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(δ,γ)-JACOBI-DUNKL LIPSCHITZ FUNCTIONS IN THE SPACE $L^2(\mathbb{R}, A_{\alpha,\beta}(x)dx)$

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Abstract. Using a generalized Jacobi-Dunkl translation, we obtain an analog of Theorem 5.2 in Younis paper [7] for the Jacobi-Dunkl transform for functions satisfying the (δ,γ)-Jacobi-Dunkl Lipschitz condition in the space $L^2(\mathbb{R}, A_{\alpha,\beta}(x)dx), \alpha \geq \beta \geq \frac{-1}{2}, \alpha \neq \frac{-1}{2}$.

1. Introduction and Preliminaries

Younis ([7], Theorem 5.2) characterized the set of functions in $L^2(\mathbb{R})$ satisfying the Dini-Lipschitz condition by means of an asymptotic estimate growth of the norm of their Fourier transforms, namely we have the following statement.

Theorem 1.1. [7] Let $f \in L^2(\mathbb{R})$. Then the following are equivalents:

(a) $\|f(x+h) - f(x)\| = O\left(\frac{h^\eta}{(\log\frac{1}{h})^\gamma}\right)$, as $h \to 0, 0 < \eta < 1, \gamma > 0$

(b) $\int_{|\lambda|\geq r} |\widehat{f}(\lambda)|^2 d\lambda = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right)$, as $r \to \infty$,

where $\widehat{f}$ stand for the Fourier transform of f.

In this paper, we obtain an analog of Theorem 1.1 for the Jacobi-Dunkl transform on the real line. For this purpose, we use a generalized Jacobi-Dunkl translation operator.

In this section , we recapitulate from [1,2,3,5] some results related to the harmonic analysis associated with Jacobi-Dunkl operator $\Lambda_{\alpha,\beta}$.

The Jacobi-Dunkl function with parameters $(\alpha,\beta), \alpha \geq \beta \geq \frac{-1}{2}, \alpha \neq \frac{-1}{2}$, defined by the formula

$$\forall x \in \mathbb{R}, \psi_\lambda^{\alpha,\beta}(x) = \begin{cases} \varphi_\mu^{\alpha,\beta}(x) - \frac{i}{\lambda}\frac{d}{dx}\varphi_\mu^{\alpha,\beta}(x) & \text{if } \lambda \in \mathbb{C}\backslash\{0\} \\ 1 & \text{if } \lambda = 0, \end{cases}$$

with $\lambda^2 = \mu^2 + \rho^2$, $\rho = \alpha + \beta + 1$ and $\varphi_\mu^{\alpha,\beta}$ is the Jacobi function given by

$$\varphi_\mu^{\alpha,\beta}(x) = F\left(\frac{\rho + i\mu}{2}, \frac{\rho - i\mu}{2}, \alpha + 1, -(\sinh(x))^2\right),$$

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F is the Gausse hypergeometric function (see [1,6]).
$\psi_\lambda^{\alpha,\beta}$ is the unique C^∞-solution on $\mathbb{R}$ of the differential-difference equation

$$\begin{cases} \Lambda_{\alpha,\beta}\mathcal{U} = i\lambda\mathcal{U} \quad , \lambda \in \mathbb{C} \\ \\ \mathcal{U}(0) = 1, \end{cases}$$

where $\Lambda_{\alpha,\beta}$ is the Jacobi-Dunkl operator given by

$$\Lambda_{\alpha,\beta}\mathcal{U}(x) = \frac{d\mathcal{U}(x)}{dx} + [(2\alpha+1)\coth x + (2\beta+1)\tanh x] \times \frac{\mathcal{U}(x) - \mathcal{U}(-x)}{2}].$$

The operator $\Lambda_{\alpha,\beta}$ is a particular case of the operator D given by

$$D\mathcal{U}(x) = \frac{d\mathcal{U}(x)}{dx} + \frac{A'(x)}{A(x)} \times \left(\frac{\mathcal{U}(x) - \mathcal{U}(-x)}{2}\right),$$

where $A(x) = |x|^{2\alpha+1}B(x)$ and B a function of class C^∞ on $\mathbb{R}$, even and positive. The operator $\Lambda_{\alpha,\beta}$ corresponds to the function

$$A(x) = A_{\alpha,\beta}(x) = 2^\rho(\sinh|x|)^{2\alpha+1}(\cosh|x|)^{2\beta+1}.$$

Using the relation

$$\frac{d}{dx}\varphi_\mu^{\alpha,\beta}(x) = -\frac{\mu^2+\rho^2}{4(\alpha+1)}\sinh(2x)\varphi_\mu^{\alpha+1,\beta+1}(x),$$

the function $\psi_\lambda^{\alpha,\beta}$ can be written in the form above (see [2])

$$\psi_\lambda^{\alpha,\beta}(x) = \varphi_\mu^{\alpha,\beta}(x) + i\frac{\lambda}{4(\alpha+1)}\sinh(2x)\varphi_\mu^{\alpha+1,\beta+1}(x), \quad x \in \mathbb{R}.$$

Denote $L^2_{\alpha,\beta}(\mathbb{R}) = L^2_{\alpha,\beta}(\mathbb{R}, A_{\alpha,\beta}(x)dx)$ the space of measurable functions f on $\mathbb{R}$ such that

$$\|f\|_{L^2_{\alpha,\beta}(\mathbb{R})} = \left(\int_{\mathbb{R}} |f(x)|^2 A_{\alpha,\beta}(x)dx\right)^{1/2} < +\infty.$$

Using the eigenfunctions $\psi_\lambda^{\alpha,\beta}$ of the operator $\Lambda_{\alpha,\beta}$ called the Jacobi-Dunkl kernels , we define the Jacobi-Dunkl transform of a function $f \in L^2_{\alpha,\beta}(\mathbb{R})$ by

$$\mathcal{F}_{\alpha,\beta}f(\lambda) = \int_{\mathbb{R}} f(x)\psi_\lambda^{\alpha,\beta}(x)A_{\alpha,\beta}(x)dx, \quad \lambda \in \mathbb{R},$$

and the inversion formula

$$f(x) = \int_{\mathbb{R}} \mathcal{F}_{\alpha,\beta}f(\lambda)\psi_{-\lambda}^{\alpha,\beta}(x)d\sigma(\lambda),$$

where

$$d\sigma(\lambda) = \frac{|\lambda|}{8\pi\sqrt{\lambda^2-\rho^2}|C_{\alpha,\beta}(\sqrt{\lambda^2-\rho^2})|}\mathbb{I}_{\mathbb{R}\backslash]-\rho,\rho[}(\lambda)d\lambda.$$

Here,

$$C_{\alpha,\beta}(\mu) = \frac{2^{\rho-i\mu}\Gamma(\alpha+1)\Gamma(i\mu)}{\Gamma(\frac{1}{2}(\rho+i\mu))\Gamma(\frac{1}{2}(\alpha-\beta+1+i\mu))}, \quad \mu \in \mathbb{C}\backslash(i\mathbb{N})$$

and $\mathbb{I}_{\mathbb{R}\backslash]-\rho,\rho[}$ is the characteristic function of $\mathbb{R}\backslash]-\rho,\rho[$.
The Jacobi-Dunkl transform is a unitary isomorphism from $L^2_{\alpha,\beta}(\mathbb{R})$ onto $L^2(\mathbb{R}, d\sigma(\lambda))$, i.e.

$$\|f\| := \|f\|_{L^2_{\alpha,\beta}(\mathbb{R})} = \|\mathcal{F}_{\alpha,\beta}(f)\|_{L^2(\mathbb{R},d\sigma(\lambda))}. \tag{1}$$

The operator of Jacobi-Dunkl translation is defined by

$$T_x f(y) = \int_{\mathbb{R}} f(z) d\nu_{x,y}^{\alpha,\beta}(z), \quad \forall x, y \in \mathbb{R}$$

where $\nu_{x,y}^{\alpha,\beta}(z), x, y \in \mathbb{R}$ are the signed measures given by

$$d\nu_{x,y}^{\alpha,\beta}(z) = \begin{cases} K_{\alpha,\beta}(x,y,z)A_{\alpha,\beta}(z)dz & \text{if } x, y \in \mathbb{R}^* \\ \\ \delta_x & \text{if } y = 0 \\ \delta_y & \text{if } x = 0 \end{cases}$$

Here, δ_x is the Dirac measure at x. And,

$$\begin{aligned} K_{\alpha,\beta}(x,y,z) &= M_{\alpha,\beta}(\sinh(|x|)\sinh(|y|)\sinh(|z|))^{-2\alpha}\mathbb{I}_{I_{x,y}} \times \int_0^\pi \rho_\theta(x,y,z) \\ &\times \ (g_\theta(x,y,z))_+^{\alpha-\beta-1}\sin^{2\beta}\theta d\theta \\ I_{x,y} &= [-|x|-|y|, -||x|-|y||] \cup [||x|-|y||, |x|+|y|] \\ \rho_\theta(x,y,z) &= 1 - \sigma_{x,y,z}^\theta + \sigma_{z,x,y}^\theta + \sigma_{z,y,x}^\theta \end{aligned}$$

$$\forall z \in \mathbb{R}, \theta \in [0,\pi], \sigma_{x,y,z}^\theta = \begin{cases} \frac{\cosh(x)+\cosh(y)-\cosh(z)\cos(\theta)}{\sinh(x)\sinh(y)} & ,\text{if } xy \neq 0 \\ \\ 0 & ,\text{if } xy = 0 \end{cases}$$

$$g_\theta(x,y,z) = 1 - \cosh^2(x) - \cosh^2(y) - \cosh^2(z) + 2\cosh(x)\cosh(y)\cosh(z)\cos\theta$$

$$t_+ = \begin{cases} t & ,\text{if } t > 0 \\ \\ 0 & ,\text{if } t \leq 0 \end{cases}$$

and,

$$M_{\alpha,\beta} = \begin{cases} \frac{2^{-2\rho}\Gamma(\alpha+1)}{\sqrt{\pi}\Gamma(\alpha-\beta)\Gamma(\beta+\frac{1}{2})} & ,\text{if } \alpha > \beta \\ \\ 0 & ,\text{if } \alpha = \beta \end{cases}$$

In [2], we have

$$\mathcal{F}_{\alpha,\beta}(T_h f)(\lambda) = \psi_\lambda^{\alpha,\beta}(h)\mathcal{F}_{\alpha,\beta}(f)(\lambda); \quad \lambda, h \in \mathbb{R}. \tag{2}$$

For $\alpha \geq \frac{-1}{2}$, we introduce the Bessel normalized function of the first kind defined by:

$$j_\alpha(x) = \Gamma(\alpha+1)\sum_{n=0}^{\infty} \frac{(-1)^n(\frac{x}{2})^{2n}}{n!\Gamma(n+\alpha+1)}.$$

Moreover, we see that

$$\lim_{x\to 0} \frac{j_\alpha(x)-1}{x^2} \neq 0,$$

by consequence , there exists $C_1 > 0$ and $\varepsilon > 0$ satisfying

$$|x| \leq \varepsilon \Rightarrow |j_\alpha(x) - 1| \geq C_1|x|^2 \tag{3}$$

Lemma 1.2. (See[8],Lemma 3.1,Lemma 3.2) The following inequalities are valid for Jacobi functions $\varphi_\mu^{\alpha,\beta}(x)$

(c) $|\varphi_\mu^{\alpha,\beta}(x)| \leq 1$,

(d) $|1 - \varphi_\mu^{\alpha,\beta}(x)| \leq x^2(\mu^2 + \rho^2)$.

Lemma 1.3. (See[4],Lemma 9) Let $\alpha \geq \beta \geq \frac{-1}{2}, \alpha \neq \frac{-1}{2}$. Then for $|\nu| \leq \rho$, there exists a positive constant C_2 such that

$$|1 - \varphi^{\alpha,\beta}_{\mu+i\nu}(x)| \geq C_2 |1 - j_\alpha(\mu x)|.$$

2. Main Results

In this section we give the main results of this paper. We need first to define (η, γ)-Jacobi-Dunkl Lipschitz class.

Definition 2.1. Let $0 < \eta < 1$ and $\gamma > 0$. A function $f \in L^2_{\alpha,\beta}(\mathbb{R})$ is said to be in the (η, γ)-Jacobi-Dunkl Lipschitz class, denoted by $Lip(\eta, \gamma, 2)$, if

$$\|T_h f(x) + T_{-h} f(x) - 2f(x)\| = O\left(\frac{h^\eta}{(\log \frac{1}{h})^\gamma}\right), \quad as \quad h \to 0.$$

Lemma 2.2. For $f \in L^2_{\alpha,\beta}(\mathbb{R})$, then

$$\|T_h f(x) + T_{-h} f(x) - 2f(x)\|^2 = 4 \int_{\mathbb{R}} |\varphi^{\alpha,\beta}_\mu(h) - 1|^2 |\mathcal{F}_{\alpha,\beta} f(\lambda)|^2 d\sigma(\lambda).$$

Proof. We us formula (2), we conclude that

$$\mathcal{F}_{\alpha,\beta}(T_h f + T_{-h} f - 2f)(\lambda) = (\psi^{\alpha,\beta}_\lambda(h) + \psi^{\alpha,\beta}_\lambda(-h) - 2)\mathcal{F}_{\alpha,\beta}(f)(\lambda),$$

Since

$$\psi^{\alpha,\beta}_\lambda(h) = \varphi^{\alpha,\beta}_\mu(h) + i\frac{\lambda}{4(\alpha+1)} sinh(2h) \varphi^{\alpha+1,\beta+1}_\mu(h),$$

$$\psi^{\alpha,\beta}_\lambda(-h) = \varphi^{\alpha,\beta}_\mu(-h) - i\frac{\lambda}{4(\alpha+1)} sinh(2h) \varphi^{\alpha+1,\beta+1}_\mu(-h),$$

and $\varphi^{\alpha,\beta}_\mu$ is even (see [2]), then

$$\mathcal{F}_{\alpha,\beta}(T_h f + T_{-h} f - 2f)(\lambda) = 2(\varphi^{\alpha,\beta}_\mu(h) - 1)\mathcal{F}_{\alpha,\beta}(f)(\lambda).$$

By Parseval's identity (formula (1)), we have the result.

Theorem 2.3. Let $0 < \eta < 1$, $\gamma > 0$ and $f \in L^2_{\alpha,\beta}(\mathbb{R})$. Then the following conditions are equivalents

(i) $\quad f \in Lip(\eta, \gamma, 2)$

(ii) $\quad \displaystyle\int_{|\lambda| \geq r} |\mathcal{F}_{\alpha,\beta} f(\lambda)|^2 d\sigma(\lambda) = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right), \quad \text{as} \quad r \to \infty.$

Proof. $(i) \Rightarrow (ii)$. Assume that $f \in Lip(\eta, \gamma, 2)$. Then we have

$$\|T_h f(x) + T_{-h} f(x) - 2f(x)\| = O\left(\frac{h^\eta}{(\log \frac{1}{h})^\gamma}\right), \quad as \quad h \to 0$$

From Lemma 2.2, we have

$$\|T_h f(x) + T_{-h} f(x) - 2f(x)\|^2 = 4 \int_{\mathbb{R}} |\varphi^{\alpha,\beta}_\mu(h) - 1|^2 |\mathcal{F}_{\alpha,\beta} f(\lambda)|^2 d\sigma(\lambda).$$

By (3) and Lemma 1.3, we get

$$\int_{\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}} |1 - \varphi^{\alpha,\beta}_\mu(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \geq C_1^2 C_2^2 \int_{\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}} |\mu h|^4 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda).$$

From $\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}$ we have

$$\begin{aligned} \left(\frac{\varepsilon}{2h}\right)^2 &- \rho^2 \leq \mu^2 \leq \left(\frac{\varepsilon}{h}\right)^2 - \rho^2 \\ &\Rightarrow \mu^2 h^2 \geq \frac{\varepsilon^2}{4} - \rho^2 h^2. \end{aligned}$$

Take $h \leq \frac{\varepsilon}{3\rho}$, then we have $\mu^2 h^2 \geq C_3 = C_3(\varepsilon)$.
So,

$$\int_{\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}} |1-\varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \geq C_1^2 C_2^2 C_3^2 \int_{\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda).$$

There exists then a positive constant C_4 such that

$$\begin{aligned} \int_{\frac{\varepsilon}{2h} \leq |\lambda| \leq \frac{\varepsilon}{h}} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) &\leq C_4 \int_{\mathbb{R}} |1 - \varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \\ &= O\left(\frac{h^{2\eta}}{(\log \frac{1}{h})^{2\gamma}}\right). \end{aligned}$$

We obtain

$$\int_{r \leq |\lambda| \leq 2r} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \leq C \frac{r^{-2\eta}}{(\log r)^{2\gamma}},$$

where C is a positive constant. Now,

$$\begin{aligned} \int_{|\lambda| \geq r} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) &= \sum_{i=0}^{\infty} \int_{2^i r \leq |\lambda| \leq 2^{i+1} r} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \\ &\leq C\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}} + \frac{(2r)^{-2\eta}}{(\log 2r)^{2\gamma}} + \frac{(4r)^{-2\eta}}{(\log 4r)^{2\gamma}} + \cdots\right) \\ &\leq C \frac{r^{-2\eta}}{(\log r)^{2\gamma}} \left(1 + 2^{-2\eta} + (2^{-2\eta})^2 + (2^{-2\eta})^3 + \cdots\right) \\ &\leq K_\eta \frac{r^{-2\eta}}{(\log r)^{2\gamma}}, \end{aligned}$$

where $K_\eta = C(1 - 2^{-2\eta})^{-1}$ since $2^{-2\eta} < 1$.
Consequently

$$\int_{|\lambda| \geq r} |\mathcal{F}_{\alpha,\beta} f(\lambda)|^2 d\sigma(\lambda) = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right), \quad as \quad r \to \infty.$$

$(ii) \Rightarrow (i)$. Suppose now that

$$\int_{|\lambda| \geq r} |\mathcal{F}_{\alpha,\beta} f(\lambda)|^2 d\lambda = O\left(\frac{r^{-2\eta}}{(\log r)^{2\gamma}}\right), \quad as \quad r \to \infty,$$

and write

$$\int_{\mathbb{R}} |1 - \varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) = I_1 + I_2$$

where

$$I_1 = \int_{|\lambda| < \frac{1}{h}} |1 - \varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda),$$

$$I_2 = \int_{|\lambda| \geq \frac{1}{h}} |1 - \varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda).$$

Firstly, we use the formula $|\varphi_\mu^{\alpha,\beta}(h)| \leq 1$ and

$$I_2 \leq 4 \int_{|\lambda| \geq \frac{1}{h}} |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) = O\left(\frac{h^{2\eta}}{(\log \frac{1}{h})^{2\gamma}}\right). \tag{4}$$

Set

$$\phi(\lambda) = \int_\lambda^\infty |\mathcal{F}_{\alpha,\beta}(f)(x)|^2 d\sigma(x).$$

An integration by parts gives

$$\begin{aligned} \int_0^x \lambda^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) &= \int_0^x -\lambda^2 \phi'(\lambda) d\lambda \\ &= -x^2\phi(x) + 2\int_0^x \lambda\phi(\lambda)d\lambda \\ &\leq 2\int_0^x \lambda^{1-2\delta}(\log\lambda)^{-2\gamma} d\lambda \\ &= O(x^{2-2\delta}(\log x)^{-2\gamma}). \end{aligned}$$

We use the formula (d) of Lemma 1.2

$$\begin{aligned} I_1 &\leq \int_{|\lambda|<\frac{1}{h}} |1-\varphi_\mu^{\alpha,\beta}(h)||\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \\ &\leq \int_{|\lambda|<\frac{1}{h}} (\mu^2+\rho^2)h^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \\ &\leq h^2 \int_{|\lambda|<\frac{1}{h}} \lambda^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) \\ &= O\left(h^2 h^{-2+2\eta}\left(\log\frac{1}{h}\right)^{-2\gamma}\right).. \end{aligned}$$

Hence,

$$I_1 = O\left(\frac{h^{2\eta}}{(\log \frac{1}{h})^{2\gamma}}\right). \tag{5}$$

Finally, we conclude from (4) and (5) that

$$\int_{\mathbb{R}} |1-\varphi_\mu^{\alpha,\beta}(h)|^2 |\mathcal{F}_{\alpha,\beta}(f)(\lambda)|^2 d\sigma(\lambda) = O\left(\frac{h^{2\eta}}{(\log \frac{1}{h})^{2\gamma}}\right).$$

And this ends the proof.

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