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OPTIMAL DECISION-MAKING IN FUZZY ECONOMIC ORDER QUANTITY (EOQ) MODEL UNDER RESTRICTED SPACE: A NON-LINEAR PROGRAMMING APPROACH

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Abstract. In this paper the concept of fuzzy Non-Linear Programming Technique is applied to solve an economic order quantity (EOQ) model under restricted space. Since various types of uncertainties and imprecision are inherent in real inventory problems they are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. The questions how to define inventory optimization tasks in such environment how to interpret optimal solutions arise. This paper allows the modification of the Single item EOQ model in presence of fuzzy decision making process where demand is related to the unit price and the setup cost varies with the quantity produced/Purchased. This paper considers the modification of objective function and storage area in the presence of imprecisely estimated parameters. The model is developed for the problem by employing different modeling approaches over an infinite planning horizon. It incorporates all concepts of a fuzzy arithmetic approach, the quantity ordered and the demand per unit compares both fuzzy non linear and other models. Investigation of the properties of an optimal solution allows developing an algorithm whose validity is illustrated through an example problem and ugh MATLAB (R2009a) version software, the two and three dimensional diagrams are represented to the application. Sensitivity analysis of the optimal solution is also studied with respect to changes in different parameter values and to draw managerial insights of the decision problem.

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1. Introduction

Although attempts were made to study the problem of control and maintenance of inventory using analytic techniques since the turn of century and its formulation in 1915, the square root formula for the economic order quantity (EOQ) was used in the inventory literature for a pretty long time. Ever since its introduction in the second decade of the past century, the EOQ model has been the subject of extensive investigations and extensions by academicians. Although the EOQ formula has been widely used and accepted by many industries, some practitioners have questioned its practical application. For several years, classical EOQ problems with different variations were solved by many researchers and had be separated in reference books and survey papers e.g. Taha [7], Urgeletti [4]. Recently, for a single product with demand related to unit price Cheng [27] and for multi products with several constraints. His treatments are fully analytical and much computational efforts were needed there to get the optimal solution.

Operations Research (OR) was first coined in 1940 by Mcclosky and Trefther in a small town, Bowdsey, in the UK. During the Second World War, this OR mathematics was used in a wider sense to solve the complex executive strategic and tactical problems of military teams. Since then the subject has been enlarged in importance in the field of Economics, Management Sciences, Public Administration, Behavioral Science, Social Work Commerce Engineering and different branches of Mathematics etc. But various Paradigmatic changes in science and mathematics concern the concept of uncertainty. In Science, this change has been manifested by a gradual transition from the traditional view, which insists that uncertainty is undesirable and should be avoided by all possible means. According to the traditional view, science should strive for certainty in all its manifestations; hence uncertainty is regarded as unscientific. According to the modern view, uncertainty is considered essential to science; it is not any an unavoidable plague but has; in fact, a great utility. But to tackle non-random uncertainty no other mathematics was developed other than fuzzy set theory and showed the intention to accommodate uncertainty in the presence of random variables. The application of fuzzy set concepts on EOQ inventory model have been proposed by many authors. Following Zadeh [14], significant contributions in this direction have been applied in many fields including production related areas. Consequently investment in introducing fuzzy is the key to avoid uncertain decision space. Many studies have modified inventory policies by considering the issues of nonrandom uncertain and fuzzy based EOQ models. Widyadana et al. [5] explain the economic order quantity model for deteriorating items and planned back order level. Hamacher et al. [6] discuss the sensitivity analysis in fuzzy linear programming. Pattnaik [16] explores some fuzzy and crisp inventory models. Vujosevic et al. [18] presented a theoretical EOQ formula when inventory cost is fuzzy. Lee et al. [9] studied an inventory model for fuzzy demand quantity and fuzzy production quantity. Tripathy et al. [21, 23, 25] introduced the concept and developed the framework for investing fuzzy in holding cost and setup cost in EOQ model. Tripathy et al. [19, 22] suggested improvements to production systems by employing entropy in the fuzzy model. Tripathy et al. [20] explains the effect of promotional factor in inventory model with units lost due to deterioration and Pattnaik [17] extends this model in fuzzy decision space with units lost due to deterioration under promotional factor.

Dutta et al. [2] studied the effect of tolerance in fuzzy linear fractional models. Sommer [3] applied fuzzy dynamic programming to an inventory and product and then withdraw from the market. Kacprzyk et al. [11] introduced the determination of optimal of firms from a global view point of top management in a fuzzy environment with fuzzy constraints improved on reappointments and a fuzzy goal for preferable inventory levels to be attained. Park [13] examined the EOQ formula in the fuzzy set theoretic perspective associating the fuzziness with the cost data. Here, inventory costs were represented by trapezoidal fuzzy numbers (TrFN) and the EOQ model was transformed to a fuzzy optimization problem. Similarly Lee et al. [9] and Vujosevic et al. [18] have applied fuzzy arithmetic approach in EOQ model without constraints.

Table-1 Summary of the Related Research

Authors	Demand	Setup cost	Holding cost	Unit cost of production	Constraint	Planning horizon	Structure of the Model	Model class
Vujosevic et al. [18]	Constant	Constant	$\frac{\tilde{c}_h c_p Q}{2 \times 100}$	Constant	No	Finite	Fuzzy	Defuzzification
Tripathy et al. [21]	Constant	Constant	$\frac{Hr^2q^2}{2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. [23]	Constant	Constant	$\frac{H\lambda q^2}{2r^2}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Tripathy et al. [25]	Constant	Constant	$\frac{Hq^2}{2r^2\lambda}$	Reliability and demand	Reliability	Infinite	Fuzzy	NLP
Present paper(2013)	Constant	Variable	$\frac{1}{2}C_1q$	Demand	Space	Infinite	Fuzzy	NLP

In this paper a single item EOQ model is developed where unit price varies inversely with demand and setup cost increases with the increase of production. In company or industry, total expenditure for production and storage area are normally limited but imprecise, uncertain, non-specificity, inconsistency vagueness and flexible. These are defined within some ranges. However, the no stochastic and ill formed inventory models can be realistically represented in the fuzzy environment. The problem is reduced to a fuzzy optimization problem associating fuzziness with the storage area and total expenditure. The optimum order quantity is evaluated by both fuzzy non linear programming (FNLP) method and the results are obtained for linear membership functions. The model is illustrated with numerical example and with the variation in tolerance limits for both shortage area and total expenditure. A sensitivity analysis is presented. The numerical results for fuzzy and crisp models are compared. The remainder of this paper is organized as follows. In section 2, assumptions and notations are provided for the development of the model and the mathematical formulation is developed. In section 3, mathematical analysis of fuzzy non linear programming (FNLP) is formulated. The solution of the FNLP inventory is derived in section 4. The numerical example is presented to illustrate the development of the model in section 5. The sensitivity analysis is carried out in section 6 to observe the changes in the optimal solution. Finally section 7 deals with the summary and the concluding remarks.

2. Mathematical Model

A single item inventory model with demand dependent unit price and variable setup cost under storage constraint is formulated as

$$\text{Min } C(D,q) = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1 q$$

s.t. $Aq \leq B$

$\forall$ $D, q > 0$ (1)

Where,

q = number of order quantity,

D = demand per unit time

C_1 = holding cost per item per unit time.

C_3 = Setup cost = C03 q ,

(C_{03} (> 0) and ν (0< < 1) are constants)

P = Unit production cost = KD^{-} K (> 0) and β (> 1) are constants. Here lead time is zero, no back order is permitted and replenishment rate is infinite. A and B are nonnegative real numbers, B is the space constraint goal. The above model in a fuzzy environment is

$$\widetilde{Min}\ C(D,q) = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1 q$$

s. t. $Aq \leq \tilde{B}$

$\forall$ $D, q > 0$ (2)

(A wavy bar (~) represents fuzzification of the parameters).

3. Mathematical Analysis of Fuzzy Non-Linear Programming (FNLP)

A fuzzy non linear programming problem with fuzzy resources and objective are defined as

$$\widetilde{Min}\; g_0(\mathrm{x})$$
$$\text{s.t. } g_i(x) \leq \tilde{b}_i \quad \text{i=1, 2, 3, ...m.} \qquad (3)$$

In fuzzy set theory, the fuzzy objective and fuzzy resources are obtained by their membership functions, which may be linear or nonlinear. Here μ_0 and μ_i (i = 1, 2 ...., m) are assumed to be non increasing continuous linear membership functions for objective and resources respectively such as

$$\mu_i\,(g_i(x)) = \begin{cases} 1 \;\; if \; g_i(x) < b_i, \\ 1 - \frac{g_i(x) - b_i}{P_i} \; if \; b_i \leq g_i(x) \leq b_i + P_{i,} \\ 0 \; if \; g_i > b_i + P_i, \end{cases} \qquad \text{i = 0, 1, 2, ..., m.}$$

In this formulation, the fuzzy objective goal is b_0 and its corresponding tolerance is P_0 and for the fuzzy constraints, the goals are b_i's and their corresponding tolerances are P_i's (i = 1, 2, ..., m). To solve the problem (3), the max - min operator of Bellman et al. [26] and the approach of Zimmermann [8] are implemented.

The membership function of the decision set, μ_D (x), is $\mu_D(x) = \min\{\mu_0(x), \mu_1(x), \ldots, \mu_m(x)\}, \forall\, x \in X$.

The min operator is used here to model the intersection of the fuzzy sets of objective and constraints. Since the decision maker wants to have a crisp decision proposal, the maximizing decision will correspond to the value of x, x_{max} that has the highest degree of membership in the decision set.

$\mu_D(x_{max}) = \max_{x\geq 0}[\min\{\mu_0(x), \mu_1(x) \ldots, \mu_m(x)\}]$. It is equivalent to solving the following crisp non linear programming problem.

Max α

s.t. $\mu_0(x) \geq \alpha$

$\mu_i(x) \geq \alpha$ (i= 1, 2, ... , m) (4)

$\forall\ x \geq 0, \alpha \in (0, 1)$

A new function, i.e the lagrangian function L (α, x λ) is formed by introducing (m +1) lagrangian multipliers $\lambda = (\lambda_0, \lambda_1, \ldots\ \lambda_m)$.

L $(\alpha, x, \lambda) = \alpha - \sum_{i=0}^{m} \lambda_i (g_i(x) - b_i - (1-\alpha)P_i)$. The necessary condition of Kuhn et al. [10] for the optimal solution to this problem implies that optimal values $x_{1,}^{*} x_2^{*}, x_3^{*}, \ldots \ldots x_n^{*}$ *and* $\lambda_{1,}^{*} \lambda_2^{*}, \lambda_3^{*}, \ldots \ldots \lambda_n^{*}$ should satisfy

$$\frac{\partial L}{\partial \alpha} = 0$$

$$\frac{\partial L}{\partial xj} = 0 \qquad , \qquad j = 1, 2, \ldots. n \tag{5}$$

$\lambda_i (g_i(x) - b_i - (1 - \alpha) P_i) = 0,$

$g_i(x) \leq b_i + (1-\alpha)P_i,$

$\lambda_i \leq 0,\ i = 0, 1, \ldots, m$

Moreover, Kuhn-Tucker's sufficient condition demands that the objective function for maximization and the constraints should be respectively colane and convex. In this formulation, it can be shown that both objective function and constraints satisfy the required sufficient conditions. Now, solving (5), the optimal solution for the FNLP problem is obtained.

4. Solution of the proposed inventory model

The proposed inventory model depicted by equation (2)

$\widetilde{Min}$ C (D,q) = $C_{03} q^{\nu-1} D + K D^{1-\beta} + \frac{1}{2} C_1$q

s. t. $Aq \leq \tilde{B}$

$\forall$ D, q > 0, reduces to following equation (4),

Max α

s.t. $C_{03} q^{\nu-1} D + K D^{1-\beta} + \frac{1}{2} C_1 q \leq C_0 + (1 - \alpha)$ P,

$Aq \leq B + (1 - \alpha)$ P, (6)

$\forall$ D, q > 0 & α $\in (0, 1)$

Here, the objective goal is C_0 with tolerance P_0 and the space constraint goal is B with tolerance P. So, the corresponding Lagrangian function is

L (α, D, q, λ_1, λ_2) $= \alpha - \lambda_1\left(C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1q - C_0 - (1-\alpha)P_0\right)$ - $\lambda_2(Aq - B - (1-\alpha)P)$

From Kuhn - Tucker's necessary conditions,

$$\frac{\partial L}{\partial \alpha} = 1 - \lambda_1P_0 - \lambda_2P \geq 0$$

$$\frac{\partial L}{\partial D} = \lambda_1\left(C_{03}q^{\nu-1} + (1-\beta)KD^{-\beta}\right) \leq 0$$

$$\frac{\partial L}{\partial q} = \lambda_1\left(C_{03}(\nu-1)q^{(\nu-2)}D + \frac{1}{2}C_1\right) - A\lambda_2 \leq 0$$

$$\frac{\partial L}{\partial \lambda_1} = C_{03}q^{\nu-1}D + KD^{1-\beta} + \frac{1}{2}C_1q - C_0 - (1-\alpha)P_0 \leq 0$$

$$\frac{\partial L}{\partial \lambda_2} = Aq - B - (1-\alpha)P \leq 0$$

and $\alpha(1 - \lambda_1P_0 - \lambda_2P) = 0$

$$\lambda_1D\left(C_{03}q^{(\nu-1)} + (1-\beta)KD^{-\beta}\right) = 0$$

$$\lambda_1q\left(C_{03}(\nu-1)q^{(\nu-2)}D + \frac{1}{2}C_1\right) - A\lambda_2q = 0$$

$$\lambda_1\left(C_{03}q^{(\nu-1)}D + KD^{1-\beta} + \frac{1}{2}C_1q - C_0 - (1-\alpha)P_0\right) = 0$$

$\lambda_2(Aq - B - (1-\alpha)P) = 0$, α, D, q ≥ 0 and λ_1, $\lambda_2 \leq 0$, solving these equations, optimum quantities are

$q^* = \frac{B+(1-\alpha^*)P}{A}$, $D^* = \left[\left(\frac{C_{03}}{(\beta-1)K}\right)\left(\frac{B+(1-\alpha)}{A}\right)^{\gamma-1}\right]^{-1/\beta}$, q = f (α) and D = f (q) where α* is a root of

$$\beta KD^{*(1-\beta)} + \frac{1}{2}C_1q^* - C_{03} - (1-\alpha^*)P_0 = 0$$

$$\Rightarrow \beta K\left[\left(\frac{C_{03}}{(\beta-1)K}\right)\left(\frac{B+(1-\alpha^*)}{A}\right)^{\gamma-1}\right]^{-(1-\beta)/\beta} + \frac{1}{2}C_1\frac{B+(1-\alpha^*)P}{A} - C_{03} - (1-\alpha^*)P_0 = 0$$

and so

$$C^*(D^*, q^*) = C_{03}q^{*\nu-1}D^* + KD^{*1-\beta} + \frac{1}{2}C_1q^*$$

So, by both FNLP and NLP techniques, the optimal values of q^* and D^* and the corresponding minimum cost are evaluated for the known values of other parameters.

5. Numerical Example

For a particular EOQ problem, let C_{03} = Rs. 200, K = 100, C_1 = Rs. 100, = 0.5, = 1.5, A = 10 units, B = 50 units, C_0 = Rs. 2000 and $P_0 = 20$ and P=15 units. For these values the optimal value of productions batch quantity q*, optimal demand rate D*, minimum average total cost C* (D*, q*), $\boldsymbol{\alpha}^*$ and Aq* obtained by FNLP are given in Table 2.

After 258 iterations Table-2 reveals the optimal replenishment policy for single item with demand dependent unit cost and dynamic setup cost. In this table the optimal numerical results of fuzzy model are compared with the results of crisp model of Pattnaik [15]. The optimum replenishment quantity q^*, the optimum quantity demand D^*, the minimum total average cost $C^*(D^*, q^*)$ and Aq^* are all 17.29%, 5.12%, 8.03% and 17.29% are more than that of other crisp model respectively. It permits the better use of present fuzzy model as compared to the crisp model and other fuzzy model. The results are justified and agree with the present model. It indicates the consistency of the fuzzy space of EOQ model from other model.

Table-2 Optimal Values for the Proposed Inventory Model

Model	Method	Iteration	q^*	D^*	$C^*(D^*, q^*)$	α^*	Aq^*
Fuzzy model	FNLP	258	6.044937	9.811542	53.93249	0.3033756	60.44937
Crisp model, Pattnaik. (2011)	NLP	23	5	9.308755	49.60392	-	50
% Change	-	-	17.286152	5.1244443	8.0259042	-	17.286152

6. Sensitivity Analysis

Now the effect of changes in the system parameters on the optimal values of q, D, C (D, q) and Aq when only one parameter changes and others remain unchanged the computational results are described in Tables 3 and 4. As a result

α^*, q^*,D^*, $C^*(D^*,q^*)$ and Aq^* are less sensitive to the parameters P_0 and P. Following Dutta et al. [1] and Hamacher et al. [4] it is observed that the effect of tolerance in the said EOQ model with the earlier numerical values and construct Tables 3 and 4 for the degrees of violation $T_0(=(1-\alpha)P_0)$ and T(=(1-$\alpha)P)$ for two constraints given by equation (7) .

From Table 3, it is seen that: (i) For higher tolerances of P_0, the value of α_{max} does not achieve 1, (ii) For higher acceptable variations P_0, the optimal solutions remain invariant and the optimal solutions are very close to the solutions (q^* =6.044937, $D^* = 9.811542$, $C^*(D^*,q^*) = 53.93249$ and $Aq^* = 60.44937$) of fuzzy model and (q^* =5, $D^* = 9.308755$, $C^*(D^*,q^*) = 49.60392$ and $Aq^* = 50)$ of the crisp model without tolerance ($\alpha = 1)$ respectively.

From Table 4 it is shown that: (i) For different values of P, degrees of violations T_0 and T are never zero, i.e. different optimal solutions are obtained. (ii) As P increases from 16, the minimum average cost $C^*(D^*,q^*)$, q^* and $D^*$are invariant in nature.

Fig. 1 represents the relationship between demand per unit time D and unit cost of production P. Similarly Fig. 2 shows the relationship between number of order quantity q and variable setup cost C_3 and Fig. 3 depicts the mesh plot of demand per unit time D, number of order quantity q and average total cost C.

Table-3 Sensitivity Analysis of the Parameters P_0

P_0	Iteration	α^*	q^*	D^*	T_0	T	$C^*(D^*, q^*)$	Aq^*
25	22	0.4170464	5.874430	9.718411	13.99089	8.744304	53.99089	58.74430
50	23	0.4398584	5.840212	9.699505	14.00354	8.402124	54.00354	58.40212
100	27	0.8570382	5.214443	9.339909	14.29618	2.144427	54.29618	52.14443
150	23	0.9044167	5.143375	9.297284	14.33750	1.433750	54.33750	51.43375
200	25	0.9282054	5.107692	9.275733	14.35893	1.076920	54.35893	51.07692
1000	26	0.9855874	5.021619	9.223348	14.41256	0.2161884	54.41256	50.21619
5000	23	0.9971153	5.004327	9.212736	14.42367	0.0432710	54.42367	50.04327
10000	23	0.9985575	5.002164	9.211408	14.42507	0.0216376	54.42507	50.02164
50000	23	0.9997115	5.000433	9.210347	14.42619	0.0432786	54.42619	50.00433
90000	24	0.9998397	5.000240	9.207241	14.42631	0.0024044	54.42631	50.00240

Table-4 Sensitivity Analysis of the Parameter P

P	Iteration	α^*	q^*	D^*	T_0	T	$C^*(D^*, q^*)$	Aq^*
16	25	0.3044350	6.112904	9.848178	13.91130	11.12904	53.91130	61.12904
20	24	0.3080997	6.383801	9.991556	13.83801	13.83801	53.83801	63.83801
23	29	0.3102921	6.586328	10.09612	13.79416	15.86328	53.79416	65.86328
36	28	0.3148201	7.329612	10.46246	13.70360	23.29612	53.70360	73.29612
38	27	0.3152590	7.602016	10.59049	13.69482	26.02016	53.69482	76.02016
40	32	0.3152770	7.670637	10.62227	13.69446	27.38892	53.69446	76.70637
42	29	0.3152770	7.670637	10.62227	13.69446	28.75836	53.69446	76.70637
150	29	0.3152770	7.670636	10.62226	13.69446	102.7084	53.69446	76.70636
160	29	0.3152770	7.670635	10.62226	13.69446	109.5557	53.69446	76.70635
162	29	0.3152770	7.670635	10.62226	13.69446	110.9251	53.69446	76.70635

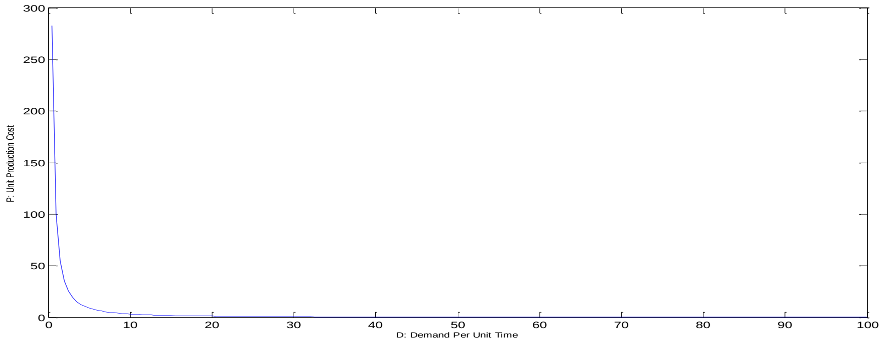

Fig. 1: Demand per Unit Time D and Unit Production Cost P.

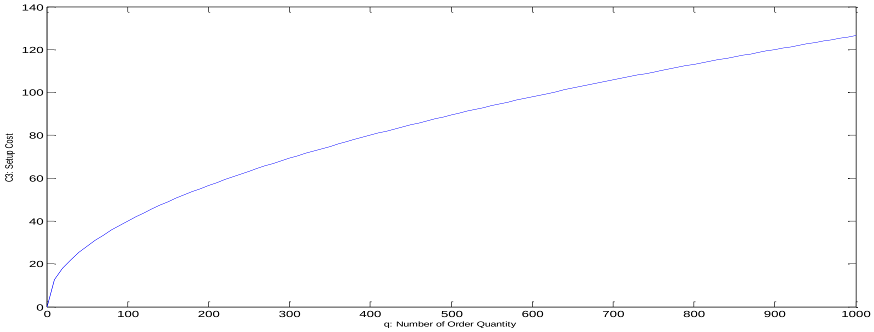

Fig. 2: Number of Order Quantity q and Dynamic Setup Cost C_3.

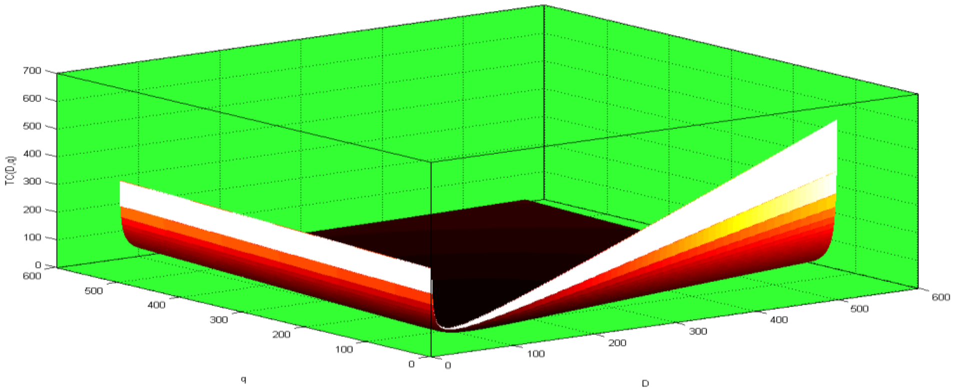

Fig. 3: Mesh plot of Demand per Unit Time D, Number of Order Quantity q and Average Total Cost C.

7. Conclusion

The approach in this paper provides solutions better than those obtained by using properties and this paper follows real life inventory model for single item in fuzzy environment by FNLP technique. Some sensitivity analyses on the tolerance limits have been presented. The results of the fuzzy model is compared with that of crisp model which reveals that fuzzy model gives better result than the usual crisp model. Inventory modelers have so far considered auto are type of setup cost that is fixed or constant. This is rarely seen to occur in the real market. In the opinion of the author, an alternative (and perhaps more realistic) approach is to consider the setup cost as a function quantity produced / purchased may represent the tractable decision making procedure in fuzzy environment. A new mathematical model with restricted space is developed and numerical example is provided to illustrate the solution procedure. The new modified EOQ model was numerically compared to the traditional EOQ model. Objective of this paper is to establish the better performance of the fuzzy technique i.e. to prove that Fuzzy Non-Linear Programming Technique minimizes the average total cost more than usual Crisp Non-Linear Programming Technique. It is also analysed in this paper that when fuzzification of the model is done in single objective case then, also average annual cost is minimized more. Finally, the effect decision space was demonstrated numerically to have an adverse affect on the total average cost per unit. This method is quite general and can be extended to other similar inventory models including the ones with shortages and deteriorate items.

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