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HOMOTOPY PERTURBATION METHOD FOR SOLVING THE FRACTIONAL FISHER'S EQUATION

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Abstract. In this paper, we apply the modified HPM suggested by Momani and al. [23] for solving the time-fractional Fisher's equation and we use the classical HPM to derive numerical solutions of the space-fractional Fisher's equation. We compared our solution with the exact solution. The results show that the HPM modified is an appropriate method for solving nonlinear fractional derivative equations.

1. Introduction

Fractional analysis is a branch of mathematics, was the first debut in 1695 with the question posed by Leibnitz as follows: what could be the derivative of order (half) of a function x?

From that date to today, the evolution of this branch of mathematics where he became a major development has many uses, among them, for example:

Fractional derivatives have been widely used in the mathematical model of the visco-elasticity of the material [1].

The electromagnetic problems can be described using the fractional integro-differential equations [2].

In biology, it was deduced that the membranes of biological organism cells have the electrical conductance of fractional order [3], and then is classified into groups of non-integer order models.

In economics, some finance systems can display a dynamic fractional order [4].

In addition to the above, we find that the development of this branch has also led to the emergence of linear and nonlinear differential equations of fractional order, which became requires researchers to use conventional methods to solve them. Among these methods there is the homotopy perturbation method (HPM). This method was established in 1998 by He ([5]-[9]) and applied by many researchers to solve various linear and nonlinear problems (see [10]- [16]). The method is a powerful and efficient technique to find the solutions of nonlinear equations. The coupling of the perturbation and homotopy method is called the homotopy perturbation method. This method can take the advantages of the conventional perturbation method while eliminating its restrictions [16].

Our concern in this work is to consider the numerical solution of the nonlinear Fisher's equation with time and space fractional derivatives of the form

$$ {}^cD_t^\alpha u = {}^cD_x^\beta u + \gamma u(1-u), \quad 0 < \alpha \leqslant 1, \ 1 < \beta \leqslant 2, \tag{1}$$

where ${}^cD_t^\alpha u = \frac{\partial^\alpha}{\partial t^\alpha}$, ${}^cD_t^\beta u = \frac{\partial^\beta}{\partial t^\beta}$.

In the case $\alpha = 1$ and $\beta = 2$, this equation become

$$u_t = u_{xx} + \gamma u(1-u), \tag{2}$$

which is a Fisher's partial differential equation.

We will extend the application of the modified HPM in order to derive analytical approximate solutions to nonlinear time-fractional Fisher's equation and we use the classical HPM to resolve the

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nonlinear space-fractional Fisher's equation. Precisely, we use the modified homotopy perturbation method described in [23] for handling an iterative formula easy-to-use for computation. Observing the numerical results, and comparing with the exact solution, the proposed method reveals to be very close to the exact solution and consequently, an efficient way to solve the nonlinear time-fractional Fisher's equation. This is the raison why we try to use it in this work.

2. Basic definitions

We give some basic definitions and properties of the fractional calculus and the Laplace transform theory which are used further in this paper. (see [18]-[20]).

Definition 1. *Let* $\Omega = [a,b]$ $(-\infty < a < b < +\infty)$ *be a finite interval on the real axis* $\mathbb{R}$. *The Riemann–Liouville fractional integrals* $I^{\alpha}_{a+}f$ *of order* $\alpha \in \mathbb{C}$ $(\mathit{Re}(\alpha) > 0)$ *is defined by*

$$(I^{\alpha}_{a+}f)(t) = \frac{1}{\Gamma(\alpha)}\int_a^t \frac{f(\tau)d\tau}{(t-\tau)^{1-\alpha}}, \quad t > 0, \mathit{Re}(\alpha) > 0, \tag{3}$$

here $\Gamma(\alpha)$ *is the gamma function.*

Theorem 2. *Let* $\mathit{Re}(\alpha) > 0$ *and let* $n = [\mathit{Re}(\alpha)] + 1$. *If* $f(t) \in AC^n[a,b]$, *then the Caputo fractional derivatives* $({}^cD^{\alpha}_t f)(t)$ *exist almost evrywhere on* $[a,b]$. *If* $\alpha \notin \mathbb{N}$, $({}^cD^{\alpha}_t f)(t)$ *is represented by:*

$$({}^cD^{\alpha}_t f)(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t \frac{f^{(n)}(\tau)d\tau}{(t-\tau)^{\alpha-n+1}}, \tag{4}$$

and if $\alpha \in \mathbb{N}$, *we obtain* $({}^cD^{\alpha}_t f)(t) = f^{(n)}(t)$.

Definition 3. *Let* $u \in C^n_{-1}$, $n \in \mathbb{N}^*$. *Then the (left sided) Caputo fractional derivative of* u *is defined (for* $t > 0$*) as*

$${}^cD^{\alpha}_t u(x,t) = \frac{\partial^{\alpha}u(x,t)}{\partial t^{\alpha}} = \begin{cases} \frac{1}{\Gamma(n-\alpha)}\int_0^t (t-\tau)^{n-\alpha-1}\frac{\partial^n u(x,t)}{\partial t^n}d\tau, n-1<\alpha<n, n \in \mathbb{N}^* \\ \frac{\partial^n u(x,t)}{\partial t^n}, \alpha = n \in \mathbb{N}. \end{cases} \tag{5}$$

According to (5), we can obtain:

${}^cD^{\alpha}K = 0$, K is a constant, and ${}^cD^{\alpha}t^{\beta} = \begin{cases} \frac{\Gamma(\beta+1)}{\Gamma(\beta-\alpha+1)}t^{\beta-\alpha}, \beta > \alpha - 1 \\ 0, \beta \leq \alpha - 1. \end{cases}$

3. The Homotopy Perturbation Method

To illustrate the basic ideas of this method, we consider the following nonlinear differential equation

$$A(u) - f(r) = 0, \quad r \in \Omega, \tag{6}$$

with the following boundary conditions

$$B(u, \frac{\partial u}{\partial n}) = 0, \quad r \in \Gamma, \tag{7}$$

where A is a general differential operator, $f(r)$ is a known analytic function, B is a boundary operator, u is the unknown function, and Γ is the boundary of the domain Ω. The operator A can be generally divided into two operators, L and N, where L is a linear, and N a nonlinear operator. Therefore, equation (6) can be written as follows:

$$L(u) + N(u) - f(r) = 0. \tag{8}$$

Using the homotopy technique, we construct a homotopy $v(r,p) : \Omega \times [0,1] \longrightarrow \mathbb{R}$, which satisfies

$$H(v,p) = (1-p)[L(v) - L(u_0)] + p[A(v) - f(r)] = 0, \tag{9}$$

or

$$H(v,p) = L(v) - L(u_0) + pL(u_0) + p[N(v) - f(r)] = 0, \tag{10}$$

where $p \in [0,\, 1]$ is an embedding parameter, and u_0 is the initial approximation of equation (6) which satisfies the boundary conditions. Clearly, from Eq. (9) and (10) we will have

$$H(v,0) = L(v) - L(u_0) = 0, \tag{11}$$
$$H(v,1) = A(v) - f(r) = 0. \tag{12}$$

The changing process of p from zero to unity is just that of $v(r,p)$ changing from $u_0(r)$ to $u(r)$. In topology, this is called deformation and $L(v) - L(u_0)$ and $A(v) - f(r)$ are called homotopic. If the embedding parameter p, $(0 \leqslant p \leqslant 1)$ is considered as a "small parameter", applying the classical perturbation technique, we can assume that the solution of equation (9) or (10) can be given as a power series in p

$$v = v_0 + pv_1 + p^2 v_2 \ + \cdots . \tag{13}$$

Setting $p = 1$, results in the approximate solution of Eq. (6)

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 \ + \cdots . \tag{14}$$

The convergence of the series (14) has been proved in ([21], [22]).

3.1. **New modification of the HPM .** Momani and al. [23] introduce an algorithm to handle in a realistic and efficient way the nonlinear PDEs of fractional order. They consider the nonlinear partial differential equations with time fractional derivative of the form

$$\begin{cases} {}^cD_t^\alpha u(x,t) = f(u, u_x, u_{xx}) = L(u, u_x, u_{xx}) + N(u, u_x, u_{xx}) + h(x,t), t > 0 \\ u^k(x,0) = g_k(x), \quad k = 0, 1, 2, ...m-1, \end{cases} \tag{15}$$

where L is a linear operator, N is a nonlinear operator which also might include other fractional derivatives of order less than α. The function h is considered to be a known analytic function and ${}^cD^\alpha$, $m - 1 < \alpha \leqslant m$, is the Caputo fractional derivative of order α.

In view of the homotopy technique, we can construct the following homotopy

$$\frac{\partial u^m}{\partial t^m} - L(u, u_x, u_{xx}) - h(x,t) = p[\frac{\partial u^m}{\partial t^m} + N(u, u_x, u_{xx}) -^c D_t^\alpha u], \tag{16}$$

or

$$\frac{\partial u^m}{\partial t^m} - h(x,t) = p[\frac{\partial u^m}{\partial t^m} + L(u, u_x, u_{xx}) + N(u, u_x, u_{xx}) -^c D_t^\alpha u], \tag{17}$$

where $p \in [0, 1]$. The homotopy parameter p always changes from zero to unity. In case $p = 0$, Eq. (16) becomes the linearized equation

$$\frac{\partial u^m}{\partial t^m} = L(u, u_x, u_{xx}) + h(x,t), \tag{18}$$

or in the second form, Eq. (17) becomes the linearized equation

$$\frac{\partial u^m}{\partial t^m} = h(x,t). \tag{19}$$

When it is one, Eq. (16) or Eq. (17) turns out to be the original fractional differential equation (15). The basic assumption is that the solution of Eq. (16) or Eq. (17) can be written as a power series in p

$$u = u_0 + pu_1 + p^2 u_2 \ + p^3 u_3 \cdots . \tag{20}$$

Finally, we approximate the solution by

$$u(x,t)=\sum_{n=0}^{\infty} u_n(x,t). \tag{21}$$

4. Numerical applications

In this section, we apply the modified homotopy perturbation method for solving Fisher's equation with time-fractional derivative and we use the classical HPM to obtain analytical solution for Fisher's equation with space-fractional derivative.

4.1. **Numerical solutions of time-fractional Fisher's equation.** If $\beta=2$, we obtain the following form of the time-fractional Fisher's equation

$${}^{c}D_t^{\alpha}u=u_{xx}+\gamma u(1-u),\quad 0<\alpha\leqslant 1, \tag{22}$$

with the initial condition $u(x,0)=f(x)$.

Application of the New modification of the HPM

In view of Eq. (17), the homotopy of Eq. (22) can be constructed as

$$\frac{\partial u}{\partial t}=p[\frac{\partial u}{\partial t}+u_{xx}+\gamma u-\gamma u^2-\ {}^{c}D_t^{\alpha}u]. \tag{23}$$

Substituting (20) into (23) and equating the terms of the same power p, one obtains the following set of linear partial differential equations

$$\begin{array}{l} p^0:\frac{\partial u_0}{\partial t}=0,\\ p^1:\frac{\partial u_1}{\partial t}=\frac{\partial u_0}{\partial t}+u_{0xx}+\gamma u_0-\gamma u_0^2-\ {}^{c}D_t^{\alpha}u_0,\\ p^2:\frac{\partial u_2}{\partial t}=\frac{\partial u_1}{\partial t}+u_{1xx}+\gamma u_1-\gamma(2u_0u_1)-\ {}^{c}D_t^{\alpha}u_1,\\ p^3:\frac{\partial u_3}{\partial t}=\frac{\partial u_2}{\partial t}+u_{2xx}+\gamma u_2-\gamma(2u_0u_2+u_1^2)-\ {}^{c}D_t^{\alpha}u_2,\\ \vdots \end{array} \tag{24}$$

with the following conditions

$$u_0(x,0)=f(x), \tag{25}$$

$$u_i(x,0)=0\quad for\quad i=1,2,.... \tag{26}$$

Case 1: Consider the following form of the time-fractional equation (for $\gamma=1$)

$${}^{c}D_t^{\alpha}u=u_{xx}+u(1-u),\quad 0<\alpha\leqslant 1, \tag{27}$$

with the initial condition

$$u(x,0)=\lambda. \tag{28}$$

Using the initial condition (28) and solving the above Eqs. (24) yields

$$\begin{array}{l} u_0\,(x,t)=\lambda,\\ u_1\,(x,t)=\lambda(1-\lambda)t,\\ u_2\,(x,t)=\lambda(1-\lambda)t+\lambda(1-\lambda)(1-2\lambda)\frac{t^2}{2!}-\lambda(1-\lambda)\frac{t^{2-\alpha}}{\Gamma(3-\alpha)},\\ u_3\,(x,t)=\lambda(1-\lambda)t+2\lambda(1-\lambda)(1-2\lambda)\frac{t^2}{2!}+\lambda(1-\lambda)(1-6\lambda+6\lambda^2)\frac{t^3}{3!}-2\lambda(1-\lambda)\frac{t^{2-\alpha}}{\Gamma(3-\alpha)}\\ +\lambda(1-\lambda)\frac{t^{3-2\alpha}}{\Gamma(4-2\alpha)}-2\lambda(1-\lambda)(1-2\lambda)\frac{t^{3-\alpha}}{\Gamma(4-\alpha)},\\ \vdots \end{array}$$

and so on. The first four terms of the decomposition series solution for Eq. (27) is given as

$$u(x,t)=\lambda+3\lambda(1-\lambda)t+3\lambda(1-\lambda)(1-2\lambda)\frac{t^{2}}{2!}+\lambda(1-\lambda)(1-6\lambda+6\lambda^{2})\frac{t^{3}}{3!}$$
$$-3\lambda(1-\lambda)\frac{t^{2-\alpha}}{\Gamma(3-\alpha)}+\lambda(1-\lambda)\frac{t^{3-2\alpha}}{\Gamma(4-2\alpha)}-2\lambda(1-\lambda)(1-2\lambda)\frac{t^{3-\alpha}}{\Gamma(4-\alpha)}. \tag{29}$$

Substituting $\alpha=1$ into (30), we get the same approximate solution of nonlinear partial diferential Fisher's equation obtained in[24] as

$$\begin{aligned} u(x,t) &= \lambda+\lambda(1-\lambda)t+\lambda(1-\lambda)(1-2\lambda)\frac{t^{2}}{2!}+\lambda(1-\lambda)(1-6\lambda+6\lambda^{2})\frac{t^{3}}{3!}+\cdots \\ &= \frac{\lambda e^{t}}{1-\lambda+\lambda e^{t}}. \end{aligned} \tag{30}$$

Case 2 Next we consider the following form of the time-fractional Fisher's equation (for $\gamma=6$)

$$^{c}D_{t}^{\alpha}u=u_{xx}+6u(1-u),\quad 0<\alpha\leqslant 1, \tag{31}$$

subject to the initial condition

$$u(x,0)=\frac{1}{(1+e^{x})^{2}}. \tag{32}$$

The use of the initial condition (32) and solving the above Eq (24), we obtain

$$\begin{aligned}
&u_0\left(x,t\right)=\tfrac{1}{(1+e^{x})^{2}},\\
&u_1\left(x,t\right)=\tfrac{10e^{x}}{(1+e^{x})^{3}}t,\\
&u_2\left(x,t\right)=\tfrac{10e^{x}}{(1+e^{x})^{3}}t+\tfrac{50e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\tfrac{t^{2}}{2!}-\tfrac{10e^{x}}{(1+e^{x})^{3}}\tfrac{t^{2-\alpha}}{\Gamma(3-\alpha)},\\
&u_3\left(x,t\right)=\tfrac{10e^{x}}{(1+e^{x})^{3}}t+\tfrac{50e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\tfrac{t^{2}}{2!}+\tfrac{250e^{x}(4e^{2x}-7e^{x}+1)}{(1+e^{x})^{4}}\tfrac{t^{3}}{3!}\\
&-\tfrac{20e^{x}}{(1+e^{x})^{3}}\tfrac{t^{2-\alpha}}{\Gamma(3-\alpha)}+\tfrac{10e^{x}}{(1+e^{x})^{3}}\tfrac{t^{3-2\alpha}}{\Gamma(4-2\alpha)}-\tfrac{50e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\tfrac{t^{3-\alpha}}{\Gamma(4-\alpha)},\\
&\vdots
\end{aligned}$$

and so on. The first three terms of the decomposition series solution for Eq. (31) is given as

$$\begin{aligned} u(x,t) &= \frac{1}{(1+e^{x})^{2}}+\frac{30e^{x}}{(1+e^{x})^{3}}t+\frac{100e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\frac{t^{2}}{2!}+\frac{250e^{x}(4e^{2x}-7e^{x}+1)}{(1+e^{x})^{4}}\frac{t^{3}}{3!} \\ &- \frac{30e^{x}}{(1+e^{x})^{3}}\frac{t^{2-\alpha}}{\Gamma(3-\alpha)}+\frac{10e^{x}}{(1+e^{x})^{3}}\frac{t^{3-2\alpha}}{\Gamma(4-2\alpha)}-\frac{50e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\frac{t^{3-\alpha}}{\Gamma(4-\alpha)}. \end{aligned} \tag{33}$$

Substituting $\alpha=1$ into (34), we obtain:

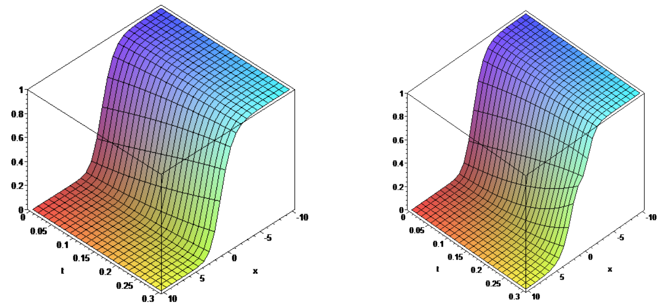
$$\begin{aligned} u(x,t) &= \frac{1}{(1+e^{x})^{2}}+\frac{10e^{x}}{(1+e^{x})^{3}}t+\frac{50e^{x}(2e^{x}-1)}{(1+e^{x})^{4}}\frac{t^{2}}{2!}+\frac{250e^{x}(4e^{2x}-7e^{x}+1)}{(1+e^{x})^{4}}\frac{t^{3}}{3!}+\cdots \\ &= \frac{1}{(1+e^{x-5t})^{2}} \end{aligned} \tag{34}$$

the same solution as presented in [24].

4.2. **Numerical solutions of space-fractional Fisher's equation.** Now we consider the following form of the space-fractional Fisher's equation with initial condition

$$u_{t}=^{c}D_{t}^{\beta}u_{xx}+\gamma u(1-u),\quad 1<\beta\leqslant 2, \tag{35}$$

$$u(x,0)=x^{2}. \tag{36}$$



solution.png

FIGURE 1. (Left): Exact solution for Eq. (31)-(32); (Right): Approximative solution of Eq. (31)-(32) with four terms for $\alpha = 1$.

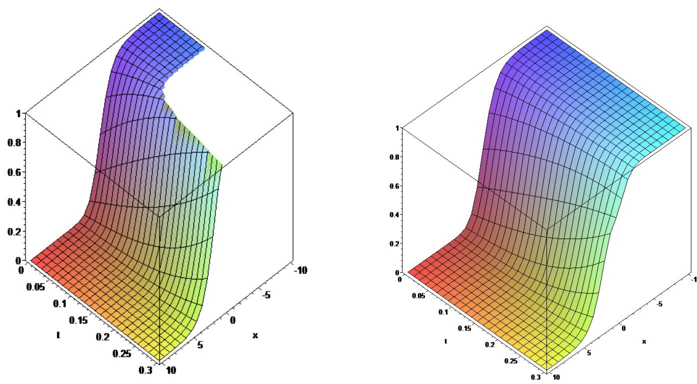


FIGURE 2. (Left): Approximative solution of Eq. (31)-(32) for $\alpha = 0.5$; (Right): Approximative solution of Eq. (31)-(32) for $\alpha = 0.9$.

The initial condition has been taken as the above polynomial to avoid heavy calculation of fractional differentiation.

Application of the HPM

According to the HPM, we construct the following homotopy

$$u_t - v_{0t} + p\left[-{}^{c}D_x^{\beta} - u(1-u) + v_{0t}\right], \quad 1 < \beta \leqslant 2, \tag{37}$$

where $p \in [0;1]$, $v_0 = u(x;0) = x^2$ and $\gamma = 1$.

In view of the HPM, substituting Eq. (20) into Eq. (37) and equating the coefficients of like powers of p, we get the following set of differential equations

$$\begin{aligned}
&p^0 : \tfrac{\partial u_0}{\partial t} = v_{0t},\\
&p^1 : \tfrac{\partial u_1}{\partial t} =^{c} D_x^{\beta} u_0 + u_0 - u_0^2 - v_{0t},\\
&p^2 : \tfrac{\partial u_2}{\partial t} =^{c} D_x^{\beta} u_1 + u_1 - 2u_0u_1,\\
&p^3 : \tfrac{\partial u_3}{\partial t} =^{c} D_x^{\beta} u_2 + u_2 - (2u_0u_2 + u_1^2),\\
&\vdots p^n : \tfrac{\partial u_n}{\partial t} =^{c} D_x^{\beta} u_{n-1} + u_{n-1} - \left(\textstyle\sum_{i=0}^{n-1} u_i u_{n-i-1}\right), \quad n \geqslant 1,
\end{aligned} \tag{38}$$

with the following conditions

$$u_0(x,0) = x^2, u_i(x,0) = 0 \quad for \quad i = 1,2,.... \tag{39}$$

Using the initial conditions (39) and solving the above Eqs. (38) yields

(40)

$u_0(x,t) = x^2,$

$u_1(x,t) = (a_1 x^{2-\beta} + x^2 - x^4)t,$

$u_2(x,t) = (a_2 x^{2-2\beta} + a_3 x^{2-\beta} + a_4 x^{4-\beta} + x^2 - 3x^4 + 2x^6)\frac{t^2}{2!},$

$u_3(x,t) = (a_5 x^{2-3\beta} + a_6 x^{2-2\beta} + a_7 x^{2-\beta} + a_8 x^{4-2\beta} + a_9 x^{4-\beta} + a_{10} x^{6-\beta} + x^2 - 6x^4 + 10x^6 - 5x^8)\frac{t^3}{3!},$

$\vdots$

where

$a_1 = \frac{2}{\Gamma(3-\beta)}$, $a_2 = \frac{2}{\Gamma(3-2\beta)}$, $a_3 = \frac{4}{\Gamma(3-\beta)}$, $a_4 = -\frac{24}{\Gamma(5-\beta)} - \frac{4}{\Gamma(3-\beta)}$, $a_5 = \frac{2}{\Gamma(3-3\beta)}$, $a_6 = \frac{6}{\Gamma(3-2\beta)}$, $a_7 = \frac{6}{\Gamma(3-\beta)}$, $a_8 = -\frac{24}{\Gamma(5-2\beta)} - \frac{4\Gamma(5-\beta)}{\Gamma(3-\beta)\Gamma(5-2\beta)} - \frac{4}{\Gamma(3-2\beta)} - \frac{4}{\Gamma^2(3-\beta)}$, $a_9 = -\frac{96}{\Gamma(5-\beta)} - \frac{16}{\Gamma(3-\beta)}$, $a_{10} = 2\frac{6!}{\Gamma(7-\beta)} + \frac{48}{\Gamma(5-\beta)} + \frac{12}{\Gamma(3-\beta)}$.

Here, setting $p = 1$, we have the following solution for three iterations

$$
\begin{aligned}
u(x,t) &= x^2 + (a_1 x^{2-\beta} + x^2 - x^4)t \\
&+ (a_2 x^{2-2\beta} + a_3 x^{2-\beta} + a_4 x^{4-\beta} + x^2 - 3x^4 + 2x^6)\frac{t^2}{2!} \\
&+ (a_5 x^{2-3\beta} + a_6 x^{2-2\beta} + a_7 x^{2-\beta} + a_8 x^{4-2\beta} + a_9 x^{4-\beta} \\
&+ a_{10} x^{6-\beta} + x^2 - 6x^4 + 10x^6 - 5x^8)\frac{t^3}{3!}.
\end{aligned} \tag{41}
$$

Case 1: substituting $\beta = 2$ into (42), we obtain

$$
\begin{aligned}
u(x,t) &= x^2 + (2 + x^2 - x^4)t + (4 - 15x^2 - 3x^4 + 2x^6)\frac{t^2}{2!} \\
&+ (-30 - 63x^2 - 90x^4 + 10x^6 - 5x^8)\frac{t^3}{3!}.
\end{aligned} \tag{42}
$$

Case 2: substituting $\beta = \frac{3}{2}$ into (42), we get

$$
\begin{aligned}
u(x,t) &= x^2 + (\frac{4}{\sqrt{\pi}}x^{\frac{1}{2}} + x^2 - x^4)t + (\frac{8}{\sqrt{\pi}}x^{\frac{1}{2}} + x^2 - \frac{104}{5\sqrt{\pi}}x^{\frac{5}{2}} - 3x^4 + 2x^6)\frac{t^2}{2!} \\
&+ (\frac{12}{\sqrt{\pi}}x^{\frac{1}{2}} - \frac{39\pi + 16}{\pi}x + x^2 - \frac{416}{5\sqrt{\pi}}x^{\frac{5}{2}} - 6x^4 + \frac{7328}{105\sqrt{\pi}}x^{\frac{9}{2}} + 10x^6 - 5x^8)\frac{t^3}{3!}.
\end{aligned} \tag{43}
$$

In the same manner, we can obtain the approximate solution of higher order of Eq. (35) by using the iteration formulas (38) and Maple.

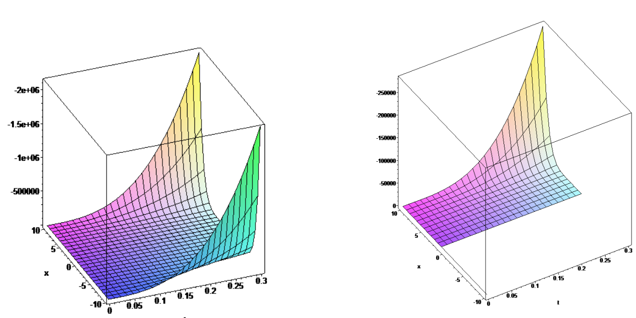


FIGURE 3. (Left): Approximative solution of Eq. (35)-(36) with $\beta = 2$; (Right): Approximative solution of Eq. (35)-(36) with $\beta = 3/2$.

5. Conclusion

In this work, the modified homotopy perturbation method was successfully used for solving Fisher's equation with time-fractional derivative, and the classical HPM has been used for solving Fisher's equation with space-fractional derivative. The final results obtained from modified HPM and compared with the exact solution shown that there is a similarity between the exact and the approximate solutions. Calculations show that the exact solution can be obtained from the third term. That's why we say that modified HPM is an alternative analytical method for solving the nonlinear time-fractional equations.

References

[1] A. Hanyga, *Fractional-order relaxation laws in non-linear viscoelasticity,* Continuum Mechanics and Thermodynamics, 19 (2007), 25-36.

[2] V. E. Tarasov, *Fractional integro-differential equations for electromagnetic waves in dielectric media,* Theoretical and Math. Phys, 158 (2009), 355-359.

[3] G. Chen and G. Friedman, *An RLC interconnect model based on Fourier analysis,* Comput. Aided Des. Integr. Circuits Syst, 24 (2005), 170-183.

[4] T. J. Anastasio, *The fractional-order dynamics of brainstem vestibule-oculumotor neurons,* Biol. Cybern, 72 (1994), 69-79.

[5] J. H. He, *Homotopy perturbation technique,* Comput. Meth. Appl. Mech. Eng, 178 (1999), 257-262.

[6] J. H. He, *Application of homotopy perturbation method to nonlinear wave equations,* Chaos Solitons Fractals, 26 (2005), 695-700.

[7] J. H. He, *A coupling method of homotopy technique and perturbation technique for nonlinear problems,* Int. J. of Nonlinear Mech, 35 (2000), 37-43.

[8] J. H. He, *Some asymptotic methods for strongly nonlinear equations,* Int. J. Modern Phys, B 20 (2006), 1141-1199.

[9] J. H. He, *A new perturbation technique which is also valid for large parameters,* J. Sound Vib, 229 (2000), 1257-1263.

[10] A. J. Khaleel, *Homotopy perturbation method for solving special types of nonlinear Fredholm integro-differentiel equations,* J. Al-Nahrain Uni, 13 (2010), 219-224.

[11] D. D. Ganji, H. Babazadeh, F. Noori, M. M. Pirouz and M. Janipour, *An Application of Homotopy Perturbation Method for Non-linear Blasius Equation to Boundary Layer Flow Over a Flat Plate,* Int. J. Nonlinear Sci, 7 (2009), 399-404.

[12] R. Taghipour, *Application of homotopy perturbation method on some linear and nonlnear parabolique equations,* IJRRAS, 6 (2011), 55-59.

[13] W. Asghar Khan, *Homotopy Perturbation Techniques for the Solution of Certain Nonlinear Equations,* Appl. Math. Sci, 6 (2012), 6487-6499.

[14] S. M. Mirzaei, *Homotopy Perturbation Method for Solving the Second Kind of Non-Linear Integral Equations,* Int. Math. Forum, 5 (2010), 1149-1154.

[15] H. El Qarnia, *Application of homotopy perturbation method to non-homogeneous parabolic partial and non linear differential equations,* W. J. Mod. Simul, 5 (2009), 225-231.

[16] A. J. Al-Saif and D. A. Abood, *The Homotopy Perturbation Method for Solving K(2,2) Equation,* J. Basrah Researches ((Sciences)), 37 (2011).

[17] M. Matinfar, M. Mahdavi and Z. Raeisy, *The implementation of Variational Homotopy Perturbation Method for Fisher's equation,* Int. J. of Nonlinear Sci, 9 (2010), 188-194.

[18] I. Podlubny, *Fractional Differential Equations,* Academic Press, New York, 1999.

[19] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations,* Elsevier, Amsterdam, 2006.

[20] K. Diethelm, *The Analysis Fractional Differential Equations,* Springer-Verlag Berlin Heidelberg, 2010.

[21] J. Biazar and H. Ghazvini, *Convergence of the homotopy perturbation method for partial dfferential equations,* Nonlinear Analysis: Real World Applications, 10 (2009), 2633-2640.

[22] J. Biazar and H. Aminikhah, *Study of convergence of homotopy perturbation method for systems of partial differential equations,* Comput. Math. Appli, 58 (2009), 2221-2230.

[23] S. Momani and Z. Odibat, *Homotopy perturbation method for nonlinear partial differential equations of fractional order,* Phys. Lett. A 365 (2007), 345-350.

[24] M. Matinfar, Z. Raeisi and M. Mahdavi, *Variational Homotopy Perturbation Method for the Fisher's Equation,* Int. J. of Nonlinear Sci, 9 (2010), 374-378.

[25] A. Bouhassoun and M. Hamdi Cherif, *Homotopy Perturbation Method For Solving The Fractional Cahn-Hilliard Equation*, Journal of Interdisciplinary Mathematics, 18 (2015), 513-524.

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