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FREE CONVECTION HEAT AND MASS TRANSFER MHD FLOW IN A VERTICAL CHANNEL IN THE PRESENCE OF CHEMICAL REACTION

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Abstract. An analysis is made to study the effects of diffusion-thermo and chemical reaction on fully developed laminar MHD flow of electrically conducting viscous incompressible fluid in a vertical channel formed by two vertical parallel plates was taken into consideration with uniform temperature and concentration. The analytical solution by Laplace transform technique of partial differential equations is used to obtain the expressions for the velocity, temperature and concentration. It is interesting to note that during the course of computation, the transient solution at large time coincides with steady state solution derived separately and the diffusion-thermo effect creates an anomalous situation in temperature and velocity profiles for small Prandtl numbers. The study is restricted to only destructive reaction and non-conducting case cannot be derived as a particular case still it is quite interesting and more realistic than the earlier one.

1. INTRODUCTION:

In many transport processes and industrial applications, transfer of heat and mass occurs simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Unsteady natural-convection of heat and mass transfer is of great importance in designing control systems for modern free convection heat exchangers.

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More recently, Jha and Ajibade [1] have studied the heat and mass transfer aspect of the flow of a viscous incompressible fluid in a vertical channel considering the Dufour effect.

Soundalgekar [2] studied the effect of mass transfer and free convection currents on the flow past an impulsively started infinite vertical plate and observed that the presence of foreign gasses in the flow domain leads to reduce the shear stress and rate of mass transfer significantly.

In nature, flow occurs due to density differences caused by temperature as well as chemical composition gradients. Therefore, it warrants the simultaneous consideration of temperature difference as well as concentration difference when heat and mass transfer occurs simultaneously. It has been found that an energy flux can be created not only by temperature gradients but by composition gradients also. This is called Dufour effect. If, on the other hand, mass fluxes are created by temperature gradients, it is called the Soret effect.

The Soret and the Dufour effects have been found to be useful as the Soret effect is utilized for isotope separation and in a mixture of gases of light and medium molecular weight, the Dufour effect is found to be of considerable order of magnitude such that it cannot be neglected. In view of importance of the diffusion-thermo effect Kafoussias and Williams [3] have studied the effects of thermal diffusion and diffusion-thermo on mixed free and forced convective and mass transfer boundary layer flow with temperature dependent viscosity. Jha and Singh [4], Kafoussias [5], and Alam *et al.* [6,7] have contributed significantly to this field of study. Recently, Beg *et al.* [8] studied chemically reacting mixed convective heat and mass transfer along inclined and vertical plates considering Soret and Dufour effect. More recently, Dursunkaya and Worek [9] have studied the diffusion-thermo and thermal diffusion effects in transient and steady natural convections from a vertical surface.

MHD flow with thermal diffusion and chemical reaction finds numerous applications in various areas such as thermo nuclear fusion, liquid metal cooling of nuclear reactions and electromagnetic casting of metals. Aberu *et al.* [10] have studied the boundary layer flows with Dufour and Soret effects. Osalusi *et al.* [11] have worked on mixed and free convective heat and mass transfer of an electrically conducting fluid considering Dufour and Soret effects. Anghel et al.[12], Postelnicu [13] obtained numerical solutions of chemical reacting mixed convective heat and mass transfer along inclined and vertical plates with the Soret and the Dufour effects and concluded that skin friction increases with a positive increase in the concentration-to-thermal-buoyancy ratio parameter.

The analysis of natural convection heat and mass transfer near a moving vertical plate has received much attention in recent times due to its wide application in engineering and technological processes. There are applications of interest in which combined heat and mass transfer by natural convection occurs between a moving material and ambient medium, such as the design and operation of chemical processing equipment, design of heat exchangers, transpiration cooling of a surface, chemical vapour deposition of solid layer, nuclear reactor, and many manufacturing processes like hot rolling, hot extrusion, wire drawing, continuous casting and fiber drawing. Gebhart and Pera [14] studied the effects of mass transfer on a steady free convection flow past a semi infinite vertical plate by the similarity method and it was assumed that the concentration level of the diffusing species in fluid medium was very low. This assumption enabled them to neglect the diffusion-thermo and thermo-diffusion effects as well as the interfacial velocity at the wall due to species diffusion. Following this assumption, Soundalgekar et al. [15] investigated the effects of simultaneous heat and mass transfer on free convection flow past an infinite vertical plate under different physical situations.

Recently, the unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion have studied by Saxena and Dubey [16]. Raveendra Babu et al. [17] studied diffusion-thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order. Sudhakar et al [18] discussed chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo.

In the present paper, the flow of an electrically conducting viscous incompressible fluid in a vertical channel formed by two vertical parallel plates in the presence of a transverse magnetic field is studied. Further, the mass transfer phenomena considered in this problem is associated with chemically reacting species. The objective of the present study is to extend the work of Jha and Ajibade [1] by incorporating the magnetic field effect as well as chemical reaction on the flow, heat and mass transfer phenomena.

2. MATHEMATICAL FORMULATION:

Let us consider the free-convective heat and mass transfer MHD flow of a viscous incompressible fluid in a vertical channel formed by two infinite vertical parallel plates in the presence of chemical reaction. A magnetic field of uniform strength B_0 is applied transversely to the plate. The induced magnetic field is neglected as the magnetic Reynolds number of the flow is taken to be very small. The convection current is induced due to both the temperature and concentration differences. The flow is assumed to be in the x' – direction which is taken to be vertically upward along the channel walls and $y'-axis$ is taken to be normal to the plates that are h distance apart.

Under the usual Boussinesq's approximations, the governing equations for flow are given by

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + g\beta(T' - T_0) + g\beta^*(C' - C_0) - \frac{\sigma B_0^2 u'}{\rho} \tag{1}$$

$$\frac{\partial C'}{\partial t'} = D\frac{\partial^2 C'}{\partial y'^2} - K_c'(C' - C_0) \tag{2}$$

$$\frac{\partial T'}{\partial t'} = \alpha \frac{\partial^2 T'}{\partial y'^2} + D_1 \frac{\partial^2 C'}{\partial y'^2} \tag{3}$$

Where $\nu, g, \beta, \beta^*, \sigma, B_0, \rho$, D , K'_c , α and D_1 are kinematic viscosity, acceleration due to gravity, coefficient of thermal expansion, coefficient of mass expansion, electrical conductivity of the fluid, uniform magnetic field, density of the fluid, chemical molecular diffusivity, the dimensional chemical reaction parameter in the diffusion equation, thermal diffusivity and dimensional coefficient of the diffusion thermo effect respectively.

Initial and boundary conditions of the problem are

$$t' \le 0 : u'(y',t') = 0,\ T'(y',t') = T_0, C'(y',t') = C_0$$

$$t' > 0 : \begin{array}{l} u'(o,t') = 0, T'(0,t') = T_w, C'(0,t') = C_w \\ u'(h,t') = 0, T'(h,t') = T_0, C'(h,t') = C_0 \end{array} \tag{4}$$

We now introduce following dimensionless quantities:

$$y = \frac{y'}{h}, t = \frac{t'\nu}{h^2}, u = \frac{u'\nu}{g\beta h^2(T_w - T_0)}, P_r = \frac{\nu}{\alpha}, S_c = \frac{\nu}{D}$$

$$T = \frac{T' - T_0}{T_w - T_0}, C = \frac{C' - C_0}{C_w - C_0}, N = \frac{\beta^*(C_w - C_0)}{\beta(T_w - T_0)}$$

$$D^* = \frac{D_1(C_w - C_0)}{\alpha(T_w - T_0)}, M = \frac{\sigma B_0^2 h^2}{\rho\nu}, K_c = \frac{K_c^* h^2}{\nu} \tag{5}$$

Here P_r, S_c, N , D^* , M and K_c are the Prandtl number, the Schmidt number,
Sustentation parameter, the Dufour number, the Magnetic field parameter and Chemical reaction parameter respectively.
The non-dimensional form of equations (1) – (3) are given by

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} + NC + T - Mu \tag{6}$$

$$\frac{\partial C}{\partial t} = \frac{1}{S_c}\frac{\partial^2 C}{\partial y^2} - K_c C \tag{7}$$

$$\frac{\partial T}{\partial t} = \frac{1}{p_r}\frac{\partial^2 T}{\partial y^2} + \frac{D^*}{p_r}\frac{\partial^2 C}{\partial y^2} \tag{8}$$

Subject to boundary conditions

$$t \le 0 : u(y,t) = 0, T(y,t) = 0, C(y,t) = 0$$

$$t > 0: \quad \begin{aligned} &u(0,t) = 0, T(0,t) = 1, C(0,t) = 1 \\ &u(1,t) = 0, T(1,t) = 0, C(1,t) = 0 \end{aligned} \tag{9}$$

3. Solution of the problem:

The solutions of equations (6)- (8) subject to the boundary conditions (9) are obtained by Laplace Transform Technique.

Case I: $(P_r \neq 1, S_c \neq 1)$

The solutions of equations (6) – (8) subject to boundary conditions (9) are given by

$$C = \sum_{n=0}^{\infty} \left[f\left(K_c, a_n\sqrt{S_c}, 1, 0, t\right) - f\left(K_c, b_n\sqrt{S_c}, 1, 0, t)\right)\right] \tag{10}$$

$$\begin{aligned} T = {} & A_1 \sum_{n=0}^{\infty} \left[f\left(0, a_n\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) - f\left(0, b_n\sqrt{P_r}, P_r - S_c, S_c K_c, t\right)\right] \\ & - A_1 \sum_{n=0}^{\infty} \left[f\left(K_c, a_n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) - f\left(K_c, b_n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right)\right] \\ & + \left(D^* + 1\right) \sum_{n=0}^{\infty} \left[f\left(0, a_n\sqrt{P_r}, 1, 0, t\right) - f\left(0, b_n\sqrt{P_r}, 1, 0, t\right)\right] - D^* C \end{aligned} \tag{11}$$

$$\begin{aligned} u = {} & A_2 \sum_{n=0}^{\infty} \left[f(M, a_n, 1 - S_c, S_c K_c - M, t) - f(M, b_n, 1 - S_c, S_c K_c - M, t)\right] \\ & - A_2 \sum_{n=0}^{\infty} \left[f\left(K_c, a_n\sqrt{S_c}, 1 - S_c, S_c K_c - M, t\right) - f\left(K_c, b_n\sqrt{S_c}, 1 - S_c, S_c K_c - M, t\right)\right] \end{aligned}$$

$$\begin{aligned} & + A_3 \sum_{n=0}^{\infty} \left[f(M, a_n, 1 - P_r, -M, t) - f(M, b_n, 1 - P_r, -M, t)\right] \\ & - A_3 \sum_{n=0}^{\infty} \left[f\left(K_c, a_n\sqrt{P_r}, 1 - P_r, -M, t\right) - f\left(K_c, b_n\sqrt{P_r}, 1 - P_r, -M, t\right)\right] \\ & + 2A_4 \sum_{n=0}^{\infty} \left[f(M, a_n, P_r - S_c, S_c K_c, t) - f(M, b_n, P_r - S_c, S_c K_c, t)\right] \\ & - A_4 \sum_{n=0}^{\infty} \left[f\left(K_c, a_n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) - f\left(K_c, b_n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right)\right] \end{aligned}$$

$$\begin{aligned} & - A_4 \sum_{n=0}^{\infty} \left[f\left(0, a_n\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) - f\left(0, b_n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right)\right] \\ & + (A_5 - A_6) \sum_{n=0}^{\infty} \left[f(M, a_n, 1, 0, t) - f(M, b_n, 1, 0, t)\right] \end{aligned}$$

$$+ A_6 \sum_{n=0}^{\infty} \left[f\left(0, a_n \sqrt{P_r}, 1, 0, t\right) - f\left(0, b_n \sqrt{P_r}, 1, 0, t\right) \right] \tag{12}$$

Where $a_n = 2n + y$ *and* $b_n = 2n + 2 - y$

The function f and erfc are given in Appendix-1. The constants A_1, A_2, A_3, A_4, A_5 and A_6 are given in Appendix-2

The rate of mass transfer (Sh), the rate of heat transfer (Nu) and the skin function (τ) at the walls of the channel are obtained as follows:

Sherwood Number (*Sh*):

$$Sh_0 = \left.\frac{\partial C}{\partial y}\right|_{y=0} = -\sqrt{S_c} \sum_{n=0}^{\infty} \left[g\left(K_c, 2n\sqrt{S_c}, 1, 0, t\right) + g\left(K_c, (2n+2)\sqrt{S_c}, 1, 0, t\right) \right] \tag{13}$$

$$Sh_1 = \left.\frac{\partial C}{\partial y}\right|_{y=1} = 2\sqrt{S_c} \sum_{n=0}^{\infty} \left[g\left(K_c, (2n+1)\sqrt{S_c}, 1, 0, t\right) \right] \tag{14}$$

Nusselt Number (*Nu*):

$$Nu_0 = -\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$= A_1\sqrt{S_c} \sum_{n=0}^{\infty} \left[g\left(K_c, 2n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) + g\left(K_c, (2n+2)\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) \right]$$

$$-A_1\sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, 2n\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) + g\left(0, (2n+2)\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) \right]$$

$$-(D^* + 1)\sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, 2n\sqrt{P_r}, 1, 0, t\right) + g\left(0, (2n+2)\sqrt{P_r}, 1, 0, t\right) \right] - D^* Sh_0 \tag{15}$$

$$Nu_1 = \left.\frac{\partial T}{\partial y}\right|_{y=1} = 2A_1\sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, (2n+1)\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) \right]$$

$$-2A_1\sqrt{S_c} \sum_{n=0}^{\infty} \left[g\left(K_c, (2n+1)\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) \right]$$

$$+ 2(D^* + 1)\sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, (2n+1)\sqrt{P_r}, 1, 0, t\right) \right] - D^* Sh_1 \tag{16}$$

Skin friction (τ):

$$\tau_0 = -\left.\frac{\partial u}{\partial y}\right|_{y=0}$$

$$= A_2 \sum_{n=0}^{\infty} \left[g\left(M, 2n, 1 - S_c, S_c K_c - M, t\right) + g\left(M, 2n+2, 1 - S_c, S_c K_c - M, t\right) \right]$$

$$- A_2 \sqrt{S_c} \sum_{n=0}^{\infty} \left[g\left(K_c, 2n\sqrt{S_c}, 1 - S_c, S_c K_c - M, t\right) + g\left(K_c, (2n+2)\sqrt{S_c}, 1 - S_c, S_c K_c - M, t\right) \right]$$

$$+ A_3 \sum_{n=0}^{\infty} \left[g\left(M, 2n, 1 - P_r, -M, t\right) + g\left(M, 2n+2, 1 - P_r, -M, t\right) \right]$$

$$- A_3 \sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(K_c, 2n\sqrt{P_r}, 1 - P_r - M, t\right) + g\left(K_c, (2n+2)\sqrt{P_r}, 1 - P_r - M, t\right) \right]$$

$$+ 2A_4 \sum_{n=0}^{\infty} \left[g\left(M, 2n, P_r - S_c, S_c K_c, t\right) + g\left(M, 2n+2, P_r - S_c, S_c K_c, t\right) \right]$$

$$- A_4 \sqrt{S_C} \sum_{n=0}^{\infty} \left[g\left(K_c, 2n\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) + g\left(K_c, (2n+2)\sqrt{S_c}, P_r - S_c, S_c K_c, t\right) \right]$$

$$- A_4 \sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, 2n\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) + g\left(0, (2n+2)\sqrt{P_r}, P_r - S_c, S_c K_c, t\right) \right]$$

$$+ (A_5 - A_6) \sum_{n=0}^{\infty} \left[g\left(M, 2n, 1, 0, t\right) + g\left((M, 2n+2, 1, 0, t\right) \right] + A_5 Sh_0$$

$$+ A_6 \sqrt{P_r} \sum_{n=0}^{\infty} \left[g\left(0, 2n\sqrt{P_r}, 1, 0, t\right) + g\left(0, (2n+2)\sqrt{P_r}, 1, 0, t\right) \right] \tag{17}$$

$$\tau_1 = -\left.\frac{\partial u}{\partial y}\right|_{y=1} = -2A_2 \sum_{n=0}^{\infty} \left[g\left(M, 2n+1, 1 - S_c, S_c K_c - M, t\right) \right]$$

$$+2A_2\sqrt{S_c}\sum_{n=0}^{\infty}\left[g\left(K_c,(2n+1)\sqrt{S_c},1-S_c,S_cK_c-M,t\right)\right]$$

$$-2A_3\sum_{n=0}^{\infty}\left[g\left(M,2n+1,1-P_r,-M,t\right)\right]+2A_3\sqrt{P_r}\sum_{n=0}^{\infty}\left[g\left(K_c,(2n+1)\sqrt{P_r},1-P_r,-M,t\right)\right]$$

$$-4A_4\sum_{n=0}^{\infty}\left[g\left(M,2n+1,P_r-S_c,S_cK_c,t\right)\right]+2A_4\sqrt{S_c}\sum_{n=0}^{\infty}\left[g\left(K_c,(2n+1)\sqrt{S_c},P_r-S_c,S_cK_c,t\right)\right]$$

$$+2A_4\sqrt{P_r}\sum_{n=0}^{\infty}\left[g\left(0,(2n+1)\sqrt{P_r},P_r-S_c,S_cK_c,t\right)\right]-2(A_5-A_6)\sum_{n=0}^{\infty}\left[g\left(M,2n+1,1,0,t\right)\right]$$

$$-2A_6\sqrt{P_r}\sum_{n=0}^{\infty}\left[g\left(0,(2n+1)\sqrt{P_r},1,0,t\right)\right]+A_5Sh_1 \tag{18}$$

Where the function g is given in appendix-1

The mass flux (volumetric flow rate) for the problem is given by

$$Q=\int_0^1 u\,dy \tag{19}$$

This is computed by using the trapezoidal rule for numerical integration.

Case – II: $(P_r = S_c = 1)$

The solutions of equations (6) – (8) subject to boundary conditions (9) are given by

$$C=\sum_{n=0}^{\infty}\left[f\left(0,a_n,1,0,t\right)-f\left(0,b_n,1,0,t\right)\right] \tag{20}$$

$$T=\frac{D^*\left(1-e^{-K_ct}\right)}{2K_ct\sqrt{\pi t}}\sum_{n=0}^{\infty}\left[a_n\exp\left(-\frac{a_n^2}{4t}\right)-b_n\exp\left(-\frac{b_n^2}{4t}\right)\right]$$
$$+(D^*+1)\sum_{n=0}^{\infty}\left[f\left(0,a_n,1,0,t\right)-f\left(0,b_n,1,0,t\right)\right]-D^*C \tag{21}$$

$$u=A_8C+\frac{D^*}{2t\sqrt{\pi t}}\left[\frac{\exp(-K_ct)}{K_c(K_c-M)}+\frac{1}{MK_c}-\frac{\exp(-Mt)}{M(K_c-M)}\right]\times$$
$$\sum_{n=0}^{\infty}\left[a_n\exp\left(-\frac{a_n^2}{4t}\right)-b_n\exp\left(-\frac{b_n^2}{4t}\right)\right]+A_6\sum_{n=0}^{\infty}$$
$$\left[f\left(0,a_n,1,0,t\right)-f\left(0,b_n,1,0,t\right)\right] \tag{22}$$

Where A_7 and A_8 are given in Appendix-2

In this case, the rate of mass transfer (Sh), the rate of heat transfer (Nu) and the skin frictions (τ) on the walls of the channel are obtained as follows.

Sherwood Number:

$$Sh_0 = -\left.\frac{\partial C}{\partial y}\right|_{y=0} = -\sum_{n=0}^{\infty}\left[g\left(K_c, 2n, 1, 0, t\right) - g\left(K_c, 2n+2, 1, 0, t\right)\right] \tag{23}$$

$$Sh_1 = -\left.\frac{\partial C}{\partial y}\right|_{y=1} = 2\sum_{n=0}^{\infty}\left[g\left(K_c, 2n+1, 1, 0, t\right)\right] \tag{24}$$

Nusselt Number:

$$Nu_0 = -\left.\frac{\partial T}{\partial y}\right|_{y=0} = -\frac{D^*\left[1-\exp(-K_c t)\right]}{2K_c t\sqrt{\pi t}}\left[\sum_{n=0}^{\infty}\left\{\exp\left(-\frac{n^2}{t}\right)+\exp\left(-\frac{(n+1)^2}{t}\right)\right\} - \right.$$

$$\left.\frac{2}{t}\sum_{n=0}^{\infty}\left\{n^2 \exp\left(-\frac{n^2}{t}\right)+(n+1)^2 \exp\left(-\frac{(n+1)^2}{t}\right)\right\}\right]$$

$$-(D^*+1)\sum_{n=0}^{\infty}\left[g\left(0, 2n, 1, 0, t\right) + g\left(0, 2n+2, 1, 0, t\right)\right] + D^* Sh_0 \tag{25}$$

$$Nu_1 = -\left.\frac{\partial T}{\partial y}\right|_{y=0}$$

$$= \frac{D^*\left[1-\exp(-K_c t)\right]}{2K_c t\sqrt{\pi t}}\left[2\sum_{n=0}^{\infty}\exp\left(-\frac{(2n+1)^2}{4t}\right) - \frac{1}{t}\sum_{n=0}^{\infty}(2n+1)^2\exp\left(-\frac{(2n+1)^2}{4t}\right)\right]$$

$$+2(D^*+1)\sum_{n=0}^{\infty}\left[g\left(0, 2n+1, 1, 0, t\right)\right] - D^* Sh_1 \tag{26}$$

Skin friction:

$$\tau_0 = -\left.\frac{\partial u}{\partial y}\right|_{y=0} = A_8 Sh_0 + \frac{D^*}{2t\sqrt{\pi t}}\left[\frac{\exp(-K_c t)}{K_c(K_c - M)} + \frac{1}{MK_c} - \frac{\exp(-Mt)}{M(K_c - M)}\right]\times$$

$$\left[\sum_{n=0}^{\infty}\left\{\exp\left(-\frac{n^2}{t}\right)+\exp\left(-\frac{(n+1)^2}{t}\right)\right\}\right.$$

$$-\frac{2}{t}\sum_{n=0}^{\infty}\left\{n^2 exp\left(-\frac{n^2}{t}\right)+(n+1)^2 exp\left(-\frac{(n+1)^2}{t}\right)\right\}\Bigg]$$

$$+A_6\sum_{n=0}^{\infty}\left[g(0,2n,1,0,t)+g(0,2n+2,1,0,t)\right]$$

$$-A_7\sum_{n=0}^{\infty}\left[g(M,2n,1,0,t)+g(M,2n+2,1,0,t)\right] \tag{27}$$

$$\tau_1=-\left.\frac{\partial u}{\partial y}\right|_{y=1}=A_8 Sh_1+\frac{D^*}{t\sqrt{\pi t}}\left[\frac{\exp(-Mt)}{M(K_c-M)}-\frac{1}{MK_c}-\frac{\exp(-K_c t)}{K_c(K_c-M)}\right]$$

$$\sum_{n=0}^{\infty}\left[\exp\left(-\frac{(2n+1)^2}{t}\right)-2\frac{(2n+1)^2}{t}\exp\left(-\frac{(2n+1)^2}{t}\right)\right]$$

$$-2A_6\sum_{n=0}^{\infty}\left[g(0,2n+1,1,0,t)\right]\ 2A_7\sum_{n=0}^{\infty}\left[g(M,2n+1,1,0,t)\right] \tag{28}$$

Steady State:

Setting $\frac{\partial}{\partial t}(\ )=0$ in equations (6) – (8), the steady state of the problem is obtained as:

$$\frac{d^2u}{dy^2}+NC+T-Mu=0 \tag{29}$$

$$\frac{d^2C}{dy^2}-S_cK_cC=0 \tag{30}$$

$$\frac{d^2T}{dy^2}+D^*\frac{d^2C}{dy^2}=0 \tag{31}$$

The solutions of the equations (29) – (31) subject to boundary conditions (9) are given by

$$C=\frac{\sinh\left(\sqrt{S_cK_c}(1-y)\right)}{\sinh\left(\sqrt{S_cK_c}\right)} \tag{32}$$

$$T = (D^* + 1)(1 - y) - D^* C \tag{33}$$

$$u = A_9 \frac{\sinh \sqrt{M}\,(1-y)}{\sinh \sqrt{M}} + A_5 C + A_6 (1-y) \tag{34}$$

Where A_9 is given in Appendix-2

In this case, the Sherwood number (Sh), the Nusselt number (Nu), and the skin friction (τ) are given by

Sherwood Number (Sh):

$$Sh_0 = \sqrt{S_c K_c} \coth \sqrt{S_c K_c} \tag{35}$$

$$Sh_1 = \sqrt{S_c K_c} \operatorname{cosech} \sqrt{S_c K_c} \tag{36}$$

Nusselt Number (Nh):

$$Nh_0 = D^* + 1 + D^* Sh_0 \tag{37}$$

$$Nu_1 = -(D^* + 1) - D^* Sh_1 \tag{38}$$

Skin friction (τ):

$$\tau_0 = -A_9 \sqrt{M} \coth \sqrt{M} + A_5 Sh_0 - A_6 \tag{39}$$

$$\tau_1 = \frac{A_9 \sqrt{M}}{\sinh \sqrt{M}} - A_5 Sh_1 + A_6 \tag{40}$$

Mass Flux:

$$Q = A_9 \sqrt{M} \frac{\cosh \sqrt{M} - 1}{\sinh \sqrt{M}} + \frac{A_5}{\sqrt{S_c K_c}} \frac{\cosh \sqrt{S_c K_c} - 1}{\sinh \sqrt{S_c K_c}} + \frac{A_6}{2} \tag{41}$$

4. RESULTS AND DISCUSSION:

Computations have been carried out by assigning the values to the pertinent parameters characterising the fluids of practical interest. The flow phenomenon is characterized by magnetic parameter (M), chemical reaction parameter (K_c), sustentation parameter (N), Dufour number (D^*), Schmidt number (S_c), and Prandtl number (P_r). The effects of various parameters on velocity, temperature and concentration profiles are shown graphically and in tabulated form for Sherwood number (S_h), Nusselt number (N_u) and skin friction (τ).

From the analytical solutions,it is observed that equation (12) for the velocity field yields a factor $\sqrt{\frac{P_r}{P_r - 1}}$ which restricts the values of P_r *to* $P_r > 1.0$ for real values of velocity field. Moreover , $K_c < 0$ results in the same situation . This suggests that oscillations in the flow field sets in for fluids with low thermal diffusivity as well as in case of generative reaction.

Further,the particular case of without magnetic field cannot be obtained from equation (12) as the magnetic parameters appears in the denominator. Another interesting point to note that diffusion in aqueous solution i.e., for higher values of S_c gives rise to oscillations in the velocity as well as temperautre field in the presence of K_c.

Fig.1 depicts the velocity profile for various values of pertinent parameters characterising flow fields . The common feature of the profiles is parabolic. This is evident from curves II and X ($t = 0.2$, Curve II; the steady state curve X) in case of steady state as well as $t \leq 0.3$, the velocity profiles remain positive throughout the flow field. This is evident from curves II and X ($t = 0.2$, Curve II; the steady state curve X) . Thus it is concluded that the time span plays a vital role for engendering back flow (Curve I and Curve II).

Further, magnetic parameter (M),chemical reaction parameter (K_c),sustentation parameter (N) decrease the velocity $|u|$ at all points. In all other cases such as S_c, P_r and D^*, the reverse effect is observed. If the mass diffusivity becomes greater than the thermal diffusivity (i.e., N, the sustentation paremeter > 1.0) then the velocity decreases. Moreover, with the stronger magnetic field Lorentz force also reduces the velocity field which is in the conformity with the result (Cramer and Pai[19]).

The increase of P_r results in the decrease of velocity which suggests that low rate of thermal diffusion leads to increase the vleocity boundary layer thickness.

Curve	*t*	*M*	*Sc*	*Kc*	*N*	*Pr*	*D**
I	0.4	0.5	0.22	2	2	1.8	2
II	0.2	0.5	0.22	2	2	1.8	2
III	0.4	2.0	0.22	2	2	1.8	2
IV	0.4	0.5	0.30	2	2	1.8	2
V	0.4	0.5	0.22	1	2	1.8	2
VI	0.4	0.5	0.22	2	4	1.8	2
VII	0.4	0.5	0.22	2	2	2.0	2
VIII	0.4	0.5	0.22	2	2	1.8	4
IX	0.4	0.5	1.00	2	2	1.0	2
X	Steady state	0.5	0.22	2	4	-	2

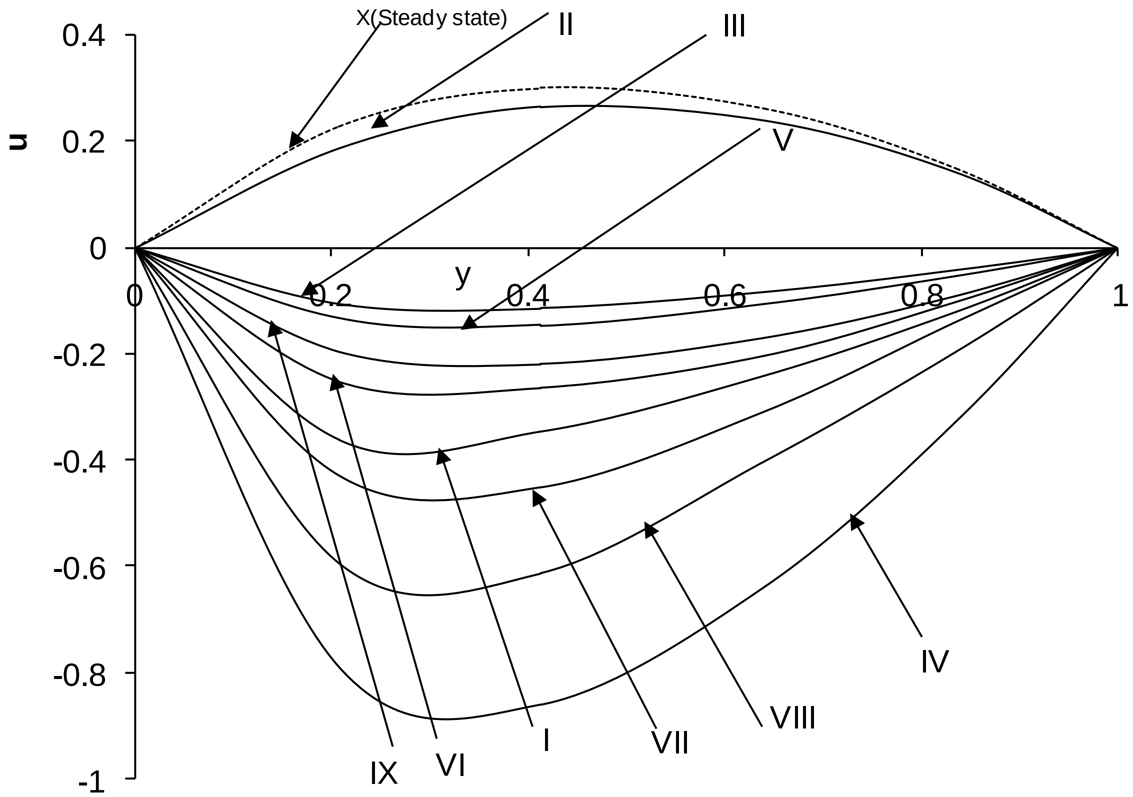


Fig.1 Velocity profile

Dufour effect measures the relative mass diffusivity with that of thermal diffusivity. The ratio of concentration to thermal buoyancy is called the sustentation parameter (N). $N = 0$ and $N \rightarrow \infty$ represent the individual cases of only thermal driven flow and only mass driven flow. When $N > 0$, both the effects are combined to drive the flow and $N < 0$ represent the case of opposition. Further, it is to note that heavier species i.e., with increasing S_c lead to flow reversal.It is most interesting to note that in the presence of destructive reaction i.e., $K_c > 0$, higher value of S_c leads to oscillatory flow and in case of diffusion in the aqueous solution,it leads to back flow. Thus, this suggests that the back flow is prevented by diffusing lighter species.

Fig.2 depicts the velocity distribution in the absence of chemical reaction. It is noteworthy to observe that the backflow occurs for higher values of S_c i.e., ($S_c = 150$ and $S_c = 200$) representing the diffusion in aqueous solution in the presence of constant magnetic field and Dufour effect. Further, it is to mention that for $S_c < 1.0$ and $K_c = 0.0$, no velocity profile could be graphed/presented.

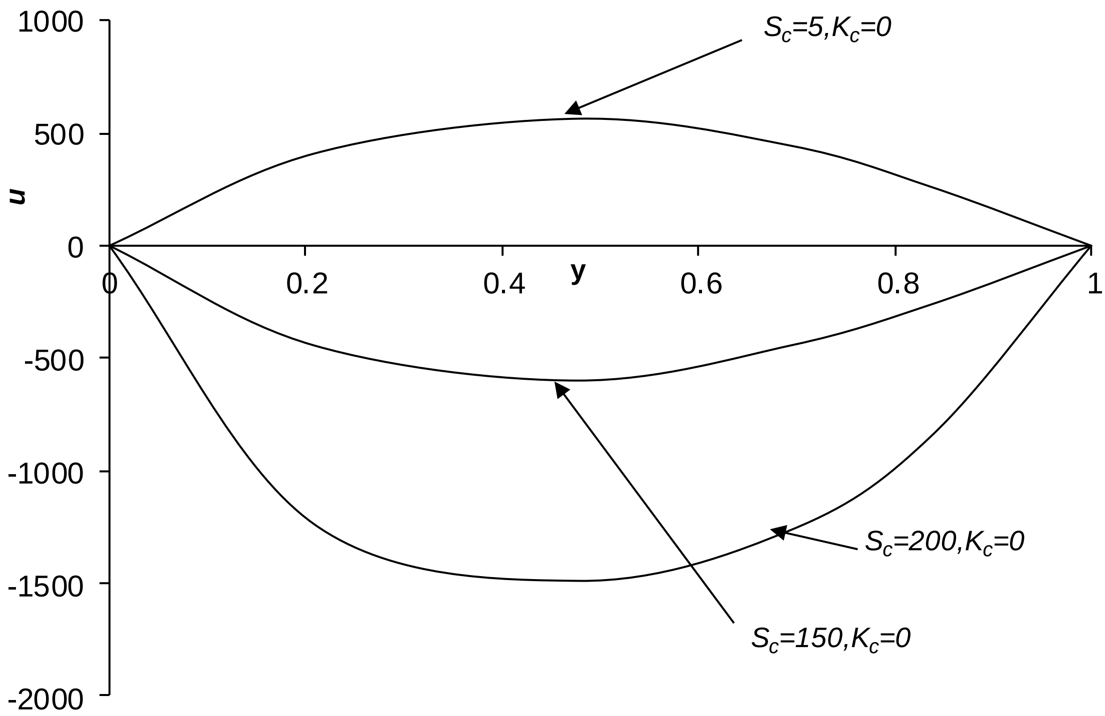


Fig.2 Velocity profile For *t=0.4,M=0.5,N=2,Pr=7 and D*=2*

Fig. 3 exhibits the steady state velocity profiles indicating no back flow irrespective of higher or lower value of mass transfer coefficient S_c and Dufour effect.

On careful observation, it is revealed that for higehr values of Dufour number D^* (curve VI), sustentation parameter N (Curve VIII) and magnetic parameter M (Curve V), steady state velocity is effected significnatly where as variation in reaction parameter and Schmidt number produces no change . Thus, it is important to note that mass transfer phenomena with chemical reaction does not effect the steady flow significantly but increase in M, N and D^* decrease the velocity (same as unsteady case), increase the velocity (opposite to unsteady case) and increase the velocity (same as unsteady case) respectively. It may be inferred that magnetic parameter and Dufour effect produce the same effect irrespective of steady or unsteady state except sustentation parameter.

Curve	*M*	*Sc*	*Kc*	*N*	*D**
I	0.5	100	1	2	2
II	0.5	0.22	2	2	2
III	0.5	152	1	2	2
IV	0.5	0.3	2	2	2
V	2.0	0.22	2	2	2
VI	0.5	0.22	2	2	30
VII	0.5	0.3	1	2	2
VIII	0.5	0.22	2	4	2

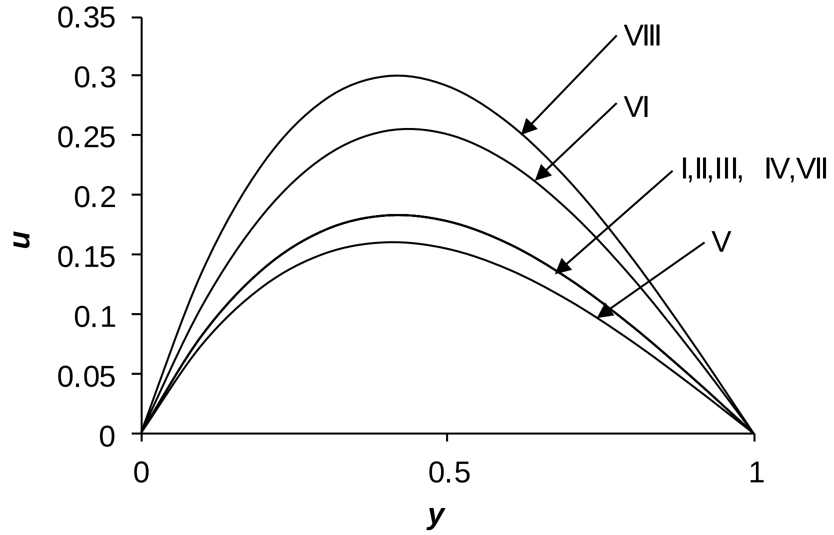


Fig.3 Velocity profile(steady state)

Figures 4, 5 and 6 present the temeprature distribution. It is seen that for an increase in destructive reaction parameter ($K_c > 0$) temperature increases whereas increase in P_r , t and S_c reduces the temperature at all points for $P_r = 7.0$(water) when $y < 0.7$.

An increase in P_r implies low rate of thermal diffusion and an increase in S_c means the diffusing species is heavier. The slow rate of diffusion leaves the heat energy spread in the fluid mass and it is enhanced for heavier species. This contributes to rise in temperature. One interesting observation is in case of $P_r = 0.71$(air). The temperature falls sharply at all points with almost linear distribution regardless of other parameters.

Thus it may be pointed out that thermal diffusivity property of the fluid plays a vital role in controlling the temperature distribution and hence contributing to thermal boundary layer thickness.

Curve	*t*	*Sc*	*Kc*	*Pr*
I	0.4	0.22	2.0	7.00
II	0.4	0.22	0.0	7.00
III	0.4	0.30	0.0	7.00
IV	0.4	0.30	2.0	7.00
V	0.2	0.30	2.0	7.00
VI	0.4	0.22	0.0	0.71
VII	0.4	0.22	0.2	0.71
VII	0.4	0.30	0.0	0.71
IX	0.4	0.30	0.2	0.71
X	0.2	0.30	0.2	0.71

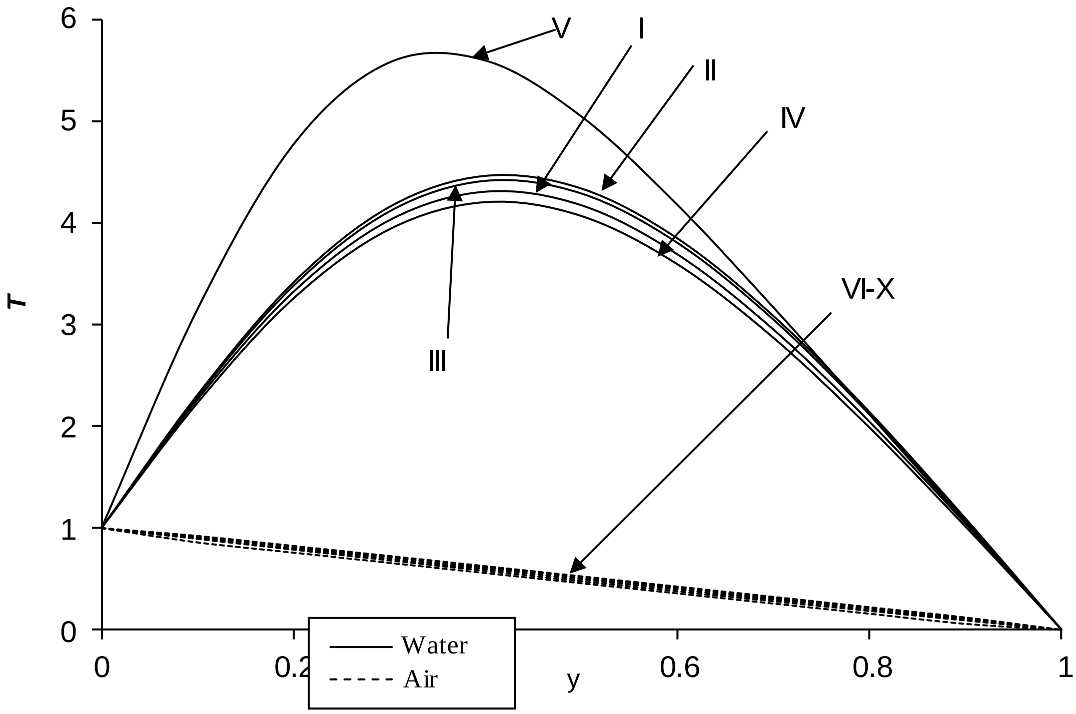


Fig.4 Temperature profile for ***D*=2.0,*** P_r***=7.0(Water)and Pr=0.71(Air)***

Fig.5 also exhibits the temperature distribution when $P_r = 1$ and $S_c = 1$ that means thermal diffusivity and mass diffusivity are at par. This contributes to the uniform variation/decrease in temperature. Increase in reaction parameter gives rise to increase in temperature whereas allowing for a large span of time reduces it.

Fig. 6 presents the non-linearity distribution of temperature due to high value of D^* i.e., for large Dufour effect. Thus, sharp rise and fall of temperature is due to low diffusivity and high Dufour effect.

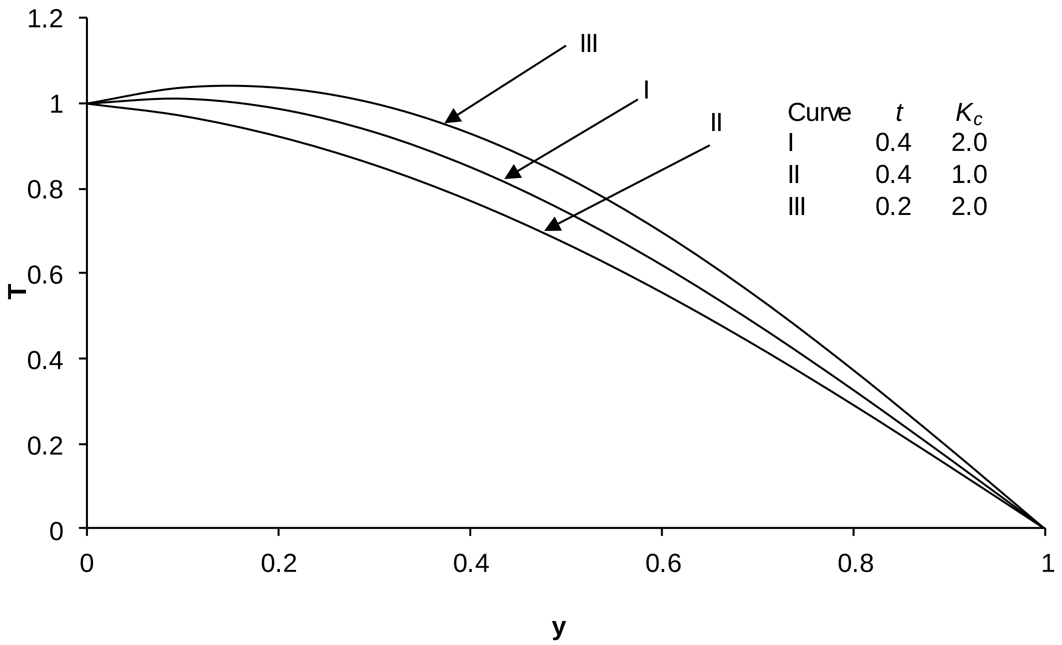
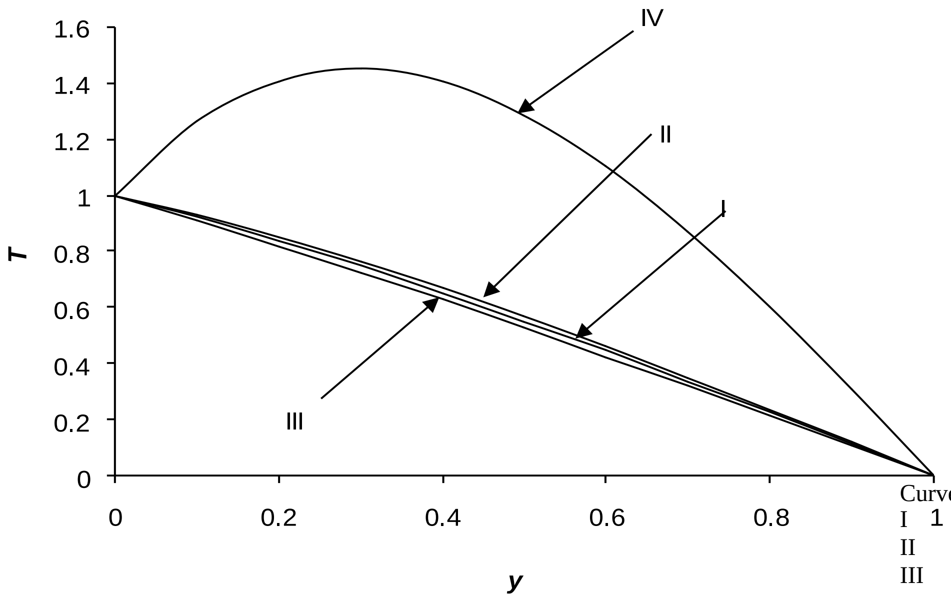


Fig.5 Temperature profile*(pr=1,sc=1)*



Curve	S_c	K_c	D^*
I 1	0.22	2	2
II	0.30	2	2
III	0.22	1	2
IV	0.22	2	30

Fig.6 Temperature profile(steady state)

Figures 7, 8 and 9 exhibit the concentration variation in the flow domain. Sharp fall of concentration is indicated in fig. 7 and 9 along with mass absorption near the plate for very high value of S_c (i.e., S_c = *617*). This means heavier species give rise to sharp fall of concentration accelerating the process of mass diffusion where as in case of higher species the variation is smooth.

Curve	t	S_c	K_c
I	0.4	100	0
II	0.4	617	0
III	0.4	100	2
IV	0.4	617	2
V	0.2	100	2

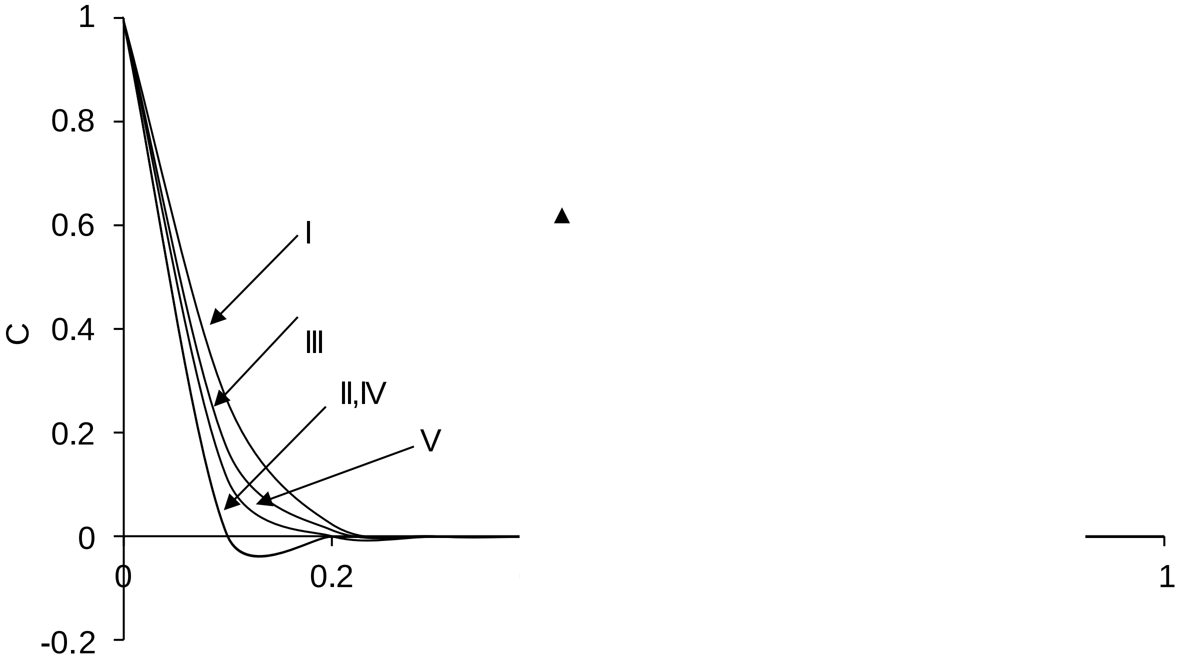


Fig.7 Concentration profile

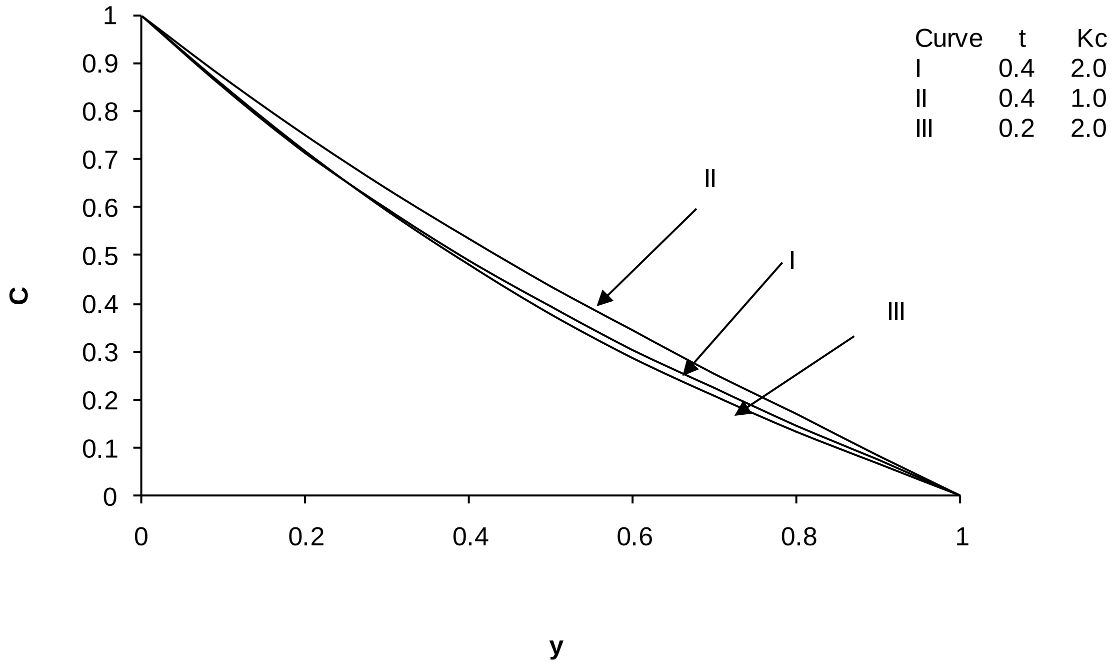


Fig.8 Concentration profile*(S_c=1)*

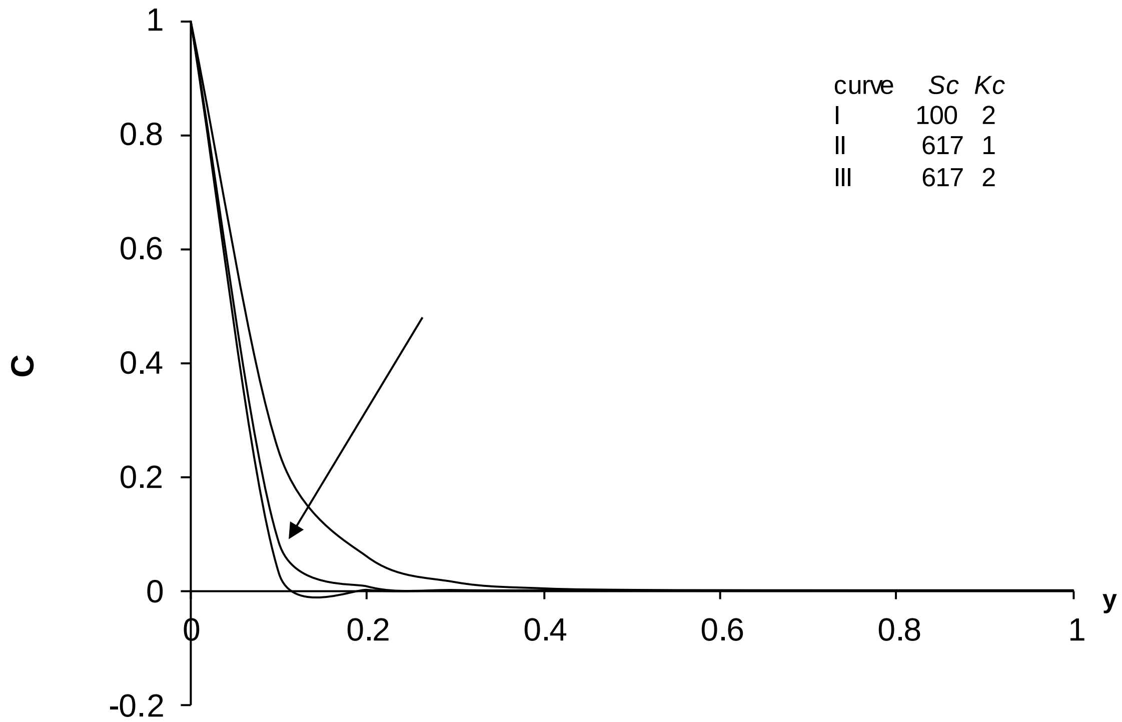


Fig.9 Concentration profile(steady state)

Skin friction is an important criteria in a flow phenomena. It measures the frictional forces encountered at the solid surfaces due to the motion of the fluid.One striking feature of the entries of the skin friction in Table1, in case of unsteady motion is that the skin frictions bear same sign in case of lower and upper plate. Further, on careful observation it is revealed that at the lower plate all are negative for $t = 0.4$ except the case when $t = 0.2$.Thus it may be concluded that time span plays an important role to modify the frictional drag due to shear stress at the plates.

Moreover, from equations (29)-(31) it is clear that the Prandtl number (P_r) has no role to play to effect the velocity, temperature and concentration fields in the steady flow and hence the skin friction. In case of unsteady flow, as P_r increases, the shearing stress increases at both the plates.

Considering Table 1 and Table 2 for both steady and unsteady cases, it is concluded that an increase in magnetic and sustentation parameters leads to reduce the magnitude of frictional drag at both the plates. Moreover an increase in S_c, K_c and D^* leads to enhance the frictional drag at the plates for steady and unsteady flow.

Table 1. Skin friction(Unsteady Case):

Sl.No.	***t***	***M***	***Sc***	***Kc***	***N***	***Pr***	***D****		
1	0.4	0.5	0.22	2	2	1.8	2	-1.376382	-0.315299
2	0.2	0.5	0.22	2	2	1.8	2	0.649408	0.466308
3	0.4	2.0	0.22	2	2	1.8	2	-0.404026	-0.122785
4	0.4	0.5	0.30	2	2	1.8	2	-2.904211	-0.982697
5	0.4	0.5	0.22	1	2	1.8	2	-0.466308	-0.158384
6	0.4	0.5	0.22	2	4	1.8	2	-0.916331	-0.286745
7	0.4	0.5	0.22	2	2	2.0	2	-1.664279	-0.334595
8	0.4	0.5	0.22	2	2	1.8	4	-2.246038	-0.344328
9	0.4	0.5	1.00	2	2	1.8	2	-0.713293	-0.277325

Table 2. Skin friction (Steady case):

Sl. No.	***M***	S_c	K_c	***N***	***D****		
1	0.5	0.22	2	2	2	1.732053	0.9325153
2	2.0	0.22	2	2	2	1.66425	0.7954359
3	0.5	0.30	2	2	2	1.732052	0.9325152
4	0.5	0.22	1	2	2	1.73205	0.932515
5	0.5	0.22	2	4	2	1.6642794	0.8692867
6	0.5	0.22	2	2	4	2.144507	1.37638192

Physically, we may interpret the above results as follows :

Lorentz force and sustentation parameter i.e., ratios of buoyancy effects due to temperature and concentration differences reduce the frictional drag which agrees well with the established result reported earlier. Moreover, presence of heavier species and exothermic reaction and Dufour effect enhance the magnitude of the shearing stress at both the plates.

From Table 3 it is seen that an increase in t, S_c and K_c leads to decrease the Nusselt number at both the platesOne interesting point is that rate of heat transfer remains negative for all the parameters at both the plates for aqueous solution $P_r = 7.0$ but in case of air i.e.,$P_r = 0.71$, Nusselt number is positive at the lower plate and negative at the upper plate.

Table 3. Nusselt Number:

Sl.No	T	S_c	K_c	P_r	D^*	Nu_0	Nu_1
1	0.4	0.22	2	7.00	2	-1.37638192	-1.150368407
2	0.2	0.22	2	7.00	2	-2.674621494	-1.213097004
3	0.4	0.30	2	7.00	2	-1.327044822	-1.110612515
4	0.4	0.22	0	7.00	2	-1.455009029	-1.191753593
5	0.4	0.22	2	0.71	2	0.113935608	-0.113935608
6	0.4	0.22	2	7.00	4	-1.569685558	-1.279941632

Table 4 presents the variation of mass transfer for various values of t, S_c and K_c. It is seen that an increase in chemical reaction parameter K_c increases the mass transfer at the lower plate and decreases it at the other. Further, it is to note that Sherwood number at both the plates increases as S_c increases i.e., for heavier species mass transfer increases at the plates. The effect of increase in time span is to reduce the rate of mass transfer at both the plates.

Table 4. Sherwood Number:

Sl.No	T	Sc	Kc	Sh_o	Sh_1
1	0.4	100	2	5.9757644	-0.612800788
2	0.2	100	2	6.313751515	0.15838444
3	0.4	617	2	7.026366229	0.470564281
4	0.2	100	0	5.67128182	-0.466307658

Tables 5 and 6 show the effects of various parameters on mass flux both for unsteady and steady cases respectively. From Table 5,it is observed that mass flux increases with increase in the values in S_c,K_c,P_r and D^* but the reverse effect is observed in case of t,M and N when the motion is unsteady.Hence, it is concluded that in case of unsteady motion more flux fluid experienced with heavier species having low thermal diffusivity and increasing Dufour effect in the presence of constructive chemical reaction.

Table 5. Mass Flux(Unsteady Case):

	t	*M*	*Sc*	*Kc*	*N*	*Pr*	*D**	Q
1	0.4	0.5	0.22	2	2	1.8	2	.197836
2	0.2	0.5	0.22	2	2	1.8	2	.176572
3	0.4	2.0	0.22	2	2	1.8	2	.057655
4	0.4	0.5	0.30	2	2	1.8	2	.568475
5	0.4	0.5	0.22	1	2	1.8	2	.091193
6	0.4	0.5	0.22	2	4	1.8	2	.166667
7	0.4	0.5	0.22	2	2	2.0	2	.264939
8	0.4	0.5	0.22	2	2	1.8	4	.364583
9	0.4	0.5	1.00	2	2	1.8	2	.120989

Now, if we analyse the steady case ,we observe that more flux is measured with higher value of N and D^* where as the reverse effect is observed in case of M .In this case S_c and K_c have no significant effect on mass flux.

Table 6. Mass Flux (Steady case)

Sl. No.	M	S_c	K_c	N	D^*	Q
1	0.5	0.22	2	2	2	.119052
2	2.0	0.22	2	2	2	.104207
3	0.5	0.30	2	2	2	.119052
4	0.5	0.22	1	2	2	.119052
5	0.5	0.22	2	4	2	.195077
6	0.5	0.22	2	2	4	.122392

Comparing both the cases steady and unsteady motion,it is observed that an increase in K_c and D^* give rise to higher flux where as magnetic field yields less amount of flux.

5. Conclusion

(i) Under the influence of dominating mass diffusivity over thermal diffusivity with stronger Lorentz force reduce the velocity at all points of the channel.

(ii) Low rate of thermal diffusion leads to increase the thickness of boundary layer.

(iii) Diffusion of heavier spcies leads to flow reversal.It is well marked in case of aqueous solution. Flow of aqueous solution in the presence of heavier species is prone to back flow.

(iv) Destructive reaction in the presence of heavier species leads to oscillatory flow.

(v) In case of steady state, no back flow occurs irrespective of high or low value of mass transfer coefficient and Dufour effects.

(vi) Sharp rise and fall of temperature is the outcome of low diffusivity and high Dufour effects.

(vii) Dufour effect and chemical reaction rate in the presence of heavier species enhance the frictional drag.

(viii) Heat transfer bears same sign at both the plates in the presence of aqueous solution but in case of air it is of opposite sign.

(ix) Mass transfer increases at both the plates due to the presence of heavier species but it reduces an with increasing time span.

(x) In case of unsteady motion more flux of fluid is experienced due to heavier
species with low thermal diffusivity. Dufour effect enhances the flux both
in steady and unsteady motion.

REFERENCES

[1] Jha, B.K., Ajibade, A.O. (2010): Free convection heat and mass transfer flow in a vertical channel with the Dufour Effect. *Proc. IMechE : J. Process Mechanical Engineering, Vol.224 PartE:pp. 91-101*.

[2] Soundalgekar, V.M. and Akolkar, S.P.(1983): Effects of free convective currents and mass transfer flow past a vertical oscillating plate. *Astrophys. Space Sci.*, Vol. 89:pp.241-254.

[3] Kafoussias, N.G. and Williams, E.W. (1995): Thermal-diffusion and diffusion-thermo effects on mixed free-forced convective and mass transfer boundary layer flow with temperature dependent viscocity. *Int. J. Eng. Sci.*,Vol.33(9):pp.1369-1384.

[4] Jha, B.K. and Singh, A.K.(1990): Soret effects on free convection and mass transfer flow in stokes problem for an infinite vertical plate. *Astrophys, Space Sci., Vol.173:pp.251-255.*

[5] Kafoussias, N.G.(1992): MHD thermal diffusion effects on free convective and mass transfer flow over an infinite vertical moving plate. *Astrophys Space Sci., Vo l.192:pp.11-19.*

[6] Alam, M.S., Rahman, M.M. and Samad, M.A.(2006): Dufour and Soret effects on unsteady MHD free convection and mass transfer flow past a vertical porous plate in a porous medium. *Nonlinear Anal, Model, Control*, Vol.11(3):pp. 217-226.

[7] Alam, M.S., Rahman, M.M., Ferdous, M., Maino, K., Mureithi, E. and Postelnicu, A. (2007): Diffusion-thermo and thermal-diffusion effects on free convective heat and

mass transfer flow in a porous medium with time dependent temperature and concentration. *Int. J. Appl. Eng. Res., Vol.2(1):pp.81-96.*

[8] Beg, O.A., Beg, T.A., Bakier, A.Y., and Prasad, V.R.,(2009): Chemically-reacting mixed convective heat and mass transfer along inclined and vertical plates with soret and Dufour effects: numerical solution. *Int. J. Appl. Math. Mech., Vol.5(2):pp. 39-57.*

[9] Dursunkaya, Z and Worek, M.W.(1992): Diffusion – thermo and thermal-diffusion effects in transient and steady natural convection from a vertical surface. *Int. J. Heat, Mass Transfer,Vol.35(8),pp.2060-2065*.

[10] Aberu, C.R.A., Alfradique, M.F.and Telles, A.S.(2006): Boundary layer flows with Dufour and Soret effects: *I: Forced and natural convection. Chemical Engineering Science,Vol. 61,pp.4282-4289*.

[11] Osalusi, E., Side, J., Haris, R.(2008): Thermal diffusion and thermo-diffusion effect on combined heat and mass transfer of a steady MHD convective and slip flow due to a rotating disk with viscous dissipation and ohmic heating, *International Communications in Heat and Mass Transfer, Vol.35(8):pp. 908-915.*

[12] Anghel,M.,Takhar,H.S.,Pop,I.(2000): Dufour and Soret effects on free convection boundary layer over a vertical surface embedded in a porous medium. *Stud.Unvi.Babes.BolyaiMath.,xlv,11-21*.

[13] Postelnicu,A.(2004): Influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering Soret and Dufour effects.*Int.J.Heat MassTransf.,47,1467-1472*.

[14] Gebhart,B.,Pera,L.,(1971):The nature of verticalnatural convection flows resulting from the combined buoyancy effects of thermal mass diffusion. *Int.J.Heat and Mass Transfer,vol.14:p.2925-2050.*

[15] Das,U.N.,Deka,R.,Soundalgekar,V.M.,(1994):Effects mass transfer on the flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction. *Forschung im Ingenieurwesen-Engineering Research,60,284-287.*

[16] Saxena, S.S., and Dubey, G.K. (2011): Unsteady MHD heat and mass transfer free convection flow of a polar fluid past a vertical moving porous plate in a porous medium with heat generation and thermal diffusion. *Advances in Applied Science Research*, 2(4), pp: 259-278.

[17] Raveendra Babu, K., Vijaya Kumar A.G., Varma, S.V.K. (2012): Diffusion-thermo and radiation effects on MHD free convective heat and mass transfer flow past an infinite vertical plate in the presence of a chemical reaction of first order.
i. *Advances in Applied Science Research, 3 (4):2446-2462.*

[18] Sudhakar, K., Srinibasa Raju, R., Rangamma, M. (2012): Chemical reaction effect on an unsteady MHD free convection flow past an infinite vertical accelerated plate with constant heat flux, thermal diffusion and diffusion thermo. *International journal of Modern Engineering Research vol.2 (5), pp. 3329-3339.*

[19] Cramer, K. R. and Pai, S.I. (1973): Magenetofluid dynamics for engineers and applied physics – *Scripta Publishing Company, Mc. Graw Hill.*

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Appendix- I

The complementary error function is given by

$$\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-\lambda^2} d\lambda,$$

$f(x_1, x_2, x_3, x_4, x_5) =$

$$\frac{1}{2} exp\left(\frac{x_4 x_5}{x_3}\right)\left[exp\left(-x_2\sqrt{\frac{x_4}{x_3}+x_1}\right) erfc\left(\frac{x_2}{2\sqrt{x_5}} - \sqrt{\left(\frac{x_4}{x_3}+x_1\right)x_5}\right) + exp\left(x_2\sqrt{\frac{x_4}{x_3}+x_1}\right) erfc\left(\frac{x_2}{2\sqrt{x_5}} + \sqrt{\left(\frac{x_4}{x_3}+x_1\right)x_5}\right)\right]$$

$g(x_1, x_2, x_3, x_4,$

$$x_5) = \frac{1}{2}\sqrt{\frac{x_4}{x_3}+x_1}\exp\left(\frac{x_4 x_5}{x_3}\right)\left[\exp\left(x_2\sqrt{\frac{x_4}{x_3}+x_1}\right) erfc\left(\frac{x_2}{2\sqrt{x_5}} + \sqrt{\left(\frac{x_4}{x_3}+x_1\right)x_5}\right) - \exp\left(-x_2\sqrt{\frac{x_4}{x_3}+x_1}\right) erfc\left(\frac{x_2}{2\sqrt{x_5}} - \sqrt{\left(\frac{x_4}{x_3}+x_1\right)x_5}\right)\right] - \frac{2}{\sqrt{\pi}}\exp\left(-\left(\frac{x_3^2}{4x_5}+x_1 x_5\right)\right)$$

Appendix-II

$$A_1 = -D^*\left[\frac{S_c}{P_r - S_c} + P_r - S_c\right],$$

$$A_2 = -\frac{1-S_c}{S_c K_c - M}\left[N + \frac{S_c D^*(K_c - M)}{S_c K_c(1-P_r) + M(P_r - S_c)}\right]$$

$$A_3 = \frac{1-P_r}{M}\left[1 + \frac{S_c D^*(K_c - M)}{S_c K_c(1-P_r) + M(P_r - S_c)}\right],$$

$$A_4 = \frac{D^* P_r (P_r - S_c)}{S_c K_c(1-P_r) + M(P_r - S_c)}, \qquad A_5 = \frac{D^* - N}{S_c K_c - M}, \qquad A_6 = \frac{D^*+1}{M},$$

$$A_7 = \frac{K_c(D^*+1) - M(N+1)}{M(K_c - M)}, A_8 = \frac{D^* - N}{K_c - M},$$

$$A_9 = \frac{-S_c K_c (D^*+1) + M(N+1)}{M(S_c K_c - M)}$$