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ps-ro FUZZY OPEN(CLOSED) FUNCTIONS AND *ps-ro* FUZZY SEMI-HOMEOMORPHISM

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Abstract. The aim of this paper is to introduce and characterize some new class of functions in a fuzzy topological space termed as *ps-ro* fuzzy open(closed) functions, *ps-ro* fuzzy pre semiopen functions and *ps-ro* fuzzy semi-homeomorphism. The interrelation among these concepts and also their relations with the parallel existing concepts are established. It is also shown with the help of examples that these newly introduced concepts are independent of the well known existing allied concepts.

1. Introduction and Preliminaries

The concept of fuzzy open (closed) functions were introduced by C.L. Chang [1] and their characterizations were studied by S.R. Malghan and S.S. Benchalli [9]. In [4], a new idea of fuzzy topology termed as pseudo regular open fuzzy topology (in short, *ps-ro* fuzzy topology) was introduced. The members of this topology are named as *ps-ro* open fuzzy sets and their complement as *ps-ro* closed fuzzy sets. Interms of above fuzzy sets, a new class of functions called *ps-ro* fuzzy continuous functions were introduced and explored in [5], [6]. In [2], a notion of *ps-ro* semiopen(closed)fuzzy sets, *ps-ro* fuzzy semiopen functions and *ps-ro* fuzzy semicontinuous functions were introduced. Also, in [3], a new idea of *ps-ro* fuzzy irresolute function was initiated and studied. The concept of fuzzy pre semiopen and fuzzy semi-homeomorphism were introduced by Yalvac [10]. In this paper, a new class of functions called *ps-ro* fuzzy open(closed) functions are defined and their different characterizations are studied. Interestingly, it is shown that the concept of *ps-ro* fuzzy open (closed) functions are independent of the well known concept of fuzzy open (closed) functions. Also, introducing a new class of functions called *ps-ro* fuzzy pre semiopen function and *ps-ro* fuzzy semi-homeomorphism, their different properties and interrelations with the existing allied concepts has been established.

To make this paper self content, we state a few known definitions and results here that we require subsequently.

Let X be a non-empty set and I be the closed interval $[0, 1]$. A fuzzy set μ on X is a function on X into I. If f is a fuction from X into a set Y and A, B are fuzzy sets on X and Y respectively, then $1 - A$ (called complement of A), $f(A)$ and $f^{-1}(B)$ are fuzzy sets on X, Y and X respectively, defined by $(1-A)(x) = 1-A(x) \forall x \in X$,

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$f(A)(y) = \begin{cases} \sup_{z\in f^{-1}(y)} A(z), when f^{-1}(y) \neq \emptyset \\ 0,\ otherwise \end{cases}$ and $f^{-1}(B)(x) = B(f(x))$ [11].
A collection $\tau \subseteq I^X$ is called a *fuzzy topology* on X if (i) $0, 1 \in \tau$ (ii) $\forall\ \mu_1, \mu_2, ..., \mu_n \in \tau \Rightarrow \wedge_{i=1}^{n} \mu_i \in \tau$ (iii) $\mu_\alpha \in \tau$, $\forall\ \alpha \in \Lambda$ (where Λ is an index set) $\Rightarrow \vee \mu_\alpha \in \tau$. Then, (X, τ) is called a fts [1]. Let f be a function from a set X into a set Y. Then the following holds:
(i) $f^{-1}(1 - B) = 1 - f^{-1}(B)$, for any fuzzy set B on Y.
(ii) $A_1 \leq A_2 \Rightarrow f(A_1) \leq f(A_2)$, for any fuzzy sets A_1 and A_2 on X. Also, $B_1 \leq B_2 \Rightarrow f^{-1}(B_1) \leq f^{-1}(B_2)$, for any fuzzy sets B_1 and B_2 on Y.
(iii) $ff^{-1}(B) \leq B$, for any fuzzy set B on Y and the equality holds if f is onto. Also, $f^{-1}f(A) \geq A$, for any fuzzy set A on X, equality holds if f is one-to-one [1]. For a fuzzy set μ in X, the set $\mu^\alpha = \{x \in X : \mu(x) > \alpha\}$ is called the strong α-level set of X. In a fts (X, τ), the family $i_\alpha(\tau) = \{\mu^\alpha : \mu \in \tau\}$ for all $\alpha \in I_1 = [0, 1)$ forms a topology on X called strong α-level topology on X [8], [7]. A fuzzy open(closed) set μ on a fts (X, τ) is said to be pseudo regular open(closed) fuzzy set if the strong α-level set μ^α is regular open(closed) in $(X, i_\alpha(\tau)), \forall \alpha \in I_1$. The family of all pseudo regular open fuzzy sets form a fuzzy topology on X called *ps-ro* fuzzy topology on X,members of which are called *ps-ro* open fuzzy sets and their complements as *ps-ro* closed fuzzy sets on (X, τ) [4]. A function f from fts (X, τ_1) to fts (Y, τ_2) is pseudo fuzzy *ro* continuous (in short, *ps-ro* fuzzy continuous) if $f^{-1}(U)$ is *ps-ro* open fuzzy set on X for each pseudo regular open fuzzy set U on Y [5]. Equivalently, f is *ps-ro* fuzzy continuous if $f^{-1}(A)$ is *ps-ro* open fuzzy set on X for each *ps-ro* open fuzzy set A on Y [6]. A fuzzy set A on a fts (X, τ) is said to be *ps-ro* semiopen fuzzy set if there exist a *ps-ro* open fuzzy set U such that $U \leq A \leq$ *ps-cl*(U), where *ps-cl*(U) is *ps*-closure of U and the complement of A is called *ps-ro* semiclosed fuzzy set [2]. The fuzzy operators termed as fuzzy *ps*-closure(interior), *ps*-semiclosure(interior) are denoted by *ps-cl*(*ps-int*) and *ps-scl*(*ps-sint*) respectively. *ps-int*(*ps-sint*) of a fuzzy subset A the union of all *ps-ro* open (*ps-ro* semiopen) fuzzy set on X contained in A and *ps-cl*(*ps-scl*) of a fuzzy subset A the intersection of all *ps-ro* closed (*ps-ro* semiclosed) fuzzy set on X containing A [5], [6], [2]. A function f from a fts (X, τ_1) to another fts (Y, τ_2) is called *ps-ro* fuzzy semiopen function [2] if $f(A)$ is *ps-ro* semiopen fuzzy set on Y for each *ps-ro* open fuzzy set A on X. The function f is called *ps-ro* fuzzy irresolute [3] if $f^{-1}(U)$ is *ps-ro* semiopen fuzzy set on X for each *ps-ro* semiopen fuzzy set U on Y. If a function f be bijective, then f is *ps-ro* fuzzy irresolute function iff for every fuzzy set A of X, *ps-sint*$(f(A)) \leq$ (*ps-sint*(A))[3]. For a function $f : X \to Y$, the following are equivalent:
$(a)$$f$ is *ps-ro* fuzzy continuous.
(b) Inverse image of each *ps-ro* open fuzzy sets on Y under f is *ps-ro* open on X.
(c)For all fuzzy set A on X, f(*ps-cl*$(A)) \leq$ *ps-cl*$(f(A))$.
(d) For all fuzzy set B on Y, *ps-cl*$(f^{-1}(B)) \leq f^{-1}$(*ps-cl*$(B))$ [6].

2. *ps-ro* FUZZY OPEN AND CLOSED FUNCTIONS

Definition 2.1. *Let (X, τ_1) and (Y, τ_2) be two fts. A function $f : (X, \tau_1) \to (Y, \tau_2)$ is said to be ps-ro fuzzy open(closed) if $f(A)$ is ps-ro open(closed) fuzzy set on Y for each ps-ro open(closed) fuzzy set A on X.*

Theorem 2.1. *If f is ps-ro continuous and ps-ro fuzzy open and A be any ps-ro semiopen fuzzy set on X then $f(A)$ is ps-ro semiopen fuzzy set on Y.*

Proof: Let A be any *ps-ro* semiopen fuzzy set on X, there exist *ps-ro* open fuzzy set U on X such that $U \leq A \leq ps\text{-}cl(U)$. So, $f(U) \leq f(A) \leq f(ps\text{-}cl(U)) \leq ps\text{-}cl(f(U))$ and $f(U)$ is *ps-ro* open fuzzy set. Hence, $f(A)$ is *ps-ro* semiopen fuzzy set on Y.

Example 2.1. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.1, A(b) = 0.2, A(c) = 0.2$ and $B(x) = 0.3\ \forall x \in X$. Let C, D and E be fuzzy set on Y defined by $C(t) = 0.3\ \forall t \in Y$, $D(x) = 0.3, D(y) = 0.3, D(z) = 0.4$ and $E(t) = 0.4\ \forall t \in Y$. Clearly, $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E\}$ are fuzzy topologies on X and Y respectively. Clearly, for $0.1 \leq \alpha < 0.2$, A is not pseudo regular open fuzzy set on (X, τ_1). Therefore, *ps-ro* fuzzy topology on X is $\{0, 1, B\}$. Again, D is not pseudo regular open fuzzy set for $0.3 \leq \alpha < 0.4$ on (Y, τ_2). So, *ps-ro* fuzzy topology on Y is $\{0, 1, C, E\}$. Define a function f from (X, τ_1) to (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = y$. B is *ps-ro* open fuzzy set on Y and $f(B)(t) = 0.3 = C(t)\ \forall\ t \in Y$. Therefore, $f(B)$ is *ps-ro* open fuzzy set on Y. Also, $F(0) = 0, f(1) = 1$. Hence, f is *ps-ro* fuzzy open. Now, $f(A)(x) = 0.1, f(A)(y) = 0.2, f(A)(z) = 0$. Clearly, $f(A)$ is not open fuzzy set on Y. Hence, f is not fuzzy open function. Again, here f is *ps-ro* fuzzy closed as $f(1 - B)(t) = 0.7 = (1 - C)(t)$, $\forall t \in Y$ is *ps-ro* closed fuzzy set on Y but f is not fuzzy closed function since $f(1 - A)(t) = 0.9, 0.8$ and 0 for $t = x, y$ and z respectively, is not fuzzy closed set on Y.

Remark 2.1. *Let f be fuzzy open (closed) from a fts (X, τ_1) to a fts (Y, τ_2) and A be a open (closed) fuzzy set on X. Then, $f(A)$ is fuzzy open (closed) on Y which is not necessarily ps-ro open (closed) fuzzy set on Y, for an example in Example(2.1), A is fuzzy open but not ps-ro fuzzy opnen on X. Hence, a fuzzy open (closed) function may not be ps-ro fuzzy open (closed). In the view of this and Example (2.1) we conclude that ps-ro fuzzy open (closed) functions and fuzzy open (closed) functions do not imply each other.*

Theorem 2.2. *Let f be a function from a fts (X, τ_1) to a fts (Y, τ_2). Then the following statements are equivalent:*
(a)f is ps-ro fuzzy open.
(b)$f(ps\text{-}int(A)) \leq ps\text{-}int(f(A))$, for each fuzzy set A on X.
(c)$f^{-1}(ps\text{-}cl(B)) \leq ps\text{-}cl(f^{-1}(B))$, for each fuzzy set B on Y.
(d)$ps\text{-}int(f^{-1}(B)) \leq f^{-1}(ps\text{-}int(B))$, for each fuzzy set B on Y.
Proof: $(a) \Rightarrow (b)$ Let f be *ps-ro* fuzzy open function. Let A be any fuzzy set on X. $f(ps\text{-}int(A))$ is *ps-ro* open fuzzy set on Y. Now, $f(ps\text{-}int(A)) = ps\text{-}int(f(ps\text{-}int(A))) \leq ps\text{-}int(f(A))$.
$(b) \Rightarrow (a)$ Let A be a *ps-ro* open fuzzy set on X. Then $A = ps\text{-}int(A)$. So, $f(A) = f(ps\text{-}int(A)) \leq ps\text{-}int(f(A)) \leq f(A)$. So, $f(A) = ps\text{-}int(f(A))$, proving $f(A)$ is *ps-ro* open fuzzy set on Y. Thus, f is *ps-ro* fuzzy open.
$(b) \Rightarrow (c)$ Let B be any fuzzy sets on Y. Let $A = f^{-1}(1-B)$ be a fuzzy set on X. We have $f(ps\text{-}int(A)) \leq ps\text{-}int(f(A)) \leq ps\text{-}int(1 - B)$. Hence, $ps\text{-}int(f^{-1}(1 - B)) \leq f^{-1}(ps\text{-}int(1 - B))$. Then, $f^{-1}(ps\text{-}cl(B)) = 1 - f^{-1}(ps\text{-}int(1 - B)) \leq 1 - ps\text{-}int(f^{-1}(1-B)) = ps\text{-}cl(1 - f^{-1}(1-B)) = ps\text{-}cl(f^{-1}(B))$. So, $f^{-1}(ps\text{-}cl(B)) \leq ps\text{-}cl(f^{-1}(B))$
$(c) \Rightarrow (d)$ Let B be any fuzzy set on Y and $C = 1 - B$. Then, C is also fuzzy set on Y. We have $f^{-1}(ps\text{-}cl(C)) \leq ps\text{-}cl(f^{-1}(C))$. So, $ps\text{-}int(f^{-1}(B)) = 1 - ps\text{-}cl(f^{-1}(C)) \leq 1 - f^{-1}(ps\text{-}cl(C)) = f^{-1}(1 - ps\text{-}cl(C)) = f^{-1}(ps\text{-}int(1 - C)) =$

$f^{-1}(ps\text{-}int(B))$. Hence, $ps\text{-}int(f^{-1}(B)) \leq f^{-1}(ps\text{-}int(B))$.
$(d) \Rightarrow (b)$ Let A be any fuzzy set on X and let $B = f(A)$. Then we have $ps\text{-}int(A) \leq ps\text{-}int(f^{-1}f(A)) = ps\text{-}int(f^{-1}(B)) \leq f^{-1}(ps\text{-}int(B))$. So, $f(ps\text{-}int(A)) \leq f(f^{-1}(ps\text{-}int(B))) \leq ps\text{-}int(B) = ps\text{-}int(f(A))$. Hence, $f(ps\text{-}int(A)) \leq ps\text{-}int(f(A))$.

Corollary 2.1. *If $f : (X, \tau_1) \to (Y, \tau_2)$ is a ps-ro fuzzy open and ps-ro fuzzy continuous then $f^{-1}(ps\text{-}cl(B)) = ps\text{-}cl(f^{-1}(B))$, for each fuzzy set B on Y.*
Proof: Straightforward and hence omitted.

Theorem 2.3. *Let f be a function from a fts (X, τ_1) to a fts (Y, τ_2). Then f is ps-ro fuzzy closed (open) iff for each fuzzy set A on Y and for any ps-ro open (closed) fuzzy set B on X such that $f^{-1}(A) \leq B$, there is a ps-ro open(closed) fuzzy set C on Y such that $A \leq C$ and $f^{-1}(C) \leq B$.*
Proof: Let f be *ps-ro* fuzzy closed(open). Let A be any fuzzy set on Y and let B be a *ps-ro* open(closed) fuzzy set on X such that $f^{-1}(A) \leq B$. Let $C = 1 - f(1 - B)$. Then C is a *ps-ro* open(closed) fuzzy set on X, since f is *ps-ro* fuzzy closed(open) and $1 - B$ is *ps-ro* closed(open) fuzzy set on X, $f(1 - B)$ is *ps-ro* closed(open) fuzzy set on Y. Hence, $1 - B \leq 1 - f^{-1}(A) = f^{-1}(1 - A)$. So, $f(1 - B) \leq f(f^{-1}(1 - A)) \leq 1 - A$. Hence, $A \leq 1 - f(1 - B) = C$. Further, $f^{-1}(C) = f^{-1}(1 - f(1 - B)) = 1 - f^{-1}(f(1 - B)) \leq 1 - (1 - B) = B$. Conversely, let f satisfies the given condition. Let B be a *ps-ro* closed(open) fuzzy set on X. Then, $A = 1 - B$ is *ps-ro* open(closed) fuzzy set on X. So, $f^{-1}(1 - f(B)) = 1 - f^{-1}(f(B)) \leq 1 - B = A$. By hypothesis, there is a *ps-ro* open(closed) fuzzy set C on Y such that $1 - f(B) \leq C$ and $f^{-1}(C) \leq A = 1 - B$. Hence, $1 - C \leq f(B)$. Also, $B \leq 1 - f^{-1}(C) = f^{-1}(1 - C)$. So, $f(B) \leq f(f^{-1}(1 - C)) \leq 1 - C$. Thus, we have $f(B) = 1 - C$, which is a *ps-ro* closed(open) fuzzy set on Y. Hence, f is *ps-ro* fuzzy closed(open).

Theorem 2.4. *Let f be a function from a fts (X, τ_1) to a fts (Y, τ_2). Then f is ps-ro fuzzy closed iff for each fuzzy set A on X, $ps\text{-}cl(f(A)) \leq f(ps\text{-}cl(A))$.*
Proof: Let f be *ps-ro* fuzzy closed and A be any fuzzy set on X. Since $ps\text{-}cl(A)$ is *ps-ro* closed fuzzy set on X and f is *ps-ro* fuzzy closed, $f(ps\text{-}cl(A))$ is *ps-ro* closed fuzzy set on Y. As, $A \leq ps\text{-}cl(A)$, $f(A) \leq f(ps\text{-}cl(A))$. So, $ps\text{-}cl(f(A)) \leq ps\text{-}cl(f(ps\text{-}cl(A))) = f(ps\text{-}cl(A))$. Conversely, let A be any *ps-ro* closed fuzzy set on X. Then $f(A) = f(ps\text{-}cl(A)) \geq ps\text{-}cl(f(A))$. As, $f(A) \leq ps\text{-}cl(f(A))$, $f(A) = ps\text{-}cl(f(A))$, i.e. $f(A)$ is *ps-ro* closed fuzzy set on Y. Hence, f is *ps-ro* fuzzy closed.

Theorem 2.5. *For a bijective function f from a fts (X, τ_1) to a fts (Y, τ_2), the following are equivalent.*
(a)$f^{-1} : Y \to X$ is ps-ro fuzzy continuous.
(b) f is ps-ro fuzzy open.
(c) f is ps-ro fuzzy closed.
Proof: $(a) \Rightarrow (b)$ Let f^{-1} be *ps-ro* fuzzy continuous. Let U be a *ps-ro* open fuzzy set on X. Since, f^{-1} is *ps-ro* fuzzy continuous, $(f^{-1})^{-1}(U) = f(U)$ is *ps-ro* open fuzzy set on Y. Hence, f is *ps-ro* fuzzy open.
$(b) \Rightarrow (c)$ Let f be bijective and *ps-ro* fuzzy open. Let V be a *ps-ro* closed fuzzy set on X. Then, $1 - V = A$ is *ps-ro* open fuzzy set on X. Since f is *ps-ro* fuzzy open and bijective, $f(A) = f(1 - V) = 1 - f(V)$ is *ps-ro* open fuzzy set on Y. Therefore, $f(V)$ is *ps-ro* closed fuzzy set on Y. Hence, f is *ps-ro* fuzzy closed.
$(c) \Rightarrow (a)$ Let f be *ps-ro* fuzzy closed and bijective. Let V be a *ps-ro* closed fuzzy

set on X. Then $f(V)$ is *ps-ro* closed fuzzy set on Y. But $f(V) = (f^{-1})^{-1}(V)$ and hence f^{-1} is *ps-ro* fuzzy continuous.

3. *ps-ro* FUZZY SEMI-HOMEOMORPHISM

Definition 3.1. *Let (X, τ_1) and (Y, τ_2) be two fts and $f : (X, \tau_1) \to (Y, \tau_2)$. Then f is said to be*
(i) ps-ro fuzzy pre semiopen function if $f(A)$ is ps-ro semiopen fuzzy set on Y, for each ps-ro semiopen fuzzy set A on X.
(ii) ps-ro fuzzy homeomorphism if f is bijective, ps-ro fuzzy continuous and ps-ro fuzzy open function.
(iii) ps-ro fuzzy semi-homeomorphism if f is bijective, ps-ro fuzzy pre semiopen and ps-ro fuzzy irresolute.

Theorem 3.1. *Let (X, τ_1) and (Y, τ_2) be two fts. If $f : (X, \tau_1) \to (Y, \tau_2)$ is ps-ro fuzzy continuous and ps-ro fuzzy open, then f is ps-ro fuzzy irresolute.*
Proof: Let f be *ps-ro* fuzzy continuous and *ps-ro* fuzzy open function. Let U be a *ps-ro* semiopen fuzzy set on Y. Then $\exists$ *ps-ro* open fuzzy set V on Y such that $V \leq U \leq$ *ps-cl*(V). Now, $f^{-1}(V)$ is *ps-ro* open fuzzy on X. Hence, $f^{-1}(V) \leq f^{-1}(U) \leq f^{-1}($*ps-cl*$(V))$. f is *ps-ro* fuzzy open and V is fuzzy set on Y, $f^{-1}($*ps-cl*$(V)) \leq$ *ps-cl*$(f^{-1}(V))$. So, $f^{-1}(V) \leq f^{-1}(U) \leq$ *ps-cl*$(f^{-1}(V))$. Thus, $f^{-1}(U)$ is *ps-ro* semiopen fuzzy set on X and hence f is *ps-ro* fuzzy irresolute.

Theorem 3.2. *Let (X, τ_1) and (Y, τ_2) be two fts. If $f : (X, \tau_1) \to (Y, \tau_2)$ is ps-ro fuzzy continuous and ps-ro fuzzy open, then f is ps-ro fuzzy pre semiopen.*
Proof: Let f be *ps-ro* fuzzy continuous and *ps-ro* fuzzy open function. Let A be a *ps-ro* semiopen fuzzy set on X. Then $\exists$ *ps-ro* open fuzzy set V on X such that $V \leq A \leq$ *ps-cl*(V). Now, since f is *ps-ro* fuzzy continuous and V is a fuzzy set on X, $f($*ps-cl*$(V)) \leq$ *ps-cl*$(f(V))$. Hence $f(V) \leq f(A) \leq f($*ps-cl*$(V)) \leq$ *ps-cl*$(f(V))$. Also, $f(V)$ is *ps-ro* open fuzzy set on Y. So, $f(A)$ is *ps-ro* semiopen fuzzy set on Y. Thus, f is *ps-ro* fuzzy pre semiopen function.

Remark 3.1. *From Theorem(3.1) and (3.2) it follows that ps-ro fuzzy homeomorphism implies ps-ro fuzzy semi-homeomorphism. However, the converse is not true follows from the example below:*

Example 3.1. Let $X = \{a, b, c\}$ and $Y = \{x, y, z\}$. Let A and B be two fuzzy sets on X defined by $A(a) = 0.2, A(b) = 0.2, A(c) = 0.3$ and $B(x) = 0.2\ \forall x \in X$. Let C, D and E be fuzzy set on Y defined by $C(x) = 0.2, C(y) = 0.3, C(z) = 0.3$, $D(x) = 0.4, D(y) = 0.4, D(z) = 0.5$ and $E(t) = 0.3\ \forall t \in Y$. Clearly, $\tau_1 = \{0, 1, A, B\}$ and $\tau_2 = \{0, 1, C, D, E\}$ are fuzzy topologies on X and Y respectively. In the corresponding topological space $(X, i_\alpha(\tau_1))$, $\forall \alpha \in I_1 = [0, 1)$, the open sets are ϕ, X, A^α and B^α, where $A^\alpha = \begin{cases} X, & \text{for } \alpha < 0.2 \\ \{c\}, & \text{for } 0.2 \leq \alpha < 0.3 \\ \phi, & \text{for } \alpha \geq 0.3 \end{cases}$ and
$B^\alpha = \begin{cases} X, & \text{for } \alpha < 0.2 \\ \phi, & \text{for } \alpha \geq 0.2 \end{cases}$
For $0.2 \leq \alpha < 0.3$, the closed sets on $(X, i_\alpha(\tau_1))$ are ϕ, X and $X - \{c\}$. Therefore, $int(cl(A^\alpha)) = X$. So, A^α is not regular open on $(X, i_\alpha(\tau_1))$ for $0.2 \leq \alpha < 0.3$. Thus, A is not pseudo regular open fuzzy set on (X, τ_1). Clearly, B^α is regular

open on $(X, i_\alpha(\tau_1))$, $\forall \alpha \in I_1$. Hence B is pseudo regular open fuzzy set on (X, τ_1). Therefore, *ps-ro* fuzzy topology on X is $\{0, 1, B\}$. Similarly, it can be seen that C and D are not pseudo regular open fuzzy set on (Y, τ_2) and thus, *ps-ro* fuzzy topology on Y is $\{0, 1, E\}$. Define a function f from the fts (X, τ_1) to the fts (Y, τ_2) by $f(a) = x, f(b) = y$ and $f(c) = y$. Then, $f^{-1}(0) = 0, f^{-1}(1) = 1$ and $f^{-1}(E)(t) = 0.3$ $\forall t \in X$. Since $f^{-1}(E)$ is not *ps-ro* open fuzzy set on X, f is not *ps-ro* fuzzy continuous. Now, *ps-cl*$(E) = 1 - E$ where, $(1 - E)(t) = 0.7$ $\forall t \in Y$. So, $E \leq D \leq$ *ps-cl*(E). Thus, D is *ps-ro* semiopen fuzzy set on Y. We have, $f^{-1}(D)(t) = 0.4$ $\forall t \in X$ and *ps-cl*$(B) = 1 - B$ where, $(1 - B)(t) = 0.8$ $\forall t \in X$. So, $B \leq f^{-1}(D) \leq$ *ps-cl*(B). So, $f^{-1}(D)$ is *ps-ro* semiopen fuzzy set on X. Again, E is *ps-ro* open and hence *ps-ro* semiopen fuzzy set on Y. $f^{-1}(E)$ is *ps-ro* semiopen fuzzy set on X, as $B \leq f^{-1}(E) \leq$ *ps-cl*(B). Hence, $f^{-1}(U)$ is *ps-ro* semiopen fuzzy set on X, for every *ps-ro* semiopen fuzzy set U on Y. Thus, f is *ps-ro* fuzzy irresolute function.

Theorem 3.3. *Let (X, τ_1), (Y, τ_2) and (Z, τ_3) be three fts and $f : (X, \tau_1) \to (Y, \tau_2)$, $g : (Y, \tau_2) \to (Z, \tau_3)$. Then the following statements are valid:*
(a) If f and g are ps-ro fuzzy pre semiopen functions then $g \circ f$ is so.
(b) If f is ps-ro fuzzy semiopen function and g is ps-ro fuzzy pre semiopen function then $g \circ f$ is a ps-ro fuzzy semiopen function.
Proof:(a) Let U be *ps-ro* semiopen fuzzy set on X. Since, f and g are *ps-ro* fuzzy pre semiopen functions, $f(U)$ and hence $g(f(U))$ are *ps-ro* semiopen fuzzy sets on Y and Z respectively. Hence, $(g \circ f)(U) = g(f(U))$ is *ps-ro* semiopen fuzzy set on Z for each *ps-ro* semiopen fuzzy set U on X. Thus, $g \circ f$ is *ps-ro* fuzzy semiopen function.
(b) Let U be a *ps-ro* open fuzzy set on X. Since, f and g are both *ps-ro* fuzzy semiopen functions, $g(f(U))$ is *ps-ro* semiopen fuzzy set on Z. Thus, $g \circ f$ is *ps-ro* fuzzy semiopen function.

Theorem 3.4. *Let a function f from a fts (X, τ_1) to a fts (Y, τ_2) be bijective. f is ps-ro fuzzy semi-homeomorphism iff f and f^{-1} are both ps-ro fuzzy irresolute functions and ps-ro fuzzy pre semiopen functions.*
Proof: Let f be *ps-ro* fuzzy semi-homeomorphism. Now, since f is bijective, f^{-1} exist. Let $f^{-1} = g$. As, f is *ps-ro* fuzzy irresolute, for each *ps-ro* semiopen fuzzy set A on Y, $f^{-1}(A)$ is *ps-ro* semiopen fuzzy set on X. But, $f^{-1} = g$, so, $g(A)$ is *ps-ro* semiopen fuzzy set on X, for each *ps-ro* semiopen fuzzy set A on Y. Thus, g is *ps-ro* fuzzy pre semiopen. Again, f is *ps-ro* fuzzy pre semiopen. Therefore, for each *ps-ro* semiopen fuzzy set B on X, $f(B)$ is *ps-ro* semiopen fuzzy set on Y. But, $f^{-1} = g$, so, $f = g^{-1}$ and $g^{-1}(B)$ is *ps-ro* semiopen fuzzy set on Y, for each *ps-ro* semiopen fuzzy set B on X. Hence, g is *ps-ro* fuzzy irresolute. Conversely, straightforward.

Theorem 3.5. *A bijective function f from a fts (X, τ_1) to a fts (Y, τ_2) is ps-ro fuzzy semi-homeomorphism iff for each fuzzy set A on X, f(ps-scl$(A)) =$ ps-scl$(f(A))$.*
Proof: Let f be *ps-ro* fuzzy semi-homeomorphism. Then, f is *ps-ro* fuzzy irresolute. So, for each fuzzy set A on X, $f($*ps-scl*$(A)) \leq$ *ps-scl*$(f(A))$. Again, since f is *ps-ro* fuzzy semi-homeomorphism, f^{-1} is *ps-ro* fuzzy irresolute. As, *ps-scl*(A) is *ps-ro* semiclosed fuzzy set on X, $(f^{-1})^{-1}($*ps-scl*$(A)) = f($*ps-scl*$(A))$ is *ps-ro*

semiclosed fuzzy set on Y. Now, $A \leq ps\text{-}scl(A)$. So, $f(A) \leq f(ps\text{-}scl(A))$, $ps\text{-}scl(f(A)) \leq f(ps\text{-}scl(A))$. Hence, $f(ps\text{-}scl(A)) = ps\text{-}scl(f(A))$. Conversely, let f be bijective and $f(ps\text{-}scl(A)) = ps\text{-}scl(f(A))$, for each fuzzy set A on X. Then, clearly $f(ps\text{-}scl(A)) \leq ps\text{-}scl(f(A))$. Hence, f is *ps-ro* fuzzy irresolute function. Let A be any *ps-ro* semiclosed fuzzy set on X. Then $B = 1 - A$ is *ps-ro* semiopen fuzzy set on X. Now, $A = ps\text{-}scl(A)$. So, $f(A) = f(ps\text{-}scl(A)) = ps\text{-}scl(f(A))$. $1 - f(A) = 1 - ps\text{-}scl(f(A))$ So, $f(1 - A) = ps\text{-}sint(1 - f(A))$(as f is bijective, $f(1 - A) = 1 - f(A)$). $f(B) = ps\text{-}sint(f(1 - A)) = ps\text{-}sint(f(B))$. This implies that $f(B)$ is *ps-ro* semiopen fuzzy set on Y. Hence, f is *ps-ro* fuzzy pre semiopen function. Therefore, f is *ps-ro* fuzzy semi-homeomorphism.

Corollary 3.1. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be bijective. f is a ps-ro fuzzy semi-homeomorphism iff for each fuzzy set B on Y, $f^{-1}(ps\text{-}scl(B)) = ps\text{-}scl(f^{-1}(B))$.*
Proof: Since, f is a *ps-ro* fuzzy semi-homeomorphism, f^{-1} is also so.

Theorem 3.6. *Let a function f from a fts (X, τ_1) to a fts (Y, τ_2) be bijective. f is ps-ro fuzzy semi-homeomorphism iff for each fuzzy set A on X, $f(ps\text{-}sint(A)) = ps\text{-}sint(f(A))$.*
Proof: Let f be *ps-ro* fuzzy semi-homeomorphism. Then, f is bijective and both f and f^{-1} are *ps-ro* fuzzy irresolute. So, for each fuzzy set A on X, $ps\text{-}sint(f(A)) \leq f(ps\text{-}sint(A))$. $ps\text{-}sint(A)$ being *ps-ro* semiopen fuzzy set on X, $(f^{-1})^{-1}(ps\text{-}sint(A)) = f(ps\text{-}sint(A))$ is *ps-ro* semiopen fuzzy set on Y. Now, $ps\text{-}sint(A) \leq A$, $f(ps\text{-}sint(A)) \leq f(A)$. So, $f(ps\text{-}sint(A)) \leq ps\text{-}sint(f(A))$. Hence, $f(ps\text{-}sint(A)) = ps\text{-}sint(f(A))$. Conversely, let f be bijective and $f(ps\text{-}sint(A)) = ps\text{-}sint(f(A))$, for each fuzzy set A on X. Then, clearly $ps\text{-}sint(f(A)) \leq f(ps\text{-}sint(A))$. Also, f is bijective. Hence, f is *ps-ro* fuzzy irresolute function. Now, let B be a *ps-ro* semiopen fuzzy set on X. Then, by given condition we have $f(ps\text{-}sint(B)) = ps\text{-}sint(f(B))$. So, $f(B) = ps\text{-}sint(f(B))$. This implies that $f(B)$ is *ps-ro* semiopen fuzzy set on Y. Hence, f is *ps-ro* fuzzy pre semiopen function. Therefore, f is *ps-ro* fuzzy semi-homeomorphism.

Corollary 3.2. *Let $f : (X, \tau_1) \to (Y, \tau_2)$ be bijective. f is a ps-ro fuzzy semi-homeomorphism iff for each fuzzy set B on Y, $f^{-1}(ps\text{-}sint(B)) = ps\text{-}sint(f^{-1}(B))$.*
Proof: Since, f is a *ps-ro* fuzzy semi-homeomorphism, f^{-1} is also so.

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