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ON THE INTEGRAL REPRESENTATION OF STRICTLY CONTINUOUS SET-VALUED MAPS

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Abstract. Let T be a completely regular topological space and $C(T)$ be the space of bounded, continuous real-valued functions on T. $C(T)$ is endowed with the strict topology (the topology generated by seminorms determined by continuous functions vanishing at infinity). R. Giles ([13], p. 472, Theorem 4.6) proved in 1971 that the dual of $C(T)$ can be identified with the space of regular Borel measures on T. We prove this result for positive, additive set-valued maps with values in the space of convex weakly compact non-empty subsets of a Banach space and we deduce from this result the theorem of R. Giles ([13], theorem 4.6, p.473).

1. Introduction

The strict topology β was for the first time introduced by R. C. Buck ([1], [2]) on the space $C(T)$ of all bounded continuous functions on a locally compact space T. He has proved among others that the dual space of $(C(T), \beta)$ is the space of all finite signed regular Borel measures on T. After a large number of papers have appeared in the literature concerned with extending the results contained in Buck's paper [1](see e.g. [4], [5], [6], [7], [8], [12],[14], [15], [17], [18], [19], [22], [25] and [27]). R. Giles has generalized this notion of the strict topology introduced by Buck for completely regular space T and has proved Buck's results, particulary the theorem 2 in [1] for an arbitrary (not necessarily Hausdorff) completely regular space T. In this paper we generalize Giles's result ([13], theorem 4.6, p.473) to additive, positive, positively homogeneous and strictly continuous set-valued maps defined on $C_+(T)$ with values in the space $cc(E)$ of all convex weakly compact non-empty subsets of a Banach space E. We deduce from this result the theorem of R. Giles.

2. Notations and definitions

Let T be a completely regular topological space and let $\mathcal{B}(T)$ be the Borel σ-algebra of T and let $C(T)$ be the space of bounded continuous real-valued functions on T. Let $C_0(T)$ be the subspace of $C(T)$ consisting of functions f vanishing at infinity i.e. for any $\varepsilon > 0$ there is a compact set $K_\varepsilon \subset T$ such that $|f(x)| < \varepsilon$ for $x \in T \backslash K_\varepsilon$. We denote by $C_+(T)$ the subspace of $C(T)$ consisting of non-negative functions and by 1_A the characteristic function of each $A \subset T$. For all $f \in C(T)$, we put $f^+ = \sup(f, 0), f^- = \sup(-f, 0)$ and $||f||_\infty = \sup\{|f(t)|;\ t \in T\}$. We denote

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by $\mathbb{R}$ the set of real numbers. Let E be a Banach space, E' its dual and $cc(E)$ be the space of all non-empty, convex weakly compact subsets of E; we denote by $\|.\|$ the norm on E and E'. If X and Y are subsets of E we shall denote by $X+Y$ the family of all elements of the form $x+y$ with $x \in X$ and $y \in Y$. The support function of X is the function $\delta^*(.|X)$ from E' to $[-\infty; +\infty]$ defined by $\delta^*(y|X) = \sup\{y(x), x \in X\}$. We endow $cc(E)$ with a Hausdorff distance, denoted by δ. For all $K \in cc(E)$ and for all $K' \in cc(E), \delta(K, K') = \sup\{|\delta^*(y|K) - \delta^*(y|K')|; y \in E', \|y\| \leq 1\}$. Recall that $(cc(E), \delta)$ is a complete metric space ([16], theorem 9, p.185) and ([21], theorem 15, p.2-2).

Definition 2.1. (1) let $m : \mathcal{B}(T) \to \mathbb{R}$ be a positive countable additive measure. We say that m is:

- (i) inner regular if for all $A \in \mathcal{B}(T)$ and $\varepsilon > 0$, there exists a compact K_ε subset of T such that $K_\varepsilon \subset A$ and $m(A \backslash K_\varepsilon) < \varepsilon$.
- (ii) outer regular if for all $A \in \mathcal{B}(T)$ and for all $\varepsilon > 0$, there exists an open subset O_ε of T such that $O_\varepsilon \supset A$ and $m(O_\varepsilon \backslash A) < \varepsilon$.
- (iii) regular if it is inner regular and outer regular.

(2) A signed measure $\mu : \mathcal{B}(T) \to \mathbb{R}$ is regular if and only if its total variation $v(\mu)$ is regular. Note that $v(\mu) : \mathcal{B}(T) \to \mathbb{R}_+$ $(A \mapsto v(\mu)(A) = \sup\{\sum\limits_i |\mu(A_i)|;$ (A_i) finite partition of $A, A_i \in \mathcal{B}(T)\})$.

Definition 2.2. A map $M : \mathcal{B}(T) \to cc(E)$ is a set-valued measure if $M(A \cup B) = M(A) + M(B)$ for every pair of disjoint sets A, B in $\mathcal{B}(T), M(\emptyset) = \{0\}$ and $M(\bigcup\limits_{n=1}^{+\infty} A_n) = \sum\limits_{n=1}^{+\infty} M(A_n)$ for every sequence (A_n) of mutually disjoint elements of $\mathcal{B}(T)$; which amounts to saying that for all $y \in E'$ the map $\delta^*(y|M(.)) : \mathcal{B}(T) \to \mathbb{R}(A \mapsto \delta^*(y|M(A)))$ is a countably additive measure ([21], corollary p. 2-25).
We say that a set-valued measure M is:

- (i) positive if for all $A \in \mathcal{B}(T), 0 \in M(A)$
- (ii) regular if for all $y \in E'$, the measure $\delta^*(y|M(.))$ is regular.

Let $\varphi \in C_0(T)$, let K be a compact subset of T. We denote by p_φ and p_K the semi-norms on $C(T)$ defined by $p_\varphi(f) = \sup\{|f(t)\varphi(t)|; t \in T\}$ and $p_K(f) = \sup\{|f(t)|;\ t \in K\}$ for every $f \in C(T)$.

Definition 2.3. The topology determined by the set of semi-norms $\{p_\varphi; \varphi \in C_0(T)\}$ (resp. $\{p_K;\ K$ belongs to the family of compact subsets of $T\}$) is called the strict (resp. the compact convergence) topology. We say that a map defined on $C(T)$ is strictly continuous if it is continuous for this topology.

Definition 2.4. A map $L : C_+(T) \to cc(E)$ is:

- (i) additive set-valued map if for all $f, g \in C_+(T)$ $L(f+g) = L(f) + L(g)$
- (ii) positively homogeneous if for $f \in C_+(T)$ and for $\lambda \geq 0$ $L(\lambda f) = \lambda L(f)$.
- (iii) positive if for every $f \in C_+(T), 0 \in L(f)$.

Definition 2.5. ([24], p. 04)
Let m be a bounded linear functional on $C(T)$, and let $B(0,1)$ be the unit ball of $C(T)$. We say that m is tight if its restriction to $B(0,1)$ is continuous for the topology of compact convergence.

3. Main result

Lemma 3.1. *Let m be a bounded linear functional on $C(T)$. If m is tight then for all $\varepsilon > 0$ there is a compact subset K_ε of T such that for all $f \in C(T)$ and $|f| \leq 1_{T\setminus K_\varepsilon}$, we have $|m(f)| < \varepsilon$.*

Proof. Assume that m is tight. Then for every $\varepsilon > 0$ there is a compact subset K_ε of T and there is $\eta > 0$ such that for all $f \in B(0,1)$ and $p_{K_\varepsilon}(f) = \sup\{|f(t)|; t \in K_\varepsilon\} < \eta$. We have $|m(f)| < \varepsilon$. In particular for all $f \in B(0,1)$ such that $|f| \leq 1_{T\setminus K_\varepsilon}$, one has $|m(f)| < \varepsilon$. □

Lemma 3.2. *Let $M : \mathcal{B}(T) \longrightarrow cc(E)$ be a positive, regular set-valued measure. Then the real-valued measure $\delta^*(y|M(.))$ are uniformly tight with respect to $y \in E', \|y\| \leq 1$ ie for every $A \in \mathcal{B}(T)$ and for every $\varepsilon > 0$ there is a compact subset K_ε of T such that $K_\varepsilon \subset A$ and $\sup\{\delta^*(y|M(A\backslash K_\varepsilon)); y \in E', \|y\| \leq 1\} \leq \varepsilon$.*

Proof. Let us consider the set $\{\delta^*(y|M(.)), y \in E', \|y\| \leq 1\}$ of countably additive real-valued measures. It is uniformly countable additive (see [9], theorem 10, p. 88–89; [28], lemma 3.1, p. 275). According to ([10], p. 443, Theorem 10.7) there is a sequence (c_n) of real numbers and there is a sequence $(\delta^*(y_n|M(.))), |y_n| \leq 1$ of measures such that $\mu(A) = \sum_{n=1}^{+\infty} c_n\delta^*(y_n|M(A))$ exists for each $A \in \mathcal{B}(T)$ and such that the series $\sum |c_n|\delta^*(y_n|M(A))$ is uniformly convergent for $A \in \mathcal{B}(T)$; moreover the countable additive measure $\nu : \mathcal{B}(T) \to \mathbb{R}(A \mapsto \nu(A) = \sum_{n=1}^{+\infty} |c_n|\delta^*(y_n|M(A)))$ verifies the following relation: $\lim_{\nu(A)\to 0}[\sup\{\delta^*(y|M(A)); y \in E', \|y\| \leq 1\}] = 0$ (*). We deduce from the uniform convergence of the series $\sum |c_n|\delta^*(y_n|M(A))$ for $A \in \mathcal{B}(T)$, that ν is regular. Indeed, given $\varepsilon > 0$ choose $n_0 \in \mathbb{N}$ such that $\sup_{A\in\mathcal{B}(T)} \left|\nu(A) - \sum_{k=1}^{n_0} |c_k|\delta^*(y_k|M(A))\right| < \varepsilon/2$.
For $A \in \mathcal{B}(T)$, choose a compact subset K of T such that $K \subset A$ and for every $k \in \{1, 2, ..., n_0\}$ $\delta^*(y_k|M(A\backslash K)) \leq \frac{\varepsilon}{2(n_0+1)r_0}$ with $r_0 = \sup\{|c_k|; k \in \{1, 2, ..., n_0\}\}$ then $\sum_{k=1}^{n_0} |c_k|\delta^*(y_k|M(A\backslash K)) \leq \varepsilon/2$, therefore $\nu(A\backslash K) \leq \varepsilon$.
The relation (*) and the inner regularity of ν show that for each $\varepsilon > 0$ and each $A \in \mathcal{B}(T)$ there exists a compact subset K of T such that $K \subset A$ and $\sup\{\delta^*(y|M(A\backslash K)); y \in E', \|y\| \leq 1\} \leq \varepsilon$. □

Let M be a positive set-valued measure defined on $\mathcal{B}(T)$. For the construction of the integral $\int fM$, with $f \in C_+(T)$ we refer to ([23], p. 17).

Lemma 3.3. *Let $M : \mathcal{B}(T) \to cc(E)$ be a positive regular set-valued measure. Then the set-valued map $L : C_+(T) \to cc(E)(f \mapsto L(f) = \int fM)$ is additive, positively homogeneous, positive and strictly continuous.*

Proof. We only prove the strict continuity. The other properties follow from the construction of the integral $\int fM, f \in C_+(T)$. For each $n \in \mathbb{N}^*$ there exists a compact subset K_n of T such that $\sup\{\delta^*(y|M(T\backslash K_n)); y \in E', \|y\| \leq 1\} \leq 2^{-2n}$ (Lemma 3.2). We then have a sequence (K_n) of compact subsets of T that we may assume monotone increasing. We repeat here the proof of R. Giles ([13], p. 471,

Lemma 4.2). Consider $\varphi = \sum_{n=1}^{+\infty} 2^{-n} 1_{K_n}$, we have $2^{-n-1} \leq \varphi(x) \leq 2^{-n}$ for all $x \in K_{n+1} \backslash K_n$. The function $1/\varphi$ is measurable and is $\delta^*(y|M(.))$ - integrable for each $y \in E', \|y\| \leq 1$. We have $\int 1/\varphi \delta^*(y|M(.)) = \int_{\cup_{n=1}^{+\infty}(K_{n+1}\backslash K_n)} 1/\varphi \; \delta^*(y|M(.)) = \sum_{n=1}^{+\infty} \int_{K_{n+1}\backslash K_n} 1/\varphi \; \delta^*(y|M(.))$
$\leq \sum_{n=1}^{+\infty} 2^{n+1} \left[\delta^*(y|M(K_{n+1})) - \delta^*(y|M(K_n))\right] \leq \sum_{n=1}^{+\infty} 2^{n+1}.2^{-2n} = 2$. Let $\varepsilon > 0$ and let $\psi_n \in C_0$ such that $\psi_n(x) = 2^{-n}$ for $x \in K_n$ and $0 \leq \psi_n \leq 2^{-n} 1_T$. Put $\psi = \sum_{n=1}^{+\infty} \psi_n$. Then $\psi \in C_0$ and $\varphi \leq \psi$. For all $f \in \{g \in C_+(T), p_{2\psi/\varepsilon}(g) < 1\}$ we have $f < \varepsilon/2\varphi$ and $\int f\delta^*(y|M(.)) < \varepsilon$ for all $y \in E'$ with $\|y\| \leq 1$. Since $\delta^*(y|\int fM) = \int f\delta^*(y|M(.))$, one has $\delta(\int fM, \{0\}) < \varepsilon$. Therefore the map $f \to \int fM$ is strictly continuous at 0. The equality $\delta^*(y|\int fM) = \int f\delta^*(y|M(.))$ for each $f \in C_+(T)$ and each $y \in E'$ enable us to prove the continuity on $C_+(T)$. □

Definition 3.4. A map $S : E' \to \mathbb{R}$ is said to be sublinear if for every $y \in E'$ and $y' \in E'$ and for every $\lambda \geq 0$ one has $S(y + y') \leq S(y) + S(y')$ and $S(\lambda y) = \lambda S(y)$.

The lemme below is a particular case of L. Hörmander's result ([16], Theorem 5, p. 182). We give here an alternative proof.

Lemma 3.5. *Let E be a Banach space, and let E' its dual space endowed with the Mackey topology $\tau(E', E)$. Let $S : E' \to \mathbb{R}$ be a sublinear map. Then S is continuous if and only if there is $C \in cc(E)$ such that $S = \delta^*(.|C)$.*

Proof. Assume that S is continuous. Let $\nabla S = \{l : E' \to \mathbb{R}$; linear *and* $l \leq S\}$. By the Hahn-Banach theorem ([11], theorem 10, p. 62), $S(y) = \sup\{l(y);$
$l \in \nabla S\}$ for each $y \in E'$. Let $l \in \nabla S$; then l is continuous for the Mackey topology $\tau(E', E)$. Therefore l determines an element $x_l \in E$ that verifies $l(y) = y(x_l)$ for each $y \in E'$. Let $\nabla_E S = \{x_l; l \in \nabla S\}$. Since ∇S is equicontinuous there is a neighborhood V of 0 in E' such that $\nabla_E S \subset V^\circ$, where V° is the polar of V in E. By the Alaoglu-Bourbaki's theorem ([20], p. 248), one has $V^\circ \in cc(E)$. Since $\nabla_E S$ is convex , its closure is one of elements of $cc(E)$ we want. The converse is obvious. Note that if S is non-negative then $0 \in \nabla_E S$. □

Theorem 3.6. *Let T be a completely regular topological space and let $C_+(T)$ be the space of bounded continuous non-negative functions defined on T endowed with the strict topology. Let E be a Banach space and $cc(E)$ be the space of convex weakly compact non-empty subsets of E endowed with the Hausdorff distance.*
Let $L : C_+(T) \to cc(E)$ be a positive, additive, positively homogeneous and strictly continuous set-valued map. Then there is a unique positive regular set-valued measure M defined on $\mathcal{B}(T)$ to $cc(E)$ such that $L(f) = \int fM$ for all $f \in C_+(T)$.
Conversely for all positive regular set-valued measure $M : \mathcal{B}(T) \to cc(E)$, the set-valued map $\theta : C_+(T) \to cc(E)$ $(f \mapsto \theta(f) = \int fM)$ is positive, additive, positively homogeneous and strictly continuous.

Proof. Let $y \in E'$. The map $\delta^*(y|L(.)) : C_+(T) \to \mathbb{R}$ $(f \mapsto \delta^*(y|L(f)))$ is additive, positively homogeneous and continuous. Then it can be extended to a continuous linear functional on $C(T)$. This extension is unique. It is denoted by $\delta^*(y|\bar{L}(.))$. Let $f \in C(T)$, one has $f = f^+ - f^-$ and $\delta^*(y|\bar{L}(.))$ is defined by $\delta^*(y|\bar{L}(.))(f) =$

$\delta^*(y|L(f^+)) - \delta^*(y|L(f^-))$. Since $\delta^*(y|\bar{L}(.))$ is strictly continuous it is tight ([26], p. 41). By the lemma 3.1 and ([3], Proposition 5, p.58) there exists a unique regular positive Borel measure μ_y on T that verifies $\delta^*(y|\bar{L}(f)) = \int f\mu_y$ for all $f \in C(T)$. Let 0 an open subset of T and let S_O the map defined on E' to $\mathbb{R}$ by $S_O(y) = \mu_y(O)$ for each $y \in E'$. We have $\mu_y(O) = \sup\{\int f\mu_y; f \in C_+(T), f \leq 1_O\} = \sup\{\delta^*(y|L(f)); f \in C_+(T), f \leq 1_O\}$, therefore S_O is a sublinear map. Let now $A \in \mathcal{B}(T)$. We denote by S_A the map defined on E' to $\mathbb{R}$ by $S_A(y) = \mu_y(A)$ for each $y \in E'$. Since the measure μ_y is regular we have $S_A(y) = \inf\{\mu_y(O); O \subset T, O$ open and $O \supset A\} = \inf\{S_O(y); O \subset T, O$ open and $O \supset A\}$. Let $y, y' \in E'$ and let $\varepsilon > 0$, there exists two open subsets O_ε and O'_ε of T containing A and such that $S_A(y) \geq \mu_y(O_\varepsilon) - \varepsilon/2$, $S_A(y') \geq \mu_{y'}(O'_\varepsilon) - \varepsilon/2$. We have $\mu_y(O_\varepsilon) + \mu_{y'}(O'_\varepsilon) \leq S_A(y) + S_A(y') + \varepsilon$, then $\mu_y(O_\varepsilon \cap O'_\varepsilon) + \mu_{y'}(O_\varepsilon \cap O'_\varepsilon) \leq S_A(y) + S_A(y') + \varepsilon$, therefore $\mu_{y+y'}(O_\varepsilon \cap O'_\varepsilon) \leq S_A(y) + S_A(y') + \varepsilon$. We have $\mu_{y+y'}(A) \leq \mu_{y+y'}(O_\varepsilon \cap O'_\varepsilon) \leq S_A(y) + S_A(y') + \varepsilon$. It follows from this $S_A(y + y') \leq S_A(y) + S_A(y')$. It is obvious that for all $\lambda \geq 0$ and for all $y \in E'$, $S_A(\lambda y) = \lambda S_A(y)$. So S_A is a non-negative sublinear map. Let us prove now that S_A is continuous for the Mackey topology $\tau(E', E)$. We have $S_A(y) \leq \mu_y(T) = \delta^*(y|L(1_T))$. Let $\widetilde{L(1_T)}$ be the closed absolutely convex cover of $L(1_T)$, one has $\widetilde{L(1_T)} \in cc(E)$ and $S_A(y) \leq \delta^*\left(y|\widetilde{L(1_T)}\right)$ for each $y \in E'$ and $A \in \mathcal{B}(T)$. We deduce that S_A is continuous for the Mackey topology for each $A \in \mathcal{B}(T)$. By the lemma 3.5 there is $C_A \in cc(E)$ such that $S_A(y) = \delta^*(y|C_A)$ for all $y \in E'$. Let $M : \mathcal{B}(T) \to cc(E)\,(A \mapsto M(A) = C_A)$. We have $\delta^*(y|M(A)) = \mu_y(A)$ for all $y \in E'$, hence the map $\delta^*(y|M(.)) : \mathcal{B}(T) \to \mathbb{R}\,(A \mapsto \delta^*(y|M(A)))$ is a positive regular countably additive measure. Then M is a regular set-valued measure. Since S_A is non-negative then M is positive. Let $f \in C_+(T)$ and let $y \in E', \int f\delta^*(y|M(.)) = \int f\mu_y = \delta^*(y|L(f))$. It follows that $L(f) = \int fM$ for all $f \in C_+(T)$ because $\int f\delta^*(y|M(.)) = \delta^*(y|\int fM)$. Let us prove that M is unique. Assume that there exist two regular positive set-valued measures M and M' which verify $\int fM = L(f) = \int fM'$. Let 0 be an open subset of T and let $y \in E'$. According to the inner regularity of $\delta^*(y|M(.))$ and ([3] Lemme 1 p. 55) we have $\delta^*(y|M(O)) = \sup\{\delta^*(y|L(f));\, f \in C_+(T),\, f \leq 1_O\} = \delta^*(y|M'(O))$. Moreover the outer regularity of $\delta^*(y|M(.))$ shows that $\delta^*(y|M(A)) = \delta^*(y|M'(A))$ for all $A \in \mathcal{B}(T)$ and $y \in E'$, hence $M(A) = M'(A)$ for all $A \in \mathcal{B}(T)$. The second assertion of the theorem is justified by the lemma 3.3. □

The following corollary is the result of R. Giles.

Corollary 3.7. *([13], Theorem 4.6) For any completely regular space T the dual of $C(T)$ under the strict topology is the space of all bounded signed Borel regular measures on T.*

Proof. Let L be a strictly continuous linear functional on $C(T)$; L is bounded. Therefore L is the difference of two non-negative linear functional. We may assume that L is non-negative. Let K_0 be an element of $cc(E)$ that contains 0 and that is subset of the unit ball of E. Consider the map $L' : C_+(T) \to cc(E)$ defined by $L'(f) = L(f)K_0 = \{L(f)k;\, k \in K_0\}$ for all $f \in C_+(T)$. The map L' is positive, positively homogeneous and strictly continuous. Let us prove that L' is additive. The inclusion $L'(f + g) \subset L'(f) + L'(g)$ for all $f, g \in C_+(T)$ is trivial. Let $u \in K_0$ and each let $v \in K_0$, $L(f)u + L(g)v = L(f + g)\left[\frac{L(f)}{L(f+g)}u + \frac{L(g)}{L(f+g)}v\right]$. Since K_0 is convex and L positive, $\frac{L(f)}{L(f+g)}u + \frac{L(g)}{L(f+g)}v \in K_0$. Then $L'(f) + L'(g) \subset L'(f + g)$.

By the Theorem 3.6, there is a unique positive regular set-valued measure $M : \mathcal{B}(T) \to cc(E)$ that satisfies the condition $\int fM = L'(f)$ for all $f \in C_+(T)$. Let $y_0 \in E'$ such that $\delta^*(y_0|L'(.)) = L$. Since $\delta^*(y_0|\int fM) = \int f\delta^*(y_0|M(.))$ for all $f \in C_+(T)$ we then have $\int f\delta^*(y_0|M(.)) = L(f)$ for all $f \in C_+(T)$ and therefore $\int f\delta^*(y_0|M(.)) = L(f)$ for all $f \in C(T)$. The uniqueness of $\delta^*(y_0|M(.))$ follows from the regularity of M. Taking the lemma 3.3 (for the scalar measures) into account we conclude that there is a bijection between the dual space of $(C(T), \beta)$ and the space of all bounded signed regular Borel measures on T. □

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