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IFαGS CONTINUOUS AND IFαGS IRRESOLUTE MAPPINGS

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Abstract. The objective of this paper is to establish intuitionistic fuzzy α-generalized semi continuous mappings and to study some of their properties. Finally we introduce intuitionistic fuzzy α-generalized semi irresolute mappings and investigate their characterizations.

1. Introduction

As a generalization of fuzzy sets, the concepts of intuitionistic fuzzy sets was introduced by Atanassov [1]. Recently, Coker and Demirci [3] introduced the basic definitions and properties of intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In 2004, M.Rajamani and K.Viswanathan [8] introduced α generalized semi continuous maps and α generalized semi irresolute maps in topological spaces. In this paper we introduce intuitionistic fuzzy α-generalized semi continuous mappings and intuitionistic fuzzy α-generalized semi irresolute mappings. Also the interconnections between the intuitionistic fuzzy continuous mappings and the intuitionistic fuzzy irresolute mappings are investigated. Some examples are given to illustrate the results.

2. Preliminaries

Definition 2.1. [1] *Let X be a non empty fixed set. An intuitionistic fuzzy set(IFS in short) A in X is an object having the form*

$$A = \{\langle\, x, \mu_A(x), \nu_A(x) \,\rangle \mid x \in X\}$$

where the function $\mu_A(x)$:$X \to [0,1]$ denotes the degree of membership(namely $\mu_A(x)$) and the function $\nu_A(x)$:$X \to [0,1]$ denotes the degree of non-membership(namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$.

IFS(X) denote the set of all intuitionistic fuzzy sets in X.

Definition 2.2. [1] *Let A and B be IFSs of the form*
$A=\{\langle\, x, \mu_A(x), \nu_A(x) \,\rangle \mid x \in X\}$ and $B=\{\langle\, x, \mu_B(x), \nu_B(x) \,\rangle \mid x \in X\}$. Then

1. *$A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$,*
2. *$A = B$ if and only if $A \subseteq B$ and $B \subseteq A$,*

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(3) $A^c = \{\langle\ x,\ \nu_A(x),\ \mu_A(x)\ \rangle \mid x \in X\}$,
(4) $A \cap B = \{\langle\ x,\ \mu_A(x) \wedge \mu_B(x),\ \nu_A(x) \vee \nu_B(x)\ \rangle \mid x \in X\}$,
(5) $A \cup B = \{\langle\ x,\ \mu_A(x) \vee \mu_B(x),\ \nu_A(x) \wedge \nu_B(x)\ \rangle \mid x \in X\}$.

For the sake of simplicity, we shall use the notation $A = \langle\ x,\ \mu_A,\ \nu_A\ \rangle$ *instead of* $A = \{\langle\ x,\ \mu_A(x),\ \nu_A(x)\ \rangle \mid x \in X\}$.

The intuitionistic fuzzy sets $0_\sim = \{\langle\ \mathrm{x},\ 0,\ 1\ \rangle : \mathrm{x} \in \mathrm{X}\}$ and $1_\sim = \{\langle\ \mathrm{x},\ 1,\ 0\ \rangle : \mathrm{x} \in \mathrm{X}\}$ are the empty set and the whole set of X respectively.

Definition 2.3. [3] *An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms:*

(1) $0_\sim,\ 1_\sim \in \tau$,
(2) $G_1 \cap G_2 \in \tau$ *for any* $G_1,\ G_2 \in \tau$,
(3) $\cup G_i \in \tau$ *for any family* $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space(IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set(IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set(IFCS in short) in X.

Definition 2.4. [3] *Let (X, τ) be an IFTS and $A = \langle\ x,\ \mu_A,\ \nu_A\ \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and the intuitionistic fuzzy closure are defined as follows:*

(1) $int(A) = \cup\{G \mid G$ *is an IFOS in X and* $G \subseteq A\}$,
(2) $cl(A) = \cap\{K \mid K$ *is an IFCS in X and* $A \subseteq K\}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ *and* $int(A^c) = (cl(A))^c$.

Definition 2.5. *An IFS $A = \langle\ x,\ \mu_A,\ \nu_A\ \rangle$ in an IFTS (X, τ) is said to be an*

(1) *intuitionistic fuzzy regular closed set(IFRCS in short) if* $A = cl(int(A))$ [3],
(2) *intuitionistic fuzzy α-closed set(IFαCS in short) if* $cl(int(cl(A))) \subseteq A$ [5],
(3) *intuitionistic fuzzy semiclosed set(IFSCS in short) if* $int(cl(A)) \subseteq A$ [3],
(4) *intuitionistic fuzzy preclosed set(IFPCS in short) if* $cl(int(A)) \subseteq A$ [3],
(5) *intuitionistic fuzzy semipreclosed set(IFSPCS in short) if there exists an IFPCS B such that* $int(B) \subseteq A \subseteq B$[14].

Definition 2.6. *An IFS $A = \langle\ x,\ \mu_A,\ \nu_A\ \rangle$ in an IFTS (X, τ) is said to be an*

(1) *intuitionistic fuzzy regular open set(IFROS in short) if* $A = int(cl(A))$[3],
(2) *intuitionistic fuzzy α-open set(IFαOS in short) if* $A \subseteq int(cl(int(A)))$[5],
(3) *intuitionistic fuzzy semiopen set(IFSOS in short) if* $A \subseteq cl(int(A))$[3],
(4) *intuitionistic fuzzy preopen set (IFPOS in short) if* $A \subseteq int(cl(A))$[3],
(5) *intuitionistic fuzzy semipreopne set (IFSPOS in short) if there exists an IFPOS B such that* $B \subseteq A \subseteq cl(B)$[14]

The family of all IFOS(respectively IFSOS, IFαOS, IFROS) of an IFTS (X, τ) is denoted by IFOS(X)(respectively IFSOS(X), IFαOS(X), IFROS(X)).

Definition 2.7. [14] *Let A be an IFS in (X, τ), then semi interior of A(sint(A) in short) and semi closure of A (scl(A) in short) are defined as*

(1) $sint(A) = \cup\{K \mid K$ *is an IFSOS in X and* $K \subseteq A\}$,
(2) $scl(A) = \cap\{K \mid K$ *is an IFSCS in X and* $A \subseteq K\}$.

Definition 2.8. [12] *Let A be an IFS in (X, τ), then semipre interior of A(spint(A) in short) and semipre closure of A (spcl(A) in short) are defined as*

(1) *spint(A)* $= \cup\{G \mid G$ *is an IFSPOS in X and* $G \subseteq A\}$,
(2) *spcl(A)* $= \cap\{K \mid K$ *is an IFSPCS in X and* $A \subseteq K\}$.

Definition 2.9. [9] *Let A be an IFS of an IFTS (X, τ). Then*

(1) $\alpha cl(A) = \cap\{K \mid K$ *is an IFαCS in X and* $A \subseteq K\}$,
(2) $\alpha int(A) = \cup\{K \mid K$ *is an IFαOS in X and* $K \subseteq A\}$.

Definition 2.10. *An IFS A of an IFTS (X, τ) is an*

(1) *intuitionistic fuzzy generalized closed set(IFGCS in short) if cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X* [13],
(2) *intuitionistic fuzzy generalized semiclosed set(IFGSCS in short) if scl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X* [11],
(3) *intuitionistic fuzzy generalized semipreclosed set(IFGSPCS in short) if sp-cl(A)* $\subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X* [12],
(4) *intuitionistic fuzzy alpha generalized closed set(IFαGCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFOS in X* [9],
(5) *intuitionitic fuzzy generalized alpha closed set (IFGαCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFαOS in X* [7].

The complements of the above mentioned intuitionistic fuzzy closed sets are called their respective intuitionistic fuzzy open sets.

Definition 2.11. [15] *An IFS A of an IFTS (X, τ) is said to be an intuitionistic fuzzy alpha generalized semi closed set(IFαGSCS in short) if* $\alpha cl(A) \subseteq U$ *whenever* $A \subseteq U$ *and U is an IFSOS in (X, τ).*
An IFS A is said to be an intuitionistic fuzzy α-generalized semi openset(IFαGSOS in short) in X if A^c *is an IFαGSCS in X. The family of all IFαGSCSs(respective IFαGSOSs) of an IFTS (X, τ) is denoted by IFαGSCS(X)(respective IFαGSOS(X)).*

Remark 2.12. [15] *Every IFCS, IFRCS, IFαCS is an IFαGSCS but their separate converses may not be true in general. Every IFαGSCS is IFGSCS, IFGαCS, IFαGCS but their separate converses may not be true in general.*

Definition 2.13. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

(1) *intuitionistic fuzzy continuous (IF continuous in short) if* $f^{-1}(B) \in$ *IFOS(X) for every* $B \in \sigma$[4],
(2) *intuitionistic fuzzy α-continuous (IFα continuous in short) if* $f^{-1}(B) \in$ *IFαOS(X) for every* $B \in \sigma$[6],
(3) *intuitionistic fuzzy pre continuous (IFP continuous in short) if* $f^{-1}(B) \in$ *IFPOS(X) for every* $B \in \sigma$[6].

Every IF continuous mapping is an IFα-continuous mapping but not conversely.

Definition 2.14. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*

(1) *intuitionistic fuzzy generalized continuous(IFG continuous in short) if* $f^{-1}(B)$ *is an IFGCS for every IFCS B of (Y, σ)*[13],
(2) *intuitionistic fuzzy generalized semi continuous(IFGS continuous in short) if* $f^{-1}(B)$ *is an IFGSCS for every IFCS B of (Y, σ)*[11],

(3) *intuitionistic fuzzy generalized semi pre continuous(IFGSP continuous in short) if $f^{-1}(B)$ is an IFGSPCS for every IFCS B of (Y, σ)*[12],
(4) *intuitionistic fuzzy α-generalized continuous(IFαG continuous in short) if $f^{-1}(B)$ is an IFαGCS for every IFCS B of (Y, σ)*[10],
(5) *intuitionistic fuzzy generalized α continuous(IFGα continuous in short) if $f^{-1}(B)$ is an IFGαCS for every IFCS B of (Y, σ)*[7].

Definition 2.15. *Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an*
(1) *intuitionistic fuzzy irresolute (IF irresolute in short) if $f^{-1}(B) \in$ IFCS(X) for every IFCS B in Y*[11],
(2) *intuitionistic fuzzy generalized irresolute(IFG irresolute in short) if $f^{-1}(B)$ is IFGCS in X for every IFGCS B in Y*[11].

3. Intuitionistic fuzzy α-generalized semi continuous mappings

In this section we introduce intuitionistic fuzzy α-generalized semi continuous mapping and study some of its properties.

Definition 3.1. *A mapping f:(X, τ) $\to$ (Y, σ) is called an intuitionistic fuzzy α-generalized semi continuous(IFαGS continuous in short) if $f^{-1}(B)$ is an IFαGSCS in (X, τ) for every IFCS B of (Y, σ).*

Example 3.2. *Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = $\langle$ x, (0.6, 0.7), (0.4, 0.3)$\rangle$ and T_2 = $\langle$ y, (0.9, 0.8), (0.1, 0.2) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαGS continuous mapping.*

Theorem 3.3. *Every IF continuous mapping is an IFαGS continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IFαGSCS, $f^{-1}(A)$ is an IFαGSCS in X. Hence f is an IFαGS continuous mapping.

Example 3.4. *IFαGS continuous mapping $\nrightarrow$ IF continuous mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = $\langle$ x, (0.4, 0.2), (0.6, 0.7)$\rangle$ and T_2 = $\langle$ y, (0.3, 0.2), (0.7, 0.8) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$y, (0.7, 0.8), (0.3, 0.2)$\rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFαGSCS but not IFCS in X. Therefore f is an IFαGS continuous mapping but not an IF continuous mapping.

Theorem 3.5. *Every IFα continuous mapping is an IFαGS continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IFαGSCS, $f^{-1}(A)$ is an IFαGSCS in X. Hence f is an IFαGS continuous mapping.

Example 3.6. *IFαGS continuous mapping $\nrightarrow$ IFα continuous mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = $\langle$ x, (0.4, 0.5), (0.6, 0.5)$\rangle$ and T_2 = $\langle$ y, (0.2, 0.4), (0.8, 0.6) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$y, (0.8, 0.6), (0.2, 0.4)$\rangle$ is IFCS in Y, $f^{-1}(A)$ is an IFαGSCS

but not IFαCS in X. Therefore f is an IFαGS continuous mapping but not an IFα continuous mapping.

Remark 3.7. *IFG continuous mappings and IFαGS continuous mappings are independent of each other.*

Example 3.8. *IFαGS continuous mapping $\nrightarrow$ IFG continuous mapping.*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $T_1 = \langle$ x, (0.4, 0.7), (0.5, 0.3)$\rangle$ and $T_2 = \langle$ y, (0.6, 0.8), (0.3, 0.2) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is IFαGS continuous mapping but not IFG continuous mapping. Since A = $\langle$y, (0.3, 0.2), (0.6, 0.8)$\rangle$ is IFCS in Y, $f^{-1}(A) = \langle$ x, (0.3, 0.2), (0.6, 0.8)$\rangle$ is not IFGCS in X.

Example 3.9. *IFG continuous mapping $\nrightarrow$ IFαGS continuous mapping.*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $T_1 = \langle$ x, (0.6, 0.8), (0.4, 0.2)$\rangle$ and $T_2 = \langle$ y, (0.3, 0.1), (0.7, 0.9) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Then f is IFG continuous mapping but not an IFαGs continuous mapping. Since A = $\langle$y, (0.7, 0.9), (0.3, 0.1)$\rangle$ is IFCS in Y, $f^{-1}(A) = \langle$ x, (0.7, 0.9), (0.3, 0.1)$\rangle$ is not IFαGSCS in X.

Theorem 3.10. *Every IFαGS continuous mapping is an IFGS continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFαGS continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ IFαGSCS in X. Since every IFαGSCS is an IFGSCS, $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

Example 3.11. *IFGS continuous mapping $\nrightarrow$ IFαGS continuous mapping.*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $T_1 = \langle$ x, (0.7, 0.8), (0.3, 0.1)$\rangle$ and $T_2 = \langle$ y, (0.2, 0), (0.8, 0.8) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$y, (0.8, 0.8), (0.2, 0)$\rangle$ is IFCS in Y, $f^{-1}(A)$ is IFGSCS in X but not IFαGSCS in X. Therefore f is an IFGS continuous mapping but not an IFαGS continuous mapping.

Remark 3.12. *IFP continuous mappings and IFαGS continuous mappings are independent of each other.*

Example 3.13. *IFP continuous mapping $\nrightarrow$ IFαGS continuous mapping*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $T_1 = \langle$ x, (0.4, 0.3), (0.6, 0.5)$\rangle$ and $T_2 = \langle$ y, (0.7, 0.8), (0.2, 0.1) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$y, (0.2, 0.1), (0.7, 0.8)$\rangle$ is IFCS in Y, $f^{-1}(A)$ is IFPCS in X but not IFαGSCS in X. Therefore f is an IFP continuous mapping but not IFαGS continuous mapping.

Example 3.14. *IFαGS continuous mapping $\nrightarrow$ IFP continuous mapping*
Let $X = \{a, b\}$, $Y = \{u, v\}$, $T_1 = \langle$ x, (0.3, 0.4), (0.7, 0.6)$\rangle$ and $T_2 = \langle$ y, (0.4, 0.5), (0.6, 0.5) $\rangle$ and $T_3 = \langle$ y, (0.7, 0.4), (0.3, 0.6) $\rangle$. Then $\tau = \{ 0_\sim, T_1, T_2, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_3, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = $\langle$y, (0.3, 0.6), (0.7,

0.4)⟩ is IFαGSCS but not IFPCS in Y, f$^{-1}(A)$ is IFαGSCS in X but not IFPCS in X. Therefore f is an IFαGS continuous mapping but not IFP continuous mapping.

Theorem 3.15. *Every IFαGS continuous mapping is an IFGSP continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFαGS continuous mapping. Let A be an IFCS in Y. Then by hypothesis f$^{-1}(A)$ is an IFαGSCS in X. Since every IFαGSCS is an IFGSPCS, f$^{-1}(A)$ is an IFGSPCS in X. Hence f is an IFGSP continuous mapping.

Example 3.16. *IFGSP continuous mapping $\nrightarrow$ IFαGS continuous mapping.*
Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = ⟨ x, (0.3, 0.1), (0.6, 0.8)⟩ and T_2 = ⟨ y, (0.7, 0.8), (0.2, 0.0) ⟩. Then τ = $\{$ $0_\sim$, T_1, $1_\sim$ $\}$ and σ = $\{$ $0_\sim$, T_2, $1_\sim$ $\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = ⟨y, (0.2, 0.0), (0.7, 0.8)⟩ is IFCS in Y, f$^{-1}(A)$ is an IFGSPCS but not IFαGSCS in X. Therefore f is an IFGSP continuous mapping but not an IFαGS continuous mapping.

Theorem 3.17. *Every IFαGS continuous mapping is an IFαG continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFαGS continuous mapping. Let A be an IFCS in Y. Since f is IFαGS continuous mapping, f$^{-1}(A)$ is an IFαGSCS in X. Since every IFαGSCS is an IFαGCS, f$^{-1}(A)$ is an IFαGCS in X. Hence f is an IFαG continuous mapping.

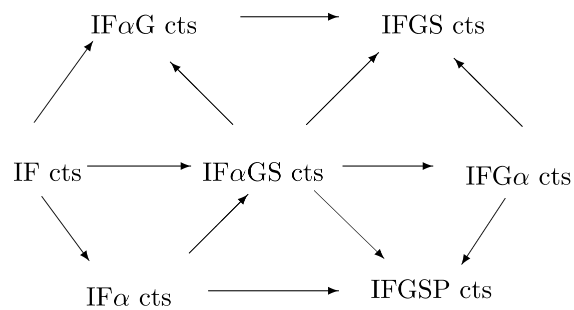
Example 3.18. *IFαG continuous mapping $\nrightarrow$ IFαGS continuous mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = ⟨ x, (0.1, 0.3), (0.7, 0.6)⟩ and T_2 = ⟨ y, (0.6, 0.5), (0.3, 0.4) ⟩. Then τ = $\{$ $0_\sim$, T_1, $1_\sim$ $\}$ and σ = $\{$ $0_\sim$, T_2, $1_\sim$ $\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = ⟨y, (0.3, 0.4), (0.6, 0.5)⟩ is IFCS in Y, f$^{-1}(A)$ is IFαGCS in X but not IFαGSCS in X. Therefore f is an IFαG continuous mapping but not an IFαGS continuous mapping.

Theorem 3.19. *Every IFαGS continuous mapping is an IFGα continuous mapping.*

Proof. Let f:(X, τ) $\to$ (Y, σ) be an IFαGS continuous mapping. Let A be an IFCS in Y. Since f is IFαGS continuous mapping, f$^{-1}(A)$ is an IFαGSCS in X. Since every IFαGSCS is an IFGαCS, f$^{-1}(A)$ is an IFGαCS in X. Hence f is an IFGα continuous mapping.

Example 3.20. *IFGα continuous mapping $\nrightarrow$ IFαGS continuous mapping*
Let X = $\{a, b\}$, Y = $\{u, v\}$, T_1 = ⟨ x, (0.4, 0.2), (0.6, 0.8)⟩ and T_2 = ⟨ y, (0.5, 0.4), (0.5, 0.6) ⟩. Then τ = $\{$ $0_\sim$, T_1, $1_\sim$ $\}$ and σ = $\{$ $0_\sim$, T_2, $1_\sim$ $\}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\to$ (Y, σ) by f(a) = u and f(b) = v. Since the IFS A = ⟨y, (0.5, 0.6), (0.5, 0.4)⟩ is IFCS in Y, f$^{-1}(A)$ is IFGαCS in X but not IFαGSCS in X. Therefore f is an IFGα continuous mapping but not an IFαGS continuous mapping.

Remark 3.21. *We obtain the following diagram from the results we discussed above.*



None of the reverse implications are not true.

Theorem 3.22. *A mapping f: X → Y is IFαGS continuous if and only if the inverse image of each IFOS in Y is an IFαGSOS in X.*

Proof. ⇒ part
Let A be an IFOS in Y. This implies A^c is IFCS in Y. Since f is IFαGS continuous, $f^{-1}(A^c)$ is IFαGSCS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IFαGSOS in X. ⇐ part
Let A be an IFCS in Y. Then A^c is an IFOS in Y. By hypothesis $f^{-1}(A^c)$ is IFαGSOS in X. Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $(f^{-1}(A))^c$ is an IFαGSOS in X. Therefore $f^{-1}(A)$ is an IFαGSCS in X. Hence f is IFαGS continuous.

Theorem 3.23. *Let f:(X, τ) → (Y, σ) be a mapping and $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IFαGS continuous mapping.*

Proof. Let A be an IFCS in Y and $f^{-1}(A)$ be an IFRCS in X. Since every IFRCS is an IFαGSCS, $f^{-1}(A)$ is an IFαGSCS in X. Hence f is an IFαGS continuous mapping.

Definition 3.24. *An IFTS (X, τ) is said to be an*

(1) *intuitionistic fuzzy $\alpha ga T_{1/2}$(in short IF$_{\alpha ga}T_{1/2}$)space if every IFαGSCS in X is an IFCS in X,*
(2) *intuitionistic fuzzy $\alpha gb T_{1/2}$(in short IF$_{\alpha gb}T_{1/2}$)space if every IFαGSCS in X is an IFGCS in X,*
(3) *intuitionistic fuzzy $\alpha gc T_{1/2}$(in short IF$_{\alpha gc}T_{1/2}$)space if every IFαGSCS in X is an IFGSCS in X.*

Theorem 3.25. *Let f:(X, τ) → (Y, σ) be an IFαGS continuous mapping, then f is an IF continuous mapping if X is an IF$_{\alpha ga}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFαGSCS in X, by hypothesis. Since X is an IF$_{\alpha ga}$T$_{1/2}$, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.26. *Let f:(X, τ) → (Y, σ) be an IFαGS continuous mapping, then f is an IFG continuous mapping if X is an IF$_{\alpha gb}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFαGSCS in X, by hypothesis. Since X is an IF$_{\alpha gb}$T$_{1/2}$, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Theorem 3.27. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS continuous mapping, then f is an IFGS continuous mapping if X is an IF$_{\alpha gc}$T$_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFαGSCS in X, by hypothesis. Since X is an IF$_{\alpha gc}$T$_{1/2}$, $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

Theorem 3.28. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS continuous mapping and g:(Y, σ) $\rightarrow$ (Z, δ) be an IF continuous, then g$\circ$f : (X, τ) $\rightarrow$ (Z, δ) is an IFαGS continuous.*

Proof. Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IFαGS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IFαGSCS in X. Hence g$\circ$f is an IFαGS continuous mapping.

Theorem 3.29. *Let f:(X, τ) $\rightarrow$ (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF$_{\alpha ga}$T$_{1/2}$ space.*

(1) *f is an IFαGS continuous mapping.*
(2) *If B is an IFOS in Y then $f^{-1}(B)$ is an IFαGSOS in X.*
(3) *$f^{-1}(int(B)) \subseteq int(cl(int(f^{-1}(B))))$ for every IFS B in Y.*

Proof. (1) $\Rightarrow$ (2): is obviously true.
(2) $\Rightarrow$ (3): Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IFαGSOS in X. Since X is an IF$_{\alpha ga}$T$_{1/2}$ space, $f^{-1}(int(B))$ is an IFOS in X. Therefore $f^{-1}(int(B)) = int(f^{-1}(int(B))) \subseteq int(cl(int(f^{-1}(B))))$.
(3) $\Rightarrow$ (1) Let B be an IFCS in Y. Then its complement B^c is an IFOS in Y. By hypothesis $f^{-1}(int(B^c)) \subseteq int(cl(int(f^{-1}(int(B^c)))))$. This implies that $f^{-1}(B^c) \subseteq int(cl(int(f^{-1}(int(B^c)))))$. Hence $f^{-1}(B^c)$ is an IFαOS in X. Since every IFαOS is an IFαGSOS, $f^{-1}(B^c)$ is an IFαGSOS in X. Therefore $f^{-1}(B)$ is an IFαGSCS in X. Hence f is an IFαGS continuous mapping.

Theorem 3.30. *Let f:(X, τ) $\rightarrow$ (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IF$_{\alpha ga}$T$_{1/2}$ space.*

(1) *f is an IFαGS continuous mapping.*
(2) *$f^{-1}(A)$ is an IFαGSCS in X for every IFCS A in Y.*
(3) *$cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS A in Y.*

Proof. (1) $\Rightarrow$ (2): is obviously true.
(2) $\Rightarrow$ (3): Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, $f^{-1}(cl(A))$ is an IFαGSCS in X. Since X is an IF$_{\alpha ga}$T$_{1/2}$ space, $f^{-1}(cl(A))$ is an IFCS in X. Therefore $cl(f^{-1}(cl(A))) = f^{-1}(cl(A))$. Now $cl(int(cl(f^{-1}(A)))) \subseteq cl(int(cl(f^{-1}(cl(A))))) \subseteq f^{-1}(cl(A))$.
(3) $\Rightarrow$ (1): Let A be an IFCS in Y. By hypothesis $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A)) = f^{-1}(A)$. This implies $f^{-1}(A)$ is an IFαCS in X and hence it is an IFαGSCS in X.Therefore f is an IFαGS continuous mapping.

Definition 3.31. *Let (X, τ) be an IFTS. The alpha generalized semi closure (αgscl(A) in short) for any IFS A is defined as follows. $\alpha gscl(A) = \cap \{K \mid K$ is an IFαGSCS in X and $A \subseteq K\}$. If A is IFαGSCS, then $\alpha gscl(A) = A$.*

Theorem 3.32. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS continuous mapping. Then the following conditions are hold.*

(1) *f(αgscl(A)) $\subseteq$ cl(f(A)), for every IFS A in X.*
(2) *αgscl(f^{-1}(B)) $\subseteq$ f^{-1}(cl(B)), for every IFS B in Y.*

Proof. (i) Since cl(f(A)) is an IFCS in Y and f is an IFαGS continuous mapping, f^{-1}(cl(f(A))) is IFαGSCS in X. That is αgscl(A) $\subseteq$ f^{-1}(cl(f(A))). Therefore f(αgscl(A)) $\subseteq$ cl(f(A)), for every IFS A in X.
(ii) Replacing A by f^{-1}(B) in (i) we get f(αgscl(f^{-1}(B))) $\subseteq$ cl(f(f^{-1}(B))) $\subseteq$ cl(B). Hence αgscl(f^{-1}(B)) $\subseteq$ f^{-1}(cl(B)), for every IFS B in Y.

Remark 3.33. *The composition of two IFαGS continuous mappings need not be IFαGS continuous as can be seen from the following example:*

Example 3.34. *Let X = $\{a, b\}$, Y = $\{u, v\}$ and Z = $\{s, t\}$. Let $\tau = \{ 0_\sim, T_1, 1_\sim \}$, $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ and $\delta = \{ 0_\sim, T_3, 1_\sim \}$ be IFTs on X, Y and Z respectively where $T_1 = \langle$ x, (0.4, 0.3), (0.6, 0.7)$\rangle$, $T_2 = \langle$ y, (0.3, 0.8), (0.7, 0.2)$\rangle$ and $T_3 = \langle$ z, (0.4, 0.9), (0.6, 0.1)$\rangle$. Define f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v and g:(Y, σ) $\rightarrow$ (Z, δ) by g(u) = s and g(v) = t. Then f and g are IFαGS continuous mappings. Since A = $\langle$z, (0.6, 0.1), (0.4, 0.9)$\rangle$ is an IFCS in Z, f^{-1}(A) is not an IFαGSCS in X. Therefore the composition map g $\circ$ f: (X, τ) $\rightarrow$ (Z, δ) is not an IFαGS continuous.*

4. Intuitionistic fuzzy α-generalized semi irresolute mappings

In this section we introduce intuitionistic fuzzy α-generalized semi irresolute mappings and study some of its characterizations.

Definition 4.1. *A mapping f:(X, τ) $\rightarrow$ (Y, σ) is called an intuitionistic fuzzy alpha-generalized semi irresolute(IFαGS irresolute) mapping if f^{-1}(A) is an IFαGSCS in (X, τ) for every IFαGSCS A of (Y, σ).*

Theorem 4.2. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS irresolute, then f is an IFαGS continuous mapping.*

Proof. Let f be an IFαGS irresolute mapping. Let A be any IFCS in Y. Since every IFCS is an IFαGSCS, A is an IFαGSCS in Y. By hypothesis f^{-1}(A) is an IFαGSCS in X. Hence f is an IFαGS continuous mapping.

Example 4.3. *IFαGS continuous mapping $\nrightarrow$ IFαGS irresolute mapping.*
Let X = $\{a, b\}$, Y = $\{u, v\}$, $T_1 = \langle$ x, (0.3, 0.4), (0.6, 0.5)$\rangle$ and $T_2 = \langle$ y, (0.7, 0.3), (0.2, 0.6) $\rangle$. Then $\tau = \{ 0_\sim, T_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, T_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f:(X, τ) $\rightarrow$ (Y, σ) by f(a) = u and f(b) = v. Then f is an IFαGS continuous. We have B = $\langle$y, (0.1, 0.5), (0.8, 0.4)$\rangle$ is an IFαGSCS in Y but f^{-1}(B) is not an IFαGSCS in X. Therefore f is not an IFαGS irresolute mapping.

Theorem 4.4. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS irresolute, then f is an IF irresolute mapping if X is an $IF_{\alpha ga}T_{1/2}$ space.*

Proof. Let A be an IFCS in Y. Then A is an IFαGSCS in Y. Therefore f^{-1}(A) is an IFαGSCS in X, by hypothesis. Since X is an $IF_{\alpha ga}T_{1/2}$ space, f^{-1}(A) is an IFCS in X. Hence f is an IF irresolute mapping.

Theorem 4.5. *Let f:(X, τ) $\rightarrow$ (Y, σ) and g:(Y, σ) $\rightarrow$ (Z, δ) be IFαGS irresolute mappings, then g$\circ$f: (X, τ) $\rightarrow$ (Z, δ) is an IFαGS irresolute mapping.*

Proof. Let A be an IFαGSCS in Z. Then $g^{-1}(A)$ is an IFαGSCS in Y. Since f is an IFαGS irresolute mapping. $f^{-1}((g^{-1}(A)))$ is an IFαGSCS in X. Hence g$\circ$f is an IFαGS irresolute mapping.

Theorem 4.6. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS irresolute and g:(Y, σ) $\rightarrow$ (Z, δ) be IFαGS continuous mappings, then g$\circ$f: (X, τ) $\rightarrow$ (Z, δ) is an IFαGS continuous mapping.*

Proof. Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFαGSCS in Y. Since f is an IFαGS irresolute, $f^{-1}((g^{-1}(A))$ is an IFαGSCS in X. Hence g$\circ$f is an IFαGS continuous mapping.

Theorem 4.7. *Let f:(X, τ) $\rightarrow$ (Y, σ) be an IFαGS irresolute, then f is an IFG irresolute mapping if X is an $IF_{\alpha gb}T_{1/2}$ space.*

Proof. Let A be an IFαGSCS in Y. By hypothesis, $f^{-1}(A)$ is an IFαGSCS in X. Since X is an $IF_{\alpha gb}T_{1/2}$ space, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG irresolute mapping.

Theorem 4.8. *Let f:(X, τ) $\rightarrow$ (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X and Y are $IF_{\alpha ga}T_{1/2}$ spaces.*

(1) *f is an IFαGS irresolute mapping.*
(2) *$f^{-1}(B)$ is an IFαGSOS in X for each IFαGSOS B in Y.*
(3) *$cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$ for each IFS B of Y.*

Proof. (1) $\Rightarrow$ (2): Let B be any IFαGSOS in Y. Then B^c is an IFαGSCS in Y. Since f is IFαGS irresolute, $f^{-1}(B^c)$ is an IFαGSCS in X. But $f^{-1}(B^c) = (f^{-1}(B))^c$. Therefore $f^{-1}(B)$ is an IFαGSOS in X.
(2) $\Rightarrow$ (3): Let B be any IFS in Y and $B \subseteq cl(B)$. Then $f^{-1}(B) \subseteq f^{-1}(cl(B))$. Since cl(B) is an IFCS in Y, cl(B) is an IFαGSCS in Y. Therefore $(cl(B))^c$ is an IFαGSOS in Y. By hypothesis, $f^{-1}((cl(B))^c)$ is an IFαGSOS in X. Since $f^{-1}((cl(B))^c) = (f^{-1}(cl(B)))^c$, $f^{-1}(cl(B))$ is an IFαGSCS in X. Since X is $IF_{\alpha ga}T_{1/2}$ space, $f^{-1}(cl(B))$ is an IFCS in X. Hence $cl(f^{-1}(B)) \subseteq cl(f^{-1}(cl(B))) = f^{-1}(cl(B))$. That is $cl(f^{-1}(B)) \subseteq f^{-1}(cl(B))$.
(3) $\Rightarrow$ (1): Let B be any IFαGSCS in Y. Since Y is $IF_{\alpha ga}T_{1/2}$ space, B is an IFCS in Y and cl(B) = B. Hence $f^{-1}(B) = f^{-1}(cl(B)) \supseteq cl(f^{-1}(B))$. But clearly $f^{-1}(B) \subseteq cl(f^{-1}(B))$. Therefore $cl(f^{-1}(B)) = f^{-1}(B)$. This implies $f^{-1}(B)$ is an IFCS and hence it is an IFαGSCS in X. Thus f is an IFαGS irresolute mapping.

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