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Applications of Hurwitz-Lerch Zeta Function to a Certain Class of Bi-Bazilevic and Pseudo-Starlike Functions

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Abstract. In this paper, we introduce a novel class of bi-univalent functions defined using the convolution of the normalized q-analogue of the Hurwitz–Lerch zeta function with the q-Srivastava–Attiya operator on the open unit disk $\mathbb{D}$. Furthermore, this class is connected with bi-Bazilevic functions, pseudo-starlike functions, and the q-analogue of the hyperbolic tangent function. The primary objective is to estimate the initial coefficients in the Taylor expansion of functions belonging to this new class and some of its subclasses. In addition, we investigate the classical Fekete–Szegö functional problem for functions in this novel class. Finally, we present several corollaries that come from particular choices of the parameters defining this class.

1. Introduction

The study of special functions is central to mathematical analysis, number theory, and mathematical physics. Among these, the Hurwitz-Lerch Zeta function stands as a monumental generalization, encompassing the Riemann Zeta function, the Hurwitz Zeta function, and polylogarithms within a single framework.

In recent decades, the field of q-calculus (or quantum calculus) has gained significant traction, for more information see the monograph [19]. By introducing a q-deformation, mathematicians have constructed "q-analogues" of classical functions. The q-analogue of the Hurwitz-Lerch Zeta function is a prime example of this evolution, offering a sophisticated tool that connects classical analytic number theory with the structures of quantum groups, see for example the articles [33], [41], [42] and the references therein.

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The q-analogue of the Hurwitz-Lerch Zeta function, denoted as $\Phi_q(z, \alpha, \lambda)$, is a bridge between two worlds. It connects the smooth, continuous world of classical analysis with the discrete, granular world of quantum calculus. By studying $\Phi_q(z, \alpha, \lambda)$, researchers gain a unified framework that preserves the beauty of classical number theory while equipping modern science with the language needed to describe complex, quantum, and fractal phenomena. The following highlights some of its applications:

- Fractional Calculus: The Hurwitz-Lerch Zeta function is deeply connected to fractional derivatives and integrals. The q-analogue extends this to q-fractional calculus, allowing researchers to model systems with "memory" effects or fractal-like structures where standard continuum calculus fails, for more information see the article [31].
- Physics and Statistical Mechanics: In physics, q-deformations are linked to non-extensive statistical mechanics (Tsallis statistics). While standard Boltzmann-Gibbs statistics apply to systems at equilibrium, complex systems like plasmas or gravitational systems often require q-deformed statistics, for more information see the article [33]. Partition functions in these contexts often take the form of q-series related to $\Phi_q(z, \alpha, \lambda)$.
- Analytic Number Theory: The function provides a testing ground for generalized number theoretic conjectures. The distribution of zeros in q-analogues of Zeta functions is a subject of active research, investigating whether the Riemann Hypothesis has a "quantum" counterpart, for more information see the article [47].

The q-Srivastava-Attiya operator is a sophisticated mathematical tool used primarily in geometric function theory. It serves as the quantum calculus generalization of the classical operator introduced by Srivastava and Attiya [36]. This operator is significant because it acts as a "unifying" operator; by adjusting its parameters, researchers can recover many other famous integral and differential operators as special cases; for more information, see the articles [10], [14], [31], [32], [39], [40], [46] and the references therein.

The evolution of the Hurwitz-Lerch Zeta function from a classical analytic tool to the q-Srivastava-Attiya operator represents a significant milestone in the synthesis of number theory and geometric function theory. By incorporating the q-deformation parameter, the operator provides a more flexible framework for characterizing the behavior of analytic functions within the unit disk, extending our reach beyond the limitations of continuous calculus.

As demonstrated throughout this discussion, the q-Srivastava-Attiya operator serves as a powerful unifying structure. Its ability to reduce to classical integral operators such as those of Bernardi, Libera, and Alexander; under specific parameter constraints highlights its role as a fundamental generalized operator in modern analysis. Furthermore, its recursive relationship with the q-derivative opens new pathways for exploring differential subordination and superordination properties, which are critical for understanding the geometric constraints of complex mappings.

Ultimately, the study of the q-analogue of the Hurwitz-Lerch Zeta function and its derivative operators is not merely a theoretical exercise in q-calculus. It provides the mathematical vocabulary necessary to describe discrete symmetries and quantum structures that are increasingly relevant in mathematical physics and complex dynamical systems. Future research into these operators may further illuminate the deep-seated connections between the distribution of Zeta zeros and the stability of q-deformed analytic functions.

In this article, we have advocated the use of convolution of the Taylor-Macluarin series representation of $f(z)$ and the q-analogue of Hurwitz-Lerch Zeta function to establish our class. Let $\mathcal{H}$ be the collection of all functions $f(z)$ that are analytic in the open unit disk $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. In this context, these functions are subject to the normalization conditions $f(0) = 1 - f'(0) = 0$. The study of such functions contributes significantly to a deeper understanding of complex analysis and its various applications. Moreover, any function f that is a member of the set $\mathcal{H}$ can be written in the following specific form

$$f(\zeta) = z + \sum_{n=2}^{\infty} a_n z^n, \quad \text{where } \ z \in \mathbb{D}. \tag{1.1}$$

It is know that the q-Srivastava-Attiya operator, denoted as $\mathcal{H}_{\alpha}^{q,\lambda}$, transforms this class $\mathcal{H}$ into itself. This allows us to study the geometric properties such as starlikeness and convexity under q-deformation, see for example the article [21].

Let f and g be analytic functions within the open unit disk $\mathbb{D}$. We say that f is subordinated to g in the open unit disk $\mathbb{D}$, denoted as $f(z) \prec g(z)$ for all $z \in \mathbb{D}$, if we can find a Schwarz function w such that $h(0) = 0$ and $|h(z)| < 1$ for every $z \in \mathbb{D}$, fulfilling the condition $f(z) = g(h(z))$ for all $z \in \mathbb{D}$. This relationship between f and g is a crucial concept in complex analysis, offering a means of comparing the behaviors of two analytic functions within the unit disk. Importantly, when the function g is univalent on $\mathbb{D}$, the condition $f(z) \prec g(z)$ is equivalent to $f(0) = g(0)$ and $f(\mathbb{D}) \subset g(\mathbb{D})$. For additional insights and in-depth discussions regarding the Subordination Principle, readers are encouraged to consult the monographs [15], [16], [26], [27] and [34]. These references offer thorough explanations and applications of this principle within the realms of complex analysis and geometric function theory.

The Hadamard product, also referred to as convolution, of two analytic functions $f(z)$ as described in equation (1.1) and $h(z) = z + \sum_{n=2}^{\infty} b_n z^n$ is expressed as follows:

$$(f * h)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n.$$

Moreover, the convolution operation provides a deeper mathematical exploration and enhances our understanding of the geometric and symmetric properties of functions within the space $\mathcal{H}$. Its significance in operator theory and geometric function theory is well-established and thoroughly discussed in the available literature. For those seeking further insights into convolution within geometric function theory, we recommend consulting the monographs [9] and [15], as well as the articles [29], [45], and the associated references therein.

In this study, the notation $\mathcal{S}$ denotes the collection of functions that are univalent within the open unit disk $\mathbb{D}$ and are members of the set $\mathcal{H}$. It is well-known that univalent functions are injective, which implies their invertibility. However, The inverse functions might not be valid across the whole unit disk $\mathbb{D}$. In particular, the Koebe one-quarter Theorem highlights that the image of $\mathbb{D}$ through any function $f \in \mathcal{S}$ includes the disk $D(0, 1/4)$, which is centered at the origin and has a radius of $1/4$. As a result, for every function $f \in \mathcal{S}$, there is an inverse function $f^{-1} = g$ that can be defined as follows

$$g(f(\zeta)) = \zeta, \quad \zeta \in \mathbb{D}$$

$$f(g(\eta)) = \eta, \quad |\eta| < r(f); \;\; r(f) \geq 1/4.$$

Moreover, the inverse function is given by

$$g(\eta) = \eta - a_2\eta^2 + (2a_2^2 - a_3)\eta^3 - (5a_2^3 - 5a_2a_3 + a_4)\eta^4 + \cdots . \tag{1.2}$$

A function $f \in \mathcal{H}$ is called bi-univalent if it maintains univalence on the unit disk $\mathbb{D}$, along with its inverse f^{-1}. Therefore, Σ is identified as the set of all bi-univalent functions within $\mathcal{H}$, as outlined in equation (1.1). The class Σ includes, for example, the following functions:

$$z(1-z)^{-1}, \quad -\log(1-z), \quad \sqrt{\log(1+z) - \log(1-z)}.$$

The Koebe function, $\dfrac{2z - z^2}{2}$ and $\dfrac{z}{1 - z^2}$, are not part of the class Σ. For those interested in learning more about univalent and bi-univalent functions, we recommend checking out the articles [24], [25], [28], as well as the monographs [15] and [18], along with the references included in those works.

Research in geometric function theory reveals complex relationships between function coefficients and their geometric properties. By examining constraints on the modulus of these coefficients, scholars improve their understanding of function behavior within the mathematical framework. This approach not only deepens comprehension of geometric function theory, but also encourages further exploration in the field. For example, in class $\mathcal{S}$, the modulus of the coefficient a_n is limited by the integer n, providing valuable information on the geometric characteristics of these functions. Specifically, restrictions on the second coefficients in class $\mathcal{S}$ yield important information about growth and distortion bounds.

Investigating the coefficient-related characteristics of functions within the bi-univalent class Σ began in the 1970s. A crucial contribution was made by Lewin in 1967 [24], who analyzed the class of bi-univalent functions and established a limit for the coefficient $|a_2|$. Subsequently, in 1969, Netanyahu's research [28] identified that the maximum value of $|a_2|$ is $\frac{4}{3}$ for functions classified under Σ. Additionally, Brannan and Clunie, in 1979 [11], proved that for functions in this category, the inequality $|a_2| \leq \sqrt{2}$ is valid. This research has established a foundation for studies on coefficient bounds of bi-univalent function subclasses. However, there remains a significant gap in understanding the general coefficients $|a_2|$ for $n \geq 4$. Estimating these coefficients, particularly

$|a_n|$, remains unresolved in geometric function theory, indicating the complexity of bi-univalent functions and the need for further investigation into their behavior in higher dimensions.

In 1933, Fekete and Szegö advanced the study of univalent functions by establishing the maximum value of $|a_3 - \lambda a_2^2|$ for λ between 0 and 1. This led to the Fekete-Szegö problem, which aims to maximize the functional $\Psi_\lambda(f) = a_3 - \lambda a_2^2$ for functions in the class $\mathcal{H}$, with λ as any complex number. The Fekete-Szegö functional and its related coefficient estimations have since attracted considerable attention from numerous researchers in the field. Notable contributions can be found in articles such as [2], [3], [4], [5], [12], [13], [17], [20], [22], [25], [44], along with the references cited therein. These investigations have significantly enhanced the comprehension of the Fekete-Szegö problem and its relevance within the domain of geometric function theory.

2. Preliminaries and Lemmas

The information presented in this section is essential to understand the key conclusions of this article. The classical Hurwitz-Lerch Zeta function, denoted as $\Phi(z, \alpha, \Lambda)$, is defined by the series:

$$\Phi(z, \alpha, \lambda) = \sum_{n=0}^{\infty} \frac{z^n}{(n+\lambda)^\alpha} \tag{2.1}$$

Here, $z, \alpha \in \mathbb{C}$, and $\lambda \in \mathbb{C} \setminus \{0, -1, -2, \dots\}$. As noted by Lerch [23], this series converges for $|z| < 1$.

To understand the q-analogue of Hurwitz-Lerch Zeta function, one must first appreciate the classical definitions and the mechanism of q-deformation. In q-calculus, we replace standard integers with q-integers $[n]_q$:

$$[n]_q = \frac{1-q^n}{1-q} \tag{2.2}$$

As $q \to 1^-$, we recover the classical integer: $\lim_{q \to 1^-} [n]_q = n$. This substitution serves as the basis for modern q-analysis, see for example [19].

Moreover, The standard definition of the q-analogue of Hurwitz-Lerch Zeta function, $\Phi_q(z, \alpha, \lambda)$, replaces the linear term $(n + a)$ with the q-analogue $[n + a]_q$. Following the family of functions generalized by Srivastava [35], the definition is given by:

$$\Phi_q(z, s, a) = \sum_{n=0}^{\infty} \frac{z^n}{([n+\lambda]_q)^\alpha} = \sum_{n=0}^{\infty} \frac{(1-q)^\alpha z^n}{(1-q^{n+\lambda})^\alpha} \tag{2.3}$$

Where $|z| < 1$ and $|q| < 1$. This structure preserves the fundamental essence of the classical function while introducing geometric progressions.

In addition, just as the classical Hurwitz-Lerch Zeta function reduces to the Riemann Zeta function, its q-analogue reduces to fundamental q-series which provides a unified framework for quantum analysis, for more information see the recent studies of meromorphic functions [30] and [38]. The following are the q-special cases:

- q-Riemann Zeta Function: Setting $z = 1$ and $\lambda = 1$, we recover the q-analogue of the Riemann Zeta function, $\zeta_q(\alpha)$:

$$\zeta_q(\alpha) = \sum_{n=1}^{\infty} \frac{1}{([n]_q)^\alpha} \tag{2.4}$$

- q-Polylogarithm: Setting $\lambda = 1$, we obtain the q-polylogarithm function:

$$\mathrm{Li}_{q,\alpha}(z) = \sum_{n=1}^{\infty} \frac{z^n}{([n]_q)^\alpha} \tag{2.5}$$

Integral representations of these q-analogues further clarify their analytic continuation to the complex plane, see for example the article [47].

Furthermore, unlike classical functions which satisfy differential equations, q-functions satisfy q-difference equations. The operator D_q replaces the standard derivative:

$$D_q f(z) = \frac{f(z) - f(qz)}{(1-q)z}. \tag{2.6}$$

The function $\Phi_q(z, \alpha, \lambda)$ satisfies specific recurrence relations involving the shift $\lambda \to \lambda + 1$, which mimic the derivative properties of the classical function but in a discrete-quantum grid format, for more information see the monograph [19].

On the other hand, we define the q−Sirvastava-Attiya operator as follows. This operator, denoted as $\mathcal{H}^\alpha_{q,\lambda}$, is defined using the Hadamard product (or convolution) of the function $f(z)$ with a specific helper function $G_{q,\alpha,\lambda}(z)$:

$$\mathcal{H}^\alpha_{q,\lambda} f(z) = G_{q,\alpha,\lambda}(z) * f(z). \tag{2.7}$$

As detailed by Srivastava [37], this helper function is derived from the q-Hurwitz-Lerch Zeta function (Φ_q):

$$G_{q,\alpha,\lambda}(z) = (1+\lambda)^\alpha \left[\Phi_q(z, \alpha, \lambda) - b^{-\alpha}\right]$$

Now, for practical computation in the geometric function theory, the series representation is preferred as follows. For a function given by equation (1.1), we define the operator $\mathcal{H}^\alpha_{q,\lambda}$ as:

$$\mathcal{H}^\alpha_{q,\lambda} f(z) = z + \sum_{n=2}^{\infty} \left(\frac{[1+\lambda]_q}{[n+\lambda]_q}\right)^\alpha a_n z^n, \tag{2.8}$$

where $[k]_q = \frac{1-q^k}{1-q}$ denotes the q-number.

Moreover, this q-operator satisfies the following recursive relationship involving the q-derivative operator D_q, which is essential to characterize subordination results:

$$[1+\lambda]_q \mathcal{H}^q_{\alpha,\lambda} f(z) = z D_q \left(\mathcal{H}^q_{\alpha+1,\lambda} f(z)\right) + q[\lambda]_q \mathcal{H}^q_{\alpha+1,\lambda} f(z). \tag{2.9}$$

The importance of the operator $\mathcal{H}^\alpha_{q,\lambda}$ lies in its versatility. The following table summarizes how classical and q-operators are nested within this framework:

- If $q \to 1$, we get the Srivastava-Attiya.

- If $\alpha = 1$, we get the q-Bernardi integral operator.
- If $\alpha = 1$ and $\lambda = 0$, we get the q-Alexander integral operator.
- If $\alpha = 1$ and $\lambda = 1$, we get q-Libera integral operator.
- If $\alpha = -1$, we get the q-Differential operator.

Now, we present a novel class comprising bi-Bazilevic and pseudo-starlike functions characterized by q- Hurwitz-Lerch Zeta function and the q-Srivastava-Attiya operator. Also, this class is associated with the q-analogue of the hyperbolic tangent function, which we denote as $\mathcal{BS}(\mu, \gamma, \beta, \alpha, \lambda, q)$. The formal definition of this class is provided below.

Definition 2.1. *A function* $f(z)$ *belonging to the family* Σ *is considered to be part of the class* $\mathcal{BS}(\mu, \gamma, \beta, \alpha, \lambda, q)$ *if it satisfies the following subordination conditions:*

$$(1-\mu)\left(\frac{z^{1-\gamma}\left(\mathcal{H}_{q,\lambda}^{\alpha} f(z)\right)'}{\left(\mathcal{H}_{q,\lambda}^{\alpha} f(z)\right)^{1-\gamma}}\right) + \mu\left(\frac{z\left(\left(\mathcal{H}_{q,\lambda}^{\alpha} f(z)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha} f(z)}\right) \prec 1 + \tanh(qz)$$

and

$$(1-\mu)\left(\frac{w^{1-\gamma}\left(\mathcal{H}_{q,\lambda}^{\alpha} g(w)\right)'}{\left(\mathcal{H}_{q,\lambda}^{\alpha} g(w)\right)^{1-\gamma}}\right) + \mu\left(\frac{w\left(\left(\mathcal{H}_{q,\lambda}^{\alpha} g(w)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha} g(w)}\right) \prec 1 + \tanh(qw),$$

where the function $g(w) = f^{-1}(w)$ *is given by the Equation (1.2), the parameters* $0 \le \mu \le 1$, $\gamma \ge 0$, $\beta \ge 1$, $0 < q < 1$, $\lambda \in \mathbb{C}$ *with* $\lambda \ne 0, -1, -2, -3, \cdots$, $\alpha \in \mathbb{C}$ *when* $|z| < 1$ *and* $\Re(\alpha) > 1$ *when* $|z| = 1$.

The lemmas presented below are extensively documented in the literature (see, for example, [22]) and are regarded a fundamental principle that contributes significantly to the research we are conducting.

Lemma 2.1. *Let* $A, B \in \mathbb{R}$ *and* $z, w \in \mathbb{C}$. *If* $|z| < r$ *and* $|w| < r$,

$$\left|(N+M)z + (N-M)w\right| \le \begin{cases} 2|N|r, & \text{if } |N| \ge |M| \\ 2|M|r, & \text{if } |N| \le |M| \end{cases}$$

Lemma 2.2. *Let* $p(z)$ *be a function of the Caratheodory class* $\mathcal{P}$. *Then for any* $z \in \mathbb{D}$ *the function* p *can be written as* $p(z) = 1 + \sum_{n=1}^{\infty} p_n z^n$. *Moreover,* $|p_n| \le 2$ *for each natural number* n. *In addition, for any complex number* ζ, *we have*

$$|p_2 - \zeta p_1^2| \le 2\max\{1, |2\zeta - 1|\}.$$

The purpose of this article is to explore a novel class of bi-Bazilevic and pseudo-starlike functions characterized by q- Hurwitz-Lerch Zeta function and the q-Srivastava-Attiya operator, which is associated with the q-analogue of the hyperbolic tangent function. The main goal is to obtain estimates for the moduli of the initial coefficients in the Taylor series expansion of functions within this category. Additionally, the paper delves into the Fekete-Szegö functional problem related to this class of functions, which improves our understanding of their essential properties.

3. Coefficient estimates

In this section, we find the bounds for the moduli of the initial coefficients of functions belonging to the class $\mathcal{BS}(\mu,\gamma,\beta,\alpha,\lambda,q)$, as indicated in equation (1.1). Furthermore, we seek to determine the coefficient bounds for several subclasses that fall under our established class. Additionally, we aim to establish the coefficient bounds for some of the subclasses within our defined class.

Theorem 3.1. *Let a function f be a bi-univalent function belonging to the class $\mathcal{BS}(\mu,\gamma,\beta,\alpha,\lambda,q)$ and is represented by equation (1.1). The following inequalities hold:*

$$|a_2| \le \frac{\sqrt{2}q[2+\lambda]_q^{\alpha}[3+\lambda]_q^{\alpha/2}}{\sqrt{2qB[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}+(qK+A^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{2\alpha}}}, \tag{3.1}$$

and

$$|a_3| \le \frac{2q^2[2+\lambda]_q^{2\alpha}}{A^2[1+\lambda]_q^{2\alpha}}+\frac{q[3+\lambda]_q^{\alpha}}{4B[1+\lambda]_q^{\alpha}} \tag{3.2}$$

where

$$A=(1-\mu)(1+\gamma)+\mu(2\beta-1),\quad B=(1-\mu)(2+\gamma)+\mu(3\beta-1)$$
$$\textit{and}\quad K=(1-\mu)(2+\gamma)(\gamma-1)+2\mu(2\beta^2-4\beta+1).$$

Proof. Let f be a function that is part of the class $\mathcal{BS}(\mu,\gamma,\beta,\alpha,\lambda,q)$. Based on Definition 2.1 and the Subordination Principle, we can find two Schwarz functions, $u(z)$ and $\eta(w)$, that are defined within the open unit disk $\mathbb{D}$ and satisfied the following equations

$$(1-\mu)\left(\frac{z^{1-\gamma}\left(\mathcal{H}_{q,\lambda}^{\alpha}f(z)\right)'}{\left(\mathcal{H}_{q,\lambda}^{\alpha}f(z)\right)^{1-\gamma}}\right)+\mu\left(\frac{z\left(\left(\mathcal{H}_{q,\lambda}^{\alpha}f(z)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha}f(z)}\right)=1+\tanh(qu(z)), \tag{3.3}$$

and

$$(1-\mu)\left(\frac{w^{1-\gamma}\left(\mathcal{H}_{q,\lambda}^{\alpha}g(w)\right)'}{\left(\mathcal{H}_{q,\lambda}^{\alpha}g(w)\right)^{1-\gamma}}\right)+\mu\left(\frac{w\left(\left(\mathcal{H}_{q,\lambda}^{\alpha}g(w)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha}g(w)}\right)=1+\tanh(q\eta(w)). \tag{3.4}$$

Now, using these Schwarz functions, we define two analytic functions $p(z)$ and $h(w)$ as follow:

$$p(z)=\frac{1+u(z)}{1-u(z)}\quad\text{and}\quad h(w)=\frac{1+\eta(w)}{1-\eta(w)}.$$

It is obvious that the functions $p(z)$ and $h(w)$ are analytic within the open unit disk $\mathbb{D}$ and are classified under the Caratheodory class. Therefore, we can express them in the following manner:

$$p(z)=\frac{1+u(z)}{1-u(z)}=1+p_1z+p_2z^2+pz^3+\cdots$$

and

$$h(w)=\frac{1+\eta(w)}{1-\eta(w)}=1+h_1w+h_2w^2+h_3w^3+\cdots$$

Moreover, $p(0) = 1 = h(0)$, they have positive real parts, $|p_i| \leq 2$ and $|h_i| \leq 2$ for all $i \in \mathbb{N}$.

Equivalently, we get the following representations of $u(z)$ and $\eta(w)$:

$$\begin{aligned} u(z) = \frac{p(z)-1}{p(z)+1} &= \frac{p_1 z + p_2 z^2 + p_3 z^3 + \cdots}{2 + p1z + p_2 z^2 + p_3 z^3 + \cdots} \\ &= \frac{p_1}{2} z + \left(\frac{p_2}{2} - \frac{p_1^2}{4}\right) z^2 + \left(\frac{p_3}{2} - \frac{p_1 p_2}{2} + \frac{p_1^3}{8}\right) z^3 + \cdots, \end{aligned} \tag{3.5}$$

and

$$\begin{aligned} \eta(w) = \frac{h(w)-1}{h(w)+1} &= \frac{h_1 w + h_2 w^2 + h_3 w^3 + \cdots}{2 + h_1 w + h_2 w^2 + h_3 w^3 + \cdots} \\ &= \frac{h_1}{2} w + \left(\frac{h_2}{2} - \frac{h_1^2}{4}\right) w^2 + \left(\frac{h_3}{2} - \frac{h_1 h_2}{2} + \frac{h_1^3}{8}\right) w^3 + \cdots, \end{aligned} \tag{3.6}$$

Hence, considering equation (3.3) and equation (3.4), then comparing the coefficients on both-sides, we get the following four equations:

$$2((1-\mu)(1+\gamma) + \mu(2\beta-1))[1+\lambda]_q^{\alpha} a_2 = q p_1 [2+\lambda]_q^{\alpha}, \tag{3.7}$$

$$\begin{aligned} &4(1-\mu)(2+\gamma) + \mu(3\beta-1)[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha} a_3 \\ &+ 2((1-\mu)(2+\gamma)(\gamma-1) + 2\mu(2\beta^2-4\beta+1))[3+\lambda]_q^{\alpha}[1+\lambda]_q^{2\alpha} a_2^2 \\ &= q(2p_2 - p_1^2)[3+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha} \end{aligned} \tag{3.8}$$

$$-2((1-\mu)(1+\gamma) + \mu(2\beta-1))[1+\lambda]_q^{\alpha} a_2 = q h_1 [2+\lambda]_q^{\alpha} \tag{3.9}$$

and

$$\begin{aligned} &4(1-\mu)(2+\gamma) + \mu(3\beta-1)[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}(2a_2^2 - a_3) \\ &+ 2((1-\mu)(2+\gamma)(\gamma-1) + 2\mu(2\beta^2-4\beta+1))[3+\lambda]_q^{\alpha}[1+\lambda]_q^{2\alpha} a_2^2 \\ &= q(2h_2 - h_1^2)[3+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}. \end{aligned} \tag{3.10}$$

In the following argument we set

$$A = (1-\mu)(1+\gamma) + \mu(2\beta-1), B = (1-\mu)(2+\gamma) + \mu(3\beta-1) \quad \text{and}$$

$$K = (1-\mu)(2+\gamma)(\gamma-1) + 2\mu(2\beta^2 - 4\beta + 1).$$

Now, on one hand, using equation (3.7) and equation (3.9) we get the following equation

$$4A^2[1+\lambda]_q^{2\alpha} a_2^2 = q^2[2+\lambda]_q^{2\alpha}(p_1^2 + h_1^2). \tag{3.11}$$

On the other hand, adding equation (3.8) to equation (3.10), we obtain the following equation

$$\begin{aligned} &4B[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha} a_2^2 + 2K[1+\lambda]_q^{2\alpha}[3+\lambda]_q^{\alpha} a_2^2 \\ &= q(p_2 + h_2)[3+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha} - \frac{q}{2}(p_1^2 + h_1^2)[3+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}. \end{aligned} \tag{3.12}$$

Now, using equations (3.11) and (3.12), we obtain the following equation

$$a_2^2 = \frac{q^2(p_2+h_2)[3+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}}{4qB[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}+(2qK+2A^2)[1+\lambda]_q^{2\alpha}[3+\lambda]_q^{\alpha}}. \tag{3.13}$$

Therefore, using constraints $|h_2| \leq 2$ and $|p_2| \leq 2$, equation (3.13) becomes

$$|a_2| \leq \frac{2q[3+\lambda]_q^{\alpha/2}[2+\lambda]_q^{\alpha}}{\sqrt{4qB[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}+(2qK+2A^2)[1+\lambda]_q^{2\alpha}[3+\lambda]_q^{\alpha}}},$$

which gives the desired inequality (3.1).

Finally, our main objective is to find a sharp bound for $|a_3|$. Subtracting equation (3.10) from equation (3.8), we get the following equation:

$$4B[1+\lambda]_q^{\alpha}(a_3-a_2^2) = q\left((p_2-h_2)+\left(\frac{h_1^2-p_1^2}{2}\right)\right)[3+\lambda]_q^{\alpha}.$$

In the light of equations (3.7) and (3.9), we have $h_1^2 = p_1^2$. Hence, the last equation becomes:

$$4B[1+\lambda]_q^{\alpha}a_3 = 4B[1+\lambda]_q^{\alpha}a_2^2 + q(p_2-h_2)[3+\lambda]_q^{\alpha}. \tag{3.14}$$

Moreover, calling in equation (3.11), the last equation can be written as follows:

$$a_3 = \frac{q^2\left(p_1^2+h_1^2\right)[2+\lambda]_q^{2\alpha}}{4A^2[1+\lambda]_q^{2\alpha}} + \frac{q\,(p_2-h_2)\,[3+\lambda]_q^{\alpha}}{4B[1+\lambda]_q^{\alpha}}. \tag{3.15}$$

Therefore, using the conditions $|p_i| \leq 2$ and $|h_i| \leq 2$ for $i \in \mathbb{N}$, we get the desired inequality for $|a_3|$. This completes the proof. □

In this article, the parameters used play crucial roles in categorizing our class $\mathcal{BS}(\mu,\gamma,\beta,\alpha,\lambda,q)$. For example, the choice of the parameters α, μ and γ leads us to identify distinct subclasses based on their values.

Example 3.1. *Let the function f be in the class Σ and represented by equation (1.1). We say f belongs to the subclass $\mathcal{BS}_1(\gamma,\beta,\alpha,\lambda,q)$ if the following conditions hold:*

$$\frac{z\left(\left(\mathcal{H}_{q,\lambda}^{\alpha}f(z)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha}f(z)} \prec 1+\tanh(qz)$$

and

$$\frac{w\left(\left(\mathcal{H}_{q,\lambda}^{\alpha}g(w)\right)'\right)^{\beta}}{\mathcal{H}_{q,\lambda}^{\alpha}g(w)} \prec 1+\tanh(qw),$$

where the function $g(w) = f^{-1}(w)$ is given by equation (1.2), the parameters $\gamma \geq 0$, $\beta \geq 1$, $0 < q < 1$, $\lambda \in \mathbb{C}$ with $\lambda \neq 0,-1,-2,-3,\cdots$, $\alpha \in \mathbb{C}$ when $|z| < 1$ and $\Re(\alpha) > 1$ when $|z| = 1$.

The following class $\mathcal{BS}^*(\gamma,q)$ has investigated by Al-Rawashdeh [6] and [7].

Example 3.2. *Let the function f be in the class Σ and represented by equation (1.1). We say f belongs to the subclass $\mathcal{BS}^*(\gamma, q)$ if the following conditions hold:*

$$\frac{z^{1-\gamma}f'(z)}{\left(f(z)\right)^{1-\gamma}} \prec 1+\tanh(qz), \tag{3.16}$$

and

$$\frac{w^{1-\gamma}g'(w)}{\left(g(w)\right)^{1-\gamma}} \prec 1+\tanh(qw), \tag{3.17}$$

where the function $g(w) = f^{-1}(w)$ is given by equation (1.2), the parameters $\gamma \geq 0$, $0 < q < 1$ and $\lambda \in \mathbb{C}$ with $\lambda \neq 0, -1, -2, -3, \cdots$.

The following Class $\mathcal{B}^*\mathcal{S}^*(q)$ has investigated by Srivastava et al [43].

Example 3.3. *Let the function f be in the class Σ and represented by equation (1.1). We say f belongs to the subclass $\mathcal{B}^*\mathcal{S}^*(q)$ if the following conditions hold:*

$$\frac{z\left(\mathcal{H}_{q,\lambda}f(z)\right)'}{\mathcal{H}_{q,\lambda}f(z)} \prec 1+\tanh(qz), \tag{3.18}$$

and

$$\frac{w\left(\mathcal{H}_{q,\lambda}g(w)\right)'}{\mathcal{H}_{q,\lambda}g(w)} \prec 1+\tanh(qw), \tag{3.19}$$

where the function $g(w) = f^{-1}(w)$ is given by equation (1.2), the parameters $0 < q < 1$ and $\lambda \in \mathbb{C}$ with $\lambda \neq 0, -1, -2, -3, \cdots$.

The following subclass $\mathcal{BS}_1^*(q)$ has investigated by Al-Amoush [1] and Al-Rawashdeh [8].

Example 3.4. *Let the function f be in the class Σ and represented by equation (1.1). We say f belongs to the subclass $\mathcal{BS}_1^*(q)$ if the following conditions hold:*

$$\left(\mathcal{H}_{q,\lambda}f(z)\right)' \prec 1+\tanh(qz), \tag{3.20}$$

and

$$\left(\mathcal{H}_{q,\lambda}g(w)\right)' \prec 1+\tanh(qw), \tag{3.21}$$

where the function $g(w) = f^{-1}(w)$ is given by equation (1.2), the parameters $0 < q < 1$ and $\lambda \in \mathbb{C}$ with $\lambda \neq 0, -1, -2, -3, \cdots$.

The following corollaries are derived directly from Theorem 3.1 and are associated with previous Examples. The techniques used to prove these corollaries are quite similar to those used in the proof of Theorem 3.1, which is why we have chosen to leave out the detailed proofs.

Corollary 3.1. *Let a function f be a bi-univalent function belonging to the class $\mathcal{BS}_1(\gamma, \beta, \alpha, \lambda, q)$ and is represented by equation (1.1). The following inequalities hold:*

$$|a_2| \leq \frac{\sqrt{2}q[2+\lambda]_q^{\alpha}[3+\lambda]_q^{\alpha/2}}{\sqrt{2q(3\beta-1)[1+\lambda]_q^{\alpha}[2+\lambda]_q^{2\alpha}+(2q(2\beta^2-4\beta+1)+(2\beta-1)^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{2\alpha}}},$$

and

$$|a_3| \leq \frac{2q^2[2+\lambda]_q^{2\alpha}}{(2\beta-1)^2[1+\lambda]_q^{2\alpha}} + \frac{q[3+\lambda]_q^{\alpha}}{4(3\beta-1)[1+\lambda]_q^{\alpha}}.$$

Corollary 3.2. *If a function $f \in \Sigma$ is represented by equation (1.1) and belongs to the subclass $\mathcal{BS}^*(\gamma, q)$, then the following hold*

$$|a_2| \leq \frac{\sqrt{2}q}{\sqrt{q(\gamma^2+3\gamma+2)+(\gamma+1)^2}},$$

and

$$|a_3| \leq \frac{2q^2}{(1+\gamma)^2} + \frac{q}{4(2+\gamma)}.$$

Corollary 3.3. *If a function $f \in \Sigma$ is represented by equation (1.1) and belongs to the subclass $\mathcal{B}^*\mathcal{S}^*(q)$, then the following hold*

$$|a_2| \leq \frac{\sqrt{2}q}{\sqrt{2q+1}},$$

and

$$|a_3| \leq \frac{16q^2+q}{8}.$$

Corollary 3.4. *If a function $f \in \Sigma$ is represented by equation (1.1) and belongs to the subclass $\mathcal{BS}_1^*(q)$, then the following hold*

$$|a_2| \leq \frac{q}{\sqrt{3q+2}},$$

and

$$|a_3| \leq \frac{6q^2+q}{12}.$$

4. FEKETE-SZEGÖ INEQUALITIES FOR THE CLASS $\mathcal{BS}(\mu, \gamma, \beta, \alpha, \lambda, q)$

In this section, we focus on the advancement of Fekete-Szegö inequalities for functions that are part of the class $\mathcal{BS}(\mu, \gamma, \beta, \alpha, \lambda, q)$. This class of bi-Bazilevic and pseudo-starlike functions is defined using the q- Hurwitz-Lerch Zeta function and the q-Srivastava-Attiya operator, which is associated with the q-analogue of the hyperbolic tangent function. Additionally, we aim to establish Fekete-Szegö inequalities for several subclasses that are included within the boundaries of our defined class.

Theorem 4.1. *Let a function f be a bi-univalent function that belongs to the class $\mathcal{BS}(\mu, \gamma, \beta, \alpha, \lambda, q)$ and is represented by equation (1.1), then for a real number ζ the following inequality holds*

$$|a_3 - \zeta a_2^2| \leq \begin{cases} \frac{q[3+\lambda]_q^{\alpha}}{[1+\lambda]_q^{\alpha}((1-\mu)(2+\gamma)+\mu(3\beta-1))} & if \quad \zeta \in [-\psi, 2+\psi] \\ \frac{4q[3+\lambda]_q^{\alpha}\Delta(\zeta,\lambda,\alpha,q)}{[1+\lambda]_q^{\alpha}} & if \quad \zeta \notin [-\psi, 2+\psi], \end{cases} \tag{4.1}$$

where

$$\Delta(\zeta, \lambda, \alpha, q) = \frac{q(1-\zeta)[2+\lambda]_q^{2\alpha}}{4qB[2+\lambda]_q^{2\alpha} + (2qK + 2A^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{\alpha}}$$

$$\psi = \frac{(qK + A^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{\alpha}}{2qB[2+\lambda]_q^{2\alpha}}$$

$$A = (1-\mu)(1+\gamma) + \mu(2\beta - 1), \quad B = (1-\mu)(2+\gamma) + \mu(3\beta - 1)$$

$$and \quad K = (1-\mu)(2+\gamma)(\gamma - 1) + 2\mu(2\beta^2 - 4\beta + 1).$$

Proof. Invoking equation (3.14), we arrive at the subsequent equation, which can be expressed as follows

$$a_3 = \frac{q(p_2 - h_2)[3+\lambda]_q^{\alpha}}{4B[1+\lambda]_q^{\alpha}} + a_2^2.$$

So, for any real number ζ, we can write the last equation as follows

$$a_3 - \zeta_2^2 = \frac{q(p_2 - h_2)[3+\lambda]_q^{\alpha}}{4B[1+\lambda]_q^{\alpha}} + (1-\zeta)a_2^2.$$

Thus, substituting a_2^2 from equation (3.13), the last equation can be written as follows:

$$a_3 - \zeta a_2^2 = \frac{q[3+\lambda]_q^{\alpha}}{[1+\lambda]_q^{\alpha}} \left\{ \begin{array}{l} \left(\Delta(\zeta, \lambda, \alpha, q) + \frac{1}{4((1-\mu)(2+\gamma)+\mu(3\beta-1))} \right) p_2 \\ + \left(\Delta(\zeta, \lambda, \alpha, q) - \frac{1}{4((1-\mu)(2+\gamma)+\mu(3\beta-1))} \right) h_2 \end{array} \right\}.$$

Applying the Lemma 2.1 to the last equation, we obtain the following inequality. Specifically, substituting the expressions and conditions provided by the lemma into the equation allows us to appropriately bound the relevant terms. Rearranging and simplifying these terms yields the following inequality.

$$|a_3 - \zeta a_2^2| \leq \begin{cases} \frac{q[3+\lambda]_q^{\alpha}}{[1+\lambda]_q^{\alpha}B} & \text{if} \quad |\Delta(\zeta, \lambda, \alpha, q)| \leq \frac{1}{4|B|} \\ \frac{4q[3+\lambda]_q^{\alpha}\Delta(\zeta,\lambda,\alpha,q)}{[1+\lambda]_q^{\alpha}} & \text{if} \quad |\Delta(\zeta, \lambda, \alpha, q)| \geq \frac{1}{4|B|}. \end{cases} \tag{4.2}$$

On the other hand, consider the inequality $|\Delta(\zeta, \lambda, \alpha, q)| \leq \frac{1}{4|B|}$. Then simple calculations give us the following inequality

$$|1 - \zeta| \leq 1 + \frac{(qK + A^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{\alpha}}{2qB[2+\lambda]_q^{2\alpha}}.$$

Taking

$$\psi = \frac{(qK + A^2)[3+\lambda]_q^{\alpha}[1+\lambda]_q^{\alpha}}{2qB[2+\lambda]_q^{2\alpha}}$$

the last inequality can be written as

$$-\psi \leq \zeta \leq 2+\psi. \tag{4.3}$$

Finally, substituting inequality (4.3) in inequality (4.2),we get the desired inequality (4.1). This completes the proof. □

The following corollaries are directly derived from Theorem 4.1. Their proofs proceed along essentially the same lines as that of the preceding theorem, relying on analogous arguments and techniques. For the sake of brevity, we therefore omit the detailed proofs.

Corollary 4.1. *Let a function $f \in \Sigma$ be represented by equation (1.1) and be part of the subclass $\mathcal{BS}_1(\gamma,\beta,\alpha,\lambda,q)$, then for a real number ζ the following inequality holds*

$$|a_3-\zeta a_2^2| \leq \begin{cases} \frac{q[3+\lambda]_q^\alpha}{[1+\lambda]_q^\alpha(3\beta-1)} & \text{if} \quad \zeta \in [-\psi^*, 2+\psi^*] \\ \frac{4q[3+\lambda]_q^\alpha \Delta^*(\zeta,\lambda,\alpha,q)}{[1+\lambda]_q^\alpha} & \text{if} \quad \zeta \notin [-\psi^*, 2+\psi^*], \end{cases}$$

where

$$\Delta^*(\zeta,\lambda,\alpha,q) = \frac{q(1-\zeta)[2+\lambda]_q^{2\alpha}}{4q(3\beta-1)[2+\lambda]_q^{2\alpha}+(4q(2\beta^2-4\beta+1)+2(2\beta-1)^2)[3+\lambda]_q^\alpha[1+\lambda]_q^\alpha}$$

$$\psi^* = \frac{(2q(2\beta^2-4\beta+1)+(2\beta-1)^2)[3+\lambda]_q^\alpha[1+\lambda]_q^\alpha}{2q(3\beta-1)[2+\lambda]_q^{2\alpha}}.$$

Corollary 4.2. *Let a function $f \in \Sigma$ be represented by equation (1.1) and be part of the subclass $\mathcal{BS}^*(\gamma,q)$, then for a real number ζ the following inequality holds*

$$|a_3-\zeta a_2^2| \leq \begin{cases} \frac{q}{\gamma+2} & \text{if} \quad \zeta \in [-\psi_1, 2+\psi_1] \\ \frac{2q^2(1-\zeta)}{(q+1)\gamma^2+(3q+2)\gamma+(2q-1)} & \text{if} \quad \zeta \notin [-\psi_1, 2+\psi_1], \end{cases}$$

where

$$\psi_1 = \frac{q(\gamma+2)(\gamma-1)+(1+\gamma)^2}{2q(\gamma+2)}.$$

Corollary 4.3. *Let a function $f \in \Sigma$ be represented by equation (1.1) and be part of the subclass $\mathcal{B}^*\mathcal{S}^*(q)$, then for a real number ζ the following inequality holds*

$$|a_3-\zeta a_2^2| \leq \begin{cases} \frac{q}{2} & \text{if} \quad \zeta \in [\frac{2q-1}{4q}, \frac{6q+1}{4q}] \\ \frac{2q^2(1-\zeta)}{2q-1} & \text{if} \quad \zeta \notin [\frac{2q-1}{4q}, \frac{6q+1}{4q}]. \end{cases}$$

Corollary 4.4. *Let a function $f \in \Sigma$ be represented by equation (1.1) and be part of the subclass $\mathcal{BS}_1^*(q)$, then for a real number ζ the following inequality holds*

$$|a_3-\zeta a_2^2| \leq \begin{cases} \frac{q}{3} & \text{if} \quad \zeta \in [\frac{-2}{3q}, \frac{6q+2}{3q}] \\ \frac{q^2(1-\zeta)}{3q+1} & \text{if} \quad \zeta \notin [\frac{-2}{3q}, \frac{6q+2}{3q}]. \end{cases}$$

5. Conclusion

This article explored a novel category of bi-Bazilevic and pseudo-starlike functions, defined using the q- Hurwitz-Lerch Zeta function and the q-Srivastava-Attiya operator, which is associated with the q-analogue of the hyperbolic tangent function. We have obtained estimates for the moduli of the initial coefficients in the Taylor series expansion of functions within this category and have formulated Fekete-Szegö inequalities that relate to functions within this category and its different subclasses. The results of this investigation are expected to provide a wide range of insights for subclasses related to orthogonal polynomials.

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