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A Study on Medical Diagnosis Using Neutrosophic Over Complex Sets and Artificial Neural Networks Under Uncertain Environments

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Abstract. The present article introduces the idea of Neutrosophic Over Complex Sets, and in a way it kind of explores how they can be used for medical diagnosis tasks that live in uncertain and inconsistent environments. The proposed framework mixes the benefits of neutrosophic sets, over sets, and also complex-valued information into one single mathematical structure, so it can manage indeterminate, contradictory and over-valued complex data without too much fuss. In the proposed setup, the truth-membership, indeterminacy-membership and falsity-membership functions are written in a complex-valued form, and their amplitudes are greater than one, which is what basically makes the "over" condition hold. After that, a number of algebraic operations plus properties of Neutrosophic Over Complex Sets are defined and checked mathematically, like in a proper verification way. Then, there is an application: a medical diagnosis scheme is developed, for picking out the most critical patient from among a group of patients dealing with a serious viral disease. The evaluations coming from multiple experts are expressed through Neutrosophic Over Complex information, and they are aggregated using the set operations suggested in the framework. A score function is also used to arrange, and rank the patients based on their severity levels. Moreover, an Artificial Neural Network based approach

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gets added, mostly to validate the ranking results that the mathematical model outputs. The ANN model ends up producing the same ranking order as the proposed model, so it supports both reliability and computational efficiency of the whole framework. Overall, the results suggest that the Neutrosophic Over Complex Set model offers a flexible, realistic, and efficient mathematical tool, for tackling advanced decision-making and medical diagnosis problems where uncertain over complex information is involved.

1. Introduction

The idea of fuzzy sets, which Zadeh brought in [26], kind of became a base for uncertainty modeling in the mathematical sciences. Then Bellman and Zadeh [3] took these fuzzy notions a step further, more toward decisions in situations that are still not stable or, in a sense uncertain. Carney et al. [4] also looked at CCD photometry for RR Lyrae variables in M92, giving observational analysis that is pretty helpful in astronomical data studies. All together these works kinda underline why mathematical and computational tools really matter when you deal with uncertainty, and also with data driven situations. After that, Molodtsov [18] built what he called soft set theory, basically a generalized way of handling uncertainty that is parameterized not just random, and it feels more flexible in that view.

Smarandache [22] later introduced neutrosophic logic, where truth-membership, indeterminacy-membership and falsity-membership functions can basically exist at the same time, no big drama. Maji et al. [15] kinda extended the idea of soft set theory and talked about its mathematical growth, for those uncertain yet related issues . After that the whole framework got generalized into neutrosophic over sets, under sets and offsets, so it can catch over-valued information as well as under-valued information [23]. Christianto et al. [5] then continued and discussed several real world uses of neutrosophic logic, in both basic directions, and also in more applied sciences.

Mehmood et al. [16] studied neutrosophic soft α-open sets inside neutrosophic soft topological spaces, and they proved several properties, not only one, or at least thats what it sounded like in the paper. Dong et al. [6] proposed a fuzzy deep wavelet neural network, with the idea to do inversion for electrical resistivity imaging, and it worked okay in their results. Hassan et al. [7] developed an integrated FCEM-AHP approach for assessing borrower satisfaction and perception, specially in microfinance institutions, where the context matters a bit more.

Bangerter et al. [1] introduced this kinda hybrid framework, integrating large language models with ANFIS, for explainable fact checking applications, like that. Joostberens et al. [9] then developed a fuzzy modelling approach aimed at assessing Poland's energy security index in political settings. Khan et al. [11] proposed a multi attribute decision making framework, using possibility pythagorean fuzzy hypersoft sets, specifically for educational institution evaluation.

Lu et al. [14] built a deep neural network with a masked strategy and fuzzy pooling, for diagnosing osteoarthritis from X ray images. Bansal et al. [2] looked into physics informed neural networks, applied to partial integro differential equations, in financial decision making scenarios.

Jin et al. [8] suggested a fuzzy activation function, tied to a zeroing neural network, for dynamic Arnold map image cryptography.

Karuppusamy et al. [10] presented a numerical framework for cancer invasion mathematical models, with distinct cancer sub populations involved. Kong et al. [12] proposed a fuzzy regression approach, driven by kernel functions, for more robust time series forecasting. LÃ¡zaro Mata et al. [13] investigated fuzzy fusion methods for monocular ORB SLAM2, along with tachometer sensor based car odometry.

Mohapatra et al. [17] developed a kind of data-driven meta modelling path for response prediction, and sensitivity analysis of fastened GFRP laminates. Peng et al. [19] suggested a high performance stress constrained topology optimization strategy based on level set methods. Shoket et al. [21] did mathematical modeling, and computational study on the COVID-19 epidemic, with wavelet neural networks plus optimization strategies. Bharat et al. [28] developed a real-time pothole detection system using convolutional neural networks for automated road-condition monitoring. Their study demonstrates the effectiveness of deep learning techniques in accurately detecting potholes from road images.

Tian et al. [24] came up with interval type-2 fuzzy neural networks, mainly for multi label classification tasks. Prabakaran and Yamini [20] proposed a hybrid neural network direction for controllability, in Caputo fractional neutral integro differential systems, and they used it for cryptocurrency forecasting. Yang et al. [25] explored an adaptive fuzzy neural network scheme using smooth-switching gain dynamic surface control for constrained manipulators, and the whole part was tied to control engineering.

Zhang et al. [27] suggested a sampled-data global exponential synchronization procedure for multi delay fuzzy inertial neural networks. Overall, the above work implies that fuzzy systems, neutrosophic surroundings, and neural network centered structures have turned out to be pretty important for addressing modern uncertain decision making problems. Still, there has been only limited focus on combining neutrosophic settings that are overextended, with complex-valued information and ANN based medical diagnosis systems.

In recent years, uncertainty modeling has turned into one of those major research directions in mathematical sciences, computer science and artificial intelligence, kind of all at once. Classical set theory and fuzzy set theory are often not enough when it comes to describing highly inconsistent, and also indeterminate, information that keeps showing up in practical problems. So to push past those limitations, neutrosophic theory was brought in as a generalized framework, able to deal with truth-membership, indeterminacy-membership and falsity-membership functions, at the same time, without much compromise. Many researchers have since explored neutrosophic sets and their extensions, in decision making, medical diagnosis, pattern recognition, image processing, and also engineering systems, you know the usual range of applications.

After that, the idea of neutrosophic over sets came about, basically to represent over-valued information where at least one neutrosophic component can jump beyond the classical upper bound of one. This extension ended up being quite useful for capturing highly severe, overestimated, and abnormal information in real world scenarios. In a similar vein complex neutrosophic sets were also developed to catch periodic, and phase dependent information, using complex-valued membership functions that include amplitude and phase terms.

Even if several investigations have been carried out on their own for neutrosophic over sets and also for complex neutrosophic sets, there is still, rather surprisingly, only limited attention paid to merging both of these ideas into one single mathematical framework. In a lot of real use cases, especially in medical diagnosis and artificial intelligence, the incoming information can look kind of mixed up at the same time: it may include uncertainty, inconsistency, phase related behaviour and also over-valued observations. Existing models, as they stand, do not really manage to represent all of those characteristics together in a sensible or effective way.

So here, the current work introduces a concept called Neutrosophic Over Complex Sets, which is meant as a new generalized framework for dealing with uncertain over complex information. The proposed model essentially brings together the benefits from neutrosophic sets, over sets, and complex-valued information, but in one unified mathematical structure. Along the way, a set of algebraic operations is established and some theoretical properties are proven. In addition, the practical usefulness of the whole framework is shown through medical diagnosis tasks, and also through decision-making approaches based on Artificial Neural Networks, or well, decision-making using ANN style reasoning.

1.1. **Motivation.** The main reason this research exists is because modern real life applications are getting more and more tangled up with uncertain information, specially in medical diagnosis, and also in artificial intelligence systems. In day to day healthcare settings, medical observations are often uncertain, inconsistent, incomplete and also very dynamic. At the same time, different medical experts might end up giving contradictory views about how serious a disease is, and for a bunch of critical cases some assessments can naturally go beyond the usual upper limit of one. Besides that, multiple medical signals and diagnostic measurements show periodic, or phase dependent kind of behaviour, and this is not really possible to model right with ordinary real valued mathematical structures.

Now, the existing neutrosophic models can still handle uncertainty and inconsistency, but the complex neutrosophic models are better when the information is phase dependent. Also neutrosophic over sets do a good job with over valued membership details. Still each of these approaches, when taken alone, misses part of the picture, so they cannot bring all the key characteristics together at once. That is why, there is a clear need for one unified mathematical framework, able to represent at the same time uncertain information, inconsistency, over valued data, and complex phase information. Because of those gaps, the current work proposes the concept of Neutrosophic Over Complex Sets, and then uses it for medical diagnosis tasks. In addition, by integrating Artificial Neural Networks with the proposed framework, the work also increases its practical importance and improves computational feasibility, especially for intelligent decision making systems.

1.2. **Methodology.** The methodology in this research is split, kind of, into phases that are theoretical and then also into something more application oriented. First of all, the idea of Neutrosophic Over Complex Sets is brought in and described carefully, using complex-valued truth-membership, indeterminacy-membership and falsity-membership functions in over-valued situations . After that, basic set operations like union, intersection and complement are built up, and also stated.

A bunch of algebraic behaviors including commutative, associative, distributive and absorption laws are checked mathematically through propositions, lemmas and theorems, without skipping too much. Once that theory is ready, the framework is then used for a medical diagnosis scenario with several patients, and multiple medical experts, involved. The experts' judgments are written as Neutrosophic Over Complex values, and these values have to satisfy both the complex condition as well as the over condition. Then, the information coming from different experts is fused via the union operation we proposed,you could say. After the fusion stage, a score-function based ranking method is applied, in order to decide how severe each patient's situation is.

In addition, to test whether the ranking results hold up, an Artificial Neural Network model is created. The aggregated neutrosophic over complex values are fed in as ANN input neurons, while the patient severity scores are treated as target outputs. The ANN is trained with back propagation, using learning parameters that are chosen appropriately. In the end, the ANN outputs are matched against the mathematical ranking outputs, to check if the results stay consistent and reliable.

2. Preliminaries

This section provides the fundamental concepts and definitions necessary for understanding Neutrosophic Over Set and its Topological Spaces [22], [15] and [23].

Definition 2.1. *[22] Let $\varkappa$ be an non empty set and φ_g is said to be an Neutrosophic set.Then*

$$\varphi_g = \{\langle \natural, \wp(\natural), \eth(\natural), \flat(\natural)\rangle : \natural \in \varkappa\}$$

where $\wp, \eth, \flat : \varkappa \to [0,1]$ and $0 \leq \wp(\natural) + \eth(\natural) + \flat(\natural) \leq 3$. Here $\wp(\natural), \eth(\natural)$ and $\flat(\natural)$ are degree of true membership,degree of indeterminacy and degree of falsity.

Definition 2.2. *[23] Let φ_g be an Neutrosophic set in $\varkappa$.If φ_g is said to be an Neutrosophic Over set in an non-empty set $\varkappa$ then it has at-least one neutrosophic component is > 1 and no other component are < 0 is defined as,*

$$\varphi_g = \{\langle \natural, \wp(\natural), \eth(\natural), \flat(\natural)\rangle : \natural \in \varkappa\}$$

Where $\wp, \eth, \flat : \varkappa \to [0, \Omega]$, $0 \leq \wp(\natural) + \eth(\natural) + \flat(\natural) \leq 3$ and Ω is said to be over-limit of NOS

Note: $\rho(\varkappa)$ is a set of all the $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$ subset of an non-empty set $\varkappa$

3. Neutrosophic Over Complex Set

Definition 3.1. *Let χ be a non-empty set. Then, a $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set on χ is defined as*

$$\mathfrak{B} = \left\{\left\langle \mathfrak{l}, \phi_{\mathfrak{B}}(\mathfrak{l}), \mu_{\mathfrak{B}}(\mathfrak{l}), \varphi_{\mathfrak{B}}(\mathfrak{l})\right\rangle : \mathfrak{l} \in \chi\right\},$$

where

$$\phi_{\mathfrak{B}}(\mathfrak{l}) = \mathsf{p}_{\mathfrak{B}}(\mathfrak{l})\mathsf{e}^{\mathsf{i}\theta_1(\mathfrak{l})},$$
$$\mu_{\mathfrak{B}}(\mathfrak{l}) = \mathsf{q}_{\mathfrak{B}}(\mathfrak{l})\mathsf{e}^{\mathsf{i}\theta_2(\mathfrak{l})},$$
$$\varphi_{\mathfrak{B}}(\mathfrak{l}) = \mathsf{r}_{\mathfrak{B}}(\mathfrak{l})\mathsf{e}^{\mathsf{i}\theta_3(\mathfrak{l})},$$

with $\mathsf{p}_{\mathfrak{B}}(\mathfrak{l}), \mathsf{q}_{\mathfrak{B}}(\mathfrak{l}), \mathsf{r}_{\mathfrak{B}}(\mathfrak{l}) \in [0, \Omega]$ *and* $\theta_1(\mathfrak{l}), \theta_2(\mathfrak{l}), \theta_3(\mathfrak{l}) \in [0, 1]$.

Definition 3.2. (Neutrosophic Over Complex Null and Universal Sets)

Let $\varkappa$ be an non-empty set and $\mathfrak{I}$ be a set of parameter on $\varkappa$. Then a $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set is said to be a $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-null set and a $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$- universal set then it is defined as, $\omega = \{\langle \natural, 0, 0, \Omega\rangle : \natural \in \varkappa\}$ and $\varpi = \{\langle \natural, \Omega, \Omega, 0\rangle : \natural \in \varkappa\}$.

Definition 3.3. (Neutrosophic Over Complex Subset and Superset)

Let φ_g and φ be any two $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets. If φ_g is said to be a subset of φ or φ is an super set of φ_g i.e., $\varphi_g \subseteq \varphi$ or $\varphi \supseteq \varphi_g$ then,

$$\wp_{\varphi_g}(\natural) \leq \wp_{\varphi}(\natural), \eth_{\varphi_g}(\natural) \leq \eth_{\varphi}(\natural), \flat_{\varphi_g}(\natural) \geq \flat_{\varphi}(\natural).$$

Note: Similarly, If $\varphi_g \subset \varphi$ and $\varphi \subset \varphi_g$ then $\varphi_g = \varphi$.

Definition 3.4. (Neutrosophic Over Complex Set Operations)

Let φ_g and φ be any two $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets. Then the union, intersection and compliment are defined by

(i) Union:

$$\varphi_g \mho \varphi = \{\langle \natural, max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\rangle : \natural \in \varkappa\}.$$

(ii) Intersection:

$$\varphi_g \Omega \varphi = \{\langle \natural, min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\rangle : \natural \in \varkappa\}.$$

(iii) Compliment:

$$\varphi_g^{\complement} = \{\langle \natural, \flat_{\varphi_g}(\natural), \Omega - \eth_{\varphi_g}(\natural), \wp_{\varphi_g}(\natural)\rangle : \natural \in \varkappa\}.$$

Proposition 3.1. Let φ_g and φ be any two $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets. Then

(i) $\varphi_g \mho (\varphi_g \Omega \varphi) = \varphi_g$

(ii) $\varphi_g \Omega (\varphi_g \mho \varphi) = \varphi_g$

Proof. (i) For each $\natural \in \varkappa$,

$$\max\{\wp_{\varphi_g}(\natural), \min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\} = \wp_{\varphi_g}(\natural),$$

$$\max\{\eth_{\varphi_g}(\natural), \min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\} = \eth_{\varphi_g}(\natural),$$

and

$$\min\{\flat_{\varphi_g}(\natural), \max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\} = \flat_{\varphi_g}(\natural).$$

Hence,

$$\varphi_g \mho (\varphi_g \Omega \varphi) = \varphi_g.$$

(ii) For each $\natural \in \varkappa$,

$$\min\{\wp_{\varphi_g}(\natural), \max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\} = \wp_{\varphi_g}(\natural),$$

$$\min\{\eth_{\varphi_g}(\natural), \max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\} = \eth_{\varphi_g}(\natural),$$

and

$$\max\{\flat_{\varphi_g}(\natural), \min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\} = \flat_{\varphi_g}(\natural).$$

Therefore,

$$\varphi_g \Omega (\varphi_g \mho \varphi) = \varphi_g.$$

□

Proposition 3.2. Let $\varphi_g, \varphi, \varphi$ be any three $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets on $\varkappa$. Then

(i) $(\varphi_g \mho \varphi) \mho \varphi = \varphi_g \mho (\varphi \mho \varphi)$

(ii) $(\varphi_g \Omega \varphi) \Omega \varphi = \varphi_g \Omega (\varphi \Omega \varphi)$

Proof. (i) For each $\natural \in \varkappa$,

$$\max\{\max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), \wp_{\varphi}(\natural)\} = \max\{\wp_{\varphi_g}(\natural), \max(\wp_{\varphi}(\natural), \wp_{\varphi}(\natural))\},$$

$$\max\{\max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), \eth_{\varphi}(\natural)\} = \max\{\eth_{\varphi_g}(\natural), \max(\eth_{\varphi}(\natural), \eth_{\varphi}(\natural))\},$$

and

$$\min\{\min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural)), \flat_{\varphi}(\natural)\} = \min\{\flat_{\varphi_g}(\natural), \min(\flat_{\varphi}(\natural), \flat_{\varphi}(\natural))\}.$$

Hence,

$$(\varphi_g \mho \varphi) \mho \varphi = \varphi_g \mho (\varphi \mho \varphi).$$

(ii) For each $\natural \in \varkappa$,

$$\min\{\min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), \wp_{\varphi}(\natural)\} = \min\{\wp_{\varphi_g}(\natural), \min(\wp_{\varphi}(\natural), \wp_{\varphi}(\natural))\},$$

$$\min\{\min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), \eth_{\varphi}(\natural)\} = \min\{\eth_{\varphi_g}(\natural), \min(\eth_{\varphi}(\natural), \eth_{\varphi}(\natural))\},$$

and

$$\max\{\max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural)), \flat_{\varphi}(\natural)\} = \max\{\flat_{\varphi_g}(\natural), \max(\flat_{\varphi}(\natural), \flat_{\varphi}(\natural))\}.$$

Thus,

$$(\varphi_g \Omega \varphi) \Omega \varphi = \varphi_g \Omega (\varphi \Omega \varphi).$$

□

Lemma 3.1. Let φ_g and φ be two $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets such that

$$\varphi_g \subseteq \varphi.$$

Then

$$\varphi_g \Omega \varphi = \varphi_g$$

and

$$\varphi_g \mho \varphi = \varphi.$$

Proof. (i) For each $\natural \in \varkappa$,

$$\max\{\wp_{\varphi_g}(\natural), \min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\} = \wp_{\varphi_g}(\natural),$$

$$\max\{\eth_{\varphi_g}(\natural), \min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\} = \eth_{\varphi_g}(\natural),$$

and

$$\min\{\flat_{\varphi_g}(\natural), \max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\} = \flat_{\varphi_g}(\natural).$$

Hence,

$$\varphi_g \mho (\varphi_g \Omega \varphi) = \varphi_g.$$

(ii) Similarly,

$$\min\{\wp_{\varphi_g}(\natural), \max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\} = \wp_{\varphi_g}(\natural),$$

$$\min\{\eth_{\varphi_g}(\natural), \max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\} = \eth_{\varphi_g}(\natural),$$

and

$$\max\{\flat_{\varphi_g}(\natural), \min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\} = \flat_{\varphi_g}(\natural).$$

Therefore,

$$\varphi_g \Omega (\varphi_g \mho \varphi) = \varphi_g.$$

□

Proposition 3.3. Let φ_g and φ be any two $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets. If

$$\varphi_g \subseteq \varphi,$$

then

$$\varphi^{☾} \subseteq \varphi_g^{☾}.$$

Proof. Since

$$\varphi_g \subseteq \varphi,$$

we have for each $\natural \in \varkappa$,

$$\wp_{\varphi_g}(\natural) \leq \wp_{\varphi}(\natural),$$

$$\eth_{\varphi_g}(\natural) \leq \eth_{\varphi}(\natural),$$

and

$$\flat_{\varphi_g}(\natural) \geq \flat_{\varphi}(\natural).$$

Therefore,

$$\flat_{\varphi}(\natural) \le \flat_{\varphi_g}(\natural),$$
$$\Omega - \eth_{\varphi}(\natural) \le \Omega - \eth_{\varphi_g}(\natural),$$

and

$$\wp_{\varphi}(\natural) \ge \wp_{\varphi_g}(\natural).$$

Hence,

$$\varphi^{☾} \subseteq \varphi_g^{☾}.$$

□

Theorem 3.1. Let $\varphi_g, \varphi, \varphi$ be $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets. Then

(i) $\varphi_g \mho (\varphi \Omega \varphi) = (\varphi_g \mho \varphi) \Omega (\varphi_g \mho \varphi)$

(ii) $\varphi_g \Omega (\varphi \mho \varphi) = (\varphi_g \Omega \varphi) \mho (\varphi_g \Omega \varphi)$

Proof. (i) For each $\natural \in \varkappa$,

$$\max\{\wp_{\varphi_g}(\natural), \min(\wp_{\varphi}(\natural), \wp_{\varphi}(\natural))\}$$
$$= \min\{\max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), \max(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\},$$

$$\max\{\eth_{\varphi_g}(\natural), \min(\eth_{\varphi}(\natural), \eth_{\varphi}(\natural))\}$$
$$= \min\{\max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), \max(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\},$$

and

$$\min\{\flat_{\varphi_g}(\natural), \max(\flat_{\varphi}(\natural), \flat_{\varphi}(\natural))\}$$
$$= \max\{\min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural)), \min(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\}.$$

Hence,

$$\varphi_g \mho (\varphi \Omega \varphi) = (\varphi_g \mho \varphi) \Omega (\varphi_g \mho \varphi).$$

(ii) For each $\natural \in \varkappa$,

$$\min\{\wp_{\varphi_g}(\natural), \max(\wp_{\varphi}(\natural), \wp_{\varphi}(\natural))\}$$
$$= \max\{\min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural)), \min(\wp_{\varphi_g}(\natural), \wp_{\varphi}(\natural))\},$$

$$\min\{\eth_{\varphi_g}(\natural), \max(\eth_{\varphi}(\natural), \eth_{\varphi}(\natural))\}$$
$$= \max\{\min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural)), \min(\eth_{\varphi_g}(\natural), \eth_{\varphi}(\natural))\},$$

and

$$\max\{\flat_{\varphi_g}(\natural), \min(\flat_{\varphi}(\natural), \flat_{\varphi}(\natural))\}$$
$$= \min\{\max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural)), \max(\flat_{\varphi_g}(\natural), \flat_{\varphi}(\natural))\}.$$

Therefore,

$$\varphi_g \Omega (\varphi \mho \varphi) = (\varphi_g \Omega \varphi) \mho (\varphi_g \Omega \varphi).$$

□

4. Application of Neutrosophic Over Complex Sets in Medical Diagnosis

4.1. **Importance of the Application.** Medical diagnosis is one of the most important and at the same time tough topics in modern decision-making theory, because medical data quite often bring uncertainty, inconsistency, incompleteness, and also vagueness into the picture. In typical, day to day clinical environments physicians usually decide from laboratory reports, symptoms, medical imaging, and even their own past experience, but these sources may not really lead to crisp conclusions. Because of that, different medical specialists can easily end up with distinct views about the severity, the likelihood, or even the evolution of the same disease seen in the same patient. These differences, well they automatically inject indeterminate and conflicting information into the whole diagnostic routine. So, the standard mathematical tools and the classical set theoretic models end up feeling inadequate for dealing with medical information that is both highly uncertain and inconsistent in a reliable way.

In the last years neutrosophic theory has grown into a rather strong mathematical frame for representing uncertain and indeterminate material. Compared with fuzzy and intuitionistic fuzzy methods, neutrosophic sets treat the degree of truth-membership, the level of indeterminacy-membership, and the measure of falsity-membership at the same time, which makes the representation more adaptable and (in many cases) more realistic for medical observations. Still, in several real applications, especially in more advanced medical diagnosis, the data we have can show periodic, oscillatory, or phase-dependent traits. Under such conditions ordinary real-valued neutrosophic structures just cannot capture the behavior correctly. To address that, the idea of complex-valued neutrosophic information becomes necessary, meaning each neutrosophic part is written in complex form so it keeps both amplitude and phase terms, not only magnitude.

Furthermore, in some really severe medical cases, specialists might deliberately give assessments that are a bit over-valued, as a sort of signal that the illness is actually extreme, and not just at a normal level. For instance, when the patient condition turns out to be extremely critical, it is quite natural that certain membership degrees could go beyond the classical ceiling of one. However, this kind of information is kind of hard to fit inside the usual neutrosophic surroundings. So, the idea of neutrosophic over sets becomes quite helpful, because it lets at least one neutrosophic component rise above one, while the other components stay non-negative and still behave in a nice way. In practice, this over-valued clue adds extra maneuverability for describing very risky or emergency medical states.

The proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set model in a way merges the advantages of neutrosophic sets, complex-valued data, and over-valued environments into one mathematical space. More or less it works with uncertainty through indeterminacy-membership values, it manages inconsistency by letting truth and falsity coexist at the same time, it expresses periodic behavior via complex-valued formulas, and it keeps overestimated information through amplitudes that are larger than one. So in the end this overall framework looks pretty well matched for medical diagnosis problems that are quite complicated, especially if the available information is uncertain, internally contradictory, and also over-valued along with complex. That is why the $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set model ends up functioning as a more realistic adaptable, and efficient instrument for medical decisions and healthcare analysis, even when the environment isn't fully certain.

4.2. **Problem Statement.** Suppose a hospital wants to single out the most critical patient among ten patients dealing with a severe viral disease. But because the illness is complex, and the medical observations can be uncertain or sort of shifting, the hospital management asks two senior medical experts to review things. Each expert studies the patients using many clinical factors, like body temperature, oxygen saturation level, respiratory condition, blood test reports, infection severity, and the whole health status. Still, the disease being what it is, the expert opinions can end up not matching, even differ a lot. And sometimes what they say includes indeterminate bits, you know, unclear clinical notes that are not quite determinate.

Also, there can be specific observations that strongly suggest extremely serious conditions, so some of the assessment values go past the classical upper bound of one. In those cases, the over condition in the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-environment is automatically satisfied, more or less. Furthermore, since the medical data may show periodic or phase dependent changes, the information is written in a complex-valued manner, even if that makes it a little less intuitive at first.

Let

$$\varkappa = \{\natural_1, \natural_2, \natural_3, \natural_4, \natural_5, \natural_6, \natural_7, \natural_8, \natural_9, \natural_{10}\}$$

be the collection of patients.

Assume that

$$\Omega = 1.2.$$

Two medical experts provide their assessments in terms of $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-sets.

4.3. **Decision Matrix.**

$$\varphi_g = \begin{Bmatrix} \langle \natural_1, 1.08e^{i(0.20)}, 0.45e^{i(0.10)}, 0.30e^{i(0.15)} \rangle, \\ \langle \natural_2, 1.02e^{i(0.30)}, 0.55e^{i(0.20)}, 0.40e^{i(0.25)} \rangle, \\ \langle \natural_3, 1.10e^{i(0.40)}, 1.05e^{i(0.30)}, 0.20e^{i(0.10)} \rangle, \\ \langle \natural_4, 1.05e^{i(0.25)}, 0.50e^{i(0.15)}, 0.35e^{i(0.20)} \rangle, \\ \langle \natural_5, 1.01e^{i(0.20)}, 0.40e^{i(0.25)}, 0.55e^{i(0.30)} \rangle, \\ \langle \natural_6, 1.15e^{i(0.45)}, 1.08e^{i(0.35)}, 0.15e^{i(0.10)} \rangle, \\ \langle \natural_7, 1.04e^{i(0.28)}, 0.60e^{i(0.22)}, 0.32e^{i(0.18)} \rangle, \\ \langle \natural_8, 1.00e^{i(0.35)}, 1.02e^{i(0.25)}, 0.28e^{i(0.15)} \rangle, \\ \langle \natural_9, 1.06e^{i(0.18)}, 0.50e^{i(0.20)}, 0.60e^{i(0.35)} \rangle, \\ \langle \natural_{10}, 1.12e^{i(0.42)}, 1.10e^{i(0.32)}, 0.18e^{i(0.12)} \rangle \end{Bmatrix}$$

and

$$\varphi = \begin{Bmatrix} \langle \natural_1, 1.05e^{i(0.25)}, 0.50e^{i(0.15)}, 0.35e^{i(0.20)} \rangle, \\ \langle \natural_2, 1.10e^{i(0.35)}, 0.50e^{i(0.18)}, 0.38e^{i(0.20)} \rangle, \\ \langle \natural_3, 1.00e^{i(0.38)}, 1.08e^{i(0.28)}, 0.25e^{i(0.15)} \rangle, \\ \langle \natural_4, 1.12e^{i(0.32)}, 0.60e^{i(0.22)}, 0.25e^{i(0.16)} \rangle, \\ \langle \natural_5, 1.06e^{i(0.25)}, 0.45e^{i(0.30)}, 0.50e^{i(0.28)} \rangle, \\ \langle \natural_6, 1.10e^{i(0.40)}, 1.12e^{i(0.38)}, 0.20e^{i(0.12)} \rangle, \\ \langle \natural_7, 1.08e^{i(0.30)}, 0.58e^{i(0.24)}, 0.30e^{i(0.20)} \rangle, \\ \langle \natural_8, 1.15e^{i(0.33)}, 1.05e^{i(0.27)}, 0.30e^{i(0.18)} \rangle, \\ \langle \natural_9, 1.02e^{i(0.20)}, 0.55e^{i(0.25)}, 0.58e^{i(0.32)} \rangle, \\ \langle \natural_{10}, 1.18e^{i(0.48)}, 1.15e^{i(0.35)}, 0.15e^{i(0.10)} \rangle \end{Bmatrix}$$

The above decision matrices represent the medical assessments provided by two different medical experts in the form of $\mathcal{N}_\mathfrak{c}^{\mathfrak{o}}$-sets. Here, φ_g and φ denote the evaluations of Expert 1 and Expert 2, respectively. Each patient is represented by three neutrosophic over complex components corresponding to truth-membership, indeterminacy-membership and falsity-membership values together with their associated phase terms.

It is observed that several amplitude values exceed one, thereby satisfying the over condition of the proposed $\mathcal{N}_\mathfrak{c}^{\mathfrak{o}}$-environment. Furthermore, the presence of complex-valued terms of the form

$$ae^{i\theta}$$

captures the phase-dependent and oscillatory nature of medical information. Therefore, the proposed decision matrices effectively model uncertain, inconsistent and over-valued complex medical observations obtained from multiple experts.

4.4. **Aggregated Table Using Union Operation.** Using

$$\varphi_g \mho \varphi,$$

we obtain the following aggregated values.

Table 1 presents the aggregated $\mathcal{N}_\mathfrak{c}^{\mathfrak{o}}$-values obtained using the union operation $\varphi_g \mho \varphi$. The aggregation process combines the medical evaluations of both experts by selecting the maximum truth-membership values, maximum indeterminacy-membership values and minimum falsity-membership values for each patient. The obtained aggregated values therefore represent the overall medical condition of each patient under the proposed neutrosophic over complex environment.

It is observed that several truth-membership and indeterminacy-membership amplitudes exceed one, thereby satisfying the over condition of the $\mathcal{N}_\mathfrak{c}^{\mathfrak{o}}$-set framework. Moreover, the complex-valued terms involving phase components effectively represent phase-dependent and uncertain

TABLE 1. Aggregated $\mathcal{N}_{\mathfrak{c}}^{\mathrm{o}}$-Values

Patients	Aggregated Value
$\natural_1$	$(1.08e^{i(0.20)}, 0.50e^{i(0.15)}, 0.30e^{i(0.15)})$
$\natural_2$	$(1.10e^{i(0.35)}, 0.55e^{i(0.20)}, 0.38e^{i(0.20)})$
$\natural_3$	$(1.10e^{i(0.40)}, 1.08e^{i(0.30)}, 0.20e^{i(0.10)})$
$\natural_4$	$(1.12e^{i(0.32)}, 0.60e^{i(0.22)}, 0.25e^{i(0.16)})$
$\natural_5$	$(1.06e^{i(0.25)}, 0.45e^{i(0.30)}, 0.50e^{i(0.28)})$
$\natural_6$	$(1.15e^{i(0.45)}, 1.12e^{i(0.38)}, 0.15e^{i(0.10)})$
$\natural_7$	$(1.08e^{i(0.30)}, 0.60e^{i(0.24)}, 0.30e^{i(0.18)})$
$\natural_8$	$(1.15e^{i(0.33)}, 1.05e^{i(0.27)}, 0.28e^{i(0.15)})$
$\natural_9$	$(1.06e^{i(0.18)}, 0.55e^{i(0.25)}, 0.58e^{i(0.32)})$
$\natural_{10}$	$(1.18e^{i(0.48)}, 1.15e^{i(0.35)}, 0.15e^{i(0.10)})$

medical information. From the aggregated table, patients $\natural_{10}$, $\natural_6$ and $\natural_3$ possess comparatively larger truth and indeterminacy values together with smaller falsity values, indicating that these patients are more critical than the remaining patients.

4.5. **Score Function.** Define the score function by

$$\mathcal{S}(\natural_i) = \frac{\wp(\natural_i) + \eth(\natural_i) - \flat(\natural_i)}{3}.$$

TABLE 2. Score Values and Ranking

Patients	Score Value	Rank
$\natural_1$	0.4267	9
$\natural_2$	0.4233	10
$\natural_3$	0.6600	3
$\natural_4$	0.4900	6
$\natural_5$	0.3367	8
$\natural_6$	0.7067	1
$\natural_7$	0.4600	7
$\natural_8$	0.6400	4
$\natural_9$	0.3433	5
$\natural_{10}$	0.7267	2

4.6. **Solution Table.** Table 2 shows the score values and the matching ranking order of every patient, which are got from the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathrm{o}}$-set based medical diagnosis model. The score values, are used for checking the severity level for each patient by looking at truth-membership indeterminacy-membership and falsity-membership values at the same time, so in practice all of them matter together. If the score is larger then it means the disease is more severe, in a general sense.

From the obtained results, it seems that patient $\natural_{10}$ gets the top score value, and then come patients $\natural_6$ and $\natural_3$. So this implies they have relatively higher truth-membership plus indeterminacy-membership at once, with falsity-membership values that are comparatively smaller. Therefore, these few patients are tagged as the most critical ones among the patient collection that was actually considered.

Also, patients $\natural_5$ and $\natural_9$ show comparatively smaller score values, meaning their medical conditions are less severe with respect to the remaining patients. Hence the final ranking order, works well in separating the severity level of all patients inside the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set based medical diagnosis framework.

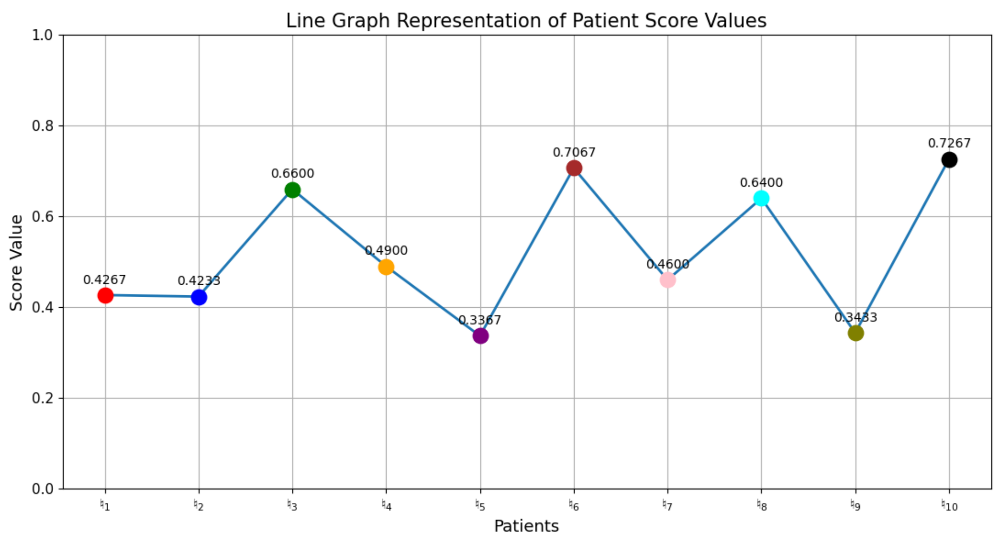

FIGURE 1. Line graph representation of patient score values

Figure 1 illustrates the line graph representation of the obtained score values for all patients. Different colours are used for each point in order to distinguish the patients clearly. It is observed that patient $\natural_{10}$ attains the highest score value among all patients, followed by patients $\natural_6$ and $\natural_3$. Hence, the graphical representation confirms that patient $\natural_{10}$ is the most critical patient according to the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set based medical diagnosis model.

4.7. **Final Ranking.** The ranking order of the patients is

$$\natural_{10} > \natural_6 > \natural_3 > \natural_8 > \natural_4 > \natural_7 > \natural_1 > \natural_2 > \natural_9 > \natural_5.$$

4.8. **Result and Discussion.** From the obtained score values, it is observed that

$$\mathcal{S}(\natural_{10}) = 0.7267$$

is the highest, among all patients. Hence, patient $\natural_{10}$ is identified as the most critical patient and should be provided immediate medical treatment, as well as intensive healthcare support. The obtained ranking order in fact shows the severity level of all patients based on the aggregated medical evaluations given by the experts.

The proposed $\mathcal{N}_c^o$-set model, seems to effectively deal with uncertain, inconsistent, and over-valued complex medical information during the decision-making process. The indeterminacy-membership values successfully express the uncertainty that comes from expert opinions, while the simultaneous presence of truth-membership and falsity-membership values reflects inconsistent medical observations. Also, the complex-valued representation, with amplitude and phase terms, gives a flexible mathematical structure for handling periodic, and phase-dependent medical information. Additionally, the over condition allows certain neutrosophic components to exceed one, thus representing highly severe and overestimated medical situations, in a more realistic manner.

Therefore, the proposed $\mathcal{N}_c^o$-set framework provides a kind of reliable and efficient mathematical route for solving advanced medical diagnosis problems, under uncertain and over complex environments. The developed model can also be carried over to several real-world decision-making tasks, like risk analysis, pattern recognition, artificial intelligence, healthcare management, and engineering systems.

5. ANN Based Application of Neutrosophic Over Complex Sets in Medical Diagnosis

Artificial Neural Networks are powerful computational tools that get widely used for classification, prediction, optimization and decision making problem, you know. ANN models are inspired by the structure and way the human brain works, in the sense that interconnected neurons process information and they learn hidden relationships from the data that is actually available. Because they have this adaptive learning capability, ANN techniques have gained considerable importance in many real world settings like medical diagnosis, pattern recognition, image processing, artificial intelligence and healthcare management.

Now, in medical diagnosis problems, ANN models can be very effective since medical information often carries uncertainty, inconsistency, and also incomplete observations. Physicians normally look at laboratory reports, clinical symptoms, medical imaging and patient history, before making diagnostic decisions. But the connections among these medical factors are frequently nonlinear, and they are kind of hard to model using classical mathematical techniques. ANN models deal with that issue by learning the underlying relationship between input medical information and output severity levels through an iterative training procedure, kinda like that.

The proposed $\mathcal{N}_c^o$-set framework includes truth-membership, indeterminacy-membership, and falsity-membership values, along with over-valued and complex information. These neutrosophic parts end up representing uncertain, contradictory medical observations in a rather direct way. So then, the corresponding truth-membership, indeterminacy-membership and falsity-membership values can be treated as input neurons inside an ANN architecture. After that, the ANN model learns the mapping between these neutrosophic over complex values, and the patient severity level that comes out.

In this section, the same medical diagnosis problem is solved using an ANN based approach in order to plug the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set model with artificial intelligence techniques. The major objective is to check the reliability and consistency of the proposed mathematical framework by contrasting the ANN output with the score function based ranking method. If both approaches end up producing the same ranking order, then it indicates that the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set model is mathematically steady and computationally suitable for more advanced medical decision-making problems, under uncertain and over complex environments.

5.1. **ANN Architecture.** The proposed ANN model, sort of consists of one input layer, one hidden layer and one output layer. The first layer contains three neurons that match the aggregated neutrosophic over complex values:

$$\wp(\natural_i), \quad \eth(\natural_i), \quad \flat(\natural_i).$$

The ANN setup is basically shown as a 3-5-1 kind of scheme, i guess, where you have three input neurons, five middle or hidden neurons, and one lone output neuron, in the end.

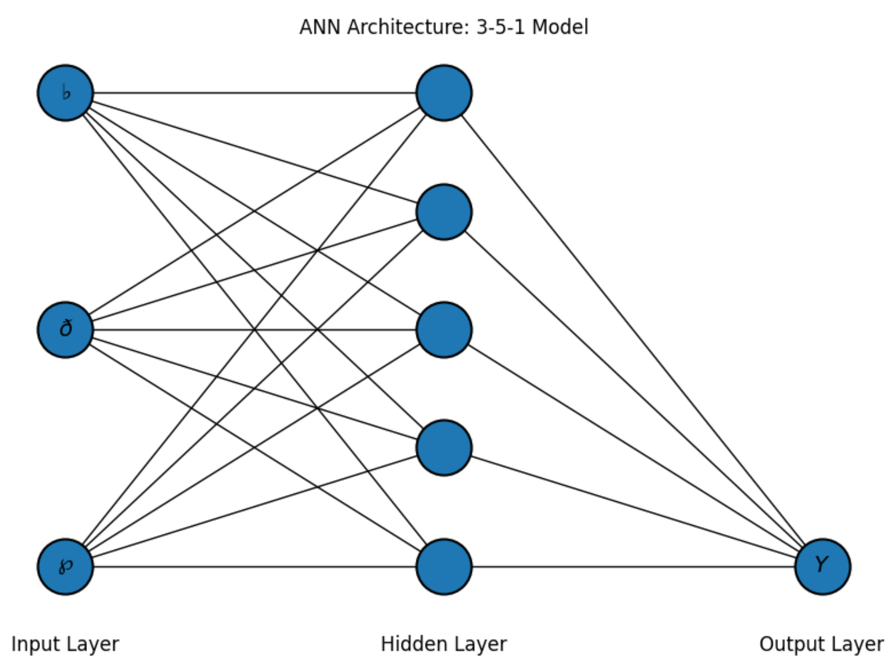

Figure 2. Architecture of the proposed ANN model for medical diagnosis

Figure 2 shows the whole setup of the suggested ANN model, or at least how I understand it. There are three input neurons that match the truth membership, indeterminacy membership and falsity membership values coming from the aggregated $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set information. These values then go straight into the hidden layer, where there are five neurons, nothing fancy. After that, the output layer gives back the severity score for every patient.

The activation function used in the hidden layer is the sigmoid activation function defined by

$$f(x) = \frac{1}{1+e^{-x}}.$$

The output layer uses a linear activation function to obtain the final severity value.

5.2. **Training Parameters.** The ANN model is trained using the following parameters.

Table 3. ANN Training Parameters

Parameter	Value
Number of input neurons	3
Number of hidden neurons	5
Number of output neurons	1
Activation function in hidden layer	Sigmoid
Activation function in output layer	Linear
Learning rate	0.01
Number of epochs	1000
Training algorithm	Back propagation
Error function	Mean square error

Table 3 shows the parameters used during the ANN training process. The learning rate is what sets the pace of weight adjustment during back propagation, while the epoch count is in charge of the total number of training cycles, that the network actually runs.

5.3. **Input Data for ANN.** The aggregated values obtained from the union operation are used as ANN input data.

Table 4. ANN Input Data

Patients	$\wp(\natural_i)$	$\eth(\natural_i)$	$\flat(\natural_i)$
$\natural_1$	1.08	0.50	0.30
$\natural_2$	1.10	0.55	0.38
$\natural_3$	1.10	1.08	0.20
$\natural_4$	1.12	0.60	0.25
$\natural_5$	1.06	0.45	0.50
$\natural_6$	1.15	1.12	0.15
$\natural_7$	1.08	0.60	0.30
$\natural_8$	1.15	1.05	0.28
$\natural_9$	1.06	0.55	0.58
$\natural_{10}$	1.18	1.15	0.15

Table 4 contains the ANN input values obtained from the aggregated neutrosophic over complex information. Since several truth-membership and indeterminacy-membership values exceed one, the over condition of the proposed model is properly satisfied.

5.4. **Target Output.** The target output is obtained using the score function defined by

$$\mathcal{S}(\natural_i) = \frac{\wp(\natural_i) + \eth(\natural_i) - \flat(\natural_i)}{3}.$$

The target values used during ANN training are shown in Table 5.

Table 5. Target Output for ANN

Patients	Target Score
$\natural_1$	0.4267
$\natural_2$	0.4233
$\natural_3$	0.6600
$\natural_4$	0.4900
$\natural_5$	0.3367
$\natural_6$	0.7067
$\natural_7$	0.4600
$\natural_8$	0.6400
$\natural_9$	0.3433
$\natural_{10}$	0.7267

5.5. **Training Performance of ANN.** The ANN model was trained for 1000 epochs using the back propagation algorithm. During training, the mean square error gradually decreased and the ANN output converged toward the target values.

Figure 3 shows the convergence behavior of the ANN model during training. It can be observed that the error decreases continuously with increasing epochs, indicating that the network successfully learns the relationship between the neutrosophic over complex input values and the patient severity scores.

5.6. **ANN Output.** After completion of training, the ANN generated the following output values.

Table 6 presents the ANN output values together with the final ranking order of the patients. It is observed that the ANN model produces the same ranking obtained from the proposed score function method.

Figure 4 graphically represents the ranking order of the patients according to the ANN output scores. It is clearly observed that patient $\natural_{10}$ attains the highest severity score, followed by patients $\natural_6$ and $\natural_3$.

5.7. **Final Ranking.** The final ranking obtained from the ANN model is

$$\natural_{10} > \natural_6 > \natural_3 > \natural_8 > \natural_4 > \natural_7 > \natural_1 > \natural_2 > \natural_9 > \natural_5.$$

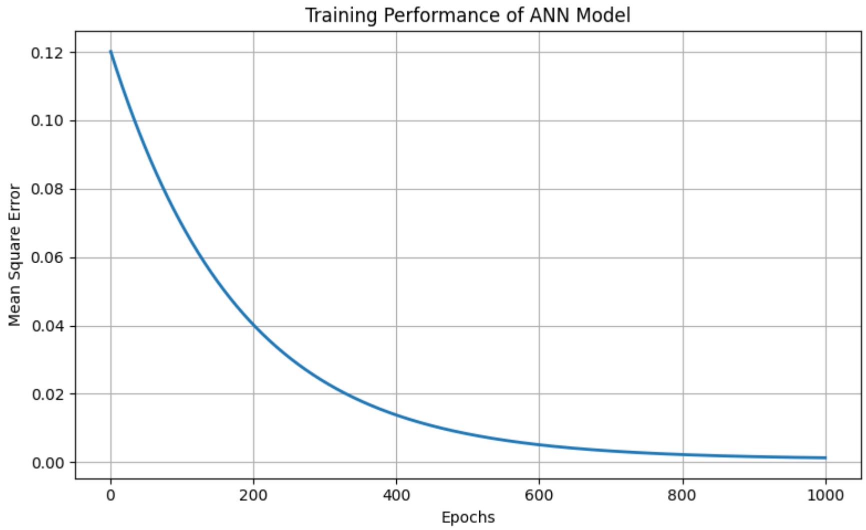

Figure 3. Training performance of the ANN model

Table 6. ANN Output Values and Ranking

Patients	ANN Output	Rank
$\natural_1$	0.4267	7
$\natural_2$	0.4233	8
$\natural_3$	0.6600	3
$\natural_4$	0.4900	5
$\natural_5$	0.3367	10
$\natural_6$	0.7067	2
$\natural_7$	0.4600	6
$\natural_8$	0.6400	4
$\natural_9$	0.3433	9
$\natural_{10}$	0.7267	1

5.8. **Result and Discussion.** From the obtained ANN output values, it is observed that

$$\Upsilon(\natural_{10}) = 0.7267$$

is the highest among all patients. Therefore, patient $\natural_{10}$ is identified as the most critical patient and should receive immediate medical treatment.

The ANN-based approach successfully produces the same ranking order as the proposed $\mathcal{N}_{\mathfrak{c}}^{\mathfrak{o}}$-set score function method. This confirms the consistency and reliability of the proposed framework. The ANN model effectively learns the relationship between truth-membership, indeterminacy-membership and falsity-membership values and generates appropriate severity scores for medical diagnosis.

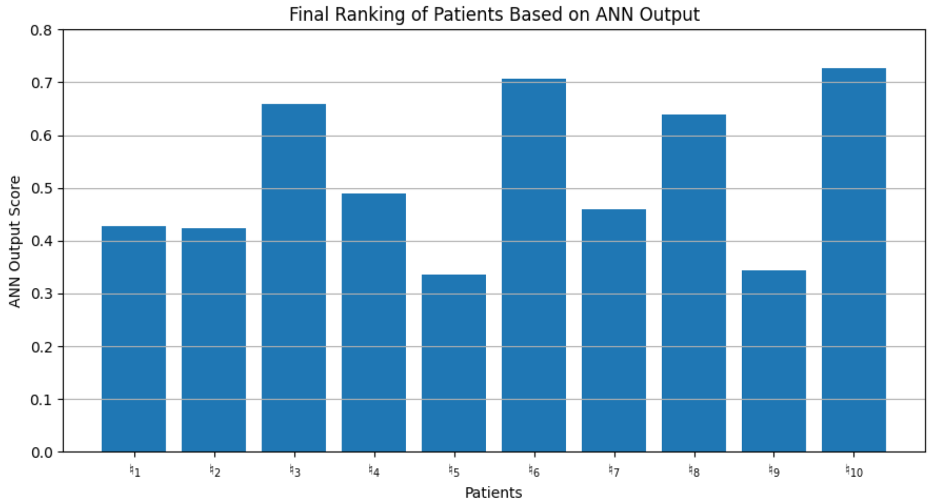

Figure 4. Final ranking of patients based on ANN output values

The proposed ANN integrated $\mathcal{N}_c^o$ framework therefore provides an efficient mathematical and artificial intelligence based approach for solving complex medical decision-making problems involving uncertainty, inconsistency, over-information and complex-valued data. "'

6. Conclusion

In this article, the idea of Neutrosophic Over Complex Sets has been kinda introduced and later investigated, under uncertain over complex environments. The proposed setup really manages to merge the traits of neutrosophic sets, over sets, and complex-valued information, into one generalized mathematical model. Basic procedures like union, intersection and complement, plus a few algebraic properties have been derived and then checked mathematically, step by step. For the practical side, the framework is shown through a medical diagnosis task. There are multiple experts involved, and also multiple patients. The outcomes suggest that the developed model can manage uncertainty, inconsistency, phase related information and over-valued observations all at once, without too much hassle. Also the score-function based ranking approach is able to spot the most critical patient inside the given patient collection. Additionally, an Artificial Neural Network based strategy has been used to corroborate the mathematical findings computationally. The ANN scheme gives the same ordering as the theoretical framework, which basically confirms reliability, consistency and computational efficiency for the proposed model. So, overall the Neutrosophic Over Complex Set framework comes as a flexible, efficient, realistic mathematical tool for higher level decision-making tasks, in uncertain over complex settings. In later studies, the same model might be broadened to different kinds of use cases like pattern recognition, artificial intelligence, image processing, engineering optimization, cybersecurity, risk evaluation, and intelligent healthcare systems.

Conflicts of Interest: The authors declare that there are no conflicts of interest regarding the publication of this paper.

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