/10.1007/bf02831952.

[16] R. Rittichuai, A. Iampan, R. Chinram, P. Singavananda, Almost Subsemirings and Fuzzifications, Int. J. Math. Comput. Sci. 17 (2022), 1491–1497.

[17] N. Sarasit, R. Chinram, A. Rattana, Applications of Fuzzy Sets for Almostity of Ternary Subsemirings, Int. J. Appl. Math. 36 (2023), 497–507. https://doi.org/10.12732/ijam.v36i4.5.

[18] L. Satko, O. Grošek, On Minimal A-Ideals of Semigroups, Semigroup Forum 23 (1981), 283–295. https://doi.org/10.1007/bf02676653.

[19] S. Suebsung, K. Wattanatripop, R. Chinram, On Almost (m,n)-Ideals and Fuzzy Almost (m,n)-Ideals in Semigroups, J. Taibah Univ. Sci. 13 (2019), 897–902. https://doi.org/10.1080/16583655.2019.1659546.

[20] S. Suebsung, W. Yonthanthum, R. Chinram, Ordered Almost Ideals and Fuzzy Ordered Almost Ideals in Ordered Semigroups, Ital. J. Pure Appl. Math. 48 (2022), 1206–1217.

[21] L. Zadeh, Fuzzy Sets, Inf. Control. 8 (1965), 338–353. https://doi.org/10.1016/s0019-9958(65)90241-x.

[22] L. Zadeh, The Concept of a Linguistic Variable and Its Application to Approximate Reasoning—I, Inf. Sci. 8 (1975), 199–249. https://doi.org/10.1016/0020-0255(75)90036-5.