Int. J. Anal. Appl. (2026), 24:54

International Journal of Analysis and Applications

Linear Diophantine Type-2 Fuzzy Sets with Applications

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Abstract. This article proposes the conventional definition and real life based applications of linear Diophantine type-2 fuzzy sets (*LDT2FS*), manifesting its supremacy over common fuzzy models. *LDT2FS* is a new mathematical framework that can address the drawbacks of existing fuzzy sets. As common fuzzy systems, however, these models hold restrictions on acceptance and rejection grades, limiting the flexibility of decision-makers in managing uncertainty. *LDT2FS* extends the previous study by relaxing these limitations, as decision-makers can specify grades-freely with reference parameters in practice in cases where decision making under uncertainty is necessary when functional relationships are unknown, or when data contain high levels of imprecision. To this end, the less demanding structure of *LDT2FS* allows for establishing the fit to allow dealing with these uncertainties in an efficient way. It also describes basic arithmetic operations on *LDT2FS* such as union, intersection, complement, containment, etc. along with their algebraic characteristics. Furthermore, two new operators, the Certainty Operator, and Feasibility Operator, are proposed to convert an *LDT2FS* into a conventional fuzzy set, simplifying mathematical processing and applications. This article proposes Haming Distance and Euclidean Distance commonly used in pattern recognition, clustering, and classification problems to quantify the differences or similarity between *LDT2FS* instances.

1. Introduction

Imprecision is an essential attribute of information. Decisions are made in an area with diverse types of uncertainty in various scientific and industrial applications. Now a days, mostly processes contain rectifying and figure out data, that's most of the time insufficient, fragmental or very often contrasting [1] . So, the miniature signalizing the actual world need being convoyed by the suitable unpredictable depiction [2] . Escorted by the emergence of soft computing (SC) methods, various effective instrument were introduced. [3].Due to indefiniteness and fuzziness present in real world problem the techniques usually used in classical mathematics are not always favorable to handle these problems. Zadeh [4] instituted the idea of fuzzy sets as an augmentation

Received: Dec. 31, 2025.

2020 *Mathematics Subject Classification.* 03E72.

Key words and phrases. type-2 fuzzy set; linear Diophantine fuzzy set; linear Diophantine type-2 fuzzy set.

https://doi.org/10.28924/2291-8639-24-2026-54

ISSN: 2291-8639

of traditional crisp set. A fuzzy set is a remarkable mathematical model to draw up an erecting of entities whose boundary is ambiguous. Since Zadeh's benefaction to fuzzy set [4], the fuzzy logic has been extended from classical logic that is defined by a acceptance function ranging in [0, 1] and gives a strong substitute to probability theory to distinguish imprecision, vagueness and ambiguities in numerous fields. The idea of linguistic variable was introduced by Zadeh [5]. According to Zadeh, a linguistic variable is a variable whose data is not numerical but contains actual language. If fuzzy sets convey these words defined over a universal set, then the variable is called a fuzzy linguistic variable [5].An ordinary fuzzy set is extended into type-2 fuzzy set $(T2FS)$ that is type-1 fuzzy set $(T1FS)$.The essential supremacy of type-2 fuzzy set upon type-1 fuzzy set is capacity of it to encapsulate acceptance of appropriate acceptance costs in which we handle ambiguity additionally precise.Type-1 fuzzy set has the acceptance value a real number in [0, 1].While the type-1 fuzzy set itself is the acceptance value of a $T2FS$. Zadeh [5–7] introduced the concept of $T2FS$. Mendel [8] ran through the type-2 fuzzy sets. Takac [9] presented that type-2 fuzzy sets are too relevant to situations where we have additional uncertainties as type-1 fuzzy sets and interval-valued fuzzy sets are specific instances of type-2 fuzzy sets. Kundu et al. [10] presented a fixed charge transportation problem with type-2 fuzzy parameters from the viewpoint of type reduction and the centroid. Mizumoto and Tanaka [11, 12] and Dubois and Prade [13] proposed the logical operations of $T2FS$. Afterwards, most of the researchers scrutinized theoretical [14–17] and application areas [18–23] of $T2FS$.Intuitionistic Type-2 Fuzzy Sets $(IT2FS)$ provide great uncertainty modeling [24] but they have crankiness and interpretability problems because of their dual-layer acceptance functions. The Linear Diophantine Fuzzy Set $(LDFS)$ was proposed to enhance uncertainty modeling by integrating a linear Diophantine acceptance structure [25]. But it has some restrictions in handling complex uncertainty, conjoining acceptances, and vigorous decision- making situations. To overcome these restrictions $LDT2FS$ expands $LDFS$ by incorporating Type-2 fuzzy logic which proposes lower and upper bounds for both the acceptance and rejection functions with reference parameters. A Linear Diophantine Type-2 Fuzzy Set $(LDT2FS)$ proposes upper and lower bounds for acceptance and rejection functions that is more flexible in uncertain conditions while in $LDFS$ we deal with single acceptance pair. This layout adequately proposes overlapping fuzzy acceptances,upgrading fuzzy computational accuracy, and reinforces multi-criteria decision-making (MCDM). By exploiting Type-2 fuzzy logic $LDT2FS$ furnishes a more vigorous and versatile approach for real-world applications that involve uncertainty like pattern recognition,expert systems, and decision support systems. This uncertainty proposes greater adaptability in presenting vague and unstable information. $LDT2FS$ reinforces fuzzy arithmetic operations,information retrieval methods, and multi-criteria decision-making (MCDM) make it appropriate for implications in pattern recognition,robotics and artificial intelligence. Here in the paper,we gave the idea of the Linear Diophantine type-2 fuzzy set $(LDT2FS)$ whose type-1 acceptance is the ordinary fuzzy acceptance, and developing type-2 includes acceptance and rejection with reference parameter as the linear Diophantine fuzzy set. The conviction of operations on basic sets and algebraic characteristics of these sets with various demonstrative examples are proposed. Subsequently two set operators of which fundamental operation has to change a *LDT2FS* into an ordinary *T2FS* are introduced and explain some characteristics of these operators. At the end we give two distance measures, the Hamming distance and Euclidian distance of *LDT2FS* which are demonstrated with an algebraic example. The major endowment of this article is:.

(i) The idea of Linear Diophantine type-2 fuzzy set is presented.

(ii) Few set-theoretic operations such as the union, intersection, and complement of *LDT2FS* are proposed.

(iii) Various Characteristics of *LDT2FS* like idempotency, commutative, associative, distributive, involution and De Morgan's law are given.

(iv) Feasibility and Certainty operators of *LDT2FS* are explained.

(v) In this article two distance measures, the Hamming distance and Euclidian distance, are proposed .

(vi) A relevant framework based on skill assessment is proposed, where we measure distance of *LDT2FS*.

We categorize the rest of the article as follow. The preliminary ideas of this article are given in Section 2. We presented linear Diophantine type-2 fuzzy set $(LDT2FS)$ with numerical examples in Section 3. Some set-theoretic operations of *LDT2FS* such as union, intersection, and complement are explained In Section 4. The Characteristics of *LDT2FS* are corroborated in Section 5. In Section 6 the Certainty and Feasibility operators of *LDT2FS* are interpreted. In Section 7, two distances of *LDT2FS* with example demonstrating implementation of these distance in a actual-life implementation grounded in skill assessment are conferred in Section 8. Eventually we have conclusion in Section 9.

2. Preliminaries

Before instituting *LDT2FS* we propose some vital concepts of *T2FS* and *LDFS*.

A fuzzy set whose acceptance degree comprises of imprecision i.e. the acceptance degree is a fuzzy set and not a crisp set is said to be a type-2 fuzzy set (T2FS). A T2FS $\mathring{\mathfrak{G}}$ is defined as

$$\mathring{\mathfrak{G}} = \{((\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) : \forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \forall \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\},$$

while $\mathfrak{o} \leq \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1$, be the secondary acceptance function and $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$ be the primary acceptance of $\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}$ that is domain of $\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})$. We can represent $\mathring{\mathfrak{G}}$ as:

$$\mathring{\mathfrak{G}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}} \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) / \widehat{z\mathring{\mathfrak{F}}} \right) / \widehat{z\dot{\mathfrak{V}}} : \forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \forall \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\},$$

whereas $\int \int$ represents union over $\widehat{z\dot{\mathfrak{V}}}$ and $\widehat{z\mathring{\mathfrak{F}}}$. We exchange $\int$ by Σ in discrete case, where secondary acceptance function $\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})$ is expressed as:

$$\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}} \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) / \widehat{z\mathring{\mathfrak{F}}}),$$

while for a specific $\widehat{z\mathring{\mathfrak{F}}} = \widehat{z\mathring{\mathfrak{F}}}' \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}')$ is known as secondary acceptance grade of $(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})$.

Example 2.1. *Suppose the set "Business Adaptability" is expressed as T2FS $\mathring{\mathfrak{G}}$. The "Agile" be primary acceptance function of $\mathring{\mathfrak{G}}$ and the "degree of Agile" be secondary acceptance function. Suppose $\ddot{Y} = \{3, 5, 7\}$ is a Business Adaptability set and has primary acceptance of the points of $\ddot{Y}$ respectively $\mathfrak{M}_3 = \{0.7, 0.6, 1.0\}$, $\mathfrak{M}_5 = \{0.4, 0.9, 0.8\}$, $\mathfrak{M}_7 = \{0.3, 0.2, 0.5\}$. Secondary acceptance function of $\ddot{Y}$ is given in table 1.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$	$\mathring{\mathfrak{G}}$
3	0.7	(0.8, 0.4, 0.5)
	0.6	(0.5, 0.2, 0.3)
	1.0	(0.6, 0.9, 0.5)
5	0.4	(0.5, 0.8, 0.3)
	0.9	(0.6, 0.4, 0.5)
	0.8	(0.7, 0.6, 0.3)
7	0.3	(0.8, 0.2, 0.5)
	0.2	(0.9, 0.4, 0.1)
	0.5	(0.1, 0.3, 0.9)

Table 1: Business Adaptability.

Now we define a linear Diophantine fuzzy set, let $\ddot{Y}$ be the universal set. A LDFS £ on $\ddot{Y}$ is proposed as follows

$$£ = \{(\widehat{z\dot{\mathfrak{V}}}, \langle \hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}}), \hat{\nu}_£(\widehat{z\dot{\mathfrak{V}}})\rangle, \langle \hat{\gamma}(\widehat{z\dot{\mathfrak{V}}}), \hat{\delta}(\widehat{z\dot{\mathfrak{V}}})\rangle) : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\},$$

whereas $\hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}}), \hat{\nu}_£(\widehat{z\dot{\mathfrak{V}}}), \hat{\gamma}(\widehat{z\dot{\mathfrak{V}}}), \hat{\delta}(\widehat{z\dot{\mathfrak{V}}}) \in [0, 1]$ such that

$$0 \le \hat{\gamma}(\widehat{z\dot{\mathfrak{V}}})\hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}}) + \hat{\delta}(\widehat{z\dot{\mathfrak{V}}})\hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}}) \le 1, \forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}$$

$$0 \le \hat{\gamma} + \hat{\delta} \le 1.$$

We can write hesitation part as

$$\xi = 1 - (\hat{\gamma}(\widehat{z\dot{\mathfrak{V}}})\hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}}) + \hat{\delta}(\widehat{z\dot{\mathfrak{V}}})\hat{\mu}_£(\widehat{z\dot{\mathfrak{V}}})),$$

where ξ is reference parameter.

3. Linear Diophantine Type-2 Fuzzy Set

Now let's initiate the idea of Linear Diophantine Type-2 Fuzzy set and type-1 acceptance is common fuzzy acceptance with secondary acceptance and rejection functions and reference parameters.

A *LDT2F* $\mathring{\mathfrak{G}}$ on $\widehat{z\dot{\mathfrak{V}}}$ is defined as an object of the following form

$$\mathring{\mathfrak{G}} = \{((\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle) : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\},$$

where the functions $\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \in [\mathfrak{o}, 1]$ such that

$$\mathfrak{o} \leq \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) + \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1, \forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1] \backslash$$

$$\mathfrak{o} \leq \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) + \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1$$

$\forall\, \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]$.

We write *LDT2F* for discrete case as:

$$\mathring{\mathfrak{G}} = \Sigma\left(\Sigma\left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \diagup \widehat{z\mathring{\mathfrak{F}}}\right) \diagup \widehat{z\dot{\mathfrak{V}}}, \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\}.$$

For a continuos case *LDT2F* is written as:

$$\mathring{\mathfrak{G}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \diagup \widehat{z\mathring{\mathfrak{F}}}\right) \diagup \widehat{z\dot{\mathfrak{V}}}, \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\}.$$

Example 3.1. *Let $\ddot{Y} = \{a, b, c\}$ be a set and the primary acceptance of the points of $\widehat{z\dot{\mathfrak{V}}}$ is $\mathfrak{M}_a = \{\mathfrak{o}.6, \mathfrak{o}.7, \mathfrak{o}.9\}$, $\mathfrak{M}_b = \{\mathfrak{o}.2, \mathfrak{o}.5, \mathfrak{o}.7\}$, $\mathfrak{M}_c = \{\mathfrak{o}.5, \mathfrak{o}.8, 1\}$, respectively. Then, the discrete LDT2FS $\mathring{\mathfrak{G}}$ is given by table 2.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$	$\mathring{\mathfrak{G}}$
a	$\mathfrak{o}.6$	$(\langle \mathfrak{o}.8, \mathfrak{o}.7\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.4, \mathfrak{o}.5\rangle, \langle \mathfrak{o}.2, \mathfrak{o}.3\rangle)$
	$\mathfrak{o}.9$	$(\langle \mathfrak{o}.6, \mathfrak{o}.9\rangle, \langle \mathfrak{o}.5, \mathfrak{o}.1\rangle)$
b	$\mathfrak{o}.2$	$(\langle \mathfrak{o}.5, \mathfrak{o}.8\rangle, \langle \mathfrak{o}.3, \mathfrak{o}.4\rangle)$
	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.6, \mathfrak{o}.7\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.7, \mathfrak{o}.6\rangle, \langle \mathfrak{o}.6, \mathfrak{o}.3\rangle)$
c	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.8, \mathfrak{o}.5\rangle, \langle \mathfrak{o}.7, \mathfrak{o}.2\rangle)$
	$\mathfrak{o}.8$	$(\langle \mathfrak{o}.9, \mathfrak{o}.4\rangle, \langle \mathfrak{o}.8, \mathfrak{o}.1\rangle)$
	1	$(\langle \mathfrak{o}.1, \mathfrak{o}.3\rangle, \langle \mathfrak{o}.9, \mathfrak{o}.\mathfrak{o}\rangle)$

Table 2: Discrete LDT2FS.

4. Operations on LDT2FS

Here we are going to introduce the operations like union, intersection and compliment to proposed *LDT2FS*.

We take two *LDT2FSs* $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ on $\ddot{Y}$.

$$\mathring{\mathfrak{G}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \diagup \widehat{z\mathring{\mathfrak{F}}}\right) \diagup \widehat{z\dot{\mathfrak{V}}},$$

and

$$\mathring{\mathfrak{B}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right) \diagup \widehat{\widehat{z\mathring{q}}}\right) \diagup \widehat{z\dot{\mathfrak{V}}},$$

whereas $\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}$ and $\mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]$ are the domains of secondary acceptance function and reference parameters. We can find union of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ as:

$$\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widetilde{\widetilde{r}} \in \mathfrak{M}^{\widetilde{\widetilde{r}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}), \hat{\nu}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}), \hat{\gamma}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}), \hat{\delta}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) \right) / \widetilde{\widetilde{r}} \right) / \widehat{z\dot{\mathfrak{V}}},$$

whereas $\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}} \cup \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}} = \mathfrak{M}^{\widetilde{\widetilde{r}}}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]$:

$$\hat{\mu}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \Phi \left(\int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) / \widehat{z\mathring{\mathfrak{F}}}\ ,\ \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) / \widehat{\widehat{z\mathring{q}}} \right)$$

applying expansion rule we get,

$$\hat{\mu}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / \Phi(\widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}})$$

where $\Phi(\widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}})$ is the t-conorm of $\widehat{z\mathring{\mathfrak{F}}}$ ans $\widehat{\widehat{z\mathring{q}}}$,

$$\hat{\mu}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}})$$

In the same way we get,

$$\upsilon_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}})$$

$$\hat{\gamma}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}})$$

$$\hat{\delta}_{\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}})$$

We can find intersection of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ as:

$$\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}} = \int_{\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}} \left(\int_{\widetilde{\widetilde{r}} \in \mathfrak{M}^{\widetilde{\widetilde{r}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\gamma}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / \widehat{\widehat{z\mathring{q}}} \right) / \widehat{z\dot{\mathfrak{V}}},$$

whereas $\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}} \cap \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}} = \mathfrak{M}^{\widetilde{\widetilde{r}}}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]$:

$$\hat{\mu}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}})$$

$$\upsilon_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}})$$

$$\hat{\gamma}_{\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widetilde{\widetilde{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \int_{\widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right) / (\widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}})$$

$$\hat{\delta}_{\mathring{\mathfrak{G}}\cap\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\overline{r}}) = \int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}}\int_{\widehat{\widehat{z\mathring{q}}}\in\mathfrak{M}^{\widehat{\widehat{z\mathring{q}}}}_{\widehat{z\mathfrak{V}}}}\left(\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}})\right)\Big/(\widehat{z\mathring{\mathfrak{F}}}\wedge\widehat{\widehat{z\mathring{q}}})$$

The compliment of $\mathring{\mathfrak{G}}$ can be expressed as:

$$\mathring{\mathfrak{G}}^c = \int_{\widehat{z\mathfrak{V}}\in\ddot{Y}}\left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}}\begin{pmatrix}\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\end{pmatrix}\Big/(1-\widehat{z\mathring{\mathfrak{F}}})\right)\Big/\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathfrak{V}}\in\ddot{Y},\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}\subseteq[\mathfrak{o},1]$$

and

$$\overline{\mathring{\mathfrak{G}}} = \int_{\widehat{z\mathfrak{V}}\in\ddot{Y}}\left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}}\begin{pmatrix}\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\end{pmatrix}\Big/\widehat{z\mathring{\mathfrak{F}}}\right)\Big/\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathfrak{V}}\in\ddot{Y},\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}\subseteq[\mathfrak{o},1]$$

or

$$\mathring{\mathfrak{G}}^c = \int_{\widehat{z\mathfrak{V}}\in\ddot{Y}}\left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}}\begin{pmatrix}\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\end{pmatrix}\Big/(1-\widehat{z\mathring{\mathfrak{F}}})\right)\Big/\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathfrak{V}}\in\ddot{Y},\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}\subseteq[\mathfrak{o},1]$$

Additionally there are some more operations on *LDT2FSs* that are given below.

$$\mathring{\mathfrak{G}}\subset\mathring{\mathfrak{B}}\ \ iff\ (\forall\widehat{z\dot{\mathfrak{V}}}\in\widehat{z\dot{\mathfrak{V}}}\ )\begin{pmatrix}\widehat{z\mathring{\mathfrak{F}}}\leq\widehat{\widehat{z\mathring{q}}},\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\leq\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}}),\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\geq\hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\leq\hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}}),\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\geq\hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}})\end{pmatrix}$$

and

$$\mathring{\mathfrak{G}}=\mathring{\mathfrak{B}}\ \ iff\ (\forall\widehat{z\dot{\mathfrak{V}}}\in\ddot{Y}\ )\begin{pmatrix}\widehat{z\mathring{\mathfrak{F}}}=\widehat{\widehat{z\mathring{q}}},\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})=\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}}),\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})=\hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})=\hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}}),\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})=\hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{\widehat{z\mathring{q}}})\end{pmatrix}$$

$$\overline{\mathring{\mathfrak{G}}} = \int_{\widehat{z\mathfrak{V}}\in\ddot{Y}}\left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}_{\widehat{z\mathfrak{V}}}}\begin{pmatrix}\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}}),\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathring{\mathfrak{F}}})\end{pmatrix}\Big/\widehat{z\mathring{\mathfrak{F}}}\right)\Big/\widehat{z\dot{\mathfrak{V}}},\widehat{z\mathfrak{V}}\in\ddot{Y},\mathfrak{M}^{\widehat{z\mathring{\mathfrak{F}}}}_{\widehat{z\mathfrak{V}}}\subseteq[\mathfrak{o},1]$$

For discrete case we replace $\int$ by Σ.

Let us take the an example to verify the Characteristics of *LDT2FS*.

Example 4.1. *Let $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ be two LDT2FSs presenting the set Stress Level. The high stress is considered as primary acceptance function of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$. Assume degree of high stress level and degree of low stress level as secondary acceptance and rejection functions of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$. Let's assume both $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ to be defined on $\ddot{Y}$ $= \{5, 1\mathfrak{o}, 15\}$, which are eventually represented in table 3 and 4:*

The set $\ddot{Y}$	*Primary acceptance $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$*	$\mathring{\mathfrak{G}}$
5	$\mathfrak{o}.6$	$(\langle\mathfrak{o}.8,\mathfrak{o}.7\rangle,\langle\mathfrak{o}.4,\mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle\mathfrak{o}.4,\mathfrak{o}.5\rangle,\langle\mathfrak{o}.2,\mathfrak{o}.3\rangle)$
	$\mathfrak{o}.9$	$(\langle\mathfrak{o}.6,\mathfrak{o}.9\rangle,\langle\mathfrak{o}.5,\mathfrak{o}.1\rangle)$
$1\mathfrak{o}$	$\mathfrak{o}.2$	$(\langle\mathfrak{o}.5,\mathfrak{o}.8\rangle,\langle\mathfrak{o}.3,\mathfrak{o}.4\rangle)$
	$\mathfrak{o}.5$	$(\langle\mathfrak{o}.6,\mathfrak{o}.7\rangle,\langle\mathfrak{o}.4,\mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle\mathfrak{o}.7,\mathfrak{o}.6\rangle,\langle\mathfrak{o}.6,\mathfrak{o}.3\rangle)$
15	$\mathfrak{o}.5$	$(\langle\mathfrak{o}.8,\mathfrak{o}.5\rangle,\langle\mathfrak{o}.7,\mathfrak{o}.2\rangle)$
	$\mathfrak{o}.8$	$(\langle\mathfrak{o}.9,\mathfrak{o}.4\rangle,\langle\mathfrak{o}.8,\mathfrak{o}.1\rangle)$
	1	$(\langle\mathfrak{o}.1,\mathfrak{o}.3\rangle,\langle\mathfrak{o}.9,\mathfrak{o}.\mathfrak{o}\rangle)$

Table 3: Stress level $\mathring{\mathfrak{G}}$.

and

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$	$\mathring{\mathfrak{B}}$
5	0.4	$(\langle 0.5, 0.6\rangle, \langle 0.3, 0.2\rangle)$
	0.1	$(\langle 0.3, 0.4\rangle, \langle 0.2, 0.7\rangle)$
	0.2	$(\langle 0.2, 0.9\rangle, \langle 0.3, 0.2\rangle)$
10	0.8	$(\langle 0.4, 0.2\rangle, \langle 0.5, 0.9\rangle)$
	0.9	$(\langle 0.5, 0.7\rangle, \langle 0.6, 0.4\rangle)$
	0.3	$(\langle 0.6, 0.4\rangle, \langle 0.4, 0.8\rangle)$
15	0.6	$(\langle 0.7, 0.6\rangle, \langle 0.7, 0.6\rangle)$
	0.7	$(\langle 0.8, 0.3\rangle, \langle 0.6, 0.4\rangle)$
	0.5	$(\langle 0.9, 0.2\rangle, \langle 0.9, 0.5\rangle)$

Table 4: Stress level $\mathring{\mathfrak{B}}$.

The union operator of $\mathring{\mathfrak{G}}$ *and* $\mathring{\mathfrak{B}}$ *is* $\hat{\mu}_{\mathring{\mathfrak{G}}\cup\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}})), \hat{\nu}_{\mathring{\mathfrak{G}}\cup\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}}), \hat{\gamma}_{\mathring{\mathfrak{G}}\cup\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}}), \hat{\delta}_{\mathring{\mathfrak{G}}\cup\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}})$, *that is represented in table 5.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$	$\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}$
5	0.6	$(\langle 0.8, 0.6\rangle, \langle 0.4, 0.2\rangle)$
	0.7	$(\langle 0.4, 0.4\rangle, \langle 0.2, 0.3\rangle)$
	0.9	$(\langle 0.6, 0.9\rangle, \langle 0.5, 0.1\rangle)$
10	0.8	$(\langle 0.5, 0.2\rangle, \langle 0.5, 0.4\rangle)$
	0.9	$(\langle 0.6, 0.7\rangle, \langle 0.6, 0.4\rangle)$
	0.3	$(\langle 0.7, 0.4\rangle, \langle 0.4, 0.3\rangle)$
15	0.6	$(\langle 0.8, 0.5\rangle, \langle 0.7, 0.2\rangle)$
	0.8	$(\langle 0.9, 0.3\rangle, \langle 0.8, 0.1\rangle)$
	1.0	$(\langle 0.9, 0.2\rangle, \langle 0.9, 0.0\rangle)$

Table 5: The union operator $\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}$.

The intersection of $\mathring{\mathfrak{G}}$ *and* $\mathring{\mathfrak{B}}$ *is* $\hat{\mu}_{\mathring{\mathfrak{G}}\cap\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}})), \hat{\nu}_{\mathring{\mathfrak{G}}\cap\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}}), \hat{\gamma}_{\mathring{\mathfrak{G}}\cap\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}}), \hat{\delta}_{\mathring{\mathfrak{G}}\cap\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}},\widetilde{\widetilde{r}})$, *that is represented in table 6.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$	$\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}$
5	0.4	$(\langle 0.5, 0.7\rangle, \langle 0.3, 0.5\rangle)$
	0.1	$(\langle 0.3, 0.5\rangle, \langle 0.2, 0.7\rangle)$
	0.2	$(\langle 0.2, 0.9\rangle, \langle 0.3, 0.2\rangle)$
10	0.8	$(\langle 0.4, 0.8\rangle, \langle 0.3, 0.9\rangle)$
	0.9	$(\langle 0.5, 0.7\rangle, \langle 0.4, 0.5\rangle)$
	0.3	$(\langle 0.6, 0.6\rangle, \langle 0.4, 0.8\rangle)$
15	0.6	$(\langle 0.7, 0.6\rangle, \langle 0.7, 0.6\rangle)$
	0.7	$(\langle 0.8, 0.4\rangle, \langle 0.6, 0.4\rangle)$
	0.5	$(\langle 0.1, 0.3\rangle, \langle 0.9, 0.5\rangle)$

Table 6: The union operator $\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}$.

The complement of $\mathring{\mathfrak{G}}$ *is* $\Big(\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\Big) \diagup (1-\widehat{z\mathring{\mathfrak{F}}})$, *that is represented in table 7.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$	$\mathring{\mathfrak{G}}^c$
5	0.4	$(\langle 0.7, 0.8\rangle, \langle 0.5, 0.4\rangle)$
	0.3	$(\langle 0.5, 0.4\rangle, \langle 0.3, 0.2\rangle)$
	0.1	$(\langle 0.9, 0.6\rangle, \langle 0.1, 0.5\rangle)$
10	0.8	$(\langle 0.8, 0.5\rangle, \langle 0.4, 0.3\rangle)$
	0.5	$(\langle 0.7, 0.6\rangle, \langle 0.5, 0.4\rangle)$
	0.3	$(\langle 0.6, 0.7\rangle, \langle 0.3, 0.6\rangle)$
15	0.5	$(\langle 0.5, 0.8\rangle, \langle 0.2, 0.7\rangle)$
	0.2	$(\langle 0.4, 0.9\rangle, \langle 0.1, 0.8\rangle)$
	0.0	$(\langle 0.3, 0.1\rangle, \langle 0.0, 0.9\rangle)$

Table 7: The complement $\mathring{\mathfrak{G}}^c$.

5. Characteristics of LDT2FS

Now we introduce Characteristics for *LDT2FS* along with related examples. Let's consider three *LDT2FS*s $\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}$ and $\widetilde{\widetilde{R}}$ we define these operations as :

(i) $\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{G}} = \mathring{\mathfrak{G}}$ (Idempotency)

(ii) $\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}} = \mathring{\mathfrak{B}} \cup \mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}} = \mathring{\mathfrak{B}} \cup \mathring{\mathfrak{G}}$ (Commutativity)

(iii) $\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big) \cup \widetilde{\mathring{\mathfrak{R}}} = \mathring{\mathfrak{G}} \cup (\mathring{\mathfrak{B}} \cup \widetilde{\mathring{\mathfrak{R}}})$ and $\big(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\big) \cap \widetilde{\mathring{\mathfrak{R}}} = \mathring{\mathfrak{G}} \cap (\mathring{\mathfrak{B}} \cap \widetilde{\mathring{\mathfrak{R}}})$ (Associativity)

(iv) $\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big) \cap \widetilde{\mathring{\mathfrak{R}}} = (\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) \cap (\mathring{\mathfrak{G}} \cup \widetilde{\mathring{\mathfrak{R}}})$ and $\mathring{\mathfrak{G}} \cap (\mathring{\mathfrak{B}} \cup \widetilde{\mathring{\mathfrak{R}}}) = (\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) \cup (\mathring{\mathfrak{G}} \cap \widetilde{\mathring{\mathfrak{R}}})$ (Distributive law)

(v) $\big(\mathring{\mathfrak{G}}^c\big)^c = \mathring{\mathfrak{G}}$ (Involution)

(vi) $\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big)^c = \mathring{\mathfrak{G}}^c \cap \mathring{\mathfrak{B}}^c$ and $\big(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\big)^c = \mathring{\mathfrak{G}}^c \cup \mathring{\mathfrak{B}}^c$(De Morgan's law)

Proof. (vi):(De Morgan's law)

$\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big)^c = \mathring{\mathfrak{G}}^c \cap \mathring{\mathfrak{B}}^c$

$\mathring{\mathfrak{G}} = \Big\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\Big\}$

and

$\mathring{\mathfrak{B}} = \Big\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\Big\}$

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$\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big) = \Big\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\Big\} \cup$
$\Big\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\Big\}$

$$\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big) = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}^c$$

$$\big(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\big)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, 1-(\widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \left(1-\widehat{z\mathring{\mathfrak{F}}}\right) \wedge (1-\widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right)^c = \left\{(\widehat{z\dot{\mathfrak{V}}}, 1-\widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\right\} \cap$$
$$\left\{(\widehat{z\dot{\mathfrak{V}}}, 1-\widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\right\}$$

$$\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right)^c = \mathring{\mathfrak{G}}^c \cap \mathring{\mathfrak{B}}^c$$

Furthermore:

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \mathring{\mathfrak{G}}^c \cup \mathring{\mathfrak{B}}^c$$

$$\mathring{\mathfrak{G}} = \left\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\right\}$$

and

$$\mathring{\mathfrak{B}} = \left\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\right\}$$

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$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\right\} \cap$$
$$\left\{(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\right\}$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}^c$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, 1-(\widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \left\{ \begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \left(1-\widehat{z\mathring{\mathfrak{F}}}\right) \vee (1-\widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \end{array} \right\}$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \left\{(\widehat{z\dot{\mathfrak{V}}}, 1-\widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle\right\} \cup$$
$$\left\{(\widehat{z\dot{\mathfrak{V}}}, 1-\widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle\right\}$$

$$\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right)^c = \mathring{\mathfrak{G}}^c \cup \mathring{\mathfrak{B}}^c.$$ □

6. Certainty and Feasibility Operator on LDT2FS

Here we are going to introduce Feasibility and Certainty of *LDT2FS*. For some situations we need a gross result. From a *LDT2FS*, in the case where we need a gross result in *T2FS*, we will have two operators which converts an *LDT2FS* into an common *T2FS*. Suppose $\mathring{\mathfrak{G}}$ is a *LDT2FS* over $\ddot{Y}$ and have primary acceptance function u, secondary acceptance function $\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}\ , \widehat{z\mathring{\mathfrak{F}}})$ and secondary rejection function $\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}\ , \widehat{z\mathring{\mathfrak{F}}})$ with reference parameters $\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}\ , \widehat{z\mathring{\mathfrak{F}}})$ and $\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}\ , \widehat{z\mathring{\mathfrak{F}}})$ then the two operators are:

(i) Certainty Operator:

$$\Delta\mathring{\mathfrak{G}} = \left\{\left\langle \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle : \widehat{z\dot{\mathfrak{V}}} \in \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1] \right\}$$

(i) Feasibility Operator:

$$\nabla\mathring{\mathfrak{G}} = \left\{\left\langle \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle : \widehat{z\dot{\mathfrak{V}}} \in \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1] \right\}$$

The operator can also be represented as:

$$\Delta\mathring{\mathfrak{G}} = \int_{\widehat{z\dot{\mathfrak{V}}}\in\ddot{Y}} \left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}} \left(\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right) \Big/ \widehat{z\mathring{\mathfrak{F}}} \right) \Big/ \widehat{z\dot{\mathfrak{V}}}$$

and

$$\nabla\mathring{\mathfrak{G}} = \int_{\widehat{z\dot{\mathfrak{V}}}\in\ddot{Y}} \left(\int_{\widehat{z\mathring{\mathfrak{F}}}\in\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}} \left(1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right) \Big/ \widehat{z\mathring{\mathfrak{F}}} \right) \Big/ \widehat{z\dot{\mathfrak{V}}}$$

We will replace $\int$ is by Σ in noncontinuous case. Definitely, if $\mathring{\mathfrak{G}}$ be a common T2FS, then $\Delta\mathring{\mathfrak{G}} = \mathring{\mathfrak{G}} = \nabla\mathring{\mathfrak{G}}$.

We will take an example for better understanding.

Example 6.1. *Let* $\ddot{Y} = \{a, b, c\}$ *primary acceptance of* $\ddot{Y}$ *is* $J_a = \{\mathfrak{o}.6, \mathfrak{o}.7, \mathfrak{o}.9\}$, $J_b = \{\mathfrak{o}.2, \mathfrak{o}.5, \mathfrak{o}.7\}$, $J_c = \{\mathfrak{o}.5, \mathfrak{o}.8, 1\}$, *respectively. The discrete LDT2F* $\mathring{\mathfrak{G}}$ *is given in table 8.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$	$\mathring{\mathfrak{G}}$
a	$\mathfrak{o}.6$	$(\langle \mathfrak{o}.8, \mathfrak{o}.7\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.4, \mathfrak{o}.5\rangle, \langle \mathfrak{o}.2, \mathfrak{o}.3\rangle)$
	$\mathfrak{o}.9$	$(\langle \mathfrak{o}.6, \mathfrak{o}.9\rangle, \langle \mathfrak{o}.5, \mathfrak{o}.1\rangle)$
b	$\mathfrak{o}.2$	$(\langle \mathfrak{o}.5, \mathfrak{o}.8\rangle, \langle \mathfrak{o}.3, \mathfrak{o}.4\rangle)$
	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.6, \mathfrak{o}.7\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.5\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.7, \mathfrak{o}.6\rangle, \langle \mathfrak{o}.6, \mathfrak{o}.3\rangle)$
c	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.8, \mathfrak{o}.5\rangle, \langle \mathfrak{o}.7, \mathfrak{o}.2\rangle)$
	$\mathfrak{o}.8$	$(\langle \mathfrak{o}.9, \mathfrak{o}.4\rangle, \langle \mathfrak{o}.8, \mathfrak{o}.1\rangle)$
	1	$(\langle \mathfrak{o}.1, \mathfrak{o}.3\rangle, \langle \mathfrak{o}.9, \mathfrak{o}.\mathfrak{o}\rangle)$

Table 8: The discrete LDT2F $\mathring{\mathfrak{G}}$.

For this set $\mathring{\mathfrak{G}}$ *the Certainty Operator* $\left(\Delta\mathring{\mathfrak{G}}\right)$ *is given in table 9.*

The set $\ddot{Y}$	*Primary acceptance* $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$	$\Delta\mathring{\mathfrak{G}}$
a	$\mathfrak{o}.6$	$(\langle \mathfrak{o}.8, \mathfrak{o}.2\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.6\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.4, \mathfrak{o}.6\rangle, \langle \mathfrak{o}.2, \mathfrak{o}.8\rangle)$
	$\mathfrak{o}.9$	$(\langle \mathfrak{o}.6, \mathfrak{o}.4\rangle, \langle \mathfrak{o}.5, \mathfrak{o}.5\rangle)$
b	$\mathfrak{o}.2$	$(\langle \mathfrak{o}.5, \mathfrak{o}.5\rangle, \langle \mathfrak{o}.3, \mathfrak{o}.7\rangle)$
	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.6, \mathfrak{o}.4\rangle, \langle \mathfrak{o}.4, \mathfrak{o}.6\rangle)$
	$\mathfrak{o}.7$	$(\langle \mathfrak{o}.7, \mathfrak{o}.3\rangle, \langle \mathfrak{o}.6, \mathfrak{o}.4\rangle)$
c	$\mathfrak{o}.5$	$(\langle \mathfrak{o}.8, \mathfrak{o}.2\rangle, \langle \mathfrak{o}.7, \mathfrak{o}.3\rangle)$
	$\mathfrak{o}.8$	$(\langle \mathfrak{o}.9, \mathfrak{o}.1\rangle, \langle \mathfrak{o}.8, \mathfrak{o}.2\rangle)$
	1	$(\langle \mathfrak{o}.1, \mathfrak{o}.9\rangle, \langle \mathfrak{o}.9, \mathfrak{o}.1\rangle)$

Table 9: The certainty operator $\Delta\mathring{\mathfrak{G}}$.

For this set $\mathring{\mathfrak{G}}$ the Feasibility Operator $\left(\nabla\mathring{\mathfrak{G}}\right)$ is given in table 10.

The set $\ddot{Y}$	*Primary acceptance $\mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}}$*	$\nabla\mathring{\mathfrak{G}}$
a	0.6	$(\langle 0.3, 0.7\rangle, \langle 0.5, 0.5\rangle)$
	0.7	$(\langle 0.5, 0.5\rangle, \langle 0.7, 0.3\rangle)$
	0.9	$(\langle 0.1, 0.9\rangle, \langle 0.9, 0.1\rangle)$
b	0.2	$(\langle 0.2, 0.8\rangle, \langle 0.6, 0.4\rangle)$
	0.5	$(\langle 0.3, 0.7\rangle, \langle 0.5, 0.5\rangle)$
	0.7	$(\langle 0.4, 0.6\rangle, \langle 0.7, 0.3\rangle)$
c	0.5	$(\langle 0.5, 0.5\rangle, \langle 0.8, 0.2\rangle)$
	0.8	$(\langle 0.6, 0.4\rangle, \langle 0.9, 0.1\rangle)$
	1	$(\langle 0.7, 0.3\rangle, \langle 1.0, 0.0\rangle)$

Table 10: The feasibility operator $\nabla\mathring{\mathfrak{G}}$.

Proposition 6.1. *Here are few characteristics of Certainty and Feasibility operators of LDT2FS. For any LDT2FS $\mathring{\mathfrak{G}}$:*

(i) $\overline{\Delta\overline{\mathring{\mathfrak{G}}}} = \nabla\mathring{\mathfrak{G}}$,

(ii) $\overline{\nabla\overline{\mathring{\mathfrak{G}}}} = \Delta\mathring{\mathfrak{G}}$,

(iii) $\Delta\Delta\mathring{\mathfrak{G}} = \Delta\mathring{\mathfrak{G}}$,

(iv)$\nabla\nabla\mathring{\mathfrak{G}} = \nabla\mathring{\mathfrak{G}}$,

(v) $\Delta\nabla\mathring{\mathfrak{G}} = \nabla\mathring{\mathfrak{G}}$,

(vi) $\nabla\Delta\mathring{\mathfrak{G}} = \Delta\mathring{\mathfrak{G}}$.

Proof. (i) $\overline{\Delta\overline{\mathring{\mathfrak{G}}}} = \nabla\mathring{\mathfrak{G}}$

$$\Delta\Delta\mathring{\mathfrak{G}} = \Delta\overline{\left\{\overline{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]}\right\}}$$

$$\overline{\Delta\overline{\mathring{\mathfrak{G}}}} = \overline{\Delta\left\{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, v_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]\right\}}$$

$$\overline{\Delta\overline{\mathring{\mathfrak{G}}}} = \overline{\left\{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, v_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-v_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]\right\}}$$

$$\overline{\Delta\overline{\mathring{\mathfrak{G}}}} = \left\{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-v_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), v_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]\right\} = \nabla\mathring{\mathfrak{G}}.$$

(ii) $\overline{\nabla\overline{\mathring{\mathfrak{G}}}} = \Delta\mathring{\mathfrak{G}}$

$$\overline{\nabla\overline{\mathring{\mathfrak{G}}}} = \overline{\nabla\overline{\left\{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{v}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{v}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]\right\}}}$$

$$\overline{\nabla\overline{\mathring{\mathfrak{G}}}} = \overline{\nabla\left\{\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{v}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \Big/ \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [0,1]\right\}}$$

$$\overline{\overline{\nabla\mathring{\mathfrak{G}}}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\} = \Delta\mathring{\mathfrak{G}}$$

(iii) $\Delta\Delta\mathring{\mathfrak{G}} = \Delta\mathring{\mathfrak{G}}$

$$\Delta\Delta\mathring{\mathfrak{G}} = \Delta\left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$$\Delta\Delta\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\} = \Delta\mathring{\mathfrak{G}}$$

(iv) $\nabla\nabla\mathring{\mathfrak{G}} = \nabla\mathring{\mathfrak{G}}$

$$\nabla\nabla\mathring{\mathfrak{G}} = \nabla\left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$$\nabla\nabla\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\} = \nabla\mathring{\mathfrak{G}}$$

(v) $\Delta\nabla\mathring{\mathfrak{G}} = \nabla\mathring{\mathfrak{G}}$

$$\Delta\nabla\mathring{\mathfrak{G}} = \Delta\left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$$\Delta\nabla\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-(1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-(1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} \right\}$$

whereas $\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}$, $\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1]$

$$\Delta\nabla\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-\hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$= \nabla\mathring{\mathfrak{G}}$

(vi) $\nabla\Delta\mathring{\mathfrak{G}} = \Delta\mathring{\mathfrak{G}}$

$$\nabla\Delta\mathring{\mathfrak{G}} = \left\{ \nabla\left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$$\nabla\Delta\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, 1-(1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ 1-(1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} \right\}$$

whereas $\widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}$, $\widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1]$

$$\nabla\Delta\mathring{\mathfrak{G}} = \left\{ \left\langle \begin{matrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \\ \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \end{matrix} \right\rangle \diagup \widehat{z\dot{\mathfrak{V}}} : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o},1] \right\}$$

$= \Delta\mathring{\mathfrak{G}}$ □

Theorem 6.1. *Take two LDT2FSs $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$, we have:*

(i) $\Delta(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cup \Delta\mathring{\mathfrak{B}}$

(ii) $\Delta(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cap \Delta\mathring{\mathfrak{B}}$

(iii) $\nabla(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cup \nabla\mathring{\mathfrak{B}}$

(iv) $\nabla(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cap \nabla\mathring{\mathfrak{B}}$

Proof. (i) $\Delta(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cup \Delta\mathring{\mathfrak{B}}$

L.H.S:

$$\Delta\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \Delta\left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle (1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \wedge (1 - \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}))\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \wedge (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\} \cup$$

$$\left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), 1 - \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), 1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\} = \Delta\mathring{\mathfrak{G}} \cup \Delta\mathring{\mathfrak{B}}$$

$\Delta(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cup \Delta\mathring{\mathfrak{B}}$

(ii) $\Delta(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cap \Delta\mathring{\mathfrak{B}}$

L.H.S:

$$\Delta\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \Delta\left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \vee (1 - \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}))\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})) \vee (1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}))\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\}$$

$$\Delta\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1 - \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\} \cap$$

$$\left\{\begin{array}{c} (\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), 1 - \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), 1 - \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1] \end{array}\right\} = \Delta\mathring{\mathfrak{G}} \cap \Delta\mathring{\mathfrak{B}}$$

$\Delta(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \Delta\mathring{\mathfrak{G}} \cap \Delta\mathring{\mathfrak{B}}$

(iii) $\nabla(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cup \nabla\mathring{\mathfrak{B}}$

L.H.S:

$$\nabla\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \nabla\left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\}$$

$$\nabla\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}} \vee \widehat{\widehat{z\mathring{q}}}), \langle \left(1 - \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \vee \left(1 - \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right)\rangle, \\ \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle (1 - \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})) \vee (1 - \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}))\rangle, \\ \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\}$$

$$\nabla\left(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}), \langle 1 - \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle 1 - \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\} \cup$$

$$\left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \langle 1 - \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle 1 - \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\} = \nabla\mathring{\mathfrak{G}} \cup \nabla\mathring{\mathfrak{B}}$$

$\nabla(\mathring{\mathfrak{G}} \cup \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cup \nabla\mathring{\mathfrak{B}}$

(iv) $\nabla(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cap \nabla\mathring{\mathfrak{B}}$

L.H.S:

$$\nabla\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \nabla\left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \wedge \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\}$$

$$\nabla\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}} \wedge \widehat{\widehat{z\mathring{q}}}), \langle \left(1 - \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \wedge \left(1 - \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right)\rangle, \\ \langle \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \\ \langle (1 - \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})) \wedge (1 - \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}))\rangle, \\ \langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \vee \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\rangle : \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\dot{\mathfrak{V}}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\}$$

$$\nabla\left(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}\right) = \left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}), \langle 1 - \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\nu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\rangle, \langle 1 - \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{z\mathring{\mathfrak{F}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\} \cap$$

$$\left\{\begin{array}{c}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \langle 1 - \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \hat{\nu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle, \langle 1 - \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\rangle \\ : \widehat{z\mathfrak{V}} \in \ddot{Y}, \widehat{\widehat{z\mathring{q}}} \in \mathfrak{M}_{\widehat{z\mathfrak{V}}} \subseteq [\mathfrak{o}, 1]\end{array}\right\} = \nabla\mathring{\mathfrak{G}} \cap \nabla\mathring{\mathfrak{B}}$$

$\nabla(\mathring{\mathfrak{G}} \cap \mathring{\mathfrak{B}}) = \nabla\mathring{\mathfrak{G}} \cap \nabla\mathring{\mathfrak{B}}$

By virtue of the above results, we can define the following relations.

Definition 6.1. *For any two LDT2FSs $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$, we define the following relations.*

$$\left[\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}\ iff\ \left(\forall \widehat{z\mathfrak{V}} \in \widehat{z\mathfrak{V}}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\right]$$

$$\left[\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}\ iff\ \left(\forall \widehat{z\mathfrak{V}} \in \widehat{z\mathfrak{V}}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \left\langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\mathfrak{V}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\mathfrak{V}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\right]$$

Theorem 6.2. *For any two LDT2FSs $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$, we have:*

(i) $\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}\ iff\ \ \Delta\mathring{\mathfrak{G}} \subset \Delta\mathring{\mathfrak{B}}$

(ii) $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}\ iff\ \ \nabla\mathring{\mathfrak{G}} \subset \nabla\mathring{\mathfrak{B}}$

(iii) $\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$ *and* $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}\ iff\ \ \mathring{\mathfrak{G}} \subset \mathring{\mathfrak{B}}$

□

Proof. (i) $\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}\ iff\ \ \Delta\mathring{\mathfrak{G}} \subset \Delta\mathring{\mathfrak{B}}$

$\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$ *then* $\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \widehat{z\dot{\mathfrak{V}}}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle$ Therefore,

$$\left\langle 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \left\langle 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle$$

hence

$$\left[\begin{array}{c}\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \\ \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\end{array}\right]$$

Therefore

$\Delta\mathring{\mathfrak{G}} \subset \Delta\mathring{\mathfrak{B}}$

Conversely if $\Delta\mathring{\mathfrak{G}} \subset \Delta\mathring{\mathfrak{B}}$ then

$$\left[\begin{array}{c}\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), 1-\hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right\rangle, \\ \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), 1-\hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq 1-\hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\end{array}\right]$$

which implies that

$\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$

(ii) $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}\ iff\ \ \nabla\mathring{\mathfrak{G}} \subset \nabla\mathring{\mathfrak{B}}$

$\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}$ *then* $\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \widehat{z\dot{\mathfrak{V}}}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle, \left\langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle$ Therefore,

$$\left\langle 1-\upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle, \left\langle 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle$$

hence

$$\left[\begin{array}{c}\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle 1-\upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle, \\ \left\langle 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\end{array}\right]$$

Therefore

$\nabla\mathring{\mathfrak{G}} \subset \nabla\mathring{\mathfrak{B}}$

Conversely if $\nabla\mathring{\mathfrak{G}} \subset \nabla\mathring{\mathfrak{B}}$ then

$$\left[\begin{array}{c}\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right)\widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle 1-\upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle, \\ \left\langle 1-\hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq 1-\hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right\rangle\end{array}\right]$$

which implies that

$\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}$

(iii) $\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$ and $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}\ iff\ \ \mathring{\mathfrak{G}} \subset \mathring{\mathfrak{B}}$

If $\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$ and $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}$ then:

$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle, \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
and
$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle, \left\langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
So,
$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle,$
$\left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
Hence,
$\mathring{\mathfrak{G}} \subset \mathring{\mathfrak{B}}$
Conversely if $\mathring{\mathfrak{G}} \subset \mathring{\mathfrak{B}}$ then
$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle,$
$\left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
i.e.
$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \hat{\mu}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\mu}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle, \left\langle \hat{\gamma}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \leq \hat{\gamma}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
and
$\left(\forall \widehat{z\dot{\mathfrak{V}}} \in \ddot{Y}\right) \widehat{z\mathring{\mathfrak{F}}} \leq \widehat{\widehat{z\mathring{q}}}, \left\langle \upsilon_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \upsilon_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \right\rangle, \left\langle \hat{\delta}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) \geq \hat{\delta}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}}) \right\rangle$
which implies that
$\mathring{\mathfrak{G}} \subset_{\Delta} \mathring{\mathfrak{B}}$ and $\mathring{\mathfrak{G}} \subset_{\nabla} \mathring{\mathfrak{B}}$ □

7. Hamming and Euclidian Distance of *LDT2FS*

Now we propose distance measures of given *LDT2FS*. Suppose $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ are two *LDT2FS* ,the distances are:

(i) Hamming distance:

$$d_H\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = \Sigma\Sigma\Sigma \begin{pmatrix} \left|\hat{\mu}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\mu}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right| \\ + \left|\upsilon^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \upsilon^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right| \\ + \left|\hat{\gamma}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\gamma}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right| \\ + \left|\hat{\delta}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\delta}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right| \end{pmatrix}$$

(i) Euclidean distance:

$$d_E\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = \left[\Sigma\Sigma\Sigma \begin{pmatrix} \left|\hat{\mu}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\mu}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right|^2 \\ + \left|\upsilon^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \upsilon^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right|^2 \\ + \left|\hat{\gamma}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\gamma}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right|^2 \\ + \left|\hat{\delta}^j_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}) - \hat{\delta}^j_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{\widehat{z\mathring{q}}})\right|^2 \end{pmatrix}\right]^{\frac{1}{2}}$$

where $\widehat{z\mathring{\mathfrak{F}}}$ and $\widehat{z\mathring{\mathfrak{F}}}'$ are primary acceptance functions of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$,whereas $\hat{\mu}_{\mathring{\mathfrak{G}}}$ and $\upsilon_{\mathring{\mathfrak{G}}}$ represent secondary acceptance and rejection functions of $\mathring{\mathfrak{G}}$, $\hat{\mu}_{\mathring{\mathfrak{B}}}$ and $\upsilon_{\mathring{\mathfrak{B}}}$ represent secondary acceptance and rejection functions of $\mathring{\mathfrak{B}}$while $\hat{\gamma}_{\mathring{\mathfrak{G}}}$ and $\hat{\delta}_{\mathring{\mathfrak{G}}}$ are reference parameters of $\mathring{\mathfrak{G}}$ and $\hat{\gamma}_{\mathring{\mathfrak{B}}}$ and $\hat{\delta}_{\mathring{\mathfrak{B}}}$ are reference parameters of $\mathring{\mathfrak{B}}$.

$L\left(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right)$ represents the distance of secondary acceptance and rejection functions of:

$$\mathring{\mathfrak{G}} = \begin{pmatrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \left(\hat{\mu}^1_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}^2_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\mu}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\upsilon^1_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \upsilon^2_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \upsilon^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\hat{\gamma}^1_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}^2_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\gamma}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\hat{\delta}^1_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}^2_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\delta}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{G}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \end{pmatrix}$$

and

$$\mathring{\mathfrak{B}} = \begin{pmatrix} \widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}, \left(\hat{\mu}^1_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\mu}^2_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\mu}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\upsilon^1_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \upsilon^2_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \upsilon^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\hat{\gamma}^1_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\gamma}^2_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\gamma}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right), \\ \left(\hat{\delta}^1_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \hat{\delta}^2_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}}), \ldots\ldots, \hat{\delta}^{\widehat{z\mathring{\mathfrak{F}}}}_{\mathring{\mathfrak{B}}}(\widehat{z\dot{\mathfrak{V}}}, \widehat{z\mathring{\mathfrak{F}}})\right) \end{pmatrix}$$

respectively. For our ease we have taken the distance of secondary acceptance and rejection functions and reference parameters of $\mathring{\mathfrak{G}}$ and $\mathring{\mathfrak{B}}$ equal. We now illustrate this concept with an example.

Example 7.1. *Let $\ddot{Y} = \{a, b, c\}$ be a universe and $\mathring{\mathfrak{G}}$,$\mathring{\mathfrak{B}}$ be two LDT2FS on $\ddot{Y}$ represented by in tables 11 and 12.*

The set $\ddot{Y}$	*Primary acceptance $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$*	$\mathring{\mathfrak{G}}$
a	0.7	$(\langle 0.4, 0.5\rangle, \langle 0.2, 0.3\rangle)$
b	0.5	$(\langle 0.6, 0.7\rangle, \langle 0.4, 0.5\rangle)$
c	0.8	$(\langle 0.9, 0.4\rangle, \langle 0.8, 0.1\rangle)$

Table 11: The LDT2FS $\mathring{\mathfrak{G}}$.

and

The set $\ddot{Y}$	*Primary acceptance $\mathfrak{M}_{\widehat{z\mathfrak{V}}}$*	$\mathring{\mathfrak{B}}$
a	0.1	$(\langle 0.3, 0.4\rangle, \langle 0.2, 0.7\rangle)$
b	0.9	$(\langle 0.5, 0.7\rangle, \langle 0.6, 0.4\rangle)$
c	0.7	$(\langle 0.8, 0.3\rangle, \langle 0.6, 0.4\rangle)$

Table 12: The LDT2FS $\mathring{\mathfrak{B}}$.

$d_H\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = |0.4 - 0.3| + |0.5 - 0.4| + |0.2 - 0.2| + |0.3 - 0.7| + |0.6 - 0.5| + |0.7 - 0.7| + |0.4 - 0.6| + |0.5 - 0.4| + |0.9 - 0.8| + |0.4 - 0.3| + |0.8 - 0.6| + |0.1 - 0.4|,$

$d_H\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = |0.1| + |0.1| + +|0.4| + |0.1| + |0.2| + |0.2| + |0.1| + |0.1| + |0.2| + |0.3|$

$d_H\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = 1.8.$

and

$$d_E\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = \begin{bmatrix} |\mathfrak{o}.4 - \mathfrak{o}.3|^2 + |\mathfrak{o}.5 - \mathfrak{o}.4|^2 + |\mathfrak{o}.2 - \mathfrak{o}.2|^2 + \\ |\mathfrak{o}.3 - \mathfrak{o}.7|^2 + |\mathfrak{o}.6 - \mathfrak{o}.5|^2 + |\mathfrak{o}.7 - \mathfrak{o}.7|^2 + \\ |\mathfrak{o}.4 - \mathfrak{o}.6|^2 + |\mathfrak{o}.5 - \mathfrak{o}.4|^2 + |\mathfrak{o}.9 - \mathfrak{o}.8|^2 + \\ |\mathfrak{o}.4 - \mathfrak{o}.3|^2 + |\mathfrak{o}.8 - \mathfrak{o}.6|^2 + |\mathfrak{o}.1 - \mathfrak{o}.4|^2 \end{bmatrix}^{\frac{1}{2}}$$

$$d_E\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = \begin{bmatrix} |\mathfrak{o}.1|^2 + |\mathfrak{o}.1|^2 + |\mathfrak{o}.4|^2 + |\mathfrak{o}.1|^2 + |\mathfrak{o}.2|^2 + \\ |\mathfrak{o}.2|^2 + |\mathfrak{o}.1|^2 + |\mathfrak{o}.1|^2 + |\mathfrak{o}.2|^2 + |\mathfrak{o}.3|^2 \end{bmatrix}^{\frac{1}{2}}$$

$$d_E\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = [\mathfrak{o}.42]^{\frac{1}{2}}$$

$$d_E\left(\mathring{\mathfrak{G}}, \mathring{\mathfrak{B}}\right) = \mathfrak{o}.65.$$

8. An Example

Example 8.1. *Many variables that humans use in reasoning are inherently imprecise, and so fuzzy set theory provides a natural framework for modeling uncertainty. Traditional fuzzy set theory provides a basis for semantic variables, where values are expressed as words rather than numerical values. This is useful, but in situations such as optimization problems simply representing a linguistic variable through a acceptance function is not sufficient. This is because real-world situations often involve elements with both acceptance and rejection grades with reference parameters, leading to uncertainty that conventional fuzzy sets cannot fully capture.*

In this sense, linear diophantine type-2 fuzzy sets $(LDT2FS)$ *provide a more rational and complete representation introducing secondary acceptance and rejection grades. LDT2FS extends the classical fuzzy set approach providing a dual-layer structure in which both the primary and secondary acceptance grades are taken into account for a given element. This extended modeling capability is particularly beneficial in complex decision-making applications.*

A skill Assessment system is used to illustrate this point, evaluating a candidate's skill levels under different degrees of uncertainty. Taking into account both primary and secondary acceptance and rejection values with reference parameters, LDT2FS allows a more precise and flexible representation of the candidate's skill level, leading to better result. This can improve decision support systems, ensuring more reliable results in domains where precise but flexible reasoning under uncertainty is required.

Let $C = C_1, C_2, C_3, C_4$ *be a set of candidates, and let S={Math,Physics,Chemistry} be a set of subjects and L={Cognition ,Ingenuity,Logic} is skill levels set. In this system a candidate's skills and their intensity are represented through primary and secondary acceptance functions along with reference parameters. Table 13 details the skills needed for each subject, including their primary acceptance functions, intensity, and secondary acceptance and rejection functions with reference parameters. Table 14 displays the skill levels of the candidates. Both the subjects and the candidates' skill levels are described using linguistic terms, which naturally involve some uncertainty. To address this uncertainty, the parameters, skill levels, and subjects for each candidate are modeled as LDT2FS. Tables 15 and 16 provide the Hamming and Euclidean distances* *for each candidate in relation to the respective subject. These distances are useful for assessing the similarity or dissimilarity between a candidate's skill levels and the skills required for the subject.*

	Math	*Physics*	*Chemistry*
Cognition	(0.68, 0.08, 0.71, 0.08) / 0.56	(0.60, 0.13, 0.64, 0.15) / 0.49	(0.69, 0.16, 0.62, 0.02) / 0.57
Ingenuity	(0.63, 0.11, 0.63, 0.05) / 0.73	(0.66, 0.18, 0.68, 0.08) / 0.60	(0.69, 0.16, 0.45, 0.03) / 0.41
Logic	(0.48, 0.07, 0.66, 0.18) / 0.77	(0.63, 0.11, 0.59, 0.07) / 0.44	(0.52, 0.03, 0.63, 0.05) / 0.46

Table 13. Subjects vs. Skill Levels

	Cognition	*Ingenuity*	*Logic*
C_1	(0.48, 0.03, 0.61, 0.05) / 0.58	(0.69, 0.16, 0.62, 0.02) / 0.42	(0.58, 0.07, 0.68, 0.08) / 0.55
C_2	(0.73, 0.21, 0.73, 0.04) / 0.23	(0.52, 0.26, 0.62, 0.02) / 0.42	(0.47, 0.06, 0.64, 0.25) / 0.34
C_3	(0.38, 0.05, 0.26, 0.68) / 0.78	(0.30, 0.43, 0.64, 0.25) / 0.72	(0.63, 0.19, 0.91, 0.06,) / 0.61
C_4	(0.91, 0.06, 0.17, 0.80) / 0.61	(0.68, 0.08, 0.71, 0.08) / 0.66	(0.62, 0.21, 0.48, 0.03) / 0.93

Table 14. Candidates vs. Skill Levels

	Math	*Physics*	*Chemistry*
C_1	0.76	0.63	0.74
C_2	0.63	1.01	0.96
C_3	2.88	2.44	3.07
C_4	3.03	1.91	2.4

Table 15. Hamming distance between Candidates and subjects.

	Math	*Physics*	*Chemistry*
C_1	0.0954	00.3657	0.4176
C_2	0.0575	0.4988	0.128
C_3	1.0186	0.8116	1.0991
C_4	1.6017	0.7696	1.0112

Table 16. Euclidian distance between Candidates and subjects.

According to the minimal distance point principle, the least distance in Tables 15 and 16 reflect appropriate skill levels of candidates. Table 15 shows that candidates C_1, C_2, and C_4 are proficient in Physics, while C_2 has specialization in Math. In contrast, Table 16 reveals that C_1 and C_2 excel in Math, while C_3 and C_4 are skilled in Physics. This indicates a potential shift or reevaluation of the candidates' skill classifications, possibly due to a reassessment of their abilities or changes in the data used for evaluation. The inconsistency underscores the need to understand how the minimum distance principle is applied and whether external factors might affect these outcomes.

Conclusion

The paper proposes a set of logical operations that allows Set-Theoretic Operations to Functional Linear Diophantine Type-2 Fuzzy Set to help improve decision making under uncertainty. This study seeks to establish different set-theoretic operations and functional laws towards Set-Theoretic

Operations to Functional Linear Diophantine Type-2 Fuzzy Set ($LDT2FS$) in an orderly manner. It also describes the introduction of two important tools or operators which are the Certainty and Feasibility Operators to facilitate the modification and use of the Type 2 Fuzzy Set. For comparing $LDT2FS$ interpretations, two distance measures are examined, specifically Hamming Distance and Euclidean Distance, which are supported by an actual implementation of a skill assessment system. The research emphasizes that generalized $LDT2FS$ has the potential to be an efficient resource in designing decision support systems in not only intelligent transportation systems but also in medicine and health care. Some notable future research endeavors are incorporation of multi-fuzzy sets and fuzzy soft sets in extending $LDT2FS$, solving its topological aspects, and devising aggregation operators for increasing its efficiency and usefulness in practical decision making scenarios.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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