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Geometric and Coefficient Estimates for Bi-Univalent Functions Defined Through Fibonacci Numbers

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Abstract. In this research work, the new subclass $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$ of bi-univalent functions related to Fibonacci numbers is presented. Our primary contributions to this for functions in this particular subclass, the study entails placing restrictions on the absolute values of the second coefficient $|a_2|$ and the third coefficient $|a_3|$. Furthermore, Fekete-Szegö functional problems are solved by us. In addition, our analysis shows interesting results from the particular parameter values applied in our primary conclusions.

1. Introduction and preliminaries

The series of integers formed by the Fibonacci numbers is recursive. The sequence is defined recursively as follows: $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$, and $F_0 = 0$, $F_1 = 1$. By combining the two preceding terms [1], this sequence is produced. First, it includes the following components: The provided sequence is in accordance with the Fibonacci sequence pattern, in which every number is equal to the sum of its two preceding numbers.

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Due to their special properties, the Fibonacci numbers are used in many different fields. Tree branching and leaf arrangement are examples of fractal patterns that are visible in the growth of biological things. Numerous branches of mathematics and science, such as number theory, physics, and geometry, use these ideas. Matrix multiplication, generating functions, and combinatorial procedures are some of the ways that the Fibonacci sequence can be produced. This series has interesting characteristics and relationships to mathematical concepts such as Lucas numbers, Pell numbers, and the golden ratio. Additionally, Fibonacci numbers are employed in technical analysis and other financial markets. To help them make well-informed trading decisions, traders and analysts use Fibonacci retracement levels to pinpoint possible support and resistance levels on financial charts. The importance of Fibonacci numbers is evident in a variety of disciplines, including biology, computer science, mathematics, and finance. This is a result of its distinct mathematical properties and organic look. Knowing and applying Fibonacci numbers can improve our awareness of the innate patterns seen in nature and gain practical analytical and problem-solving skills. [2].

When it comes to the study or implementation of bi-univalent functions, one method that could be taken is to make use of mathematical analysis or computing approaches that capitalise on the characteristics of the Fibonacci numbers. In many cases, Fibonacci numbers

In the context of sequences, series, and mathematical structures, demonstrate some fascinating mathematical features through demonstration. In the analysis of functions, particularly Bi-Univalent Functions, these qualities could be utilised. This is especially true in situations when comprehending the behaviour or properties of these functions requires the utilisation of complex mathematical patterns or sequences. As an additional benefit, the study of Fibonacci numbers has the potential to inspire the development of new mathematical methods or approaches that could be of assistance in the analysis or manipulation of bi-univalent functions. It is not uncommon for mathematical ideas and methods that are established in one area of mathematics to have unexpected applications in domains that appear to be unrelated to mathematics. In conclusion, the investigation of mathematical connections and the utilization of the qualities of Fibonacci numbers have the potential to result in the development of insights, methodologies, or approaches that enhance the comprehension of bi-univalent functions in the context of mathematical computing and analysis. Let $\mathcal{A}$ denote the class of analytic functions in the open unit disk $\nabla = \{\xi : |\xi| < 1\}$ with $f(0) = 0$, $f'(0) = 1$ and having the form:

$$f(\xi) = \xi + \sum_{n=2}^{\infty} a_n \xi^n \qquad (\xi \in \nabla). \tag{1.1}$$

Let $\boldsymbol{Q}$ denote the class of functions of the form:

$$q(\xi) = 1 + q_1\xi + q_2\xi^2 + q_3\xi^3 + \cdots \qquad (\xi \in \nabla). \tag{1.2}$$

$q(\xi)$ is an analytic function, called a Carathéodory function, characterized by

$$\mathrm{Re}\{q(\xi)\} > 0 \quad (\xi \in \nabla), \text{ see [4].}$$

If there exists a Schwarz function w such that

$$q(\xi) = 1 + w(\xi), \quad |w(\xi)| < 1,$$

then q belongs to the set Q. Let $\boldsymbol{Q}(\alpha)$, with $0 \le \alpha$, denote the class of analytic functions q in ∇ that satisfy $q(0) = 1$ and $\mathrm{Re}\{q(\xi)\} > \alpha$.

Recently, Sokół [10] and Dziok et al. [5] investigated the classes

$$\mathcal{SL}(\widetilde{q}) = \left\{ f : \frac{\xi f'(\xi)}{f(\xi)} \prec \widetilde{q}(\xi) \right\}, \quad \mathcal{KSL}(\widetilde{q}) = \left\{ f : 1 + \frac{\xi^2 f''(\xi)}{f'(\xi)} \prec \widetilde{q}(\xi) \right\}.$$

Here, the function $\widetilde{q}(\xi)$ is defined by

$$\widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\xi - ß^2\xi^2}, \quad |\xi| < \frac{3-\sqrt{5}}{2} \approx 0.382,$$

where $ß = \frac{1-\sqrt{5}}{2} \approx -0.618$.

Note that $\widetilde{q}(\xi)$ has a unique value for $|\xi| < \frac{3-\sqrt{5}}{2}$, but not for all $\xi \in \nabla$. Since ß satisfies

$$ß^2 = 1 + ß,$$

higher powers $ß^n$ can be expressed as a linear combination of ß and 1. This recursive relation generates the Fibonacci numbers, denoted by σ_n (see [6,9]).

$$ß^{\,n} = \sigma_n ß + \sigma_{n-1}.$$

As an additional point of interest, Raina and Sok [9] demonstrated that the equation $ß\xi = t$ is correct.

$$\widetilde{q}(\xi) = \frac{1 + ß^{\,2}\xi^2}{1 - ß\,\xi - ß^{\,2}\xi^2} = \sum_{n=1}^{\infty} (\tau_{n-1} + \tau_{n+1})\, ß^{\,n}\xi^n, \tag{1.3}$$

where

$$\tau_n = \frac{(1 - ß\,)^n - ß}{\sqrt{5}}, \quad ß = \frac{1-\sqrt{5}}{2}, \quad n = 1, 2, \cdots.$$

This proves that:

$$\tau_0 = 0, \quad \tau_1 = 1, \quad \tau_{n+2} = \tau_n + \tau_{n+1}, \quad n = 0, 1, 2, \cdots. \tag{1.4}$$

Hence,

$$\begin{aligned}
\widetilde{q}(\xi) &= 1 + \sum_{n=1}^{\infty} \widetilde{q}_n \xi^n \\
&= 1 + (\tau_0 + \tau_2)ß\,\xi + (\tau_1 + \tau_3)ß^{\,2}\xi^2 + \sum_{n=3}^{\infty} (\tau_{n-3} + \tau_{n-2} + \tau_{n-1} + \tau_n)\, ß^{\,n}\xi^n \\
&= 1 + ß\,\xi + 3ß^{\,2}\xi^2 + 4ß^{\,3}\xi^3 + 7ß^{\,4}\xi^4 + 11ß^{\,5}\xi^5 + \cdots.
\end{aligned} \tag{1.5}$$

We observe that $\widetilde{q}$ is contained within $\boldsymbol{Q(\alpha)}$, where α is equal to $\frac{\sqrt{5}}{10}$, with α being around 0.2236, see [9]

Given two analytic functions f and g in the field of mathematics ∇, if there is an analytic function w that guarantees the following:

$$w(0)=0, \quad \left|w(\xi)\right|<1 \quad \text{and} \quad f(\xi)=g(w(\xi)) \qquad (\xi\in\nabla)$$

When f is subordinate to g, it is represented by $f \prec g$ or $f(\xi)\prec g(\xi)$..

Also, when g is univalent in ∇,

$$f(0)=g(0) \text{ and } f(\nabla)\subset g(\nabla) \Leftrightarrow f\prec g \qquad (\xi\in\nabla).$$

Furthermore, for all functions f that are univalent in ∇, we shall assign the symbol $\mathcal{S}$ to such functions. Therefore, every function f that belongs to the set S has an inverse f^{-1}, which is defined

$$f^{-1}(f(\xi))=\xi \qquad (\xi\in\nabla)$$

and

$$f(f^{-1}(w))=w \qquad (|w|<r_0(f);\ r_0(f)\geq \frac{1}{4})$$

where

$$f^{-1}(w)=w-a_2w^2+(2a_2^2-a_3)w^3-(5a_2^3-5a_2a_3+a_4)w^4+\cdots. \tag{1.6}$$

If both $f(\xi)$ and $f^{-1}(\xi)$ are univalent within ∇, then the function $f\in\mathcal{A}$ is considered a member of Σ, which encompasses all bi-univalent functions within Σ. Additional information on the features of the class Σ can be found in ([11]- [39]) and ([44]- [48]).

The following will be presented as a result of the research conducted by Ali et al. [3] and Orhan et al. [7].

Definition 1.1. *The class* $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$ *contains the function* $f\in\mathcal{A}$, *where* $m\geq 0$, $-\pi<\varphi\leq\pi$, $\eth$, $\mathcal{L}\geq 0$, $\Im\geq 1$, *if fulfilled:*

$$\left(1+me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{f(\xi)}{\xi}\right)^{\mathcal{L}}+\Im\left(f'(\xi)\right)^{1-\mathcal{L}}+\eth\xi f''(\xi)\right\}-me^{i\varphi}\prec\widetilde{q}(\xi)=\frac{1+ß^2\xi^2}{1-ß\xi-ß^2\xi^2} \quad (\xi\in\nabla) \tag{1.7}$$

and g is the inverse of f given by (1.6),

$$\left(1+me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{g(\omega)}{\omega}\right)^{\mathcal{L}}+\Im\left(g'(\omega)\right)^{1-\mathcal{L}}+\eth\xi g''(\omega)\right\}-me^{i\varphi}\prec\widetilde{q}(\xi)=\frac{1+ß^2w^2}{1-ßw-ß^2w^2} \quad (w\in\nabla) \tag{1.8}$$

where $ß=\frac{1-\sqrt{5}}{2}\approx -0.618.$

If we specify the values of $\Im$, $\eth$ and $\mathcal{L}$ *where* $=\frac{1-\sqrt{5}}{2}$, the class $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$ is a mathematical function, which can then be broken down into many subclasses. Take, for instance:

i. If $\Im = 1$; then a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{1,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$, where $\eth$, $\mathcal{L} \geq 0$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\left\{(f'(\xi))^{1-\mathcal{L}} + \eth\xi f''(\xi)\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6)

$$\left(1 + me^{i\varphi}\right)\left\{(g'(\omega))^{1-\mathcal{L}} + \eth\xi g''(\omega)\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2 w^2}{1 - ß\,w - ß^2 w^2} \qquad (w \in \nabla).$$

ii. If $\Im = 1$ and $\eth = 0$; then a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{1,0,\mathcal{L},m,\varphi}(\widetilde{q})$, where $\mathcal{L} \geq 0$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\left\{(f'(\xi))^{1-\mathcal{L}}\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6)

$$\left(1 + me^{i\varphi}\right)\left\{(g'(\omega))^{1-\mathcal{L}}\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2 w^2}{1 - ß\,w - ß^2 w^2} \qquad (w \in \nabla).$$

iii. If $\Im = 1$ and $\eth = \mathcal{L} = 0$; than a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the$\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{1,0,0,m,\varphi}(\widetilde{q})$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\{f'(\xi)\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6)

$$\left(1 + me^{i\varphi}\right)\{g'(w)\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2 w^2}{1 - ß\,w - ß^2 w^2} \qquad (w \in \nabla).$$

iv. If $\eth = 0$; than a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{\Im,0,\mathcal{L},m,\varphi}(\widetilde{q})$, where $\Im \geq 1$ and $\mathcal{L} \geq 0$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\left\{(1 - \Im)\left(\frac{f(\xi)}{\xi}\right)^{\mathcal{L}} + \Im\,(f'(\xi))^{1-\mathcal{L}}\right\} - me^{i\varphi}$$
$$\prec \widetilde{q}(\xi) = \frac{1 + ß^2\,\xi^2}{1 - ß\,\xi - ß^2\,\xi^2}, \qquad (\xi \in \nabla)$$

and for g given by (1.6),

$$\left(1 + me^{i\varphi}\right)\left\{(1 - \Im)\left(\frac{g(\omega)}{\omega}\right)^{\mathcal{L}} + \Im\,(g'(\omega))^{1-\mathcal{L}}\right\} - me^{i\varphi}$$
$$\prec \widetilde{q}(\omega) = \frac{1 + ß^2\,\omega^2}{1 - ß\,\omega - ß^2\,\omega^2}, \qquad (\omega \in \nabla)$$

v. If $\eth = 0$ and $\mathcal{L} = 1$; than a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{\Im,0,1,m,\varphi}(\widetilde{q})$, where $\Im \geq 1$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{f(\xi)}{\xi}\right) + \Im\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6),

$$\left(1 + me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{g(\omega)}{\omega}\right) + \Im\right\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2w^2}{1 - ß\,w - ß^2w^2} \qquad (w \in \nabla),$$

vi. If $\eth = 0$ and $\mathcal{L} = 0$; than a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{\Im,0,0,m,\varphi}(\widetilde{q})$, where $\Im \geq 1$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\{(1-\Im) + \Im\,(f'(\xi))\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6),

$$\left(1 + me^{i\varphi}\right)\{(1-\Im) + \Im\,(g'(\omega))\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2w^2}{1 - ß\,w - ß^2w^2} \qquad (w \in \nabla),$$

vii. If $\mathcal{L} = 0$; than a function $f(\xi) = \xi + a_2\xi^2 + a_3\xi^3 + \cdots$ belongs to the $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q}) = \Upsilon_{\Sigma}^{\Im,\eth,0,m,\varphi}(\widetilde{q})$, where $\eth \geq 0, \Im \geq 1$, if satisfied:

$$\left(1 + me^{i\varphi}\right)\{(1-\Im) + \Im\,(f'(\xi)) + \eth\xi f''(\xi)\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2\xi^2}{1 - ß\,\xi - ß^2\xi^2} \qquad (\xi \in \nabla)$$

and for g given by (1.6),

$$\left(1 + me^{i\varphi}\right)\{(1-\Im) + \Im\,(g'(\omega)) + \eth\xi g''(\omega)\} - me^{i\varphi} \prec \widetilde{q}(\xi) = \frac{1 + ß^2w^2}{1 - ß\,w - ß^2w^2} \qquad (w \in \nabla),$$

To prove our results we need the following lemma.

Lemma 1.1. *[8] If $c \in C$, then $|c_i| \leq 2$ for each i, where C is the family of all functions c analytic in ∇ for which*

$$Re\{c(\xi)\} > 0,\ c(\xi) = 1 + c_1\xi + c_2\xi^2 + \cdots \quad (\xi \in \nabla).$$

2. Fekete-Szegö inequality and coefficient estimates for the function class the formula $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$

The Taylor-Maclaurin coefficients and the Fekete-Szegö inequality are estimated in this section for functions in the class $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$.

Theorem 2.1. *Let $f(\xi)$ given by (1.1) be in the function class $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$, where $m \geq 0, -\pi < \varphi \leq \pi, \eth$, $\mathcal{L} \geq 0, \Im \geq 1$. Then*

$$|a_2| \leq \sqrt{\frac{2}{F}}\left|ß\right|, \quad |a_3| \leq \frac{\left|ß\right|\left(F + ß\,(1 + me^{i\varphi})\,[2\mathcal{L}\,(1-4\Im) + 6\Im + 12\eth]\right)}{(1 + me^{i\varphi})\,[\mathcal{L}\,(1-4\Im) + 3\Im + 6\eth]\,F} \tag{2.1}$$

and for $\varepsilon \in \mathbb{R}$

$$\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{|\beta|}{(1+me^{i\varphi})(\mathcal{L}(1-4\Im)+3\Im+6\eth)}; & \left|\epsilon(\varepsilon)\right| \leq \frac{|\beta|}{4(1+me^{i\varphi})[\mathcal{L}(1-4\Im)+3\Im+6\eth]} \\ 4\left|\epsilon(\varepsilon)\right|; & \left|\epsilon(\varepsilon)\right| \geq \frac{|\beta|}{4(1+me^{i\varphi})[\mathcal{L}(1-4\Im)+3\Im+6\eth]}. \end{cases}, \tag{2.2}$$

where $F = \left(1 + me^{i\varphi}\right)\beta\left[\mathcal{L}^2(3\Im+1) + \mathcal{L}(1-11\Im) + 6\Im + 12\eth\right]$
$+ 2(1-3\beta)\left[\left(1+me^{i\varphi}\right)\mathcal{L}(1-3\Im) + 2(\Im+\eth)\right]^2$
and $\epsilon(\varepsilon) = \frac{(1-\varepsilon)\beta^2}{2F}$.

Proof. Due to the fact that f is a member of the $\Upsilon <_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$, it follows that there are two functions, τ and τ, which are analytic in ∇.using the equation $\tau(0) = \beta(0) = 0$, $|\tau(\xi)| < 1$, and $|\beta(w)| < 1$ for all ξ, w to be within the range of ∇. The following is what we get from (1.7) and (1.8):

$$\left(1+me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{f(\xi)}{\xi}\right)^{\mathcal{L}} + \Im\left(f'(\xi)\right)^{1-\mathcal{L}} + \eth\xi f''(\xi)\right\} - me^{i\varphi} = \widetilde{q}(\tau(\xi)) \tag{2.3}$$

and for $f^{-1} = g$,

$$\left(1+me^{i\varphi}\right)\left\{(1-\Im)\left(\frac{g(\omega)}{\omega}\right)^{\mathcal{L}} + \Im\left(g'(\omega)\right)^{1-\mathcal{L}} + \eth\xi g''(\omega)\right\} - me^{i\varphi} = \widetilde{q}(\beta(\xi)). \tag{2.4}$$

Given that $q \prec \widetilde{q}$ and $q(\xi) = 1 + q_1\xi + q_2\xi^2 + q_3\xi^3 + \cdots$, define

$$t(\xi) = \frac{1+\tau(\xi)}{1-\tau(\xi)} = 1 + t_1\xi + t_2\xi^2 + \cdots.$$

Note that $t(\xi) \in \boldsymbol{Q}$. As a consequence,

$$\tau(\xi) = \frac{t(\xi)-1}{t(\xi)+1} = \frac{t_1}{2}\xi + \left(\tau_2 - \frac{\tau_1^2}{2}\right)\frac{\xi^2}{2} + \left(\tau_3 - \tau_1\tau_2 + \frac{\tau_1^3}{4}\right)\frac{\xi^3}{2} + \cdots$$

and

$$\begin{aligned}
\widetilde{q}(\tau(\xi)) = 1 &+ \widetilde{q}_1\left(\frac{\tau_1}{2}\xi + \left(\tau_2 - \frac{\tau_1^2}{2}\right)\frac{\xi^2}{2} + \left(\tau_3 - \tau_1\tau_2 + \frac{\tau_1^3}{4}\right)\frac{\xi^3}{2} + \cdots\right) \\
&+ \widetilde{q}_2\left(\frac{\tau_1}{2}\xi + \left(\tau_2 - \frac{\tau_1^2}{2}\right)\frac{\xi^2}{2} + \left(\tau_3 - \tau_1\tau_2 + \frac{\tau_1^3}{4}\right)\frac{\xi^3}{2} + \cdots\right)^2 \\
&+ \widetilde{q}_3\left(\frac{\tau_1}{2}\xi + \left(\tau_2 - \frac{\tau_1^2}{2}\right)\frac{\xi^2}{2} + \left(\tau_3 - \tau_1\tau_2 + \frac{\tau_1^3}{4}\right)\frac{\xi^3}{2} + \cdots\right)^3 + \cdots \\
&= 1 + \frac{\widetilde{q}_1\tau_1}{2}\xi + \left(\frac{1}{2}\left(\tau_2 - \frac{\tau_1^2}{2}\right)\widetilde{q}_1 + \frac{\tau_1^2}{4}\widetilde{q}_2\right)\xi^2 \\
&+ \left(\frac{1}{2}\left(\tau_3 - \tau_1\tau_2 + \frac{\tau_1^3}{4}\right)\widetilde{q}_1 + \frac{\tau_1}{2}\left(\tau_2 - \frac{\tau_1^2}{2}\right)\widetilde{q}_2 + \frac{\tau_1^3}{8}\widetilde{q}_3\right)\xi^3 + \cdots
\end{aligned} \tag{2.5}$$

Likewise, there is a function $\beta \in \mathcal{A}$ such that $q(w) = \widetilde{q}(\beta(w))$ and $|\beta(w)| < 1$ in . Consequently, the function

$$d(w) = \frac{1+\beta(w)}{1-\beta(w)} = 1 + d_1 w + d_2 w^2 + \cdots \in \boldsymbol{Q}.$$

It follows that

$$\beta(w) = \frac{d(w)-1}{d(w)+1} = \frac{\beta_1}{2}w + \left(\beta_2 - \frac{\beta_1^2}{2}\right)\frac{w^2}{2} + \left(\beta_3 - \beta_1\beta_2 + \frac{\beta_1^3}{4}\right)\frac{w^3}{2} + \cdots$$

and

$$\begin{aligned}\widetilde{q}(\beta(w)) &= 1 + \widetilde{q}_1\left(\frac{\beta_1}{2}w + \left(\beta_2 - \frac{\beta_1^2}{2}\right)\frac{w^2}{2} + \left(\beta_3 - \beta_1\beta_2 + \frac{\beta_1^3}{4}\right)\frac{w^3}{2} + \cdots\right)\\
&\quad + \widetilde{q}_2\left(\frac{\beta_1}{2}w + \left(\beta_2 - \frac{\beta_1^2}{2}\right)\frac{w^2}{2} + \left(\beta_3 - \beta_1\beta_2 + \frac{\beta_1^3}{4}\right)\frac{w^3}{2} + \cdots\right)^2\\
&\quad + \widetilde{q}_3\left(\frac{\beta_1}{2}w + \left(\beta_2 - \frac{\beta_1^2}{2}\right)\frac{w^2}{2} + \left(\beta_3 - \beta_1\beta_2 + \frac{\beta_1^3}{4}\right)\frac{w^3}{2} + \cdots\right)^3 + \cdots\\
&= 1 + \frac{\widetilde{q}_1\beta_1}{2}w + \left(\frac{1}{2}\left(\beta_2 - \frac{\beta_1^2}{2}\right)\widetilde{q}_1 + \frac{\beta_1^2}{4}\widetilde{q}_2\right)w^2\\
&\quad + \left(\frac{1}{2}\left(\beta_3 - \beta_1\beta_2 + \frac{\beta_1^3}{4}\right)\widetilde{q}_1 + \frac{\beta_1}{2}\left(\beta_2 - \frac{\beta_1^2}{2}\right)\widetilde{q}_2 + \frac{\beta_1^3}{8}\widetilde{q}_3\right)w^3 + \cdots . \end{aligned} \tag{2.6}$$

By (2.3), (2.4), (2.5) and (2.6), we have

$$\left(1 + me^{i\varphi}\right)\left[\mathcal{L}\left(1-3\mathfrak{I}\right) + 2(\mathfrak{I} + \eth)\right] a_2 = \frac{\tau_1 ß}{2}, \tag{2.7}$$

$$\begin{aligned}&\left(1 + me^{i\varphi}\right)\left[\frac{\mathcal{L}\left(\mathcal{L}-1\right)\left(3\mathfrak{I}+1\right)}{2}\right]a_2^2\\
&+\left(1 + me^{i\varphi}\right)\left[\mathcal{L}\left(1-4\mathfrak{I}\right) + 3\mathfrak{I} + 6\eth\right] a_3 = \frac{1}{2}\left(\tau_2 - \frac{\tau_1^2}{2}\right)ß + \frac{3\tau_1^2}{4}ß^2,\end{aligned} \tag{2.8}$$

$$-\left(1 + me^{i\varphi}\right)\left[\mathcal{L}\left(1-3\mathfrak{I}\right) + 2(\mathfrak{I} + \eth)\right] a_2 = \frac{\beta_1 ß}{2} \tag{2.9}$$

and

$$\begin{aligned}&\left(1 + me^{i\varphi}\right)\left[\frac{\mathcal{L}^2\left(3\mathfrak{I}+1\right) + \mathcal{L}\left(3-19\mathfrak{I}\right) + 12\mathfrak{I} + 24\eth}{2}\right]a_2^2\\
&-\left(1 + me^{i\varphi}\right)\left[\mathcal{L}\left(1-4\mathfrak{I}\right) + 3\mathfrak{I} + 6\eth\right] a_3 = \frac{1}{2}\left(\beta_2 - \frac{\beta_1^2}{2}\right)ß + \frac{3\beta_1^2}{4}ß^2.\end{aligned} \tag{2.10}$$

From (2.7) and (2.9), we get

$$\tau_1 = -\beta_1 \tag{2.11}$$

and

$$2\left[\left(1 + me^{i\varphi}\right)\mathcal{L}\left(1-3\mathfrak{I}\right) + 2(\mathfrak{I} + \eth)\right]^2 a_2^2 = \frac{(\tau_1^2 + \beta_1^2)ß^2}{4}. \tag{2.12}$$

Adding (2.8) to (2.10), we have

$$\left(1+me^{i\varphi}\right)\left[\mathcal{L}^2\left(3\mathfrak{I}+1\right)+\mathcal{L}\left(1-11\mathfrak{I}\right)+6\mathfrak{I}+12\eth\right]a_2^2 = \frac{1}{2}(\tau_2+\beta_2)ß-\frac{1}{4}(\tau_1^2+\beta_1^2)ß+\frac{3}{4}(\tau_1^2+\beta_1^2)ß^2. \tag{2.13}$$

By substituting (2.12) in (2.13), we get

$$a_2^2 = \frac{(\tau_2+\beta_2)\,ß^2}{2\Big((1+me^{i\varphi})\,ß\Big(\mathcal{L}^2(3\mathfrak{I}+1)+\mathcal{L}(1-11\mathfrak{I})+6\mathfrak{I}+12\eth\Big) + 2(1-3ß)\Big[(1+me^{i\varphi})\mathcal{L}(1-3\mathfrak{I})+2(\mathfrak{I}+\eth)\Big]^2\Big)}. \tag{2.14}$$

Now, using Lemma 1.1, we get

$$|a_2| \leq \frac{\sqrt{2}\,|ß|}{\sqrt{(1+me^{i\varphi})\,ß\Big(\mathcal{L}^2(3\mathfrak{I}+1)+\mathcal{L}(1-11\mathfrak{I})+6\mathfrak{I}+12\eth\Big) + 2(1-3ß)\Big[(1+me^{i\varphi})\mathcal{L}(1-3\mathfrak{I})+2(\mathfrak{I}+\eth)\Big]^2}}. \tag{2.15}$$

Next, by subtracting (2.10) from (2.8), we get

$$a_3 = \frac{(\tau_2-\beta_2)ß}{2(1+me^{i\varphi})\left[2\mathcal{L}\left(1-4\mathfrak{I}\right)+6\mathfrak{I}+12\eth\right]}+a_2^2. \tag{2.16}$$

Hence, by Lemma 1.1, we have

$$\begin{aligned}|a_3| &\leq \frac{(|\tau_2|+|\beta_2|)\,|ß|}{2(1+me^{i\varphi})\left[2\mathcal{L}(1-4\mathfrak{I})+6\mathfrak{I}+12\eth\right]}+|a_2|^2 \\ &\leq \frac{2\,|ß|}{(1+me^{i\varphi})\left[2\mathcal{L}(1-4\mathfrak{I})+6\mathfrak{I}+12\eth\right]}+|a_2|^2\,. \end{aligned} \tag{2.17}$$

By (2.15), we obtain

$$|a_3| \leq \frac{|ß|\Big(F+(1+me^{i\varphi})ß\left[2\mathcal{L}\left(1-4\mathfrak{I}\right)+6\mathfrak{I}+12\eth\right]\Big)}{(1+me^{i\varphi})\left[\mathcal{L}\left(1-4\mathfrak{I}\right)+3\mathfrak{I}+6\eth\right]F},$$

where $F = (1+me^{i\varphi})ß\left[\mathcal{L}^2\left(3\mathfrak{I}+1\right)+\mathcal{L}\left(1-11\mathfrak{I}\right)+6\mathfrak{I}+12\eth\right]$
$+\,2(1-3ß)\left[(1+me^{i\varphi})\mathcal{L}\left(1-3\mathfrak{I}\right)+2(\mathfrak{I}+\eth)\right]^2.$

From (2.16), we get

$$a_3-\varepsilon a_2^2 = \frac{(\tau_2-\beta_2)ß}{2(1+me^{i\varphi})\left[2\mathcal{L}\left(1-4\mathfrak{I}\right)+6\mathfrak{I}+12\eth\right]}+(1-\varepsilon)a_2^2. \tag{2.18}$$

By substituting (2.14) in (2.18), we get

$$
\begin{aligned}
a_3 - \varepsilon a_2^2 &= \frac{(u_2 - v_2)\text{ß}}{2(1+me^{i\varphi})\left[2\mathcal{L}(1-4\mathfrak{I}) + 6\mathfrak{I} + 12\partial\right]} + \frac{(1-\varepsilon)(\tau_2 - \beta_2)\text{ß}^2}{2F} \\
&= \left(\epsilon(\varepsilon) + \frac{|\text{ß}|}{2(1+me^{i\varphi})\left[2\mathcal{L}(1-4\mathfrak{I}) + 6\mathfrak{I} + 12\partial\right]}\right)\tau_2 \qquad (2.19) \\
&\quad + \left(\epsilon(\varepsilon) - \frac{|\text{ß}|}{2(1+me^{i\varphi})\left[2\mathcal{L}(1-4\mathfrak{I}) + 6\mathfrak{I} + 12\partial\right]}\right)\beta_2.
\end{aligned}
$$

where $\epsilon(\varepsilon) = \frac{(1-\varepsilon)\text{ß}^2}{2F}$.

Taking the modulus of (2.19), we have

$$
\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{2|\text{ß}|}{(1+me^{i\varphi})[2\mathcal{L}(1-4\mathfrak{I})+6\mathfrak{I}+12\partial]}; & \left|\epsilon(\varepsilon)\right| \leq \frac{|\text{ß}|}{2(1+me^{i\varphi})[2\mathcal{L}(1-4\mathfrak{I})+6\mathfrak{I}+12\partial]} \\ 4\left|\epsilon(\varepsilon)\right|; & \left|\epsilon(\varepsilon)\right| \geq \frac{|\text{ß}|}{2(1+me^{i\varphi})[2\mathcal{L}(1-4\mathfrak{I})+6\mathfrak{I}+12\partial]}. \end{cases}
$$

□

3. A few functions in the subclass $\Upsilon_{\Sigma}^{\mathfrak{I},\partial,\mathcal{L},m,\varphi}(\widetilde{q})$ that are corollaries

In this section, we will present particular instances of subclasses that are contained within the $\Upsilon_{\Sigma}^{\mathfrak{I},\partial,\mathcal{L},m,\varphi}(\widetilde{q})$.

Corollary 3.1. *Let $f(\xi) \in \Upsilon_{\Sigma}^{\partial,\mathcal{L},m,\varphi}(\widetilde{q})$, where $m \geq 0, -\pi < \varphi \leq \pi, \partial, \mathcal{L} \geq 0$.Then*

$$
|a_2| \leq \sqrt{\frac{2}{F_1}}\left|ß\right|, \quad |a_3| \leq \frac{\left|ß\right|\left(F_1 + 6ß\,(1+me^{i\varphi})\,[2\partial - \mathcal{L} + 1]\right)}{3(1+me^{i\varphi})\,[2\partial - \mathcal{L} + 1]\,F_1},
$$

and for $\varepsilon \in \mathbb{R}$

$$
\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{|ß|}{3(1+me^{i\varphi})[2\partial-\mathcal{L}+1]}; & \left|\epsilon(\varepsilon)\right| \leq \frac{|ß|}{12(1+me^{i\varphi})[2\partial-\mathcal{L}+1]} \\ 4\left|\epsilon(\varepsilon)\right|; & \left|\epsilon(\varepsilon)\right| \geq \frac{|ß|}{12(1+me^{i\varphi})[2\partial-\mathcal{L}+1]}. \end{cases},
$$

where $F_1 = 2ß\,(1+me^{i\varphi})\left[2\mathcal{L}^2 - 5\mathcal{L} + 6\partial + 3\right] + 8(1-3ß)\left[(1+me^{i\varphi})(\partial - \mathcal{L} + 1)\right]^2$
and $\epsilon(\varepsilon) = \frac{(1-\varepsilon)ß^2}{2F_1}$.

Corollary 3.2. *Let $f(\xi) \in \Upsilon_{\Sigma}^{\mathcal{L},m,\varphi}(\widetilde{q})$, where $m \geq 0, -\pi < \varphi \leq \pi, \mathcal{L} \geq 0,$. Then*

$$
|a_2| \leq \sqrt{\frac{2}{F_2}}\left|ß\right|, \quad |a_3| \leq \frac{\left|ß\right|\left(F_2 + 6ß\,(1+me^{i\varphi})\,[1 - \mathcal{L}]\right)}{3(1+me^{i\varphi})\,[1 - \mathcal{L}]\,F_2},
$$

and for $\varepsilon \in \mathbb{R}$

$$
\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{|ß|}{3(1+me^{i\varphi})[1-\mathcal{L}]}; & \left|\epsilon(\varepsilon)\right| \leq \frac{|ß|}{12(1+me^{i\varphi})[1-\mathcal{L}]} \\ 4\left|\epsilon(\varepsilon)\right|; & \left|\epsilon(\varepsilon)\right| \geq \frac{|ß|}{12(1+me^{i\varphi})[1-\mathcal{L}]}. \end{cases},
$$

where $F_2 = 2ß\,(1+me^{i\varphi})\left[2\mathcal{L}^2 - 5\mathcal{L} + 3\right] + 8(1-3ß)\left[(1+me^{i\varphi})(1 - \mathcal{L})\right]^2$
and $\epsilon(\varepsilon) = \frac{(1-\varepsilon)ß^2}{2F_2}$.

Corollary 3.3. *Let* $f(\xi) \in \Upsilon_{\Sigma}^{\Im,\mathcal{L},m,\varphi}(\widetilde{q})$, *where* $m \geq 0, -\pi < \varphi \leq \pi$, $\mathcal{L} \geq 0, \Im \geq 1$. *Then*

$$|a_2| \leq \sqrt{\frac{2}{F_3}}\left|ß\right|, \quad |a_3| \leq \frac{\left|ß\right|\left(F_3 + ß\left(1+me^{i\varphi}\right)\left[2\mathcal{L}\left(1-4\Im\right)+6\Im\right]\right)}{\left(1+me^{i\varphi}\right)\left[\mathcal{L}\left(1-4\Im\right)+3\Im\right]F_3},$$

and for $\varepsilon \in \mathbb{R}$

$$\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{2\left|ß\right|}{\left[(1+me^{i\varphi})2\mathcal{L}(1-4\Im)+6\Im\right]}; & \left|\epsilon\left(\varepsilon\right)\right| \leq \frac{\left|ß\right|}{2\left[(1+me^{i\varphi})2\mathcal{L}(1-4\Im)+6\Im\right]} \\ 4\left|\epsilon\left(\varepsilon\right)\right|; & \left|\epsilon\left(\varepsilon\right)\right| \geq \frac{\left|ß\right|}{2\left[(1+me^{i\varphi})2\mathcal{L}(1-4\Im)+6\Im\right]}. \end{cases},$$

where $F_3 = ß\left(1+me^{i\varphi}\right)\left[\mathcal{L}^2\left(3\Im+1\right)+\mathcal{L}\left(1-11\Im\right)+6\Im\right]$
$+2(1-3ß)\left[\mathcal{L}(1+me^{i\varphi})\left(1-3\Im\right)+2\Im\right]^2$ *and* $\epsilon\left(\varepsilon\right) = \frac{(1-\varepsilon)ß^2}{2F_3}$.

Corollary 3.4. *Let* $f(\xi) \in \Upsilon_{\Sigma}^{\Im,m,\varphi}(\widetilde{q})$, *where* $m \geq 0, -\pi < \varphi \leq \pi$, $\Im \geq 1$. *Then*

$$|a_2| \leq \sqrt{\frac{2}{F_4}}\left|ß\right|, \quad |a_3| \leq \frac{\left|ß\right|\left(F_4 + (1+me^{i\varphi})2ß\left[1-\Im\right]\right)}{\left(1+me^{i\varphi}\right)\left[1-\Im\right]F_4},$$

and for $\varepsilon \in \mathbb{R}$

$$\left|a_3 - \varepsilon a_2^2\right| \leq \begin{cases} \frac{\left|ß\right|}{(1+me^{i\varphi})(1-\Im)}; & \left|\epsilon\left(\varepsilon\right)\right| \leq \frac{\left|ß\right|}{4(1+me^{i\varphi})[1-\Im]} \\ 4\left|\epsilon\left(\varepsilon\right)\right|; & \left|\epsilon\left(\varepsilon\right)\right| \geq \frac{\left|ß\right|}{4(1+me^{i\varphi})[1-\Im]}. \end{cases},$$

where $F_4 = ß\left(1+me^{i\varphi}\right)\left[\mathcal{L}^2\left(3\Im+1\right)+\mathcal{L}\left(1-11\Im\right)+6\Im\right]$
$+2(1-3ß)\left[(1+me^{i\varphi})\mathcal{L}\left(1-3\Im\right)+2\Im\right]^2$ *and* $\epsilon\left(\varepsilon\right) = \frac{(1-\varepsilon)ß^2}{2F_4}$.

4. Conclusions

Within the scope of this investigation, we presented a novel subclass of standardised analytic functions and bi-univalent functions that are connected to the Fibonacci numbers. This particular subclass is represented by the notation $\Upsilon_{\Sigma}^{\Im,\eth,\mathcal{L},m,\varphi}(\widetilde{q})$ at the moment. Bounds for the Taylor coefficients $|a_2|$ and $|a_3|$ were calculated, and solutions to Fekete-Szegö functional problems were presented for functions falling under this category. This discovery has the potential to pave the way for further investigation into the development of new categories of analytic and bi-univalent functions that are related with the Fibonacci numbers.

In the course of future research, the study has the potential to be extended to encompass a variety of conic regions. These regions include three leaf domains, Ozaki-Type Bi-Close-to-Convex, and Bi-Concave Functions. These functions use a Modified Caputo's Fractional Operator that is coupled with a Three-Leaf Function [49].

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