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FIXED POINT THEOREMS FOR $\alpha-\psi-$QUASI CONTRACTIVE MAPPINGS IN METRIC-LIKE SPACES

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Abstract. In this paper, we give fixed point theorems for $\alpha-\psi-$quasi contractions and $\alpha-\psi-p-$quasi contractions in complete metric-like spaces.

1. Introduction and preliminaries

Fixed point theory became one of the most interesting area of research in the last fifty years. Many authors studied contractive type mappings on a complete metric space which are generalizations of Banach contraction principles. Recently, Samet et al. [17] introduced the notion of $\alpha-\psi$ contractive mappings and established some fixed point theorems in complete metric spaces. Later some other authors generalized $\alpha-\psi$ contractions ([5-7][9-14],[18]).

In last years, many generalizations of the concept of metric spaces are defined and some fixed point theorems was proved in these spaces.In particular, in 1994, Matthews introduced the notion of a partial metric space as a part of the study of denotational semantics of dataflow networks and showed that the Banach contraction principle can be generalized to the partial metric context for applications in program verification ([15]). Later on, many researchers studied fixed point theorems in partial metric spaces ([1],[2],[8],[16],[20]). Recently, Amini-Harandi generalized the partial metric spaces by introducing the metric-like spaces and proved some fixed point theorems in such spaces ([3]). After authors gived some fixed point theorems in metric-like spaces ([19]).

In this paper, we introduce the notion of $\alpha-\psi-$quasi contractive mappings in complete metric-like spaces and in last parts we give $\alpha-\psi-p-$quasi contraction in metric like spaces. Our results are generalisations of the many existing results in the literature.

First we give some definitions and facts about metric-like spaces.

Definition 1. *([3]) A mapping $\sigma : X \times X \to \mathbb{R}^+$, where X is a nonempty set, is said to be metric-like on X if for any $x, y, z \in X$, the following three conditions hold true:*

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($\sigma 1$) $\sigma(x,y)=0 \Rightarrow x=y$;
($\sigma 2$) $\sigma(x,y)=\sigma(y,x)$,
($\sigma 3$) $\sigma(x,z)\leq\sigma(x,y)+\sigma(y,z)$.

The pair (X,σ) is called a metric-like space.Then a metric-like on X satisfies all of the conditions of a metric except that $\sigma(x,x)$ may be positive for $x\in X$. Each metric-like σ on X generates a topology τ_σ on X whose base is the family of open $\sigma-$balls

$$B_\sigma(x,\varepsilon)=\{y\in X:\quad |\sigma(x,y)-\sigma(x,x)|<\varepsilon\}$$

for all $x\in X$ and $\varepsilon>0$.

Then the sequence $\{x_n\}$ in the metric-like space (X,σ) converges to a point $x\in X$ if and only if $\lim\limits_{n\to\infty}\sigma(x_n,x)=\sigma(x,x)$.

Let (X,σ) and (Y,τ) be metric-like spaces and let $f:X\to Y$ be a continuous mapping. Then

$$\lim_{n\to\infty}x_n=x \implies \lim_{n\to\infty}f(x_n)=f(x).$$

A sequence $\{x_n\}_{n=0}^{\infty}$ of elements of X is called $\sigma-$Cauchy if $\lim\limits_{n,m\to\infty}\sigma(x_n,x_m)$ exists and is finite.The metric-like space (X,σ) is called complete if for each $\sigma-$Cauchy sequence $\{x_n\}_{n=0}^{\infty}$, there is some $x\in X$ such that

$$\lim_{n\to\infty}\sigma(x_n,x)=\sigma(x,x)=\lim_{n,m\to\infty}\sigma(x_n,x_m).$$

Every partial metric space is a metric-like space. Below we give another example of a metric-like space.

Example 1. *([3]) Let $X=\{0,1\}$, and let*

$$\sigma(x,y)=\begin{cases} 2, & \text{if } x=y=0 \\ 1, & \text{otherwise}\end{cases}$$

Then (X,σ) is a metric-like space, but since $\sigma(0,0)\nleq\sigma(0,1)$, then (X,σ) is not a partial metric space.

2. Fixed Point Results For $\alpha-\psi$ Contractive Mappings

Denote by Ψ the family of nondecreasing functions $\psi:[0,\infty)\to[0,\infty)$ such that $\lim_{n\to\infty}\psi^n(t)=0$ for all $t>0$.

Lemma 1. *If $\psi\in\Psi$, then the following are satisfied.*

(a) $\psi(t)<t$ for all $t>0$
(b) $\psi(0)=0$
(c) ψ is right continuous at $t=0$.

Remark 1. *(a) If $\psi:[0,\infty)\to[0,\infty)$ is nondecreasing such that $\sum\limits_{n=1}^{\infty}\psi^n(t)<\infty$ for each $t>0$, then $\psi\in\Psi$.*

(b) If $\psi:[0,\infty)\to[0,\infty)$ is upper semicontinuous such that $\psi(t)<t$ for all $t>0$, then $\lim_{n\to\infty}\psi^n(t)=0$ for all $t>0$.

Definition 2. *([16])Let $T : X \to X$ and $\alpha : X \times X \to [0, \infty)$. We say that T is α-admissible if*

$$x, y \in X, \quad \alpha(x, y) \geq 1 \Rightarrow \alpha(Tx, Ty) \geq 1.$$

Definition 3. *Let (X, σ) be a complete metric-like space and $T : X \to X$ be a given mapping. We say that T is an $\alpha - \psi-$quasi contractive mapping if there exist $\alpha : X \times X \to [0, \infty)$ and $\psi \in \Psi$ such that*

$$\alpha(x, y) \sigma(Tx, Ty) \leq \psi(M(x, y)) \tag{1}$$

for all $x, y \in X$ where

$$M(x,y) = \max\{\sigma(x, y), \sigma(x, Tx), \sigma(y, Ty), \sigma(x, Ty), \sigma(y, Tx), \sigma(x, x), \sigma(y, y)\}.$$

Theorem 2. *Let (X, σ) be a complete metric-like space and $T : X \to X$ be an $\alpha - \psi-$quasi contractive mapping. Assume that there exists $x_0 \in X$ such that $O(x_0, \infty) = \{T^n x_0 : n = 0, 1, 2...\}$ is bounded and*

(i) $\alpha\left(T^i x_0, T^j x_0\right) \geq 1$ *for all $i, j \geq 0$ with $i < j$,*
(ii) *T is $\sigma-$continuous or*

$\lim_{n\to\infty} \inf \alpha(T^n x_0, x) \geq 1$ for any cluster point x of $\{T^n x_0\}$.
Then T has a fixed point.

Proof. Let $x_0 \in X$ be such that $O(x_0, \infty) = \{T^n x_0 : n = 0, 1, 2...\}$ is bounded and $\alpha\left(T^i x_0, T^j x_0\right) \geq 1$ for all $i, j \geq 0$ with $i < j$. Define the sequence $\{x_n\}$ in X by $x_{n+1} = Tx_n$ for all $n \in \mathbb{N} \cup \{0\}$.

If $x_n = x_{n+1}$ for some $n \in \mathbb{N}$, then $x^* = x_n$ is a fixed point of T.

Assume that $x_n \neq x_{n+1}$ for all $n \in \mathbb{N} \cup \{0\}$.

Now we shall show $\{x_n\}$ is a $\sigma-$Cauchy sequence.

Let $\delta(x_n) = diam(\{Tx_n, Tx_{n+1}, ...\})$ for $n = 0, 1, 2, ...$ Since $\delta(x_n) \leq \delta(x_0)$ and $\delta(x_0) < \infty$

We assert that for $n = 0, 1, 2, ...$

$$\delta(x_n) \leq \psi^n(\delta(x_0)). \tag{2}$$

For $n = 0$, (2) holds. Suppose that (2) holds for $n = k$. We will show that (2) holds when $n = k + 1$. Let $Tx_{r-1}, Tx_{s-1} \in \{Tx_k, Tx_{k+1}, ...\}$ for any $r, s \geq k + 1$. Then

$$\begin{aligned} \sigma(x_r, x_s) &= \sigma(Tx_{r-1}, Tx_{s-1}) \\ &\leq \alpha(x_{r-1}, x_{s-1}) \sigma(Tx_{r-1}, Tx_{s-1}) \\ &\leq \psi(M(x_r, x_s)) \end{aligned} \tag{3}$$

where

$$\begin{aligned} M(x_r, x_s) &= \max\left\{ \begin{array}{c} \sigma(x_{r-1}, x_{s-1}), \sigma(x_{r-1}, Tx_{r-1}), \sigma(x_{s-1}, Tx_{s-1}), \\ \sigma(x_{r-1}, Tx_{s-1}), \sigma(x_{s-1}, Tx_{r-1}), \\ \sigma(x_{r-1}, x_{r-1}), \sigma(x_{s-1}, x_{s-1}) \end{array} \right\} \\ &= \max\left\{ \begin{array}{c} \sigma(Tx_r, Tx_s), \sigma(Tx_r, Tx_{r+1}), \sigma(Tx_s, Tx_{s+1}) \\ , \sigma(Tx_r, Tx_{s+1}), \sigma(Tx_s, Tx_{r+1}), \\ \sigma(Tx_r, Tx_r), \sigma(Tx_s, Tx_s) \end{array} \right\} \\ &\le \delta(x_k). \end{aligned}$$

Then by (3),

$$\begin{aligned} \sigma(x_r, x_s) &= \sigma(Tx_{r-1}, Tx_{s-1}) \le \psi(\delta(x_k)) \\ &\le \psi\left(\psi^k(\delta(x_0))\right) \\ &= \psi^{k+1}(\delta(x_0)). \end{aligned}$$

Thus (2) is proved for $n = 0, 1, 2, ...$

Hence from (2) we have $\lim_{n\to\infty} \delta(x_n) = 0$. Thus $\{x_n\}$ is a $\sigma-$Cauchy sequence in (X, σ). By the completeness of X , there exists $z \in X$ such that $\lim_{n\to\infty} x_n = z$, that is,

$$\lim_{n\to\infty} \sigma(x_n, z) = \sigma(z, z) = \lim_{n,m\to\infty} \sigma(x_n, x_m) = 0. \tag{4}$$

If T is $\sigma-$continuous,

$$\lim_{n\to\infty} \sigma(Tx_n, Tz) = \lim_{n\to\infty} \sigma(x_{n+1}, Tz) = \sigma(z, Tz) = 0.$$

This proves z is a fixed point.

If $\lim_{n\to\infty} \inf \alpha(T^n x_0, x) \ge 1$ for any cluster point x of $\{T^n x_0\}$, there exists $n_0 \in \mathbb{N}$ such that $\alpha(x_n, z) \ge 1$, for all $n > n_0$. Thus,

$$\begin{aligned} \sigma(x_{n+1}, Tz) &\le \sigma(Tx_n, Tz) \\ &\le \alpha(x_n, z)\, \sigma(Tx_n, Tz) \\ &\le \psi(M(x_n, z)) \end{aligned} \tag{5}$$

where

$$M(x_n, z) = \max\left\{ \begin{array}{c} \sigma(x_n, z), \sigma(x_n, Tx_n), \sigma(z, Tz), \sigma(x_n, Tz), \\ \sigma(z, Tx_n), \sigma(x_n, x_n), \sigma(z, z) \end{array} \right\}$$

If $\sigma(z, Tz) > 0$, using upper semicontinuity of ψ,

$$\begin{aligned} \sigma(z, Tz) &= \lim_{n\to\infty} \sup \sigma(x_{n+1}, Tz) \\ &\le \lim_{n\to\infty} \sup \psi(M(x_n, z)) \\ &\le \psi(\sigma(z, Tz)) \\ &< \sigma(z, Tz) \end{aligned}$$

which is a contradiction. Thus, we obtain $\sigma(Tz, z) = 0$.So $Tz = z$. □

Example 2. *Let* $X = \{0, 1, 2\}$ *. Define* $\sigma : X \times X \to \mathbb{R}^+$ *as follows:*

$$\begin{array}{lll} \sigma(0,0) & = & 0 \qquad \sigma(1,1) = 3 \qquad \sigma(2,2) = 1 \\ \sigma(0,1) & = & \sigma(1,0) = 7 \qquad \sigma(0,2) = \sigma(2,0) = 3 \\ \sigma(1,2) & = & \sigma(2,1) = 4. \end{array}$$

Then (X, σ) *is a complete metric-like space. Define the mapping* $T : X \to X$ *by*

$$T0 = 0, \ T1 = 2 \ \text{and} \ T2 = 0$$

and $\alpha : X \times X \to [0, \infty)$ *by*

$$\alpha(x, y) = \begin{cases} \frac{1}{4}, & \text{if } (x, y) \neq (0, 0) \\ 1, & (x, y) = (0, 0) \end{cases}$$

Then T *is an* $\alpha-$ ψ*-quasi contractive mapping with* $\psi(t) = \frac{t}{1+t}$.

Moreover, there exists $x_0 \in X$ such that $\alpha\left(T^i x_0, T^j x_0\right) \geq 1$, for all $i, j \geq 0$ with $i < j$. So for $x_0 = 0$, we have

$$\alpha\left(T^i 0, T^j 0\right) = \alpha(0, 0) = 1.$$

Obviously (1) is satisfied for all $x, y \in X$.

All hypotheses of Theorem 2 are satisfied. Consequently T has a fixed point. And $x_0 = 0$ is fixed point of T.

Taking in Theorem 2, $\alpha(x, y) = 1$ for all $x, y \in X$,we obtain immediately the following corollaries.

Corollary 3. *Let* (X, σ) *be a complete metric-like space and* $T : X \to X$ *be a given mapping. Suppose that there exists a function* $\psi \in \Psi$ *such that*

$$\sigma(Tx, Ty) \leq \psi(M(x, y))$$

where

$$M(x,y) = \max\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty), \sigma(x,Ty), \sigma(y,Tx), \sigma(x,x), \sigma(y,y)\}$$

for all $x, y \in X$*. Then* T *has a unique fixed point.*

Corollary 4. *Let* (X, σ) *be a complete metric-like space and* $T : X \to X$ *be a given mapping. Suppose that there exists a constant* $c \in (0, 1)$ *such that*

$$\sigma(Tx, Ty) \leq cM(x, y)$$

where

$$M(x,y) = \max\{\sigma(x,y), \sigma(x,Tx), \sigma(y,Ty), \sigma(x,Ty), \sigma(y,Tx), \sigma(x,x), \sigma(y,y)\}$$

for all $x, y \in X$*. Then* T *has a unique fixed point.*

3. Fixed Point Results For $\alpha-\psi-p-$Quasi Contractive Mappings

In this section we give $\alpha-\psi-p-$quasi contraction in consideration of Amini-Harandi [4].

Definition 4. *Let (X,σ) be a complete metric-like space and $T:X\to X$ be a given mapping. We say that T is an $\alpha-\psi-p-$quasi contractive mapping if there exist $\alpha:X\times X\to[0,\infty)$ and $\psi\in\Psi$ such that*

$$\alpha\left(x,y\right)\sigma\left(T^{p}x,T^{p}y\right)\leq\psi(M(x,y)) \tag{6}$$

for all $x,y\in X$ where

$$M(x,y)=\max\left\{\sigma\left(T^{i}u,T^{j}v\right):u,v\in\{x,y\},\ 0\leq i,j\leq p\text{ and }i+j<2p\ \right\}.$$

Theorem 5. *Let (X,σ) be a complete metric-like space and let $p\in\mathbb{N}$. Suppose that $T:X\to X$ be an $\alpha-\psi-p-$quasi contractive map. Assume that there exists $x_0\in X$ such that $O\left(x_0,\infty\right)=\{T^{n}x_0:\ n=0,1,2...\}$ is bounded and*

(i) there exists $x_0\in X$ such that $\alpha\left(T^{i}x_0,T^{j}x_0\right)\geq1$ for all $i,j\in\mathbb{N}\cup\{0\}$,
(ii) $T^{m}:X\to X$ is $\sigma-$continuous for some $m\in\mathbb{N}$.

Then T has a fixed point.

Proof. Let $x_0\in X$ be such that $O\left(x,\infty\right)=\{x,Tx,...\}$ is bounded and $\alpha\left(T^{i}x_0,T^{j}x_0\right)\geq1$ for all $i,j\in\mathbb{N}\cup\{0\}$. Define the sequence $\{x_n\}$ in X by $x_{n+1}=Tx_n$ for all $n\geq0$. Let n be a positive integer with $n\geq p$, and let $i,j\in\{p,p+1,...,n\}$. Since T is an $\alpha-\psi-p-$quasi contractive map, then

$$\begin{aligned}\sigma\left(T^{i}x,T^{j}x\right)&=\sigma\left(T^{p}T^{i-p}x,T^{p}T^{j-p}x\right)\\&\leq\alpha\left(T^{i-p}x,T^{j-p}x\right)\sigma\left(T^{p}T^{i-p}x,T^{p}T^{j-p}x\right)\\&\leq\psi\{\max\left\{\sigma\left(T^{k}T^{i-p}x,T^{l}T^{j-p}x\right):\ 0\leq k,l\leq p\text{ and }k+l<2p\right\}\}\\&\leq\psi(\delta\left[O\left(x,n\right)\right])\end{aligned} \tag{7}$$

Hence by lemma 1 (a) ,

$$\sigma\left(T^{i}x,T^{j}x\right)<\delta\left[O\left(x,n\right)\right].$$

Thus for sufficiently large $n\in\mathbb{N}$ there exist positive integers k,l with $k<p$ and $p\leq l\leq n$ such that

$$\sigma\left(T^{k}x,T^{l}x\right)=\delta\left[O\left(x,n\right)\right].$$

we show that $\{T^{n}x\}$ is a $\sigma-$Cauchy sequence. Without loss of generality assume that $p\leq n<m$.Then, from (7)

$$\begin{aligned}\sigma\left(T^{n}x,T^{m}x\right)&=\sigma\left(T^{p}T^{n-p}x,T^{m-n+p}T^{n-p}x\right)\\&\leq\alpha\left(T^{n-p}x,T^{m-p}x\right)\sigma\left(T^{p}T^{n-p}x,T^{m-n+p}T^{n-p}x\right)\\&\leq\psi\left(\delta\left[O\left(T^{n-p}x,m-n+p\right)\right]\right).\end{aligned}$$

by (7), there exists positive integers k_1 and l_1 with $k_1<p$ and $p\leq l_1\leq m-n+p$ such that

$$\delta\left[O\left(T^{n-p}x,m-n+p\right)\right]=\sigma\left(T^{k_1}T^{n-p}x,T^{l_1}T^{n-p}x\right).$$

Similarly we have

$$\begin{aligned}\sigma\left(T^{k_1}T^{n-p}x, T^{l_1}T^{n-p}x\right) &= \sigma\left(T^{k_1+p}T^{n-2p}x, T^{l_1}T^{n-p}x\right)\\ &\leq \psi(\delta\left[O\left(T^{n-2p}x, m-n+2p\right)\right]).\end{aligned}$$

Thus,

$$\sigma\left(T^n x, T^m x\right) \leq \psi(\delta\left[O\left(T^{n-p}x, m-n+p\right)\right]) \leq \psi^2(\delta\left[O\left(T^{n-2p}x, m-n+2p\right)\right]).$$

Proceeding in this manner, we obtain

$$\sigma\left(T^n x, T^m x\right) \leq \psi^{\left[\frac{n}{p}\right]}\left(\delta\left[O\left(T^{n-\left[\frac{n}{p}\right]p}x, m-n+\left[\frac{n}{p}\right]p\right)\right]\right) \leq \psi^{\left[\frac{n}{p}\right]}\left(\delta\left[O\left(x, m+p\right)\right]\right).$$

Hence

$$\sigma\left(T^n x, T^m x\right) \leq \psi^{\left[\frac{n}{p}\right]}\left(\delta\left[O\left(x, \infty\right)\right]\right).$$

By definition of ψ, $\lim_{n\to\infty}\sigma\left(T^n x, T^m x\right) = 0$. Hence we conclude that $\{T^n x\}$ is a $\sigma-$Cauchy sequence. By the completeness of X, there is some $u \in X$ such that

$$\lim_{n\to\infty}\sigma\left(T^n x, u\right) = \lim_{n\to\infty}\sigma\left(T^n x, T^m x\right) = \sigma\left(u, u\right) = 0 \quad \text{for each } x \in X.$$

Now we show that $Tu = u$. By the continuity of T^m,

$$\lim_{n\to\infty}\sigma\left(T^{m+n}x, Tu\right) = \sigma\left(u, Tu\right) = 0.$$

Hence, $u = Tu$. □

Example 3. *Let $X = [0,\infty)$ and $\sigma : X \times X \to [0,\infty)$ be defined*

$$\sigma\left(x, y\right) = \begin{cases} 0, & x = y \\ \max\left\{x, y\right\}, & \text{otherwise}\end{cases}.$$

Then, (X, σ) is a complete metric-like space. Let $\mathbb{Q}$ and $\mathbb{Q}'$denote respectively the set of rational numbers and irrational numbers. Let $T : [0,\infty) \to [0,\infty)$ and $\alpha : X \times X \to [0,\infty)$ be defined by

$$T\left(x\right) = \begin{cases}\sqrt{2}, & x \in \mathbb{Q} \\ \sqrt{3}, & \text{otherwise}\end{cases}$$

$$\alpha\left(x, y\right) = \begin{cases} 1, & x \in \mathbb{Q}' \\ 0, & \text{otherwise}\end{cases}$$

Then T is an $\alpha-$ ψ-2$-$contractive mapping with $\psi\left(t\right) = \frac{t}{1+t}$.

Then $T^2\left(x\right) = \sqrt{3}$ for each $x \in X$. Moreover T is discontinuous and T^2 is continuous. Then all conditions of Theorem 5 are satisfied. And $x = \sqrt{3}$ is fixed point of T.

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